

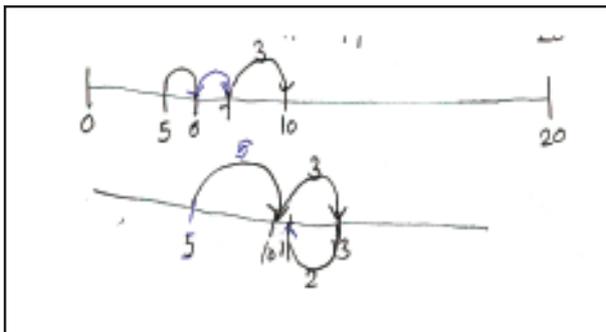
The Teaching of Place Value – Cognitive Considerations

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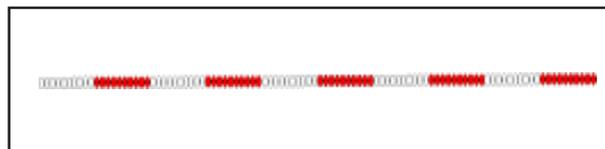
This paper considers the different approaches to the teaching of place value. It is argued that current practices involve a heavy emphasis on place value. Both mainstream and alternative approaches attempt to make children understand the meaning of the notational system for the writing of numbers. In mainstream cases, children are asked to write the numbers in columns marked out as units, tens and hundreds. In the alternative approaches prevalent in India, different concrete materials such as Dienes blocks and match-stick bundles are used to make children understand the *logic* behind the place-value system. In this sense, there is a commonality in both the approaches.

These methodologies assume that the learning of number as such is comparatively straightforward and that the real issue is that of understanding the *conceptual representation* of number or of number as the sum of products of powers of ten.



A different approach developed in the Netherlands delays the teaching of place value and focuses on the development of number sense. The activities chosen,

such as counting on the ten-structured bead string (see figure) and the jumps on the empty number line keep the numbers whole without differentiating them on the basis of place value.



This method is strongly rooted in the conception of Freudenthal that there was more than one concept of number, which included counting number, numerosity number and measuring number. He argued very strongly that the numerosity number was 'mathematically insufficient', 'mathematically unimportant' and 'didactically insufficient' for the teaching of natural numbers (Freudenthal, 1973, pp. 179-194). He argued in favour of focusing on the counting number and the measuring number and specifically for the use of the number line. This advocacy of number line is related to the emphasis Freudenthal put on the constitution of the mental object by children (Freudenthal, 1983 p. x and pp. 28-33).

Theoretical aspects

I would argue that this RME (Realistic Mathematics Education) approach is significant both in terms of current theoretical advances in cognitive sciences *and* in terms of practical applications. In the framework of cognitive linguistics, mathematical concepts are seen as image schemas and conceptual metaphors. Accord-

ing to Lakoff and Nunez arithmetic is based on the four grounding metaphors of Object Collection, Object Construction, Measuring Stick and Motion Along a Path. (2000, pp. 50- 103) The logic of arithmetic is seen to emerge from the inferential logic of these four source domains by preserving the image-schema structure. The development of the mental object of the number line can be understood as a metaphorical blend in this framework.

In RME, activities with concrete materials are used not to embody the formal algorithmic operations but to facilitate domain-specific *level raising*. For example, in early arithmetic this involves, going from adding by counting to adding by structuring. The materials used are to be seen, not as devices for cognising the numbers that exist out there, but as functional extensions of the human body that literally lay the basis for new forms of cognitions. This can be understood in terms of Vygotsky's understanding of functional learning systems as mediational devices leading to changes in one's behaviour. Specifically on the question of arithmetic thinking, Vygotsky discussed how counting on the fingers served as a bridge between natural arithmetic (subitizing) and cultural arithmetic. Vygotsky argued, "the quantitative characteristic of any group of objects is perceived initially as one of the qualitative characteristics.The matter changes as soon as man, in reacting to the quantitative aspect of any situation, resorts to his fingers as a tool to aid in carrying out the counting operation". (Vygotsky, 1997, p.52). Counting on the structured string and the use of the empty number line can be seen as the creation of new functional learning systems for the child, aiding to create a new mental object of number. This approach is also consistent with the studies on the relationship between procedural and conceptual competencies (Gelman and Meck, 1987).

In the bead string the colour cue functions as a neutral stimulus to organise counting. The colour cue gets recruited to solve the demands of the situation. This later gets internalised to form an understanding of the ten-based structure of the number system. The cognitive processes can be understood in terms of the 'methodology of double stimulation' that Vygotsky put forward (Vygotsky, 1999, p. 59).

The RME approach of mathematisation as a human activity also takes it beyond that of mere embodiment as seen from cognitive semantics. The importance accorded to the goal and affect in the designing of the activities brings it in consonance with the perspective of Activity Theory and the work of Vygotsky.

Practical Implications

The case for postponing the teaching of place value

also comes from evidences that are emerging from countries outside the Netherlands. In a recent study from U.K., Ian Thompson has argued that place value should be seen to comprise of 'quantity value' and 'column value' and that the concept of 'quantity value' develops before that of 'column value' and pointed to the possible negative consequences of the excess emphasis on place value (Thompson, 2002, p. 10).

Analysis of Indian textbooks shows that most of the activities focus on embodying the conceptual representation in concrete materials for teaching of place-value. This emphasis on place-value goes to such an extent that children are taught numbers up to 9 and not up to 10 or up to 99 and not up to 100. The unnaturalness of teaching up to 9 when children are using the ten fingers of their hands to count is obvious.

A counting based, number line using method as in RME would differ very much from the approaches prevalent in India. Yet experience in the last four years shows that teachers very warmly welcome a teaching aid such as the structured bead string/ *ganit mala*. This can be understood in terms of the traditional rote practices in which the emphasis is on the chanting of numbers forward and backward as well as on various forms of skip counting. In the traditional practice any form of visual support is given only up to number ten. The development of number sense in the traditional practices can perhaps be understood in terms of the structuring implicit in the number names. Yet these practices can be considered to have the potential to blend with those for developing number sense using the empty number line as a new functional learning system.

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