Diagrammatic Knowledge in Mathematics in the Information Age

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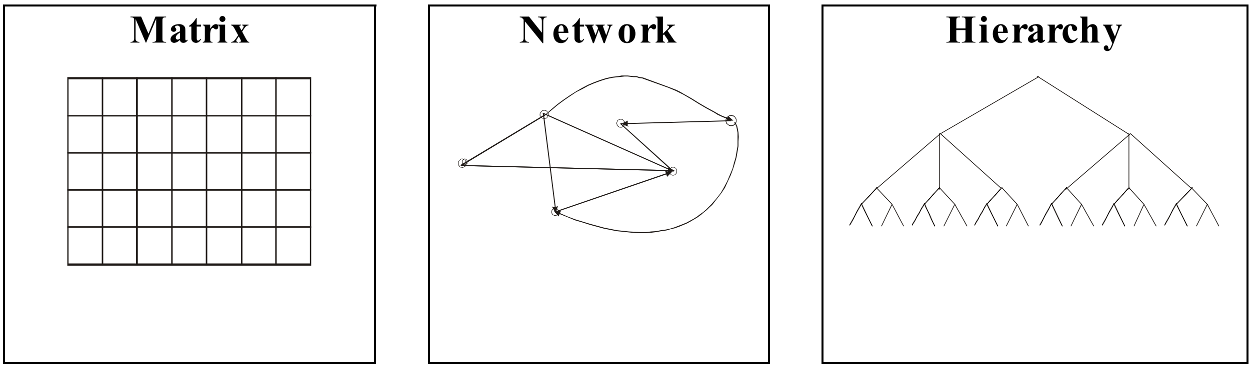
Mathematicians have long recognized the value of diagrams as cognitive tools and the use of diagrammatic representation has resulted in major mathematical breakthroughs, such as the Pythagorean discovery of irrational numbers. However, in the Information Age society, all citizens need to be able to create and interpret general purpose diagrams, such as the matrix, network, and hierarchy (Diezmann & English, 2001). Diagrams have three key cognitive advantages. First, diagrams facilitate the conceptualisation of the problem structure, which is a critical step towards a successful solution. Second, diagrams are an inference-making knowledge representation system that has the capacity for knowledge generation. Third, diagrams support visual reasoning, which is complementary to, but differs from, linguistic reasoning. However, students of all ages are reluctant to employ diagrams, experience difficulty using diagrams or lack the expertise to use diagrams effectively. Thus, students' use of diagrams can inhibit rather than facilitate their mathematical understanding and performance.

The ability to identify the properties of diagrams is a fundamental issue for diagrammatic research because the selection of an appropriate representation is a critical step in reasoning about information. As students encounter diagrammatic representations from their early primary years, it is essential to understand their knowledge of the properties of diagrams in order to promote mathematical learning and inform teaching practice. While diagrammatic knowledge and use have been studied intermittently over the past three decades, it is only recently that Novick (2001) has developed a cohesive framework of three general purpose diagrams (i.e., matrix, network, hierarchy) (see Figure 1) and their properties (e.g., were global structure, number of sets, item/link constraints, link type, linking relations and transversal).

This paper reports on an "in progress" longitudinal study of primary students' knowledge of the properties of diagrams using Novick's (2001) framework. Students' knowledge was explored through a series of scenario-based tasks that required them to (1) select a diagram that best suited the given problem information and to (2) justify their selection and (3) non-selection of particular diagrams. Scenario-based tasks are more appropriate for measuring knowledge of diagrammatic representation than problem-solving tasks because the former specifically elicit knowledge about representation, whereas the latter also involve reasoning.

The first sentence or two of the scenario tasks set up a cover story. The same broad scenario of "The Amusement Park" was used for all tasks to avoid students selecting their responses on the basis of the cover stories rather than the structural information. The next sentence or two focuses on a particular property of a diagram. The final sentence indicates that someone wants a diagram for a purpose relevant to the cover story. The students' task was to select the appropriate diagram from two labelled diagrams (matrix or network or hierarchy). In one of these diagrams, the property was represented, and in the other diagram the property was not represented. Only two (correct/incorrect)

Figure 1. Three general purpose diagrams.



spatially-oriented diagrams were presented for each scenario. Students were asked to justify their selection and also to explain why they did not select the remaining diagram. Examples of the tasks and the types of student responses will be presented on the poster. What was notable in the study was some students' reliance on the pictorial components of the diagram. For example, some students accepted or rejected a diagram on the basis of what could or could not physically fit on the diagram or the shape of spaces within the diagram. Thus, these students considered the diagrams as concrete representations of particular aspects of the story, rather than as abstract representations of the relationships between information.

The development of students' knowledge about diagrams was explored through interviews about their everyday (in school and out-of-school) experiences. The students' responses indicated that, as anticipated, there was limited attention to diagrams at school and no systematic effort to develop knowledge of diagrams. Generally, students' experiences with diagrams were *ad hoc* and one-off. Sometimes there was passing reference to particular diagrams in class or students encountered diagrams on their homework.

Outside school, students encountered diagrams in various forms, such as knockout sporting competitions represented on hierarchies. Some students were able to draw detailed reconstructions of these diagrams and talk about what they meant. Thus, these students had referents to which they could relate similar diagrams. The intent was to document these everyday situations that provide authentic contexts for interpreting diagrams and propose that these be embedded into the curriculum. However, there were also many students who had presumably seen the same information in common everyday situations as the diagram-aware students but had no recollection of these diagrams. This raises the question of what is it that makes something memorable or meaningful for some students but not for others?

In summary, this study contributes to our understanding of the way in which we think by extending knowledge of diagrammatic representation and cognition in mathematics. This has been achieved by investigating which diagrams and which properties are easier or harder for students. Students' explanations of their selections of particular diagrams to represent specific problem information provided insight into what information students perceived was represented on the diagrams and what they thought were the characteristics of diagrams. Knowledge of diagrams was further enhanced by a consideration of students' in-school and out-of-school experiences with diagrams. This exploration revealed that despite the heavy use of diagrams in print and electronic forms many students had scant experience or awareness of these important visual representations. Thus, the results of the study have broad applicability to all disciplines that use diagrams in print or electronic forms. If diagrams are to be used in learning, teaching or assessment, students must be diagram literate.

References

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