Development and evaluation of a concept inventory in rotational kinematics

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I. INTRODUCTION

This synopsis summarises the original thesis work done towards the development and evaluation of a concept inventory (CI) in rotational kinematics at the higher secondary school level. A CI essentially comprises of a carefully crafted set of multiple choice questions on a concept or a topic aimed at probing student difficulties, misconceptions or alternative conceptions, and eliciting their ill-suited reasoning patterns (Singh, 2011). They constitute a major trend in the field of physics education research (PER) (Kumar, 2011). Our concept inventory comprises of three parts as stated below and developed broadly in the same order.

1. Rotational kinematics of a particle (19 questions) (Mashood and Singh, 2012a).

2. Rotational kinematics of a particle in rectilinear motion (7 questions) (Mashood and Singh, 2012b, 2012c).

3. Rotational kinematics of a rigid body rotating about a fixed axis (13 questions) (Mashood and Singh, 2013b).

The questions that served as the basis of our research are the following.

1. What are the difficulties faced by students in rotational kinematics at the higher secondary school level?

2. Do the pitfalls in student reasoning in the topic exhibit patterns? Can they be categorised into broader themes within physics education research?

3. Are there any parallels between our findings in rotational kinematics and the documented research in linear kinematics?

The thesis consists of eight chapters. We begin with a literature review of concept inventories, with focus on physics. We discuss the relevance of CI's in the Indian context and why we chose rotational kinematics as the topic of our investigation. The second chapter describes the methodological details involved in the systematic and iterative construction and administration of our inventory (Mashood and Singh, 2013a). In chapter 3 we discuss the first part of our inventory comprising 19 questions on rotational kinematics of a particle (Mashood and Singh, 2012a). The content evolution of items along with our findings is presented. Misconceptions and pitfalls were identified and categorised into broader themes within PER. Chapter 4 similarly discusses the next part of the inventory, namely rotational kinematics of a particle in rectilinear motion (Mashood and Singh, 2012b). This consists of 7 questions. Chapter 5 describes an interesting pedagogical spin-off related to the case of a particle moving in rectilinear motion (Mashood and Singh, 2012c). The non-intuitive variation of the angular velocity and the angular acceleration with associated extremum is the theme of the chapter. The final part of the inventory focusing on rotational kinematics of a rigid body about a fixed axis (13 questions) is discussed in chapter 6. We administered the entire inventory to around thousand students in five cities spread across the country. The data from this large scale administration served as the basis for item level and whole test statistical analyses. This includes item response curve analyses and calculation of the Kuder-Richardson reliability index and other indices for the inventory. These statistical analyses constitute the subject of chapter 7. Chapter 8 constitutes a brief conclusion.

II. CHAPTER 1: CONCEPT INVENTORIES: A LITERATURE SURVEY

The history of concept inventories in science education can be traced back to the Force Concept Inventory (FCI) published in 1992 along with the Mechanics Baseline Test (Hestenes, Wells, and Swackhammer, 1992; Hestenes and Wells, 1992; Richardson, 2004; Hake, 2011). The prequel to these inventories appeared earlier in 1985 (Halloun and Hestenes, 1985a, 1985b). CI's played a significant role in stimulating research driven educational reforms in physics (Richardson, 2004; Hake 2011). Hake (2011) has provided a review of the impact inventories had on physics education and related disciplines. He characterized the pre-inventory period of physics education research (PER) as the 'dark ages of post secondary physics education' in the United States. Singh (2011) has provided an informative expository article on concept inventories with emphasis on the Indian context. The effectiveness and success of FCI led to the development of inventories in other areas of science and engineering. In physics these include the Test of Understanding Graphs in Kinematics (TUG-K), Force and Motion Conceptual Evaluation (FMCE), Conceptual Survey of Electricity and Magnetism (CSEM), Brief Electricity and Magnetism Assessment and Student Understanding of Rotational and Rolling motion concepts, among others (Beichner, 1994; Thornton and Sokoloff, 1998; Maloney et al, 2001; Rimoldini and Singh, 2005; Ding et al, 2006). A useful list has been provided by Biechner (2007). The topic-wise subcategories include graphing, force, mechanics, energy, thermodynamics, electricity and magnetism, light and optics, quantum mechanics, astronomy and waves. Allen (2007) catalogued a similar list of inventories in the domains of engineering, chemistry, maths, geo-sciences etc. A review of concept inventories in biology was carried out by D' Avanzo (2008). Fisher and Williams (2011) have provided a list of CI's in various sub-disciplines of biology which include natural selection, genetics, introductory biology, and molecular and cell biology.

The decision to craft our research in the mode of a concept inventory was motivated by the fact that CI's uniquely blend research and dissemination. Its potential for large scale application is particularly relevant to the Indian educational scenario owing to our huge and diverse student population. A well developed CI, whose validity and reliability has been established, serves as a ready to use diagnostic and assessment tool for teachers. They can be administered to a large number of students at a time and evaluated easily and objectively. The fact that historically inventories played a significant role in stimulating research driven education reforms in US also motivated us (Hake, 2011). We chose rotational motion because the topic has not yet received the attention it deserves from the physics education research community. This is despite the fact that it is one of the most difficult topics at the higher secondary level, as revealed by our interactions with both students and the teachers. The work by Rimoldini and Singh (2005) was the first major effort to address this lacuna. There also exist scattered work on student understanding of the dynamics of rigid body rotation and rolling motion (Lopez, 2003; Carvalho and Sousa, 2005; Ortiz et al, 2005; Singh and Pathak, 2007; Unsal, 2011; Close and Heron, 2011; Close et al, 2013). The study by Rimoldini and Singh (2005) is a broad spectrum inventory on rotation and rolling motion. In-depth studies on concepts of angular velocity ($\vec{\omega}$) and angular acceleration ($\vec{\alpha}$) are missing. On the other hand their linear counterparts, velocity ($\vec{v}$) and acceleration ($\vec{a}$) have been the subject of repeated investigations (Trowbridge and McDermott, 1980, 1981; Halloun and Hestenes, 1985a, 1985b; Reif and Allen, 1992; Hestenes and Wells, 1992; Thornton and Sokoloff, 1998; Shaffer and McDermott, 2005). Our observation of the existence of difficulties among students as well as teachers regarding $\vec{\omega}$ and $\vec{\alpha}$, led us to deal with them in a focused manner.

III. CHAPTER 2: METHODOLOGY

Inventory development is a systematic and iterative process. The various steps involved in the construction of our inventory on rotational kinematics are schematically sketched in figure 1. These steps were formulated after reviewing the methodologies employed by prominent inventories in PER (Hestenes et al. 1992; Hestenes and Wells, 1992; Beichner, 1994; Maloney et al, 2001; Ding et al, 2006). The processes involved in the construction phase can be categorised into (a) theoretical analysis, which constitute content mapping, cognitive analysis and literature review. (b) empirical investigations which include interactions with students and teachers. The theoretical analyses led to a preliminary draft which evolved iteratively in the course of our interactions with students and teachers. Interactions were aimed at obtaining insights so that appropriate items and distractors could be framed and inappropriate ones discarded. In this section we briefly discuss the sequential steps depicted in figure 1. The methodological aspects pertaining to our interactions with students and teachers are also described.

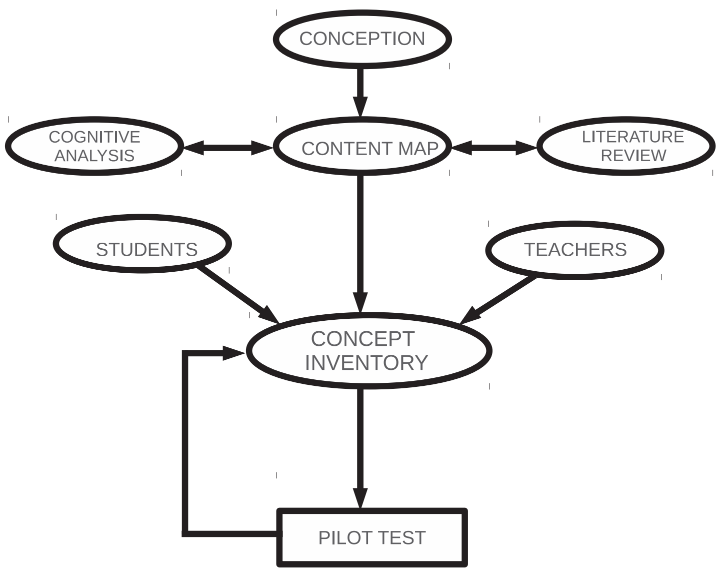

FIG. 1. Initial steps involved in the development of our concept inventory (Mashood and Singh, 2013a).

Theoretical analyses: Content mapping involved chalking out the aspects of $\vec{\omega}$ and $\vec{\alpha}$ covered at the higher secondary level. For this we analysed presentations of rotational motion by popularly used text books. Five introductory level (Reif, 1995; Halliday et al., 2001; Young and Freedman, 2004; Giancoli, 2005; NCERT, 2006), one undergraduate (Kleppner and Kolenkow, 2007) and an advanced level (Goldstein et al., 2004) texts were analysed. In addition two books in the vernacular language (Hindi) were also consulted (Singh, 1988; Kumar and Mittal, 1991). Questions from national level tests, namely the Indian Institute of Technology - Joint Entrance Examinations, spanning twenty years were reviewed. This was supplemented by a cognitive analysis of $\vec{\omega}$ and $\vec{\alpha}$ akin to that done by Reif and Allen (1992) for linear acceleration.

Literature review was focused on basic mechanics, particularly linear kinematics (Trowbridge and McDermott, 1980, 1981; Halloun and Hestenes, 1985a, 1985b; Reif and Allen, 1992; Hestenes and Wells, 1992; Thornton and Sokoloff, 1998; Shaffer and McDermott, 2005;) and rotational motion (Lopez, 2003; Carvalho and Sousa, 2005 Rimoldini and Singh, 2005; Ortiz et al, 2005; Close and Heron, 2011). Possible pitfalls were identified by literature review combined with our analysis. Reasoning errors such as indiscriminate usage of equations (Reif and Allen, 1992), position - velocity confusion (Trowbridge and McDermott, 1980; Hestenes et al., 1992), among others reported in linear kinematics are likely to have parallels in rotational kinematics. In addition, we made a conscious effort to invoke physically relevant situations like pendulum, elliptical or planetary motion wherein questions could be posed. Analysis of the Indian text books, National Council of Education Research and Training - Physics I and II (NCERT 2006) and the vernacular text books (Kumar and Mittal, 1991; Singh, 1988) along with our own experiences helped in identifying contexts familiar to the Indian students such as the wall clock, the potters wheel, the giant wheel, etc for posing the questions. On the basis of the theoretical analyses, a preliminary draft of questions was crafted and this served as the basis for our empirical investigations.

Empirical investigations: Interaction with students and teachers is one of the important aspects involved in the process of developing an inventory. A knowledge of the thought processes of novice students helps significantly in constructing good questions/items (term questions and items are used interchangeably throughout) and distractors. We interacted with around 50 students and 12 practising teachers before administering the test to a larger sample. The students comprised of 21 from the higher secondary level, 14 doing their bachelors degree, 6 at the post graduate level and 9 pursuing their Ph D's. Some of the interactions were with small groups (2 - 6 students) while others were individual. Details have been documented elsewhere (Mashood and Singh, 2013a) and will be discussed in chapter 2. Verbal data was collected, primarily through the following modes.

1. Think aloud protocol: This involved of candidates answering the questions by thinking aloud. This was often followed by clarifications which progressed into discussions.

2. Retrospective probing: This involved students solving the questions and being probed by us at the end of the task (Young, 2005). Some students were more comfortable with this mode rather than the think aloud protocol in which they have to simultaneously solve and verbalise.

3. Semi-structured interviews: This involved candidates taking the test, marking their confidence level to each answer and then being interviewed.

All three of these were employed in the initial phase of development whereas only semi-structured interviews were employed later. The think aloud data, interviews and discussions were audio recorded and analysed. The insights obtained were successively incorporated at each stage thereby refining our questions and distractors. Some of the intricacies involved in the above mentioned methodologies are noted here. Think aloud protocol essentially comprises of the subject articulating their thoughts while solving a given problem. The method is particularly useful for providing insights during the early phases of investigation (Young, 2005). This makes it apt to be used in the developmental phase of a test. Cognitively it aims at capturing what is held in the short term memory (Ericsson and Simon, 1993). The primary aim is to elicit the sequence of thoughts as the subject is processing the information. As such the researcher should restrict oneself to minimal intervention so that the stream of thought is not cued or influenced. We, like others, limited ourselves to minimal 'prompts' or 'proddings' such as 'keep talking', whenever the subject turned quiet (Rimoldini and Singh, 2005; Young, 2005). It is also important that the problems should be of optimal cognitive load (Young, 2005). A highly demanding problem makes it difficult for the participant to simultaneously attend to solving it and verbalizing. An extremely easy task may be performed reflexively and the subject may not be able to describe a sequence of steps. We tried to make our questions optimal in terms of difficulty and ensured that they could be answered without resorting to any lengthy algebraic manipulations. These issues are significant for retrospective probing as well. The individual differences in the ability to verbalize was taken into account. Students who found it difficult to solve and verbalise simultaneously opted for retrospective probing. The cognitive analysis of the concepts helped structure our interviews. We tabulated a list of probable methods and arguments participants may invoke.

Details of our interactions and the insights derived from them have been documented (Mashood and Singh, 2013a). The preliminary draft of questions evolved to a multiple choice format based on our interactions and pilot studies. The questions were then validated. A review of PER literature reveals that the usually employed validities with regard to concept inventories in physics are face validity and content validity (Halloun and Hestenes, 1985a; Ding et al, 2006; Wuttiprom et al, 2009;). Face validity is a prima facia assessment of the test and its appropriateness by the subjects (e.g., students and non experts who take test). The purpose is to ensure the clarity of statements of the questions and the distractors so that they are not misinterpreted (Adams et al, 2006). Our inventory was face validated by 10 practising teachers, 10 higher secondary students and 8 graduate students. Content validity refers to assessing whether all relevant aspects of the concepts were adequately covered by the items. This is carried out by content experts. Our inventory was content validated by 8 experts which included senior professors, associated professors and highly experienced undergraduate lecturers in physics. All of them had experience in designing various types of physics tests. They carefully analysed each item and the corresponding distractors. We carried out a semi-quantitative approach employed by Maloney et al which requires the experts to rate each item on a 5 point scale for reasonableness and appropriateness (Maloney et al, 2006). Suggested modifications and changes were made to ensure that the inventory measures what it purports to measure. The multiple choice questions were again pilot tested to a group of 58 undergraduate students. They were asked to write down brief explanations for their answer choices. The construction of an inventory consists of development (as discussed above), administration, analysis and evaluation. Note once again that this process is iterative. Having discussed the development part we move on to the discussion of samples to which it was administered and the methodology of evaluation.

Samples: The inventory was administered to four groups of students and two groups of teachers. We first discuss the students. In the first stage of administration the student groups comprised of 79 and 74 candidates respectively. All the students were at the higher secondary school level. The age range was 16-18. They had been taught rotational motion. We call the N=79 group as S1. They were from schools in Mumbai and it was a convenient sample. The percentage of boys and girls was almost equal in this sample. The group N = 74, which we denote as S2 were among the finalists who appeared for selection tests to represent India in the international olympiads in physics, chemistry and mathematics. The number of girls in S2 was 8 (11 %). The entire inventory was further administered to a large sample of over nine hundred students from 5 urban centres (Jaipur, Patna, Mumbai, Hyderabad and Bangalore) spread across the country. We denote this sample of 905 students as S3. The number of girls in this sample was around 360 (40 %). Requests were sent to schools in 7 urban centres and, among those who volunteered, 12 schools were selected. Our selection was influenced by (a) the geographical spread (b) variety in terms of certifying government boards (state vs central) and (c) administrative set up (private vs government schools). A part of the inventory was also administered to 384 introductory level students at the University of Washington, Seattle. We call this group as S4. A subset of 7 students from S2 and 35 students from S3 were interviewed.

The teachers taught physics at the HSS or undergraduate level. They were attending an exposure camp in physics olympiad in our institute and therefore constitute a convenient sample. The selection to the camp involved minimal screening. However they were from across the country and hailed mainly from semi-urban areas. The sample in the first stage consisted of 26 teachers. We denote the group as T1. A second group of 25 teachers (T2) participated in a later stage. Their demographics was similar to T1. The number of female teachers in both groups was around 30 %. Informal conversational interviews were carried out with a subset of 5 teachers each from T1 and T2.

Evaluation of the inventory: The performance in the inventory was gauged by assigning one mark to the correct answer. There was no penalty for wrong choices. No strict time restriction was imposed. Participants were asked to answer all items. Item level and whole test statistical analyses including item response curve analysis were carried out. The statistical indices calculated included difficulty level, index of discrimination, point biserial coefficient, Ferguson's delta and Kuder Richardson reliability index among others (Ding et al., 2006; Ding and Beichner, 2009). Difficulty level is defined as the ratio of the number of correct responses to the total number of students who attempted the item. The index of discrimination measures the extent to which an item can discriminate between low and high scoring students. The point biserial coefficient is a measure of correlation of students' score on the item and the score in the test. A high value indicates that students whose total scores are high are more likely to answer the item correctly. Ferguson's delta (δ) is a measure of the discriminatory power of the whole test. Kuder Richardson reliability (r_{test}) index measures the self consistency of the test. Item response curves (IRC) are a visually rich versatile tool for analysing student responses. IRC involves a plot of the percentage of students $P_i(\theta)$ selecting a choice i to an item vis-a-vis their ability θ. A detailed description of the technique, its merits and theoretical underpinnings can be found elsewhere (Morris et al., 2006; Ding and Beichner, 2009; Singh et al., 2009)

IV. CHAPTER 3: ROTATIONAL KINEMATICS OF A PARTICLE

In this chapter we discuss how the physics content of the items that constitutes the first part of our inventory evolved. It is followed by an analysis of the response patterns obtained after administration of the test and interviews with a subset of students. Here we give illustrative examples of a few questions and the summary of our major findings. Detailed description of all the items and analysis has been documented (Mashood and Singh, 2012a) and will be presented in chapter 3.

Example 1: During our study we noted that the concept of limit or the instantaneous aspect associated with the definitions of $\vec{\omega}$ and $\vec{\alpha}$ presents difficulties to students. This is related to the non-discrimination between position and velocity where the students think that bodies moving in parallel have the same velocity when they reach the same position (Trowbridge and McDermott, 1980; Hestenes et al, 1992). Students tend to focus on the 'perceptually obvious phenomenon of passing' rather than the procedure for identifying instantaneous speed (Trowbridge and McDermott, 1980). Similar lack of discrimination between position and acceleration was also observed (Trowbridge and McDermott, 1981). We devised the following question and its distractors to probe rotational parallels to these pitfalls. The item was posed in the familiar context of a wall clock and pertained to the angular velocities of the tips of the second hand A and the minute hand C.

1. At an instant t, tips of both the second hand (A) and the minute hand (C) of the clock are at 12 O'clock position. Regarding the angular velocities of A and C at time t which of the following statements is true ?

   (a) Angular velocity of A is greater than angular velocity of C.

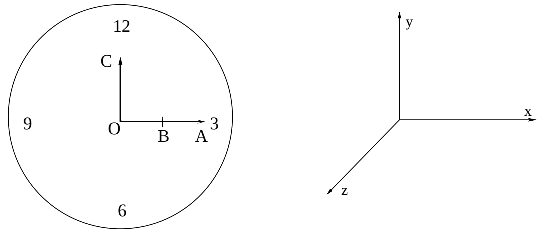

FIG. 2.

(b) Angular velocity of A is less than angular velocity of C.

(c) Angular velocity of A is equal to angular velocity of C.

(d) Angular velocity at an instant cannot be defined.

Example 2: Use of equations ignoring their associated validity conditions is a characteristic of novice thinking (Reif and Allen, 1992). We observed an instance of this pitfall among students regarding the relation $\vec{v} = \vec{\omega} \times \vec{r}$. Many students believed that this equation is valid for curvilinear motion in general, whereas it is valid only for circular motion. We probed this issue by crafting the following question in the context of elliptical motion of a planet around the sun.

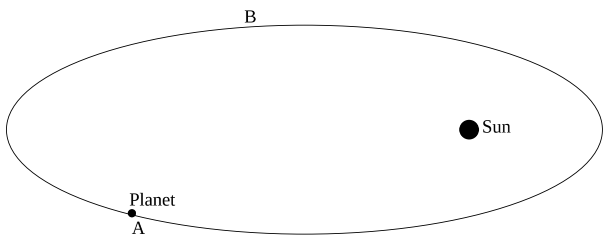

FIG. 3.

2. A planet is moving in an elliptical orbit with the sun at one of its foci. Let $\vec{v}_{\mathrm{B}}$ be the linear velocity, $\vec{\omega}_{\mathrm{B}}$ the angular velocity and $\vec{r}_{\mathrm{B}}$ the position vector of the planet at B (all about sun). Which of the following statements is correct?

   (a) $\vec{v}_{\mathrm{B}} = \vec{\omega}_{\mathrm{B}} \times \vec{r}_{\mathrm{B}}$ from the definition of angular velocity.

   (b) $\vec{v}_{\mathrm{B}} = \vec{\omega}_{\mathrm{B}} \times \vec{r}_{\mathrm{B}}$ because the planet is in rotational motion.

   (c) $\vec{v}_{\mathrm{B}} \neq \vec{\omega}_{\mathrm{B}} \times \vec{r}_{\mathrm{B}}$ because the motion is not circular.

   (d) $\vec{v}_{\mathrm{B}} \neq \vec{\omega}_{\mathrm{B}} \times \vec{r}_{\mathrm{B}}$ because $\vec{\omega}_{\mathrm{B}}$ is not perpendicular to the plane of motion.

Examples 3 and 4: The angular velocity (like any other vector) can vary either with a change in magnitude or direction or both. We noted that students ignore or forget one of these aspects. Often it was the directional aspect possibly because it is less familiar. Understanding and distinguishing both aspects of a vector clearly is important. We designed a set of questions to probe student understanding of variation in magnitude and direction of $\vec{\omega}$ and $\vec{\alpha}$. An oscillating simple pendulum provided us with a rich context for this investigation. In the same context we also probed the lack of differentiation between angular velocity and angular acceleration among students. The non-discrimination between linear velocity and linear acceleration had been documented in earlier works (Trowbridge and McDermott, 1981; Shaffer and McDermott, 2005). Below we provide examples of two questions probing the above mentioned ideas.

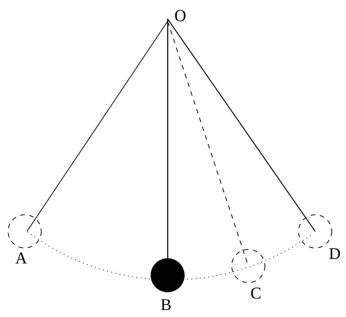

FIG. 4.

3. Figure 4 shows a simple pendulum oscillating about the mean position B. A and D are the left and right extreme positions respectively. The angular velocities of the bob at a point C on the trajectory, when going from A to D and from D to A are
   (a) equal in magnitude, but differ in direction.
   (b) the same in direction, but differ in magnitude.
   (c) different in both magnitude and direction.
   (d) equal in both magnitude and direction.
4. Regarding the angular acceleration of the bob at the instant when it is at the extreme position A on the left, which of the following statements is true?
   (a) Angular acceleration is zero.

(b) Angular acceleration at a single instant is undefined.

(c) Angular acceleration at a single position is undefined.

(d) Angular acceleration is non zero.

Administration and analysis: The inventory was administered to groups S1, S2 and T1. The average of the number of questions correctly answered by candidates of each group is given in table I. The average performance in percentage is given in brackets. We also calculated the corresponding standard deviations and the average difficulty level (averaged over items) for all the three groups. The difficulty level (DL) of an item for a given group is defined as the ratio of the number of correct responses to the number of candidates who attempted the question. It may be noted from the definition that a more meaningful word for the index would be 'easiness level' (Ding et al, 2006).

TABLE I. The average performance of candidates of each group, the associated standard deviation and the average difficulty level. The corresponding percentage performances are in brackets. Total number of questions was 18 for students and 17 for teachers.

	S1 (N=79)	T1 (N=26)	S2 (N=74)
Average score	6.49 (36.06 %)	6.69 (39.35 %)	15.57 (86.48 %)
Standard deviation	2.69	2.87	1.83
Average Difficulty Level	0.37	0.41	0.87

As can be inferred from table I the S2 group answered most of the items correctly. The response pattern of S1 as well as T1 exhibit pitfalls in understanding. In what follows we mainly discuss these two groups. Henceforth, student refers to S1 unless mentioned otherwise. Analysis of the frequency with which distractors were chosen to each item suggested pitfalls in reasoning. This was confirmed later by interviewing a subset of students. For example, to question 1 described above, 33 % of students incorrectly chose the distractor *c*. This indicates that many students think the angular speeds of particles rotating about the same centre are equal when their angular positions overlap. We similarly analysed the detailed response pattern of all the groups. The analyses of the distractors and the interviews helped us categorise the pitfalls and difficulties under the following broad themes.

1. Fixation with inappropriate prototypes

   Students and teachers had difficulty comprehending the non intuitive direction of $\vec{\omega}$ and correspondingly $\vec{\alpha}$. Many were unable to grasp the idea that direction of $\vec{\omega}$ of a particle is always perpendicular to the plane of motion. We repeatedly encountered the notion that $\vec{\omega}$ is in the plane of motion. It had been noted earlier that students' performance is impeded by 'fixation' to prototypical notions (Reif and Allen, 1992). What distinguishes the present case is that the prototype is a formally learnt one. The notion that a vector should always be in the plane of motion is a 'hangover', particularly from linear kinematics and not a preconception acquired from everyday life. We also observed that students are often unable to think beyond the circular motion framework when it comes to rotational motion. We found an extreme case of this 'fixation' where students and teachers were reluctant to ascribe rotational motion concepts to a particle in rectilinear motion (origin not on the path). This will be discussed in detail in the next chapter.

2. Indiscriminate use of equations

   We found instances of indiscriminate use of the equation $\vec{v} = \vec{\omega} \times \vec{r}$. A significant portion of students and teachers did not appreciate that the equation is valid only for circular motion. Similar pitfall for angular acceleration was observed concerning the use of $\vec{a} = \vec{\alpha} \times \vec{r}$. Many students were not aware that $\vec{a}$ in the relation denotes only the tangential component of the acceleration and not the total acceleration. Even a section of S2 harboured this misconception.

3. Pitfalls paralleling those found earlier in linear kinematics

We identified the rotational parallel to the the position-velocity non-discrimination reported for one-dimensional motion (Trowbridge and McDermott, 1980; Hestenes et al., 1992). Prior work on linear kinematics documented confusion concerning $\vec{v}$ and $\vec{a}$ among students (Trowbridge and McDermott, 1981; Hestenes et al., 1992; Shaffer and McDermott 2005). We found a similar confusion concerning $\vec{\omega}$ and $\vec{\alpha}$. The phrase 'as $\vec{\omega}$ behaves so does $\vec{\alpha}$' succinctly captures the essence of a reasoning pattern we uncovered. Many considered angular acceleration of an oscillating pendulum bob to mimic the behaviour of angular velocity. When $\vec{\omega}$ is zero at the extreme position they think $\vec{\alpha}$ is also zero. The case of a ball thrown vertically up is a popular example illustrating the corresponding pitfall in linear mechanics. Students think that the acceleration of the ball is zero at maximum height since its velocity is zero. As the pendulum swings from one extreme to the other the angular velocity is perpendicular to the plane of motion and remains unchanged in direction. Students tend to think that angular acceleration also remains in the same direction not knowing that it flips direction at the mean position. Here the magnitude of $\vec{\omega}$ first increases and then decreases. However the magnitude of $\vec{\alpha}$ first decreases and then increases. Also at the mean position angular velocity is maximum while the angular acceleration is zero. The angular velocities of the pendulum bob at an intermediate point as it oscillates to and fro remain equal in magnitude but differ in direction. The angular accelerations however remain same in both magnitude and direction which once again revealed the existence of erroneous notion that $\vec{\alpha}$ tracks $\vec{\omega}$.

Other difficulties and pitfalls which could not be incorporated in any of the above themes include student difficulties with ratios and simple visualizations. A significant number of candidates thought that the angular velocity of the tip of the clock (transparent) hand would change direction depending on whether we are looking at it from the front or the rear, since the motion switches from clockwise to counter-clockwise.

V. CHAPTER 4: ROTATIONAL KINEMATICS OF A PARTICLE IN RECTILINEAR MOTION (7 QUESTIONS)

This chapter constitutes the discussion of the second part of our inventory on rotational kinematics of a particle moving in a straight line (origin not along the line of motion). We devised 7 multiple choice questions probing aspects of angular velocity, angular acceleration and components of linear velocity. The context is depicted in figure 5. The rotational kinematics of the particle, content evolution of the items and our findings has been documented (Mashood and Singh, 2012b) . Here we give examples of two questions and the summary of major findings based on the analyses of the response patterns and interviews with a subset of students.

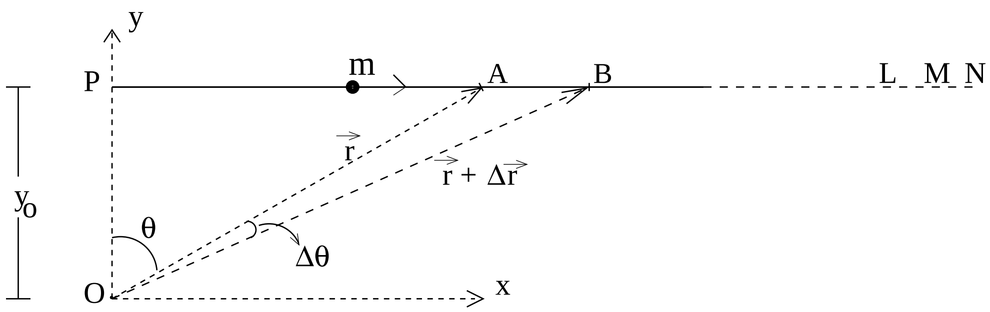

FIG. 5. A particle of mass m moving in a straight line path PABN with constant speed v_0. The origin O is not on the path. The particle travels from A to B in time Δt (Mashood and Singh, 2012b).

Example 1: A particle in rectilinear motion has zero angular velocity only in the special case when the origin is located on the line of motion. We observed that novices are reluctant to ascribe rotational motion concepts to the particle since it is moving in a straight line. Close and Heron (2011) had earlier noticed this for angular momentum. We devised question 1 to probe student understanding of angular velocity of the particle in this context.

1. The magnitudes of angular velocities of the particle (about O) at points A and B are ω_{A} and ω_{B} respectively. Which of the following is true?

   (a) $\omega_{\mathrm{A}} > \omega_{\mathrm{B}}$ because $v = \omega r$, v is constant and $r_{\mathrm{A}} < r_{\mathrm{B}}$.

   (b) $\omega_{\mathrm{A}} > \omega_{\mathrm{B}}$ because the angular displacement at A is greater than angular displacement at B for the same interval of time.

   (c) $\omega_{\mathrm{A}} = \omega_{\mathrm{B}}$ because there is no acceleration.

   (d) $\omega_{\mathrm{A}} = \omega_{\mathrm{B}} = 0$ because the motion is linear.

   Initially the distractors probed only relative magnitudes of angular speeds at two distinct points on the trajectory. We found out that many students selected the correct choice but employed a wrong reasoning. They used the relation $v = \omega r$ which is valid only for circular motion. As a result we reframed the distractors so that this error was explicitly addressed.

   Example 2: The current context provides an interesting situation where there exists an angular acceleration without a net torque ($\vec{\tau}$). Most of the students strongly held

TABLE II. The average performance of candidates of each group, the associated standard deviation and the average difficulty level. The corresponding percentage performances are in brackets. Total number of questions was 7 for students and 5 for teachers.

	S1 (N=79)	T1 (N=26)	S2 (N=74)
Average score	1.29 (18.43 %)	1.19 (23.8 %)	4.62 (66 %)
Standard deviation	1.44	1.49	1.70
Average Difficulty Level	0.19	0.24	0.67

the view that an angular acceleration should always imply a torque. The influence of analogy from basic mechanics where an acceleration of a particle is impossible without a net force (when mass remains constant) was evident. In addition the oft used relation $\tau = I\alpha$ is not valid here. We devised question 2 to probe student understanding of these aspects.

2. Torque τ on the particle at any instant is

   (a) zero because force acting on the particle is zero.

   (b) non zero because there exists an angular acceleration.

   (c) zero because $\tau = I\alpha$ and $\alpha = 0$ (I is the moment of inertia and α is the angular acceleration).

   (d) undefined since the motion is linear.

Administration and analysis: We administered the questions to S1, S2 and T1. These were the same groups as described in the previous chapter (see table 1). Table 2 gives the average of the number of questions correctly answered by candidates of each group. The average performance in percentage is given in brackets. We also calculated the corresponding standard deviations and the average difficulty level. As can be inferred from the average scores, the performances of groups S1 and T1 exhibit pitfalls in understanding. Even a section of S2 found the questions difficult as indicated by their drop in average relative to the first part of the inventory.

We found a tendency among students and teachers alike to consider angular velocity and angular acceleration to be zero since the particle is moving in a straight line. A significant portion among those who maintained that an angular velocity exists based their reasoning on the equation $v = \omega r$ which is not valid here. Those who considered existence of an angular acceleration thought that torque on the particle should thereby be non zero. A significant majority from all groups thought torque on the particle to be zero. However their reasoning was flawed as they considered it to follow from the equation $\tau = I\alpha$ and the condition that $\alpha = 0$. The students (N=79) seem to think that one (radial) or both of the components of the velocity remain constant as the particle moves.

Visual appraisal and asymptotic reasoning helped experts answer some of the questions easily without resorting to algebra. Also whenever there was a doubt they checked for consistency in their answers by arguing in different ways. However such reasoning patterns were rarely present among students and teachers, except for a few students from S2. We have suggested an operational definition for the angular velocity of a particle as part of remedial measures to be undertaken.

VI. CHAPTER 5: VARIATION OF ANGULAR VELOCITY AND ANGULAR ACCELERATION OF A PARTICLE IN RECTILINEAR MOTION

A closer look at the problem described in the previous chapter (see figure 5) prompted an interesting pedagogical spin-off. We studied the variation of angular velocity and angular acceleration for two cases namely particle moving with (i) constant positive acceleration $a\hat{i}$ ($a > 0$) along the x-axis, (ii) constant velocity $v_o\hat{i}$ ($v_o > 0$) (Mashood and Singh, 2012c). When the particle is moving with a constant acceleration the magnitude of angular velocity ω initially increases and then decreases, as given by

$$\omega = \frac{y_o\sqrt{{v_o}^2 + 2a\sqrt{r^2 - {y_o}^2}}}{r^2}. \tag{1}$$

Here r is the distance of the particle from the origin O as shown in figure 5. Figure 6 provides the numerical plot for ω. It may be noted that unit values were taken for y_o, v_o and a while plotting all the graphs and calculating extrema. The point of maximum lies at $r = \sqrt{1}0/3$. The magnitude of angular acceleration of the particle in the same case decreases initially and then increases. The variation is given by

$$\alpha = \frac{ay_o(4{y_o}^2/r^2 - 3)}{r^2} - \frac{2y_ov_o{}^2\sqrt{r^2 - {y_o}^2}}{r^4}. \tag{2}$$

Figure 7 provides the numerical plot for α. The minimum lies at $r = \sqrt{2}$ or $\theta = \pi/4$ rad.

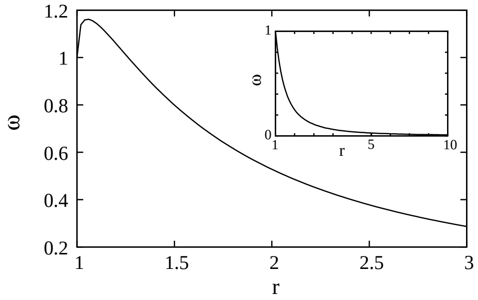

FIG. 6. Variation of ω with r when the particle is moving with constant acceleration. The inset shows the special case of constant velocity. (Mashood and Singh, 2012c).

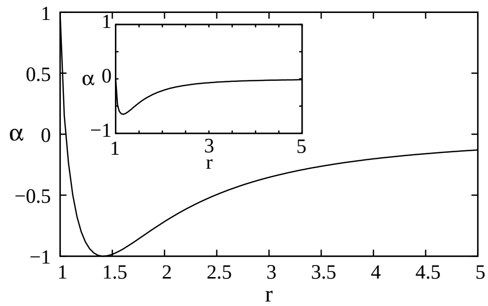

FIG. 7. Variation of α with r when the particle is moving with constant acceleration. The inset shows the special case of constant velocity (Mashood and Singh, 2012c).

On the other hand when the particle is moving with a constant velocity, ω monotonously decrease as shown in the inset of figure 6. However α of the particle in the case initially decreases and then increases. The inset of figure 7 shows the numerical plot. The minimum lies at $r = 2/\sqrt{3}$ which gives $\theta = \pi/6$ rad. The formal derivation and other details can be found in Mashood and Singh (2012c). The non-monotonous behaviour of ω and α with associated extremum is surprising and non intuitive. As mentioned earlier this was a pedagogical spin-off.

VII. CHAPTER 6: ROTATIONAL KINEMATICS OF A RIGID BODY ABOUT A FIXED AXIS (13 QUESTIONS)

The final part of our inventory comprises of 13 questions on rotational kinematics of a rigid body rotating about a fixed axis. This part followed as a natural progression of our work on a particle. Like the chapters on the first two parts of our inventory, this chapter discusses how the physics content of the items evolved, followed by a discussion of the administration and analysis. Here we provide illustrative examples of a few questions and the summary of our major findings based on the analyses of the response patterns and of interviews.

Example 1: We observed that many students had a vague understanding of the angular velocity of a rigid body. Questions 1 was devised to probe student understanding of the operational definition of the angular velocity of a rigid body. In other words to investigate how the angle $\Delta\theta$ in the definition $\omega = \Delta\theta/\Delta t$ is identified. The notion that an angle is traced by the position vector of some particle, led many to incorrectly think that $\Delta\theta$ in $\omega = \Delta\theta/\Delta t$ is the angle traced by the position vector of the centre of mass. Distractors incorporated this notion. Another related but erroneous idea was that $\Delta\theta$ is traced by the position vector of any particle with respect to a specified origin, which was incorporated as another distractor to the item. This example is listed below.

1. The magnitude of the angular velocity ω of a rigid body rotating about a fixed axis is given by $\omega = \Delta\theta/\Delta t$. The angle $\Delta\theta$ here is the angle traced by

   (a) the position vector of the centre of mass of the rigid body from a specified origin.

   (b) the position vector of any particle on the body from a specified origin.

   (c) a line perpendicular to the axis from any particle on the body.

   (d) a line perpendicular to the axis from the centre of mass only.

   Example 2: The notion that angular velocity is the same for all particles of a rotating rigid body unlike their linear velocities is very important. Nevertheless, our interactions revealed that there are students who think angular velocity to be distinct for each particle. Some students also think that a particle closer to the axis moves faster thereby having greater linear and angular speeds. Their conviction was based on their visualization of this motion. At times, the idea that the particle closer to the axis has lesser distance to cover which enables them to do it faster supplemented their wrong appraisal. Question 2 (see below) was devised to probe these errors in student understanding.

2. Figure 8 shows a potter's wheel rotating uniformly about a fixed axis. P and Q are two points on the wheel. P is closer to the axis than Q. The particle P compared to the particle Q has a

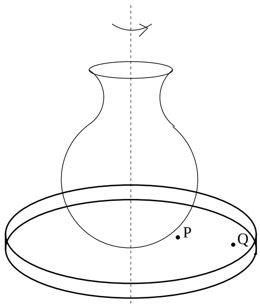

FIG. 8. A potter's wheel

(a) greater angular speed.

(b) smaller angular speed.

(c) greater linear speed.

(d) smaller linear speed.

Example 3: One of our interesting findings while studying rotational kinematics of a particle in rectilinear motion was the fact that an angular acceleration can exist despite zero torque (chapter 4, example 2). Even S2 group found this surprising. Many were convinced by a kinematic analysis that an angular acceleration exists. A cognitive conflict on whether to assert an angular acceleration based on kinematical deductions or to negate it based on non existence of a torque ensued. We observed that similar confusion exists in the case of rigid bodies as well. Item 8 addresses this confusion which incorporates a case of indiscriminate use of the equation $\tau = I\alpha$ as well (Mashood and Singh, 2012b). Most students failed to observe that this equation is invalid if the moment of inertia varies with time.

3. A girl is sitting on a stool with her arms outstretched and is rotating with constant angular velocity. The axis of rotation is fixed throughout the motion as shown in Figure 9 (by a dotted line). As she folds her arms toward her body she acquires an angular acceleration. Which of the following statements is true regarding the rotational motion of the girl-stool system?

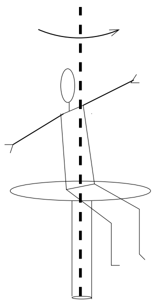

FIG. 9. A girl sitting on a stool, rotating about a fixed axis.

(a) There exists an angular acceleration because both the magnitude and direction of the angular velocity of the system changes.

(b) There exists an angular acceleration because only the direction of the angular velocity of the system changes.

(c) A torque ($\vec{\tau}$) acts on the system which results in the angular acceleration $\vec{\alpha}$ as per $\vec{\tau} = I\vec{\alpha}$ (where I is the moment of inertia).

(d) Torque acting on the system is zero.

Administration and analyses: The questions were administered to groups T2, S2 and S3. Table III shows the average performance, associated standard deviations and the average difficulty level for all the three groups. Similar to the response to part one of our inventory average score and the difficulty level of S2 is high indicating expertise. As such the focus of our analysis in what follows will be mainly on T2 and S3. In the remainder of our discussion, student refers to S3 unless mentioned otherwise.

Analyses of the detailed response patterns of the students and the teachers based on the frequency with which distractors to each items were chosen indicate pitfalls and reasoning errors. Further analyses aided by insights from interviews indicate that most of them could be categorized under the following four broad themes:

TABLE III. The average performance of each group along with their corresponding standard deviation and average difficulty level. Performance in percentages are given in parentheses. Total number of questions was 13 for S3 , 11 for S2 and 10 for T2.

	S3 (N=905)	S2 (N=74)	T2 (N=25)
Average score	6.28 (48.1 %)	3.08 (30.80 %)	9.72 (88.36 %)
Standard deviation	3.87	1.75	1.37
Average difficulty level	0.48	0.31	0.88

1. *Inappropriate extension of familiar procedural practices*: Repeated application of procedural practices in a topic results in habituation. This acquired familiarity at times led to students extending the same procedures to contexts where they are inappropriate. In our study on rigid body such practices can be traced to basic mechanics or rotational kinematics of a particle. Many incorrectly think that $\Delta\theta$ in the definition of $\vec{\omega}$ of a rigid body rotating about a fixed axis is traced by the position vector of any particle on the body with respect to a specified origin. Even a section of the S2 made the same mistake. A significant majority also considered $\vec{\omega}$ to be the vector sum of the angular velocities of all the particles constituting the pulley. The practice of summing a quantity over all particles is appropriate only for dynamical quantities like energy, momentum, rotational kinetic energy, angular momentum etc. Extending it to angular velocity is inappropriate.

2. *Reasoning cued by primitive elements in thought*: Concepts or ideas which resonate with more primitive ideas can influence our reasoning. Close and Heron (2011) describe an example of such a case where energy conservation appears to assume a higher status in student thinking over other conservation laws. We found a similar pattern wherein students ascribe a special status to the concept of centre of mass (CoM). Students extrapolated the notion of CoM as a representative point beyond what is warranted by physics. The special status of CoM seems to have influenced their answers to our questions. They thought that $\Delta\theta$ appearing in $\vec{\omega}$ of a rigid body is the angle traced by the position vector of the centre of mass of the body. The choice could also have been made to circumvent the difficulty in analysing a rigid body by reducing the analysis to a more comfortable choice of a single particle (CoM) (Ortiz et al., 2005). Another

striking case is the example of a rotating pulley. Here one can easily see that the centre of mass of the pulley is its geometric centre and it is at rest. Nonetheless students and teachers thought that the angular velocity of the pulley is equal to the angular velocity of its CoM. Another instance of the apparent cuing influence of the concept of CoM is evident when candidates ascribe a circular trajectory to an arbitrary particle on a rotating rigid body only if the axis passes through the centre of mass.

3. *Lack of differentiation between related but distinct concepts* : Novice thinking is often characterized by a failure to differentiate between related but distinct concepts. We discussed this earlier for a particle. In the case of rigid body motion we find a similar confusion between $\vec{v}$ and $\vec{\omega}$. Students do not realize that the angular velocity is the same for all particles unlike linear velocity for a rotating rigid body. We found at various instances, a tendency among students and teachers to ascribe aspects of linear velocity and acceleration to their angular counterparts. This holds for the magnitude as well as the direction.

Some of the difficulties observed could not be incorporated in any of the above four themes. This includes some students thinking that $\vec{a}$ and $\vec{\alpha}$ are unrelated. Another issue concerns the origin dependence of $\vec{\omega}$ of a particle and a rigid body. Many do not realize that the choice of origin is unimportant for $\vec{\omega}$ of a rigid body rotating about a fixed axis unlike that for a particle. The origin dependence of $\vec{\omega}$ of a particle is discussed elsewhere (Mashood and Singh, 2012c).

We administered the questions to 384 calculus based introductory level physics students at the University of Washington (UW). The administration was done after the lectures on rotational motion. The analysis revealed that popular distractors to most items were similar to the Indian population. This suggests that the pitfalls in reasoning corresponding to these distractors may be cross-national. Further research in this direction would be fruitful. However there exist significant differences between the Indian and the American students when it comes to differentiation between linear and angular variables. We noted that the UW students exhibit a better understanding of the magnitude of angular velocity of a rigid body. They in general seem to differentiate angular velocity from linear velocity. This may be because they have undergone tutorials in basic mechanics which addresses similar pitfalls experienced by students in linear mechanics (McDermott et al., 2002). However, when it came to the direction of the angular velocity, a significant majority of the UW students consider it to be clockwise or counter-clockwise. Such a response from the students may result from a tendency of instruction (in high school and even at the introductory level) to gloss over the vector nature of the angular velocity and to describe rotation as being either clockwise or counter-clockwise, without reference to direction.

VIII. CHAPTER 7: EVALUATION OF THE INVENTORY - STATISTICAL AND ITEM RESPONSE CURVE ANALYSIS

Inventory construction is an iterative process. To verify the inferences we made in the earlier administrations, further validate the items and subject the inventory to detailed statistical scrutinies the inventory was administered to a larger sample S3 of over nine hundred higher secondary students. A subset of around 35 students were interviewed. Here we describe the summary of the analyses and major findings along with illustrative examples. The analysis included the difficulty level, index of discrimination and point biserial coefficient for each item. We also calculated the whole test statistics, namely Ferguson's delta and Kuder Richardson reliability index. Item response curve (IRC) analyses were carried out for all the items constituting the inventory. These indices and techniques were briefly discussed in section 2 on methodology.

Administration and analyses: The analysis of the earlier administrations and the associated interactions with students led to minor modifications of a few items and distractors. A new item was added which brought the total number of items to 39. This revised inventory was employed in the large scale administration discussed in this chapter.

The whole test statistics we calculated include the average score, standard deviation, median, Ferguson's delta and Kuder Richardson reliability index. They are given in table IV. The average score of the students was 18.4 (47.18 %) with a standard deviation of 10.1. Note that the average is around 50 % which is optimal. The high standard deviation indicates a broad distribution of the total scores. The median was 15. Ferguson's delta (δ) for the inventory was 0.99. The desired value is ≥ 0.9. The value of . Kuder Richardson reliability (r_{test}) for the inventory was 0.93. The desired value for r_{test} is ≥ 0.8. The inventory did well on these indices.

Item wise statistics we calculated include the difficulty level, index of discrimination and the point biserial coefficient (Ding et al., 2006; Ding and Beichner, 2009). The average value of these indices for all items is given in table IV. The average difficulty level for the 39 items was 0.48. The difficulty level was above the desired value of 0.3 for all items except two. We observed an increase in difficulty level to most of the items compared to S1. This implies that the number of students getting the answers right went up in general. The average value of the index of discrimination to all items was 0.65. The desired value is ≥ 0.3. The value of the index of discrimination of all items was above 0.3 except for one. The average value of point biserial coefficient of all items was 0.53. The desired value for the index is ≥ 0.2. All items except one had a value above 0.2. The items for which some of the indices were below the desired values were further analysed using IRCs.

TABLE IV. Summary of the calculated test statistics. There were 39 items. Evaluation was done by giving 1 mark to the correct answer and there was no penalty for wrong choices. The total number of students N=905 (S3).

Test statistics	value
Average score	18.40 (47.18 %)
Standard deviation	10.01
Median	15
Ferguson's delta	0.99 (desired value, ≥ 0.9)
Kuder Richardson reliability index	0.93 (desired value, ≥ 0.8)
Average difficulty level	0.48 (desired value, ≥ 0.3)
Average of index of discrimination	0.65 (desired value, ≥ 0.3)
Average of point biserial coefficient	0.53 (desired value, ≥ 0.2)

In the previous administrations we made our inferences about patterns of student reasoning based on the frequency with which particular distractors were chosen to each item. The analysis was supplemented by the insights obtained from interactions with students. These patterns will be described in detail in chapters 3, 4 and 6 of the thesis. (e. g., fixation with inappropriate prototypes, indiscriminate use of equations etc.) The response patterns of the students in the large scale administration (N=905) discussed in this chapter is broadly consistent with those observed in earlier phases. The consistency of these patterns was also verified by interviewing a subset of students. Consider item 4 described earlier in section 4 as an illustrative example. The item probes the angular acceleration of an oscillating pendulum bob at the extreme position when its velocity is zero. The most popular distractor in phase 1 was the choice which stated that the angular acceleration is zero. This pitfall is similar to one observed in linear kinematics where students erroneously think that the linear acceleration of the bob at the extreme position is zero when the linear velocity is zero. In phase 2 the same distractor remained the most popular choice. In the interviews eight students who chose the distractor were asked about the linear acceleration at the point of maximum height of a ball thrown vertically upward. Seven of them said that linear acceleration at that point was zero thus indicating consistency.

Item response curve analyses:. We plotted the IRCs for all the 39 items. The total score of the students in the test was considered to represent their ability level. Here we discuss two items as illustrative examples on how we gainfully learned from the IRCs.

Example 1: The item given below probes students' ability to visualise the motion (clockwise - anticlockwise) of the second hand of a transparent clock (see figure 2) from the front and rear side. Also they have to ascertain the direction of angular velocity of the tip of the clock hand. Direction $\hat{k}$ is out of the plane of the paper.

1. A wall clock is transparent and the second hand can be viewed from both the front and the back side. Then,

   (a) if viewed from the front, the second hand moves anticlockwise with angular velocity in the - $\hat{k}$ direction.

   (b) if viewed from the front, the second hand moves clockwise with angular velocity in the $\hat{k}$ direction.

   (c) if viewed from the back, the second hand moves anticlockwise with angular velocity in the $\hat{k}$ direction.

   (d) if viewed from the back, the second hand moves anticlockwise with angular velocity in the - $\hat{k}$ direction.

Figure 10 depicts the IRCs plotted for all choices to the item. Fluctuations apart, the correct choice d to the item correlates positively with the ability level as can be seen. To facilitate a clear interpretation we modelled it by the logistic response function (see Fit in figure 10).

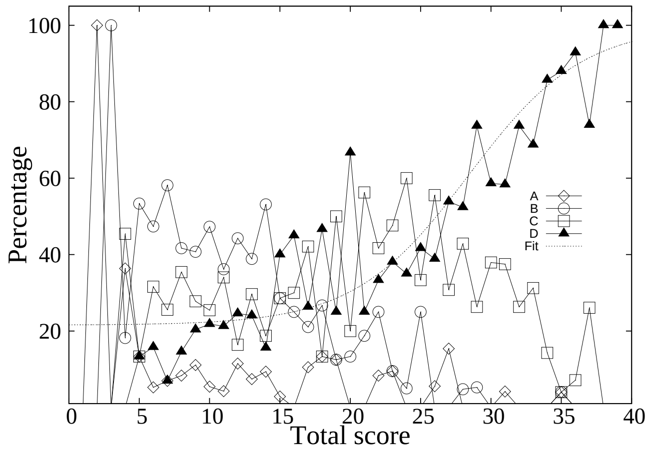

FIG. 10. Item response curves for the question on the angular velocity of the tip of the seconds hand of a wall clock.

The fit to the correct choice d to the item is a sigmoid which remains flat till the ability level 18 . The percentage of students opting d steadily increases after that. The slope starts decreasing after a score of 34. The choice a is insignificant as can be inferred from the low lying curve close to the x-axis. A closer look at the item response curves to the distractors reveals that b and c constitute the prominent wrong choices. However IRCs help us to meaningfully distinguish between these two distractors. IRC of distractor b is prominent in the ability range [3:15] while that of c is more popular among ability levels greater than 17. Thus the distractors b and c exhibit discriminatory power. Analysis of the content of these distractors reveals that choice c is more proximate to the correct answer than b. Choice c demands visualization of movement of the hand of a transparent clock from the rear side and also ascertaining the direction of its angular velocity. On the other hand b requires the same analysis by looking at the clock directly.

Example 2: Consider the item that probes the angular acceleration of the pendulum bob at the extreme position as the second example. The item can be found in section IV as example 4. We also discussed the item earlier in this section as revealing an instance of a pitfall paralleling one found in linear kinematics.

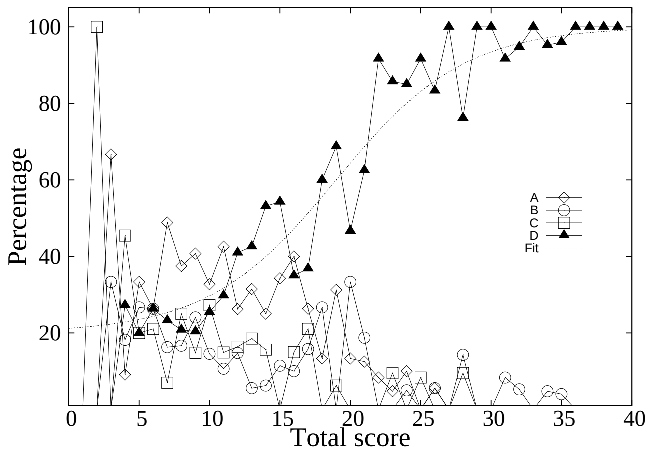

FIG. 11. Item response curves for the question on the angular acceleration of a pendulum bob at the extreme position

The most popular incorrect notion was that the acceleration of the pendulum bob is zero at the extreme position when its velocity is zero, given by distractor *a*. The IRC of the distractor *a* is significant in the region [5:17] and is a visual confirmation of this (see figure 11). In addition to the visual display, it may once again be noted that IRC provides the ability range of students harbouring this notion. This information is not provided by a tabulation of the frequency of the response patterns of students to the item. The other distractors *b* and *c* are relatively less popular as revealed by their IRCs. Their curves are overlapping, almost flat and close to the x-axis throughout the range from 2 to 35. Distractor *b* states that the angular acceleration at a single instant is undefined while *c* states that the angular acceleration at a single position is undefined. The curve corresponding to the correct choice *d* follows a sigmoid. The sigmoid is flat till the score 10 and then steadily rises till 30. The slope of the curve is indicative of its relatively good discriminatory power. After score 30 the slope decreases and the curve once again turns flat.

As mentioned earlier IRCs were plotted for all the 39 items. IRCs of the correct choice to all items had positive slope. Fits to most of them were clear sigmoids. Also, most distractors were chosen by a significant portion of students. The items for which the statistical indices were below their desired values were further analysed based on the IRCs. For these items we found that the curves to the correct answers were sigmoids and the distractors discriminating. Thus these items are useful and need not be dropped.

IX. CHAPTER 8: CONCLUDING REMARKS

Students as well as teachers find rotational motion as one of the more difficult topics in introductory physics. Our work on rotational kinematics has addressed this inadequately researched topic in PER. We have developed a concept inventory (CI) for this purpose. A CI apart from its utility as a ready to use diagnostic and assessment tool may serve other purposes significant to improvements in science education. The items can be adapted as clicker questions which are important ingredients of the peer instruction pedagogy (Mazur, 1997). It may be noted that peer instruction is one of the most popular research driven pedagogies. CIs also play a crucial role in large scale science education reforms as exemplified by the Carl Wieman Science Education Initiative (Weiman, 2007; Adams and Wieman, 2010). The fact that two among the three authors of the Force Concept Inventory are high school teachers indicates the potential of inventory development in teacher professional development. We have made modest attempts to encourage motivated teachers to think along these lines.

Taking into account the considerable potentials and possibilities we ensured that all salient aspects involved in the construction of our inventory were documented. This may help or serve as a guideline to those who would like to take up similar endeavours in the Indian context. We have demonstrated that interactions with students, teachers and experts constitute a core aspect in the development of an inventory. Analysis of Indian textbooks and review of problems in national level exams helped us to pose our items in contexts familiar to an Indian student audience. The systematic and iterative nature of inventory construction was illustrated. In the initial phase detailed verbal data was collected from around 50 students and 12 teachers. Think aloud protocol and semi-structured interviews were employed. Questions at this stage were open ended and explanatory type, among others. The resulting multiple choice questions were validated by experts and students before each stage of administration. The first phase of administration included two groups of students S1 (N=79), S2 (N=74) and teachers T1 (N=26), T2 (N=25). In the second phase the inventory was administered to 905 students from five urban centres spread across the country (denoted as S3). This large scale administration enabled us to perform detailed statistical tests and item response curve analysis. In addition, results from earlier administrations were verified and items were further validated. A subset of students were interviewed at each stage of the administration. A part of the inventory was also administered to 384 introductory level students, S4 at the University of Washington, Seattle.

Popularly employed statistical indices were calculated to evaluate the quality of individual items as well as the whole test. Item-wise statistics include the difficulty level, index of discrimination and the point biserial coefficient(Ding et al., 2006; Ding and Biechner, 2009). The whole test statistics Ferguson's delta (δ) and Kuder Richardson reliability index (r_{test}) were calculated for the inventory. The average difficulty level, index of discrimination and point biserial coefficient for the 39 items were 0.48, 0.65 and 0.63 respectively. Ferguson's delta for the test was 0.99 while Kuder Richardson reliability index was 0.93. The test did well on all these indices which were well above their desired values testifying to the quality of items and the reliability of the inventory. In addition to the statistical analysis we also carried out item response curve (IRC) analysis. IRCs were plotted for all 39 items. IRCs to the correct choices of all items correlated positively with the ability level and were mostly sigmoids. The curves provided a visual confirmation of the insights provided by the statistical indices. IRCs also provided additional information like the ability range of students partial to a particular distractor in an item.

The frequency of response patterns of students were analysed in detail. This analysis along with the interviews with a subset of students enabled us to uncover broader patterns of thinking prevalent among students. These include fixation with inappropriate prototypes, indiscriminate use of equations, pitfalls paralleling those found earlier in linear kinematics, inappropriate extension of familiar procedural practices, reasoning cued by primitive elements in thought, and lack of differentiation between related but distinct concepts. These patterns were connected to findings made by researchers in other topics. Administration of part of the inventory to students at the University of Washington, Seattle indicated similarities between the response patterns of Indian and American students. The extent to which the corresponding pitfalls in reasoning are cross-national is worth investigating in future.

We emphasise that the inventory underwent a series of quality checks. The items were content validated by experts and face validated by students. The statistical indices and the item response curve analysis attested to the quality of items as well as the whole test. We communicated results at appropriate stages of our research to international peer-reviewed journals in the discipline. The critical comments from the referees provided valuable insights. The stage-wise publication of our work ensured that the research was proceeding in the right direction. During the large scale administration we also administered two internationally standardised inventories namely the Force Concept Inventory and the Conceptual Survey on Electricity and Magnetism (Hestenes et al., 1992; Maloney et al., 2001). The indices δ and r_{test} were calculated for these standardised inventories. For FCI they were 0.99 and 0.95 respectively. Similarly for CSEM the values were 0.98 and 0.96 respectively. These values are consistent with the values obtained for our inventory.

Our study revealed that students as well as teachers experience an array of difficulties in rotational kinematics. We found that rotational kinematics of a particle is inadequately dealt with in most text books (Mashood and Singh, 2012c). We suggest emphasising operational definitions and procedural specifications as immediate corrective measures. We provided an operational definition for angular velocity of a particle (Mashood and Singh, 2012b). Our work as well as the study by Rimoldini and Singh (2005) pointed out that part of the difficulties can be traced to related difficulties in basic mechanics. This suggests that discussion of linear and rotational concepts in the same contexts will help students better differentiate between them. We suggest two pedagogically rich contexts for such a purpose - the oscillating simple pendulum and the particle in rectilinear motion.

The performance of the teachers is a matter of grave concern (Mashood and Singh, 2012a). We interacted with them in this regard. One of the plausible reasons was the lack of meaningful teacher training programs. The prevalent, unimaginative evaluation system which fosters rote learning exercises is another reason. Consequently teachers are pressurised into teaching to tests that form the basis of grades. These exams hardly assess conceptual knowledge or genuine problem solving skills. This, over a period of time, has blunted their ability to answer conceptual questions. Other socio-economic issues such as simultaneously managing family, administrative tasks in school and personal health, among others, may also have burdened them. We note that the teachers were from the hinterlands and semi-urban areas, whereas the students were from major urban centres. In other words these were not the teachers of the students surveyed.

A limitation of the concept inventories is their multiple choice format. The extent to which conceptual nuances can be covered by a limited number of simple statements (the distractors) is debatable. We do not have detailed access to what a student is thinking while making a particular choice to an item. There can be cases of false negatives and false positives (Hestenes and Halloun, 1995). The items are closed-ended (Smith and Tanner, 2010). If all possible modes of responses are not represented by the distractors the test may be imposing an answer on the student. It may be possible that distractors to items relevant to a population may not be appropriate for another. We tried to minimise some of these effects by constructing the inventory after considering open ended and free responses of students. The context dependence of novice thinking is another issue of concern (Bao and Redish, 2006). Novices may correctly answer a question about a concept in one context and incorrectly in another. As a result, when a student answers an item it becomes difficult to delineate the effect of understanding from the effect of context. It has been shown that the timing of administration of a CI and the incentives for the participants can affect the results (Ding et al., 2008). What precisely a concept inventory measures is another issue of concern. This has been extensively debated in case of the Force Concept Inventory (Huffman and Heller, 1995). Apart from the phenomenological insights provided, a cognitive science perspective of what CIs actually measure is unclear.

In future we plan to administer the inventory to larger and varying samples which include rural India. This is a daunting task given the linguistic and cultural diversities of India. We have initiated this effort in Hindi, the national language. Thus far we have confined ourselves to the concepts of angular velocity and angular acceleration. We also plan to investigate more complex motions like rolling with and without slipping along with other rotational concepts like angular momentum. We have learnt that other researchers are investigating related areas. Beichner is studying student understanding of graphs in the context of rotational motion (private communication). Physics education group at the University of Washington intends to revise the chapter on rotational motion in the Tutorials in Introductory Physics (McDermott et al., 2002). We also plan to work on an Indian adaptation of the same.

We conclude by mentioning some of the possible avenues that our work opens up. Quality science education to a larger populace is important for the economic and societal progress of the country. We need to promote research driven education reforms to achieve this. Our work points to the stark necessity of meaningful teacher professional development in the country. Scientific communities in our universities collaborating with practising teachers in developing CIs is one possible way to achieve this. Textbook writing in rotational motion should take into account our research findings. Rotational kinematics of a particle needs to be dealt with in detail before addressing rigid bodies. We think that such a sequencing would be pedagogically more prudent (Mashood and Singh, 2012a). We need to promote a culture of learning where alternative conceptions and pitfalls are considered as resources to achieve a better understanding. Instructors who are in a state of denial regarding the findings of science education research need to be made aware of the importance of research in education. CIs can play a facilitating role in this regard. Interactive pedagogies like peer instruction (Mazur, 2007) should supplement traditional modes of instruction. CI items can be adapted as clicker questions. Furthermore research driven curriculum like the Tutorials in Introductory Physics (McDermott et al., 2002) needs to be developed to address the pitfalls in understanding. In all these efforts a synergistic collaboration of practising teachers, scientific communities in the universities and researchers in education is important. We have made modest efforts in this regard. Further, analysis of the misconceptions and pitfalls we identified from the perspective of cognitive science would be fruitful. Such an endeavour would throw light on why these misconceptions and pitfalls arise. It may then help us to address them better.

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