Transparent Objects and Processes in Learning Mathematics

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This paper examines the notion of *transparency* in learning and understanding mathematics. The notion of transparency may be associated with mathematical objects as well as thought processes. It carries two related meanings: Seeing-through and being visible. It has been addressed with respect to learning mathematics in a number of different contexts. In dealing with the roles and nature of mathematical examples, Mason & Pimm (1982) discuss the idea of a generic example, which basically is a specific example that conveys a more general case. That is, a generic example is *transparent* to a general case. The authors (ibid) suggest that often students fail to see in an example what the teacher had in mind. In particular, a generic example that is meant to demonstrate a general case or principle may be perceived by the students as a specific instance, overlooking its generality. When this is the situation, we consider the example to be non-transparent (or *opaque*) to the learner. Movshovitz-Hadar's (2002) approach to *transparent proofs* can be viewed as an extension of this kind of transparency. A transparent proof, according to Movshovitz-Hadar (ibid), is a proof of a particular case that is "small enough to serve as a concrete example, yet large enough to be considered a non-specific representative of the general case. One can see the general proof through it because nothing specific of the case enters the proof".

In the context of generating counter-examples Peled and Zaslavsky (1997) distinguish between counter-examples that only disprove a statement, and those that also reflect an explanatory feature regarding how it was generated and why it, in fact, refutes the statement. The latter can be seen as *transparent counter-examples*.

Another aspect of transparency in mathematics education is related to thought processes. More specifically, to the extent to which teachers' authentic mathematical thought processes are made transparent to the learners. Schoenfeld (1983) raises this point in the following excerpt:

"Part of the difficulty in teaching mathematical thinking skills is that we've gotten so good at them (especially when we teach elementary mathematics) that we don't have to think about them; we just do them, automatically. We know the right way to approach most of the problems that will come up in class. But the students don't, and simply showing them the right way doesn't help them avoid all the wrong approaches they might try themselves. For that reason we have to unravel some of our thinking, so that they can follow it." (ibid, p. 8).

In this sense, unraveling one's thinking is actually making it a *transparent process* to the learner.

The above discussion of the notion of transparency focuses on its meaning in terms of seeing-through. We also consider transparent objects and properties in terms of being visible. The proposed presentation will elaborate on the notion of transparency in mathematics education, and adopt it to examine a number of studies in which the extent to which an object or process is transparent plays a role in mathematical understanding.

Among the examples that will be presented are the following:

Transparent algorithms: There are several deep mathematical principles that are embedded in the basic arithmetic algorithms. Apprently, the algorithms are not equally transparent to each principle. For example, the long division algorithm, is highly transparent to the notion of periodicity, however, it is rather opaque to the place value principle. Empirical data that was collected indicate that these differences may account for some limtations and strengths in understanding this algorithm and carrying it out correctly (Zaslavsky, 2003).

Transparent definitions: In a study that aimed at identifying students' and teachers' conceptions of a mathematical definition, Shir and Zaslavsky (2002) found that the participants were reluctant to accept definitions of geometric figures based on their latent parts (e.g., a diagonal of a quadrilateral). This phenomenon is explained by the tendency to consider an element as part of a figure only if it is visible. That is, there is an expectation that the connection between parts of a figure and the figure itself needs to be transparent;

Transparent features: In sorting tasks that were given to mathematics teachers as group assignments, it was found that visible features of mathematrical objects were dominant in the sorting criteria, while more deep structured ones that were not transparent, were either not identified or were used at a later stage (Zaslavsky & Leikin, 2004).

Transparent problem solving processes: In an attempt to characterize a special learning environment that evolved within a project, the goal of which was to provide after-school mathematics tutorial sessions by engineers in an informal setting, some distinctive elements that enhanced students learning were identified (Zodik & Zaslavsky, 2004). One of the most salient characteristic that was identified in the engineer's tutoring was the way in which he made his thinking transparent to his students. He unraveled his thought processes by sharing with them each step, including his doubts and barriers in an apprenticeship like manner. He felt comfortable to turn to them for help when he felt stuck. This in return allowed them to become true contributing participants, leading to the emergence of a community engaging in collaborative problem solving activities.

To summarize, the claim that there is a strong connection between the extent to which an object or process are transparent to the learner and the understanding s/he develops, will be supported by findings from a number of studies.

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