Supporting In-service Professional Development of Mathematics Teachers: The Role of Beliefs and Knowledge

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By

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Declaration

This thesis is a presentation of my original research work. Wherever contributions of others are involved, every effort has been made to indicate this clearly, with due reference to the literature, or acknowledgement of collaborative research and discussions.

The work was done under the guidance of Professor K. Subramaniam, at the Homi Bhabha Centre for Science education, Tata Institute of Fundamental Research, Mumbai.

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In my capacity as the supervisor of the candidate's thesis, I certify that above statements are true to the best of my knowledge.

Date: 7-05-2018

Prof. K. Subramaniam

Thesis advisor

Dedicated to

Tauji

whose life inspired me

&

Eshaan

who loves maths

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1
Introduction

1.1 The educational reform context

The purpose of this chapter is to set the context and describe the motivation for the research study. Efforts in India have been undertaken for over a decade to implement the National Curriculum Framework 2005 (NCF 2005) that prioritizes learning with understanding and child-centered teaching (National Council of Educational Research & Training [NCERT], 2005). The NCF 2005 has major implications for preparing both pre-service and in-service teachers to understand and implement the new vision of education. It advocates a shift away from a textbook centered rote learning approach, to one that emphasizes the link between school learning and life outside school. It stresses that the knowledge that students bring to the classroom from their life outside, and the diversity of ability and ways of thinking within the classroom are resources for teaching and learning and not hindrances. Specifically with regard to mathematics, it gives precedence to the goal of mathematical thinking or mathematization, rather than knowing mathematics as a set of rules and facts. Clarity of thought, pursuing assumptions to logical conclusions, the ability to handle abstractions, and problem solving are considered to be central to mathematics and worthwhile aims of mathematics teaching and learning (NCERT, 2006a). The new curriculum, arguably expects from the teacher a deeper understanding of subject matter as well as the teaching learning process, rather than merely adopting new techniques. Teachers in the elementary and middle grades are expected to not only make their students fluent in computational mathematics but also address process goals in the learning of mathematics, such as reasoning, using multiple ways to solve problems, justifying their solutions, making generalizations and conjectures, analyzing the mathematical work of others, etc. (NCERT, 2006a). The efforts to focus on the quality of classroom teaching and the pedagogy used for teaching has been reinforced by policy related efforts to universalize elementary education through strong legislation in the form of the Right to Education (RTE) Act (Government of India[GOI], 2009).

Although ideas such as child-centered learning are not new, NCF 2005 has been effective in changing the discourse on education in a system-wide manner. Teachers are now more open to the idea that their teaching approach needs to undergo fundamental change. However, there is very little clarity about what this change really amounts to in terms of classroom teaching and learning, and schools and teachers continue to look for help as they try to interpret the message of the new curriculum framework in terms of teaching in particular domains. In terms of implementation of NCF 2005, besides a significant change in the textbooks, administrators of major school systems have tried to implement reform measures through directives and circulars.

Circulars have played a major role in the implementation of the curricular reforms after NCF 2005. The directives communicated through the circulars are treated as orders by principals and teachers and set the expectations for what is to be projected and displayed to authorities during inspections. In the circular dated Jan 3, 2007, by the Central Board of Secondary Education (CBSE[1]) board, schools were asked to implement the new curriculum package by asking teachers to use the textbooks with "hands on" and "activity oriented learning" through following a "constructivist approach". A series of teleconferences were also held on this theme. The excerpt from a circular given below illustrates how the main thrust areas of reforms were communicated.

> Teachers must understand the pedagogical orientation to the course materials and organize the children's classroom experiences in a manner that permits them to construct knowledge on their own. There is a great thrust to distinguish knowledge from information and to perceive teaching as a professional activity, not as coaching for memorization or as transmission of facts. Schools have to make every attempt to empower the teachers accordingly so that the curriculum transaction truly reflects the shift in educational paradigm as envisaged in NCF-2005. Simultaneously schools will have to prepare themselves for implementing new syllabi and textbooks for classes II, IV, VII, X and XII during 2007-08. (CBSE, 2007)

The measures promoted by NCF 2005 included the thrust on connecting mathematics with daily life and the use of manipulative. These were consistent with the step of making a mathematics laboratory compulsory in all schools affiliated to CBSE board. A circular was also issued for reducing the weight of school bag by "not compelling students to bring textbook" daily and "avoid reading from the textbook" as well as "reducing the dependence" on the textbook for teachers (Kendriya vidyalaya Sangathan[KVS], 2009).

These circulars convey the voice of the authorities, who ultimately depend on teachers to implement the vision of curricular reform. A dependence on authority to effect curricular reform can result in teachers adopting the discourse of reform while not actually incorporating the changes in their practice. There is insufficient recognition that as teachers attempt to implement the ideas inherent in the curriculum and legislation reform, they face multiple challenges. The challenges

1 The Schools in this study are affiliated to the CBSE board.

range from lack of resources and large numbers of students in the classroom, to lack of subject matter and pedagogical content knowledge, knowledge of how to relate school subjects with the daily lives and contexts of children. NCF 2005 has been criticized for being silent on how teachers are supposed to bring about the change in their classroom and for not addressing the much needed teacher development to support curriculum renewal (Batra, 2005). Efforts undertaken like changing textbooks and issuing directives to schools and teachers sidestep the issue of addressing beliefs and developing adequate knowledge amongst teachers, which is needed to realize the vision portrayed in the new curriculum framework. Although workshops have been conducted to "orient" the teachers to the new curriculum and textbooks, their impact on classroom teaching is doubtful. A recent study by NCERT (Yadav, 2012) acknowledged that even after curriculum reform teachers' practice has largely remain unchanged in their classroom.

1.2 The in-service teacher education challenge

A robust infrastructure and well placed programs for in-service teacher education would have greatly supported the curriculum reform spearheaded by NCF 2005. Several national committees have over the years recognized the need for the continuing professional education of teachers and recommended "at least two or three months of in-service education in every five years of service" (Government of India [GoI], 1966, Ch. IV, Para 4.56). The New Education Policy of 1986 recommended a rapid expansion of the infrastructure for education of teachers at the elementary level through the setting up of institutions at the district and block levels, which would deal with both pre-service and in-service teacher education (GoI, 1986). While the teacher education infrastructure has indeed expanded vastly, issues of poor quality and low relevance of teacher preparation remain (Sharma & Ramachandran, 2009). Further, teacher education institutions have tended to focus more on pre-service education leading to the neglect of in-service education (MHRD, 2009). Studies have indicated that the present teacher education, pre-service as well as in-service, fails to address the needs of the teachers as they do not consider the stark realities that teachers face in their schools like large class size, multilingual students and incomplete understanding of the content in the textbook (Ramachandaran et al., 2009). The problems in the design and mode of teacher education and professional development programs have been recognized and change has been recommended by several national committees, but little change has come over the years (Batra, 2005).

The National Focus Group on Teacher Education remarked on the inadequacy of teacher education and how despite various recommendations of commissions they have remained unchanged in terms of their "substance, experience offered and modalities adopted" (NCERT, 2006b, p.3). It recommended "recognizing the active 'agency' in institutionalizing the process of school curriculum renewal" by creating "reflective practitioners" (p. 25). The position paper by National Focus Group on Teaching of Mathematics recognized the problem of inadequate teacher preparation leading to primary teachers reproducing techniques experienced in their schooling, the pedagogy adopted rarely "resonating with findings of children's psychology" and inability to link formal mathematics with experiential learning (NCERT, 2006c, p.7).

1.2.1 Recognizing the agency of the teacher in in-service teacher education

It has been suggested that pro-active engagement of the school teacher in the process of curriculum design needs to be ensured to make curriculum reform a success (Batra, 2006). Batra talks about why teachers are central to the process of educational reform in the following excerpt.

> Connecting knowledge to life outside the school and enriching the curriculum by making it less textbook-centered are two important concerns of the NCF [2005]. In order to help children move away from rote learning, teachers will need to be prepared to give children the opportunity to derive meaning from what they read, see, hear and experience. This is possible only when teachers are able to play an active role in the design of learning material, and have the knowledge and skills to organize meaningful learning experiences and to use evaluation as a means to improve their own performance and children's learning. For this to happen, the teacher needs several support mechanisms, including a pool of learning resources to choose from, the skills to identify developmentally appropriate text materials, a critical and analytic mind and the opportunity to engage children with learning resources outside the classroom (Batra, 2009, p.8)

Most pre and in-service programs view teachers as mere agents of the state, and teachers as implementers of curricular and reform directives. Hence they do not directly address the teacher"s own conceptions of teaching, learning and mathematics gained from her experience. Thus in-service teacher education modules have over the years acquired the character of reform focused "add-ons" that do not aim at fundamental reflection and awareness in teachers beliefs and attitudes. A more detailed review of in-service teacher development initiatives is presented in the next chapter. The creation of spaces where teachers as well as other stakeholders articulate and reflect on the beliefs that they hold is still an unfulfilled need of in-service teachers. Professional development programs need to include opportunities to share and discuss views held by teachers while respecting their identity as professionals. In my view therefore TPD programs need to have a broader vision of the needs of a teacher as a developing professional, and must address issues of knowledge, beliefs, attitudes and practices in a comprehensive manner, rather than in the narrow context of a particular reform. An important criteria that in-service TPD programs need to meet is to bring the workshop goals closer to the actual work of teaching and help in developing teachers as professionals, creating opportunities to put them in-charge of their own learning.

1.2.2 The problem of inadequate content knowledge

The typical educational experiences of a teacher in school or in the university do not prepare her or him to engage with mathematics, to struggle to find a solution to a problem, to examine a concept from different points of view, to make connections, to reason and provide justifications, all of which are stressed by the new curriculum framework (NCERT, 2006c). In a typical B.Ed. program, the focus is almost entirely on pedagogical technique, and content is assumed to have been mastered earlier. The fact that such education leads to a grossly inadequate preparation of the mathematics teacher with regard to her understanding of mathematics is well recognized (Ravindra, 2011). These is consistent with the findings of other studies such as Banerji & Kingdon (2010), Ravindra (2007), and Dewan (2009), which have revealed the unsatisfactory status of knowledge of mathematics of regular school teachers. This state is a reflection of the teachers' own education which valued only rote memorization of procedures on the one hand and lack of opportunities to re-learn mathematics in a meaningful way during professional education and during the course of their career on the other hand. A detailed review of the teacher education curriculum for preparing teachers of mathematics is presented in the next chapter.

In the present context of curriculum revision, secondary teachers are faced with content in which they are not confident and thus unable to make connection within and across mathematics while relying on notes/guides available in the market. The Focus Group recommended that professional development have a specific focus on mathematics as opposed to "generic" teacher training. A recommendation was also made for the generation of a large number of freely available resources and networking among teachers, college teachers and research mathematicians to enhance their pedagogic competence.

The Justice Verma Commission (GOI, 2012) set up to review the problems in teacher education pointed out that subject knowledge is not focused in most of the pre-service teacher education program and that there is urgent need to develop comprehensive programs for the continuing professional development of secondary teachers. It identified up-gradation of the knowledge of the subject as one of the important goals of in-service teacher education. Subsequent to NCF 2005, an initiative to reform teacher education was undertaken in the form of the new National Curriculum Framework for Teacher Education ("NCFTE 2009"; National Council of Teacher Education [NCTE], 2009). NCFTE 2009 seeks to promote a balanced teacher education curriculum that pays attention to not only pedagogy, but to deep understanding of the subject of choice.

While the teacher education policy documents and the curricula of some innovative programs acknowledge the importance of content knowledge, actual policy measures suggest the opposite. With the passing of the Right to Education Act, and the consequent pressure to universalize elementary education, most states are faced with a shortage of teachers. This situation has led to multiple cadres of teachers and the appointment of para-teachers without the requisite teacher qualification (Govinda & Josephine, 2004). This policy measure, which reiterates the assumption that a primary teacher does not need to know mathematics beyond the level that he/she is going to teach, stands in contrast to policy documents that stress the importance of content knowledge. Thus, in practice, there are very low expectations by policy makers regarding the level of content knowledge required of a primary teacher.

1.2.3 Addressing teachers' beliefs and knowledge to develop practice

In India most teachers' teaching is shaped by what is expected of them in the system as well as the kind of folk ideas of teaching that teachers as a community have a shared understanding of (Ramachandaran et al., 2009). The teaching of mathematics in India is largely focused on teaching of procedures for solving textbook based tasks. Clarke (2001) found that Indian teachers expect one right answer, emphasize repetition, listening carefully to examples shown by the teacher and "explaining" the steps of the procedure again when a student gives a wrong answer. The curriculum review focus group on teaching of mathematics raised concerns about the "tyranny of procedure and memorization of formulas in school mathematics" (NCERT, 2006c, p. 6) and suggested that mathematics teaching should focus on conceptual aspects and processes of mathematics like reasoning and communication. A study by Dewan (2009) indicates that beliefs held by not only teachers but even administrators, faculty members and directors of teacher education institutions are far removed from the ones envisioned in NCF 2005, thereby indicating the extent of challenge to implement the new framework. A recent study (NCERT, 2015) indicated that mathematics teachers were positive about using problem solving activity (85%), thought that their students understand geometry (74%) and were confident that their students would be able to explain how they got their answers (58%). However, only 36% were confident that students can solve the problems on their own, suggesting that they had low expectations from students for the kind of mathematical abilities advocated by NCF 2005.

Although research studies across many countries have demonstrated how it is important to address teachers' beliefs and knowledge to bring about substantive change in teachers' practices, there have been few Teacher Professional Development (TPD) programs in India, which have focused on the beliefs and knowledge required to facilitate the kind of teaching as envisioned in the curriculum reform documents like NCF 2005 (Kumar, Dewan & Subramaniam, 2012). Studies elsewhere in the world have indicated that focus on change in teaching strategies without taking *teacher thinking* into consideration leads to teachers making superficial changes without any significant change in student learning opportunities (Cohen & Ball, 1990). It is therefore important to first understand the beliefs and practices that are prevalent among teachers in order to support reform in teaching that is not superficial. Research is needed to understand how the workshop based interventions, typical of in-service programs, support the reflection on beliefs and building on teachers' knowledge. Studies identifying teachers' beliefs and how they are held will help to design TPD programs to allow teachers to engage with reflecting on and questioning their beliefs, rather than just superficially adopting the pedagogies proposed. Thus, in the Indian context as elsewhere, the goals that TPD programs need to focus on include providing opportunities to make teachers' knowledge and beliefs explicit and building on them through reflection and engagement in tasks as well as fostering communities of practice where spaces for developing shared understanding for these issues is provided. One needs to understand the beliefs, conceptions and practices of teachers as well as understand the contexts in which they are placed, in order to design a program that would be effective and useful.

1.2.4 Need to redesign in-service teacher education

According to a recent report (Mehta, 2011), 35% of all elementary school teachers in India received some form of in-service training in the year 2007-08. However, in-service programs do not follow a well thought out structure and there is no regulatory mechanism that ensures the relevance, quality and suitability of the training provided.

In India, workshops are an important component of TPD programs on which the greatest time, effort and resources of the state are spent. In my experience, and as reported elsewhere, TPD workshops are often organized in an *ad hoc* manner on the basis of expediency, sometimes driven by the need to utilize funds (MHRD, 2009, p. 2,15-16). There is no clear consensus about what needs to be done in these workshops and how it is to be done. Resource persons, who are chosen by the coordinators on the basis of availability and willingness, design and conduct their sessions without considering how their session fits into the workshop as a whole. In structured large-scale programs, TPD is sought to be achieved through the "cascade model" of training (MHRD, 2009, p.15), where master resource teachers are trained first, who in turn train other teachers. The design and content of the modules, which are used repeatedly at each tier of the cascade training, is generally not based on empirical evidence. The vision underlying most of these programs restrict teachers' agency to implementing a new textbook, a pre-designed pedagogy or a prescribed assessment technique.

The document for curriculum framework for teacher education, NCFTE 2009, observed that there is very little research on the effectiveness of in-service teacher programs and the research that exists does not provide a thorough understanding of the interventions reported. The research reported has been anecdotal and impressionistic and there has been reporting of even contradictory findings depending on who is doing the research (NCTE, 2009). The pedagogy adopted by teacher educators in most teacher education institutions is mostly the lecture method (Walia 2004; Ravindra, 2007; NCERT, 2016). The recent research on evaluation of in-service teacher education (Yadav, 2012) describes the strategies adopted in Sarva Shiksha Abhiyaan (SSA) in-service teacher education program and found that in the states where there is "adequate planning and activity based reflective training transaction, and is reinforced by further training in CRC and with onsite professional help during school visits, the transfer of training to classroom practice is evident" (Parvin Sinclair, Foreword in Yadav, 2012). However, the training is largely described in terms of forms of interaction and strategies used for discussion with teachers; the tasks worked on by teacher and the content focused is not elaborated. The changes observed in the classrooms of participating teachers is also described through teacher behavior and use of materials and activities which keeps the content invisible as also the pedagogical useful ways in which teachers use their content knowledge for teaching (Yadav, 2012). Further, research is needed to understand what ideas from the workshop contexts are taken up by the teachers in the classroom and what kinds of supports are needed to facilitate teachers in making the intended changes in their teaching practices.

As mentioned above, a renewed attempt to address the problems of pre-service and in-service teacher education was made by the NCFTE 2009, which re-affirms the importance of in-service teacher development, and puts forth several principles that need to govern the design of in-service teacher education programs (NCTE, 2009). Of these, the following principles are highlighted as they are focused in the study:

- Designing programs with clarity about aims and strategies for achieving these aims,
- Allowing teachers to relate the content of the program to their experiences and also to find opportunities to reflect on their experiences,
- Need to respect the professional identity and knowledge of a teacher and to work with and from it,
- Creating spaces for sharing of teachers' experiences,
- Addressing teachers' needs, and
- Extended interaction with a group of teachers (NCTE, 2009, pp. 66-67).

NCFTE emphasizes that there is a need to develop clear vision of the goals that programs must achieve and the means by which they can be achieved. It cautions against compromising interactivity especially through the use of electronic media, aiming at quick fixes, over-training, and routinized and superficial training. The principles highlighted above are especially important for the focus on in-service teacher professional development (TPD) workshops. However, there is a dearth of studies that show the use of these principles and their relation to the teachers' development of beliefs and knowledge or adoption of student-centered practices.

1.2.5 Revisiting the goals of teacher education

It is pertinent at this point to draw together the implications of the discussion above for what the goals of pre or in-service mathematics teacher education must be. Several components of knowledge that are needed to teach mathematics remain so far inadequately addressed in the educational trajectory of teachers. An important need is strengthening teachers' knowledge of mathematics, which includes not only an understanding of the concepts involved but also an appreciation of the nature of the discipline and its specific nuances. A second aspect that teachers need to feel assured about is the need for children to learn mathematics, and the kind of mathematics that they need to learn. A third aspect that the teacher needs to know involves the learners: what strengths and experiences do they bring to the classroom and how do these shape their capability to learn? A fourth aspect is understanding how mathematics needs to be addressed and engaged with in the classroom keeping in mind the above.

Teachers' beliefs and attitudes are also crucial. They are related to the components of knowledge described above, but are also independently directed. These attitudes, which may arise from prejudices, include their notions about the nature of mathematics, about children, their background and learning capability, about classroom processes and about what the purpose of education including of mathematics education can be and should be. It is quite common for educators and administrators to believe that children from disadvantaged socio-economic backgrounds are incapable of learning mathematics, either because of an inherent lack of ability or because they do not have the cultural preparation and attitude to learning. The teacher also needs to have confidence in her own ability to do mathematics in order to encourage students and to give space for their thinking.

There is a need to reformulate in-service teacher education programs to address the issues discussed above including in particular, the capabilities of all children and the strengths specific to the group, how children learn mathematics and what conceptual understanding of mathematics means. The challenge of the divorce of pedagogy from mathematical content and mathematical thinking is one of the deep structural problems that needs to be addressed. Some efforts in this direction have been made by innovative programs of pre – service teacher education such as the integrated 4-year programs combining a University degree together with a teacher qualification and the Bachelor of Elementary Education program of the Delhi University. Several innovative in-service teacher training programs have also addressed the issues described above such as those by Vidya Bhavan, Eklavya, Digantar and HBCSE. (These programs are discussed in the next chapter.) What is not addressed in these in-service teacher education initiatives is the systematic study of what teachers take up as ideas, processes and resources from the programs into their classrooms which have an impact on the type of opportunities students get to engage with mathematics in the classroom. This study aims to address this gap by studying the impact of a teacher professional development (PD) intervention on the beliefs and knowledge of participating in-service teachers and studying what is taken up from the workshops into the teachers' classrooms.

1.3 Overview of the thesis

The study reported in the thesis was located in a project aimed at promoting change in teachers' practice towards teaching that is more responsive to the development of students' understanding. The broad question that was investigated in the study is:

In the context of the classroom teaching, what factors support teachers in adopting learner centered practices and what factors inhibit or constrain them in doing so.

This question was interpreted in terms of a framework that took teachers' beliefs, knowledge and goals as the core components of teacher learning. The larger study was composed of four sub-studies having specific research questions arising from this larger question. Following are the main research questions addressed in each of the four sub-studies:

Sub-study 1: What are the preferred practices and beliefs held by the participating teachers and how do these interact with one another?

Sub-study 2: What principles of teacher professional development underlie the design and enactment of tasks during the PD workshop? What are the significant features of the participation (agency) by the teachers and teacher educators in the workshop?

Sub-study 3: What challenges does a responsive teacher face in implementing the intended change in practice?

Sub-study 4: How does teachers' participation in regular topic study group meetings support the development of their topic specific specialized content knowledge and how does it influence their teaching in classrooms?

The first two sub-studies were located in the setting of a PD workshop and focused respectively on teachers' beliefs and design and interaction in the workshop during year 1. Sub-study 3 focused on one teacher's attempts to change her practice and was located in the classroom collaboration setting across the two years. Sub-study 4 in year 2 includes studied teachers' engagement with beliefs and knowledge in a topic study group through use of a framework for meanings and representations of integers, and the take up in their classroom teaching.

This chapter (Chapter 1 of the thesis) introduces the context in which the study was undertaken, identifies the motivation for the study, the research gap which the study addresses and the limitations of the study.

Chapter 2 reviews relevant research and attempts to meet multiple goals. Through support from prior research, it elaborates the philosophy, theory and framework used in the study for designing in-service teacher education. It presents a review of the research about the beliefs, knowledge and practice of mathematics teachers and their interaction. Research on strategies used for effective mathematics teacher professional development and assessing change in teachers' practice is reviewed. A review is also presented of the methodological aspects of investigating in-service teacher professional development and teachers' beliefs, knowledge and practice.

Chapter 3 discuss the research study and the methodology used in the overall study. The methods used in the different sub-studies for data collection and analysis are discussed along with the role the researcher and the participating teachers played in different sub-studies. The methodological approach followed was participant observation and the methods of analysis were qualitative including case studies, supplemented with quantitative analysis for the belief questionnaire. The participants of the study were selected through purposive sampling, and included mathematics teachers, who participated in the Professional Development (PD) workshop for ten days (inclusive of a non working Sunday) in Year 1. A few additional teacher participants, selected on the basis of convenience and similarity of school system with the main sample, also responded to the questionnaire. The professional development activities during the two years of the study are summarized in Table 1.

Table 1: Timeline of the study

Sub-study	Year of the study	Data collected	Professional development activity
Sub-study 1	Year 1 (May-June, 2009)	Questionnaire and interviews	10 day professional development workshop
Sub-study 2	Year 1 (May-June, 2009)	Video records of workshop sessions	10 day professional development workshop
Sub-study 3	Year 1 and 2 (2009-2010)	Audio records of lessons, researcher's notes of lessons and post-lesson discussions	Collaboration in the classroom
Sub-study 4	Year 2 (2010)	Video and audio records of workshop sessions; audio record and researcher's notes of teaching of integers	Topic focused workshop + collaboration in the classroom

Chapter 4 discusses the teachers' preferred practices as well as beliefs at the beginning of the study, which is the focus of Sub-study 1. The findings of this Sub-study serve as a background to the findings of the other sub-studies, in which teachers engaged in professional development activities.

Chapter 5 presents Sub-study 2, which describes teachers' engagement in a professional development workshop by analyzing the tasks as well as interactions that occurred in the workshop.

Sub-study 3 described in Chapter 6 is a case study of a teacher who participated in the PD workshop and showed inclination to change her practices towards teaching mathematics with understanding. The Sub-study highlights the challenges that arise when a teacher may agree with the philosophy of curriculum reform but still needs effort and relevant knowledge to engage students in developing an understanding of mathematics.

Sub-study 4 in Chapter 7 illustrates the importance and feasibility of developing specialized content knowledge among a group of four teachers by engaging in topic-focused professional development on the topic of integers. The teachers developed their knowledge of the meaning of integers and integer operations to construct and use tasks for teaching integers.

In the concluding chapter, I provide a summary of findings from the four sub-studies and draw conclusions across them about teachers' practices, beliefs and knowledge as well as the impact of professional development initiatives on participant teachers' beliefs and practices. Implications for professional development initiatives of the study are discussed. Recommendations for further studies are also presented.

1.4 Limitations of the study

This research study being exploratory in nature is bound to have limitations, some of which are listed below.

1. The sample selected for the study is small and primarily from Mumbai city. For sub-studies 2, 3 and 4 smaller samples were selected from the initial sample as the nature of the research question required getting an in-depth understanding of the challenges faced by teachers while teaching and the intervention in the form of a topic focused workshop. The qualitative nature of the study as a whole and the sub-studies justifies the small sample.
2. A large part of the data in this study is in form of audio recordings which required transcribing. To limit the scope of the study and time required to transcribe and analyze, there were several lessons which had to be dropped from the analysis. For Sub-study 3, only lessons related to fractions were considered since it was one of the topics focused

during Sub-study 2 and the data included lessons on this topic across both years. For Sub-study 4, lessons related to integers were selected since they provided information about the impact of the topic study workshops on teachers' practice.

3. It was not possible to observe the teaching of participating teachers before Sub-study 1 and 2 because of the timing of the study and school schedules. The first point of contact with the teachers was during the TPD workshop. Data about the initial preferred practices of teachers was thus collected through questionnaire and interviews rather than through lesson observation.

4. The claims for teachers' learning and changes in practice have been made from observations by the researcher and the self-reports of the teachers. No standard instrument was used to assess the development of teachers' knowledge. However, in-depth analysis of teachers' discourse in workshops and their teaching provide evidence for their learning.

2

Review of Literature

2.1 Introduction

The objective of this chapter is to set the research context for the study presented in this thesis. This review provides the background for the four sub-studies reported in this thesis and the constructs that have been explored in the sub-studies. Specific literature related to Sub-study 3 and 4 on the learning of fractions and integers is reported in Chapters 6 and 7.

The review of research literature has been guided by following questions:

1. What are the efforts that have been made in India towards improvement in teacher education?
2. What are the different theoretical positions that guide the design of professional development for teachers' professional growth?
3. How has teachers' professional growth been characterized in terms of development or revision in their beliefs, knowledge and practice?
4. What mechanisms or processes have been identified in the research literature as contributing towards teachers' professional growth in terms of beliefs, knowledge and practice and interactions between them?

2.2 Policy and efforts to improve teacher education in India

In this section, review of some of the recommendations pertaining to teacher education in India of the landmark policy documents over the years has been done. These recommendations form the background in which current policy debates and measures are undertaken.

The Kothari commission (Government of India[GOI], 1966), which examined at all aspects and all levels of education in a comprehensive manner, is considered one of the most important policy documents in education in India. Recognizing the importance of teachers in improving the quality of education in India, the commission recommended "securing a sufficient supply of high quality recruits to the teaching profession" by increasing the status of teachers, "providing them with the best possible professional preparation" and "creating satisfactory conditions of work" (GOI, 1966, Ch 3, Sec 3.01). To improve teacher education in the country, the commission recommended professionalization of teacher education and urged that isolation of teacher education institutes from university life, from schools and from one another be removed. It recommended reorganization of teacher education programs at all levels, including the reorientation of subject knowledge and improvement in methods of teaching and evaluation. It recognized problems in teacher preparation programs like set pattern and rigid techniques for practice teaching done for a few isolated lessons, which was unsupervised or ill supervised. It recommended increasing the duration of teacher education programs at primary and secondary levels from 1 to 2 years to allow deep study of fundamental concepts in the subject matter. For in-service teacher education, the commission called for research based inputs in "the organization of a large scale, systematic and coordinated program of in-service education, so that every teacher would be able to receive at least two or three months of in-service education in every five years of service" (GOI, 1966, Ch. IV. Para 4.56). Many of the recommendations of the Kothari commission remained unimplemented until much later (Naik, 1982).

The National Commission on Teachers (GOI, 1983-85), among other recommendations, advocated a 4-year integrated course after 12th grade for the professional qualification of teachers, while for in-service teacher education it recommended establishing school complexes for school based professional development. These complexes include schools within the radius of 5-10 miles having 1-2 higher secondary schools, 6-7 middle schools and 30-35 primary schools. While acknowledging the woeful inadequacy of in-service education, the commission recommended that classroom and practical needs of teachers should be identified by surveys and studies. The programs should be announced well in advance and feedback from schools and teachers should be taken after in-service courses. Resource persons for teacher professional development were recommended to be from diverse backgrounds – university professors, people from industry and agriculture, practicing teachers and supervisors. The in-service course should be in the workshop mode for developing materials that teachers can take with them for directly using in classrooms. The commission noted that what teachers need most "is a change in the climate of schools, an atmosphere conducive to educational research and enquiry". Key recommendations again remained unimplemented. For example, very few 4-year integrated teacher education programs exist in the country and attempts are being made now to initiate them.

The New Education Policy of 1986 recommended a rapid expansion of the infrastructure for education of teachers at the elementary level through the setting up of institutions at the district and block levels, which would deal with both pre-service and in-service teacher education (GOI, 1986). NPE 1986 attempted to break the separation between pre and in-service teacher education by considering both as phases of a continuous process thus acknowledging the need for career long professional development of teachers. The NPE led to a rapid and vast expansion of the number of teacher education institutions. However, these were overwhelmingly privately owned, delivering poor quality education (Batra, 2013).

The Acharya Ramamurthy committee (GOI, 1990) set up to evaluate the progress on National policy of Education, 1986 emphasized the role of actual field experience during internship to foster professional growth of teachers. The Committee explicitly stated that "in-service and refresher courses should be related to the specific needs of the teachers. In-service education should take due care of the future needs of teacher growth; evaluation and follow up should be part of the scheme" (as cited in NCERT, 2006c, pp. 4) It recommended adoption of "internship model" by having brief theoretical orientation followed by 3-5 year supervised teaching under mentors.

The Yashpal committee report titled "Learning without burden" (GOI, 1993), reviewed the school curriculum and had a major impact on its revision. With regard to teacher education, it recommended restructuring of the course content to serve the changing needs of school education and making teacher education more practice oriented. The National curriculum framework 2005, which attempted to implement the recommendations of the "Learning without burden" report in a systemic manner, acknowledged the problems in teacher preparation as teachers are prepared for disseminating information rather than fostering reasoning in mathematics. It acknowledged that while the teacher education infrastructure has indeed expanded vastly, issues of poor quality and low relevance of teacher preparation remain. Further, teacher education institutions have tended to focus more on pre-service education leading to the neglect of in-service education.

The Teacher Eligibility Test (TET), now made an essential qualification by the Right to Education act of 2009 to secure a teaching job in any school, acknowledges the importance of content knowledge. There are different tests for primary and middle school level aspiring teachers having 150 multiple choice questions. Out of the 150 questions in the primary level test, 30 questions are devoted to Mathematics, of which 15 questions are based on the content in the school textbooks and the remaining 15 on pedagogical issues like error analysis and related aspects of teaching and learning, and understanding children's reasoning and thinking patterns and strategies for making meaning and learning. For the elementary level mathematics teachers (Grades 6 to 8), again 30 questions are devoted to mathematics, of which 20 are devoted to content and 10 to pedagogical issues. (A similar pattern is followed for science.) Teachers need to get 60% correct answers to pass the test. The recent results show an extremely low pass percentage of 5.5% for primary teachers and 6.5% for middle school teachers. The Human resource development minister ascribed the poor results to the mushrooming of private teacher education institutions (12689 private institutions as compared to 1178 government teacher training institutes), whose quality of teacher preparation may be poor (Teacher tests results...Kapil Sibal, 2012).

2.2.1 Teacher education in India

Pre-service teachers of primary school in India qualify to teach by completing a Diploma in Education (D.Ed.), while secondary teachers complete the B.Ed. Degree. The mathematics component in the D.Ed. programs, like in school, emphasizes remembering known solutions to problems, and does not encourage a genuine engagement with the content. While recognizing this the National Curriculum Framework for Teacher Education (NCFTE, mentioned in the previous chapter, NCTE, 2009) recommended enhancement of entry qualification and duration of training making the D.Ed. equivalent to a degree program and bringing these isolated institutions under universities for their management. It must be noted that the teacher and the teaching profession in India has a low social status and becoming a teacher is the last choice for most entrants into the profession.

Among the graduates and post-graduates who complete the B.Ed. program, the capability of even those who have studied mathematics at the University level is limited, since most University mathematics programs do not give the learner any confidence in the subject, fostering a view of mathematics as a limited set of problems that have been already solved (Dewan, 2009). The tasks that students learn to complete is not one of formulating and solving problems that cannot be solved by using known principles but of solving problems that can only be solved with a known trick. It is possible that this attitude to mathematics and learning, and their lack of confidence in mathematics leads them, as school or college teachers, to shun dialogue in the classrooms.

2.2.1.1 Teacher education curriculum and its revision

In the post independence era from the 1950's to the 1970's, pre -service teacher education mainly emphasized theoretical aspects like discussing aims of mathematics education, inductive and deductive method, analytic and synthetic method, focus on Herbatian steps of preparation, and presentation and application for planning lessons (Chel, 2011). The mathematics method paper had a weightage of 10% of the total marks. The student teachers were expected to make charts and other teaching aids but there was no emphasis on relating mathematics to out of school experience or to other subjects.

The comprehensive curriculum framework for teacher education was released in 1978 which adopted a task oriented approach to teacher education viewing teaching as a series of concrete and hierarchically graded tasks. It had practical aspects of teaching as its focus as it suggested that student teachers should be put through a series of micro teaching situations as simulations before being pushed into actual classrooms. The weightage of the mathematical component was raised to 22.5%. The assessment of content was made by asking student teachers to solve problems from different content areas of school mathematics up to class 12. In the earlier syllabus, there was no separate evaluation of mathematics content. However in 1990's there was criticism of this move as teachers and teacher educators felt that testing of content separately without integration with methods is redundant since teachers have already been tested for it in their undergraduate degree program (Chel, 2011).

Following a major revision of the school curriculum, through the "National Curriculum for Elementary and Secondary Education" (NCERT,1988), which recommended integration of theoretical understanding with practical application and recommended more weightage to practical application, a revision was also made of the teacher education curriculum. A major watershed development in teacher education was the establishment of the National council for Teacher Education (NCTE) as a statutory body in 1993. The NCTE brought out a "Curriculum Framework for Quality Teacher Education" in 1998, which was the first to provide stage specific guidelines for teacher education. It defined several areas of commitment, competence and performance to serve as guiding principles for teacher education programs (NCTE, 1998). The competencies for teachers were established with a view to supporting the achievement of the minimum levels of learning for students in classrooms as laid down in a document on the "Minimum levels of learning" (GoI, 1990). It expected teachers to express learning outcomes in the form of constituent competencies and behaviors that indicated mastery learning. It was assumed that minimum levels of learning are to be achieved uniformly across students. There was focus on developing diagnostic tests and therefore construction of "Achievement test" was assigned additional weightage in the B.Ed. course. The questions in the achievement test were categorized as focusing on knowledge, skills, understanding and application. Remedial teaching was recommended after diagnosis of mistakes through the test, but it was not clarified as to how remediation is to be done in order to help students learn.

In the 1998 teacher education curriculum, integration of content with methodology was introduced in the form of "pedagogical analysis of concepts" having weightage both in theory and practical papers. The purpose of pedagogical analysis was to make a student teacher "conversant with the objectives of teaching a unit, the entry behavior of the pupils, the classroom management and evaluation strategies" and thus make him/her more "effective and confident in his/her interventions in the classroom" (NCTE, 1998). The total weightage of mathematics in the B.Ed. Curriculum was raised to 28.5% of the total marks.

The National Curriculum Framework for Teacher Education (NCTE, 2009) is the most recent attempt at a thorough overhaul of the teacher education curriculum. It contains many new proposals, but is yet to be implemented across universities in the country. It advocates teacher education to be open and flexible, emphasizing dialogical exploration rather than didactic communication, diversity of social contexts and learning spaces as sources of inspiration, and teacher education based on reflective practice rather than on a fixed knowledge base (NCTE, 2009). Major revisions in curricular areas are recommended and attempts have been made to draw upon theoretical and empirical knowledge as well as student teachers' experiential knowledge. The attempt is to focus on the learner, develop teachers' understanding of self as well as the social context, critically examine disciplinary knowledge and develop professional skills and pedagogic approaches to address needs of learners. Each curricular area has a theory and related "field based units of study" (practicum) in which the student teacher is expected to undertake projects, field work, clinical interviews, observation and analysis and interpretation of qualitative data to generate knowledge and continually seek clarity of ideas. The teaching of the subject is now conceived as "pedagogic studies" under which linkages among learner, context, subject discipline and the pedagogical approach has to be established. The shift in view of what is considered as knowledge is evident through inclusion of a course like "knowledge as construction through experiences" as compared to the earlier focus on disciplinary content in textbooks as knowledge. Another important aspect is the emphasis on research related to student learning in different areas, studies on addressing learners' misconceptions and engagement with epistemological questions. These indicate an important shift in recognizing centrality of the student and her learning in teacher education. The practicum course work includes "hands-on experience at developing curriculum and learning materials, designing appropriate activities and formulating questions to facilitate learning" (NCTE, 2009, p. 38).

2.2.1.2 Analysis of the B.Ed. curriculum

In an analysis of six B.Ed. course syllabi from different colleges across India (Kumar, Dewan & Subramaniam, 2012), it was found that the mathematics method course in the B.Ed. Course has broadly three foci. The first comprises the nature of mathematics, its aims, its connections to other subjects and contributions of great mathematicians. The second focus is on specific methods and maxims of teaching like "inductive–deductive method", which in a few universities include "models of teaching" like advanced organizer model, concept attainment model, etc. Out of school activities for mathematics as well as development of math clubs, math laboratory design have also been are included in some syllabi. The third focus is on content enrichment for which some universities prescribe study from school textbooks, while others expect students to formulate specific methodologies for teaching a particular topic and in some rare cases frequently ask student teachers to critically analyze school textbooks. To assess content some universities (for e.g., Mumbai university) have a "content enrichment" component wherein students are expected to do self study of the subject they have chosen for the special methods course. Tests are conducted internally by colleges based on the syllabus of the state board for Grades 9 to 12. This reflects a concern for building proficiency in mathematics at that level. But focus on school textbooks for developing understanding of content might make it difficult for student teachers to go beyond textbooks while teaching. As a teacher one needs to have proficiency of developing problems to enable student learning, which does not find a place in the teacher education curriculum. Also the kind of mathematics that is needed for teaching of mathematics is different from what is typically learnt in school as identified by several researchers (for example, Ball, Thames & Phelps, 2008).

Pedagogical content analysis, which includes identification of concepts, listing behavioral outcomes, listing activities and experiences and listing evaluation techniques, is included in 3 of the 6 syllabi. However it is not clear how it leads to construction of knowledge that is useful in classroom teaching since there is no indication of students teaching a topic after doing pedagogical content analysis and getting some insight about student learning.

What has not changed over the years (since perhaps the 1950's) in the B.Ed. syllabus is discussion of aims and objectives of mathematics education, maxims of teaching, methods of teaching like "inductive, deductive, analysis, synthesis" methods, techniques of teaching like "oral work, drill work, brain storming, self study" and preparation of teaching aids like charts, models and lately "power point presentations". Most of these topics adopt a view of teaching without considering student thinking thereby preparing teachers for transmitting information in different ways (NCERT, 2006b). Clearly the teaching of methods is unlikely to effect a change in the way mathematics is taught in classrooms and developing students' understanding and reasoning in mathematics as envisioned in NCF 2005, even though though there is a substantial component of practice teaching.

What is lacking in the syllabi is a perspective of teaching that makes the child the centre, and views her conceptions and sense making process as an important part of the teaching and thus the teacher preparation process. In contrast, teaching is fragmented into its components which are dealt separately with a hope that this will impact teaching in classrooms. Its not clear if the B.Ed. program allows opportunity for students to think critically about their own mathematics learning, teaching practices prevalent in schools, curriculum and textbooks.

The NCFTE 2009, in its radical departure from earlier teacher education curricula, has recognized the importance of developing an understanding of the learner, and classroom based teaching and research work as important tools to such understanding. The proposed syllabus for B.Ed. based on NCFTE 2009 has incorporated many interesting features. The pedagogy for teacher education has been proposed to include "focused reading and reflection, observation-documentation-analysis, seminars, case studies and school based practicals and workshops" besides lecture-demonstration. The assessment of student teachers has been recommended to include reflective journals, products like lesson plans and observation of student teachers in various contexts of teacher education. The school based experience has been aimed at preparing teachers for "understanding and developing meaningful learning sequences appropriate to the specificity of different levels of learning and also mobilize appropriate learning resources for them". Pedagogical analysis of content now includes content analysis, identification of various content categories and skills, task analysis with reference to learning objectives, student capabilities and learning approaches, learning resources, possible assessment modes, visualizing learning situations, organizing learning sequences and contextualizing learning. The integration between theoretical and practical aspects of teaching has been proposed through designing learning situations which allow teachers to scaffold learning, clarify fallacies and misconceptions, and reconstruct meaning that teacher has to facilitate in classroom. Comparative textbook analysis has also been proposed. While the major recommendation of increasing the duration of the B.Ed. program from one to two years has been implemented across the country, the changes in the curricula and course content are yet to be implemented in spirit. At present, many teacher education institutions are struggling to manage the logistical demands arising from a doubling of the duration of the program.

A unique, innovative program Bachelor of elementary education (B.El.Ed.) is run by Delhi university as a 4 year integrated course for preparing elementary teachers. The program is based on a more wholistic vision of teaching as compared to the B.Ed. program discussed earlier where different aspects of teaching were dealt with separately and then student teachers were expected to incorporate them in their classroom teaching. The aggregation view of teaching in the B.Ed. syllabi assumes that any method, teaching aid can be useful in teaching any concept. There is no scope of exploring how a particular teaching aid helps in concept formation. Understanding this might contribute more towards building knowledge for teaching of mathematics rather than knowing how to make different teaching aids without consideration of content in the teaching learning process. Unlike some B.Ed. syllabi which include “drill work” in techniques of teaching, the B.El.Ed. syllabus is progressive in giving an opportunity to engage student teachers in critical study of practices like drill but also to critically examine concepts that have a propensity for being used as buzz words like “zone of proximal development”. Further, the course attempts to connect teacher education with research in education, making efforts to bridge the gap between research and practice. As compared to the B.Ed. Syllabus, this course keeps the child at the centre of the teaching-learning process and assumes a view of teaching which encourages construction of knowledge through investigations and using students’ ideas and strategies in teaching.

2.2.2 In-service programs in India

In-service training is provided by a large network of government owned teacher training institutions at various levels of hierarchy. The National Council of Educational Research and Training (NCERT) along with its six Regional Institutes of Education undertake design and implementation of in-service programs for both teachers and teacher educators. At the state level, the state councils of educational research and training (SCERT) prepare modules for and conduct teacher training for teachers and teacher educators. The colleges of teacher education and Institutes for advanced learning in Education (IASE) provide pre-service (B.Ed) and in-service training to secondary teachers and teacher educators, develop materials for teachers and conduct surveys and Research. The District Institutes of Education and Training (DIETs) provide in -service and pre-service education for elementary teachers.

As emphasized in the policy documents, the central and state governments in India have made efforts to include in-service Teacher Professional Development (TPD) as an integral part of the school education system. There have been major initiatives in the in-service training of teachers over the last few decades, in roughly two phases. The first phase began with two programs initiated in the 1980s at the national level called program of mass orientation of School teachers (PMOST) and Special orientation of primary teachers (SOPT) (NCERT, 1991; 1995). The emphasis in these programs was on methodology and how to teach in the classrooms, rather than on the content of mathematics. The SOPT also saw the beginning of the idea of Minimum Levels of Learning (MLL) in education, which was further reinforced by the report on MLL published by the Ministry of Human Resource Development in 1991 (GOI, 1990). The document viewed learning as occurring in separate small chunks, each of which could be mastered separately by repeated practice. In the SOPT and subsequent MLL based programs, teacher training was seen merely as a forum where teachers would be given activities and materials that they could use in the classroom.

In a typical SOPT program of 7 days, 3 sessions would be on mathematics. The training modules included detailed descriptions of what kind of activities could be done with children. The modules assumed that children have similar views and follow similar ways of learning and therefore suggested how an activity could proceed with a group of children. The emphasis was on activity and use of materials. The key words were hard spots, MLLs, competencies, assessment, diagnostic testing and remedy as well as activities, modules and demonstrations (NCERT, n.d.).

These efforts were followed in the second phase by the capacity building programs under the District Primary Education program (DPEP) and similar projects supported by many multi-lateral partnerships. In-service training in these programs centered around "joyful learning" and presentation of activities to teachers. The orientations were marked by an attempt to introduce games and other interesting devices into classrooms without necessarily looking at the nature of the concepts to be transacted or the nature of mathematics. The activities that were developed involved a lot of movement, play, singing and use of materials but there was little thought about how this could be related to conceptual development in mathematics. The time spent on mathematical thinking in relation to these tasks was much smaller than the total time required for the activity and most of the effort was spent on ensuring that children had fun. The pattern of training that was evolved in the DPEP continues to influence newer initiatives such as the Sarva Shiksha Abhiyaan (Education for all mission) and the Rashtriya Madhyamik Shiksha Abhiyaan (National secondary school mission) (World Bank, 2004). Thus, a continuing influence of these programs has been the emphasis on technique or activity and a reduced emphasis on mathematical understanding or thinking.

Some in-service TPD initiatives have introduced significant and important elements. The Shiksha Karmi (Education worker) initiative of the 1990s in the state of Rajasthan emphasized the autonomy of the teacher in its in -service programs, and developed a critique of the top-down "transmissionist" model of in-service training (Sharma & Ramachandran, 2009). The main features of the program supported by village education committees involved selecting local youths to act as "grassroot" teachers in dysfunctional schools and providing them continuous and intensive training through out the year. The "Shikshak Samaksha" (Teachers' empowerment) project in Madhya Pradesh involved teachers meeting once a month in the resource centers to discuss their problems, experiences and suggestions to make their teaching interesting. The teachers were provided regular academic support (Mohanty, 1994). The Andhra Pradesh Primary Education project (APPEP) included the use of demonstration lessons given to a group of children to illustrate new pedagogic techniques and making the classroom interesting by displaying and organizing children's work. Teachers planned and generated activities for teaching at the teacher centers established at sub-district level in 23 districts (Mohanty, 1994). So the major departure in these innovations, in comparison to previous ones, were providing regular academic support and discussion of teachers' experiences in the classroom. The influence of these and similar initiatives have led the new National Curriculum Framework for Teacher Education to stress the need to respect the professional identity and knowledge of a teacher and to work with and from it (NCTE, 2009, p. 66-67).

The Project in Science and Mathematics (PRISM) initiated in year 2000 involved collaboration of the Homi Bhabha Centre for science Education (HBCSE) with the Bombay Municipal Corporation. The objective was to strengthen teachers' understanding of fundamental principles, creating an environment in the classroom for students to ask questions and helping teachers and students to go beyond the textbook. HBCSE members worked directly with 50 resource teachers for a year to develop their capacities to train the larger group of teachers working in about 250 schools. There was focus on developing conceptual understanding through discussing usefulness of teaching aids (for e.g., bundles of matchsticks of 10 or 100 for place value concepts and operations). Activities were done with teachers to challenge the belief that all mathematics problems have only one correct answer by asking teachers to formulate open-ended questions. The approach adopted for the resource teachers included planning of lessons followed by one teacher teaching students while other colleagues observed the lesson, in a manner similar to and inspired by the Japanese "lesson study" model. The lesson would be followed by intensive discussion focused on the teaching as well as student responses and thinking, followed by planning for subsequent lessons. "Model lessons" by HBCSE team members, problem solving, observing simulated teaching and teaching in schools of participant teachers was part of the program (Burte, 2005).

The "Prashika" experiment in primary education was an innovative program launched by Eklavya, a leading voluntary organization working in the area of elementary education for many decades. This program included a teacher education component, for which the description "orientation program" was used instead of “teacher training”. The word “orientation” reflected the Prashika standpoint that teacher education cannot be completed in a 20 day contact period program, which serves only as an initiation into engaging with teaching, trying out things and "learning from experiences” (Agnihotri, Khanna & Shukla, 1994, p.127). Thus the teacher development was conceived as "gradual, ongoing, interactive and collaborative process of change" (Agnihotri et al., 1994, p.122). The major objectives of the program were to

- Create an awareness of the learning process and bring about attitudinal changes,
- Cultivate skill and confidence,
- Help teachers acquire knowledge,
- Develop those operational skills that are needed to put curriculum in practice, and
- Help teachers in a sense to become their own informal researchers (Agnihotri et al. 1994, p. 126).

The Prashika approach focused on building teachers’ understanding of the child, curricular understanding for creating appropriate activities and enhancing creativity of the teacher by overcoming inhibitions and engaging in activities like drawing, singing and role play. The expectation was that the teacher will function as a “partial source of information and knowledge” while being able to “plan a multiplicity of activities, observe carefully their implementation and analyze the feedback to modify and change the activities” (Agnihotri et al., 1994, p. 120). The pedagogy adopted during teacher orientation emphasized establishing equality among resource persons and teachers by realizing that much can be learnt from teachers,. The approach emphasized flexible plans for the program, which could be modified based on the needs of the group and getting feedback from teachers, resource persons and observers for revising materials for classrooms and deciding teacher orientation agenda.

Recognizing the limitations of teachers’ knowledge of mathematics, Prashika placed emphasis on enhancing conceptual knowledge of teachers. “A large number of them know rules and formulas, but they are often incapable of handling questions like why and how a particular algorithm works” (Agnihotri et al., 1994, p. 135). One of the principles behind teacher orientation activities was to let teachers enjoy mathematics to ensure that at least some of it is taken up in the classroom. The vision of teaching mathematics involved using concrete materials at early stages and then moving to abstract concepts, opportunities for children to articulate their understanding, opportunities to make hypothesis and make their own problems, allowing expression and exploration of alternative procedures and an attempt to understand why children make mistakes. Over the course of the engagement, teachers arrived at important realizations like "reciting numbers upto 100 is not counting", students appear to understand and solve sums correctly in classroom when the topic is being done but not later, and students face problems in developing functional understanding of concepts like place value even when student are able to understand their abstract nature (Agnihotri et al., 1994, p. 131).

Another well known voluntary educational organization working with teachers, Digantar, offers a "Certificate course in foundations of education". The mathematics component of the course emphasizes that teachers must be involved in "doing mathematics" to understand the nature of mathematics through emerging patterns and rules. In the contact sessions, teachers engage in problem solving followed by discussion on how general rules can be derived by comparing the approaches used by participants. Teachers are involved in discussing theoretical aspects of mathematics teaching through discussing readings and papers (for e.g., absolutist and conceptual change view of mathematics discussed in the writings of Paul Ernest (Ernest, 1995)). Teachers are encouraged to speak about areas of mathematics where their understanding is weak. Other colleagues are urged to help their peers in overcoming these weaknesses. Group work and presentations by groups is central to the pedagogy adopted for teacher orientation (Digantar, 2010).

Recent initiatives by NCERT have focused on developing a range of resources useful for teacher training including the development of an "in-service teacher professional development program" having 5 day workshops every year for in-service teachers and heads of schools. The Training package of the program for mathematics includes mathematics kits, source book for assessment and ICT Kits (Pattanayak, 2009). NCERT has been promoting Mathematics laboratories for a number of years. The need for a maths lab has been mentioned in the school curriculum frameworks (NCERT, 2000; NCERT, 2005). As a result, the Central Board of Secondary Education has introduced Maths lab as a part of the curriculum for secondary school. Maths Lab Manuals containing suggestions for various activities for different concepts and instructions on how to do them have been developed by NCERT. Some educationists have cautioned against the excessive promotion of the idea of a maths lab since it may foster an incorrect epistemology of mathematics (accepting verification in a few cases as a substitute for proof), and may encourage drawing a sharp distinction between classroom teaching and "activities" done in the lab (Dhankar, n.d.).

The Department of Education in Science and mathematics in the NCERT organizes orientation programs for teachers and master trainers (who teach teachers) to strengthen the teaching of Science and mathematics, for e.g., orientation program on "activity based teaching" in mathematics. The draft of a textbook on pedagogy of mathematics has been prepared for use in teacher preparation in line with NCF 2005 recommendations for moving from content to process and "transformation of procedural level understanding to conceptual level understanding" (NCERT, 2011). It includes experimentation and activity with low cost materials and teaching of mathematics through games, puzzles and visuals along with curriculum construction in mathematics at various stages with examples. Enrichment material has been prepared in collaboration with practicing teachers at the higher secondary stage on themes like conceptual understanding, applications and misconceptions. A teacher training manual for class 1 and 2 teachers has also been developed by the "Group arithmetic" cell established in NCERT for strengthening early mathematics development programs (NCERT, 2011). For the promotion of mathematics several programs have been started at state levels like Metric Melas, Math festivals, Math forum, Math clubs and even Maths Marathon. At "Ganit Melas" (Math Fairs) alternative teaching learning materials, activities and methods of assessment are presented to participants, i.e., teachers and students. The development of self learning and interactive learning material by teachers has also been undertaken by various states (Pattanayak, 2009).

The initiatives discussed above are transforming the landscape of in-service teacher education in India. However, systematic frameworks that can guide the design and implementation of TPD programs for mathematics teachers in India are lacking. Further, the engagement with content together with pedagogy in the programs for "grassroots" teachers is often limited by the low qualifications or insufficient educational background of the teachers. Where content is engaged, it may not involve a deep and sustained engagement with specific topic areas in the curriculum. Although these initiatives indicate efforts to focus on developing resources and competency of teachers to focus on development of concepts, often very little information is available of teachers' use of these ideas and research is needed to identify teachers' take up.

2.3 Teachers' professional development: Theories, frameworks and models

In this section, discussion focuses on the research literature in mathematics education across countries that focuses on theories and frameworks for teacher professional development. Glatthorn (1995) defines professional development as "professional growth a teacher achieves as a result of gaining increased experience and examining his or her teaching systematically" (p. 41). "Professional development" is generally understood as a process that is "intentional, ongoing and systemic" process (Guskey, 2000, p.6), which results in teacher empowerment (Peter,

1995) and change in knowledge, beliefs, practices (Clarke, 1994) and professional identity (Hodgen & Askew, 2007). It includes both formal and informal experiences encountered in different professional development settings like workshops, school setting and professional communities, etc, (Borko, 2004) as well as in diverse organizational contexts and culture in which they are situated (Stigler & Hiebert, 1999).

Though the term professional development is used mostly for in-service teachers' development, it has also been used to designate both pre-service and in-service education (Sowder, 2007). The coining of this term and extensive usage in research and teaching community points to the realization that the teacher's journey is one of lifelong learning to develop professional expertise. However, the in-service teacher education efforts in India till recent times reflected the assumption that pre-service education is the main focus of learning since in-service "training" was conceptualized for implementing curricular reforms. On the other hand, countries like Japan have developed an integrated system of professional learning for teachers within the school system by providing opportunities to study and improve their practice through events like lesson study and encouraging interactions with the university. The term has thus come to represent various organized or spontaneous efforts to engage the individual teacher or a group of teachers in learning about and through practice, while participating in several processes and events held in professional development spaces like workshops or in the context of teaching and planning in school.

The debate over teaching as an art versus a skill finds a place in the research literature (Jarvis, 2006). This debate has implications for the way teacher education is designed. The idea of teaching as an art is often associated with viewing teaching as a talent restricted to a few and thus may not account the expertise in teaching as being related to teacher education significantly. On the other hand, research in teacher education has helped in identifying the knowledge and skills which can support the development of a novice teacher into an expert (Shulman, 1987; Borko & Livingston, 1989; Hill, Schilling & Ball, 2004) These studies have contributed to the image of the teacher as a professional having an expertise in the field of teaching by developing and integrating knowledge of content, pedagogy, curriculum and learning. However, teachers have comparatively less collective autonomy (Hoyle, 1995) as compared to other professions in decision making, rational use of acquired knowledge through training and determining the policies of practice which are considered as one of the important characteristic for defining a 'profession' (Weiler, 1995). In India, teacher unions have existed for a long time but their focus has been on collective action to improve for teacher salaries and working conditions rather than development of the profession (Kingdon & Teal, 2010).

In this study, I adopt the view of the teacher as an "active learner" who is in-charge of her own learning through professional development opportunities that are meaningful. I acknowledge the knowledge and understanding that in-service teachers might have acquired through years of teaching, though they might not have thought critically about their own practices developed over the years. Many studies on teacher education have and continue to have a deficit view of teachers leaving them little opportunity in professional development contexts to participate as active learners and to connect what is being discussed to what they think and know. My views align with the view of professional development as promoting teacher's change as "growth" by recognizing the agency of the teacher (Day, 1999; Hannula, Liljedahl, Kaasila, & Rösken, 2007). However, this does not mean that efforts to introduce new ideas of philosophy and pedagogy of teaching should be avoided, rather there is need to strike a balance between addressing teachers needs and introducing new ideas in the education system (Cochran-Smith & Lytle, 2001; Krainer, 2005).

In this study, the approach towards in-service teacher professional development involves respecting teachers' identity and focuses on interventions which can support them in developing their professional identity further through a combination of workshops and collaboration in the classroom. Teachers' engagement in the professional development contexts and interactions in the classroom has been analyzed to throw light on the way teachers' beliefs, knowledge and practice are influenced by their participation in the study. Chapter 5, 6 and 7 have frameworks which were developed and used for design of professional development opportunities of teachers as well as for their analysis.

2.3.1 Theoretical assumptions informing design of teacher professional development

Theoretical development in teacher education has followed the development of theoretical literature about "how people learn". The theoretical perspectives developed across the continuum from psychological to socio-cultural perspectives. While the psychological perspective focuses on the behavioral and cognitive changes that occur in an individual teacher leading to change in practice, the socio-cultural perspective focuses on how a teacher's participation in various social situations like that of a school micro-culture, a community of practice or learning communities contributes towards learning about and for teaching.

In this study, a situative perspective is adopted on teacher learning and professional development, where learning is seen as both a process through which an individual actively builds on his/her knowledge while also acknowledging learning as a social process of enculturation into social practices of the community (Lave & Wenger, 1991;Borko, 2004; Cobb, 1994; Adler, 2000; Putnam & Borko, 2000). This perspective provide tools for analyzing learning using multiple units of analysis. Psychological frameworks inform the analysis taking the individual as a unit of analysis while sociocultural frameworks inform the analysis of patterns of participation in the social context, describing how learning is mediated in a group by interaction between members and how knowledge is constructed or reified in a group through these interactions. Thus teacher learning can be analyzed in multiple contexts, be it an individual teacher learning in the classroom or learning as a result of collegial discussion in a professional development context. The situative perspective adopts the view that teachers learn best by engaging in activities 'situated' in teaching like analyzing teaching live or records of teaching or through artifacts of teaching referred to as "records of practice" (Borko, 2004). Considering learning as "enculturation" of teachers into teaching practice, several researchers have emphasized the use of *practice based* tasks or engaging teachers in closely observing and analyzing practice as a road to teacher learning (Cohen & Ball, 1990; Cochran-Smith & Lytle, 1999). The use of lesson study, case studies, video analyzes, analyzes of student work, lesson plans, etc. are different ways in which teachers' practice is opened up for reflection and learning. How the situative perspective has influenced the selection of features and tasks for the design of professional development activities for the teachers will be discussed in Section 2.4 on features and factors contributing to professional development.

2.3.2 Models for supporting teacher change in practice

Several models for supporting in-service teachers' professional development in the context of curriculum reform have been proposed across the world which aim to "change" teachers' practices. The training model currently used in India assumes that after training teachers can change their behaviors to adopt new practices proposed by authorities as worthy of replication (Sparks & Loucks – Horsley, 1990). Tirosh and Graeber (2003) note that "top-down approaches involve little teacher participation in either selecting the innovation or in planning how to implement it" and result in teacher anxiety, anger and frustration leading to little change in practice. Curricular reforms enforced in top down manner fail as a result of not involving teachers in making a decision to change (Ponte, Matos, Guimaraes, Leal & Canavarro, 1994).

In response to the criticism of "top down" approaches, different models of "bottom up" approaches have been devised and studied. However, the idea of what the bottom up approach involves are very diverse. Clarke (1994) identified addressing teachers' concerns and soliciting teachers' conscious commitment to participate actively as one of the key elements of professional development. An example of the bottom-up strategy given by Tirosh and Graeber (2003) is the summer math program by Schifter and Fosnot (1993) which was followed by support in classrooms in which an alternative vision of the mathematics classroom was shared with the teachers but the initiative for change was left to teachers. Similarly, Richardson (1998) found that issues of teachers changing in practice are related to power in terms of who drives the change. She argues that teachers resist change when enforced by authorities, which they fail to make sense of, but continuously undergo voluntary change. Thus, she engaged teachers in a collaborative study to help develop their identity as autonomous teachers who make informed decisions and chart their own trajectory of change. In these studies mentioned above, the implicit model of professional development involves active teacher participation through shared goals and collaboration for change in practice. However, some studies assuming teachers in the role of curriculum designers found that teachers need to be provided with adequate support in the form of knowledge and strategies to adopt the role, without which teachers perceived this role as a new set of demands imposed on them through forced autonomy (Skott, 2000). Thus, when there is lack of adequate support the expectations of teacher autonomy are perceived as imposition rather than empowerment. Loucks-Horsley, Hewson, Love and Stiles (1998) noted the challenges faced by teachers who did not get support in schools post their professional development for change in practice. These challenges were in the form of lack of time, activities and development of content knowledge to bring meaningful change in practice. They advocate that teachers should be engaged in meaningful discussions, planning and practice as part of the professional development itself.

Another way in which "bottom up" approaches have been visualized is to connect professional development initiatives to teachers' practice in the classrooms through "practice-based professional development" (Cohen & Ball, 1990; Cochran-Smith & Lytle, 1999; Matos, Powell, Sztajn, Ejersbo, Hovermill & Matos, 2009). The training model is associated with the idea of "expanding individual repertoire of well defined and skillful classroom practice" (Little, 1993), while practice based professional development involves reflection and discussion on the artifacts related to teaching to develop specific aspects of the knowledge related to the "work of teaching" mathematics (Ball & Forzani, 2009). Building on situated learning theory, Matos et. al. (2009) elaborate how practice based professional development is promising as "the text of teaching serves as the context for teachers to learn about the specific aspects of their labor and reflection is expected to increase teachers' awareness of practice, allowing them to make thoughtful decisions in the immediacy of classroom work" (p. 169). Thus, learning experiences which involve activities that replicate or resemble teaching practice and all its complexity, help teachers in identifying the nuances that goes into decision making in teaching (Ball & Bass, 2003; Little, 1993). Silver, Clark, Ghousseini, Charalambous and Sealy (2007) identified the different types of opportunities that arise when using practice-based professional learning tasks (Ball & Cohen, 1999) for working and learning about mathematics. The embedded activity segments in the tasks included opening activity of problem solving followed by individual reading and analysis of the case. This individual engagement is followed by collaborative engagement of case analysis and discussion followed by collaborative lesson planning and debriefing. They consider both acquisition and participation metaphors (Sfard, 1998) as complementary in their project. They discuss how individual learning is stimulated by reflection and cognitive conflict while conversations within the group with others mediate the development of shared knowledge within the group. The authors describe how teachers engaged with problems not only as doers of mathematics but also as teachers thereby thinking of possible student responses to problems. The collaborative conversations helped to connect different pedagogical and student related issues, while the facilitator helped in focusing on mathematical aspects in discussing student errors when teachers only expressed pedagogical concerns.

One shot workshops have been termed as ineffective for professional development basically because they are unrelated to needs of teachers and have no follow up. Extended workshops of about one to two weeks have been found to be useful when accompanied by other professional development opportunities like supporting feedback or mentoring of a collaborative nature from facilitators in teachers' classrooms. For e.g., Borko and Putnam (1995) had 4 weeks of summer institute followed by ongoing support in school, which were evaluated by teachers as very valuable. However, Markovits and Even (1999) illustrated how even one shot workshops of two days duration can be helpful in making teachers reflect on the beliefs held by them about students' mistakes, although it may take time for the idea to sink in and to develop into the form of functional pedagogical content knowledge that can be used recognizing that teachers' knowledge is situated in classroom experiences and that teachers need to critically reflect on them in order to change practices, Putnam and Borko (2000) discuss how combining multiple contexts like workshop and ongoing support during school year is promising for teacher learning. They argue that workshops can promote "developing different conceptions of mathematics and deeper understanding of mathematical learning and teaching" (p.7), while teachers' own classrooms can be sites to explore enactment of specific practices. Studies indicate on site teacher professional development contexts in school usually grounded teacher's learning in their own teaching experiences which were reflected upon (Borko, Mayfield, Marion, Flexer & Combo, 1997). Teachers' self reports indicated that interactions in the on-site setting helped them specify the focus of observations of students' thinking and use them for assessment (Borko et al., 1997). Many studies used summer workshops in university institutes as settings for professional development followed by on-site support. However, it is challenging for teachers to integrate ideas learnt outside classrooms into their practice and they need further support. Many discourse communities of which teachers are participants, may provide confirmatory experiences of beliefs held rather than encouraging critical reflection on practice. However, communities have been successfully established where each participant is considered to be contributing to the knowledge of the community by bringing in their unique beliefs, expertise, knowledge of subject, pedagogy and students (Thomas, Wineburg, Grossman, Myhre & Woolworth, 1998). In another study Saunders, Goldenberg and Hamann (1992) used the idea of instructional conversations to engage teachers in meaning making discussions and envision it for their student involvement in classroom. In these studies teachers brought their understanding of their own classrooms and worked on it. The role of teacher educators was pertinent in developing norms of critical and reflective stance towards teaching. Putnam and Borko (2000) thus suggest multiple settings as promising for teacher learning since workshops can develop new insights and ideas in teachers while on-site contexts facilitate enactment of and reflection on specific practices.

Based on the setting where professional development experience is provided to teachers, these studies can be categorized as those focusing on out-of-school contexts like universities or educational institutions where workshops for teachers take place, and those focusing on professional development experience within school itself. The interventions held in out-of-school settings may include examples or artifacts of practice in various ways as is common in practice based professional development. There exist several studies in which both out-of-school as well as follow ups within schools have been designed to support continued teacher professional development and implementation of ideas discussed in workshops. Here, the role of the teacher can be seen as that of a learner in out of school contexts where teacher educators engage teachers in tasks to promote their learning about teaching. Within school professional development contexts, the teacher's own teaching can become the object of analysis, reflection and learning. Besides engaging in tasks, teachers are engaged in various practices to promote their professional development that may include planning, designing tasks, collaboration, exploration, reflection and inquiry, etc.

In this study, different frameworks have been integrated for designing and analyzing teacher professional development opportunities. Mainly socio-cultural frameworks have been used to design professional development opportunities in the workshop along with situative perspective to design practice based professional development opportunities. This is in alignment with the belief of the workshop design team that reflecting and analyzing practice and artifacts of practice in social settings helps in eliciting knowledge of in-service teachers while also offering opportunities for challenge and revision towards developing practices for conceptual understanding.

Adler, Ball, Krainer, Fou-Lai Lin and Novotna (2005) surveyed the research in mathematics teacher education from 1999 to 2003 in key journals and conference proceedings and identified four important themes for reflection on the current state of the field of mathematics teacher education along with commentaries by five experts from across the world. They found that small scale qualitative research were more in number since it is an emerging field and "particularization precedes generalization" (p.369). Preponderance of case studies was explained by complexity due to several factors involved, slow and difficult process of change and focus on understanding the phenomena by in depth studies of individual teachers. Further, small case studies are necessary to show the complexity of teacher learning, are more easily related to teachers' actual practice and for busting the myth of there being some "best practices" for promoting teacher learning. These studies help in identifying critical factors which may constrain or support teachers' learning. Experimental and quasi-experimental studies though few in number provide insights about conditions and processes which contribute to teacher development along with school improvement and student learning. However, these studies need to take the knowledge generated from case studies about teachers and their cultural conditions. The authors noted that few studies report analysis of "pedagogic discourse" in teacher education or analyze how teachers come to learn knowledge and practice in professional development contexts. They found that research conducted on teachers who have collaborated with teacher educators, play a role in improving as well as understanding the phenomenon of teacher education. This type of research may help bridge the research-practice divide when teachers engage in inquiry about practices used by them for teaching, while teacher educators study the support and constraints in teachers' growth and engage in self evaluation. They identified the need for developing theories and understanding of how mathematics and teaching get integrated in teachers' professional development "to constitute both mathematics and teaching identities" (p.378).

2.4 What develops in teacher professional development? – Characterizing professional growth

Professional development has been characterized as the development or change in teachers' beliefs, knowledge, attitudes and practices and in terms of development of teachers' identity in the sense of "becoming" a mathematics teacher. In this study beliefs, knowledge and practices of teachers have been focused as teachers experience professional development opportunities in different settings of workshops and collaboration in classrooms. More specifically, thefocus is on the role of beliefs and knowledge in influencing teachers' practices which are discussed in the subsequent sections.

2.4.1 The role of beliefs in influencing practice

2.4.1.1 What are beliefs?

Beliefs about the nature of mathematics and the nature of teaching and learning are among the most researched beliefs of teachers in relation to their practice. Knowledge, beliefs and emotions have been found to play an important role in shaping teachers' thinking. Teacher's thinking influences what happens in the classroom (Wilson & Cooney, 2002; Grootenboer, 2008). This is further supported by the finding that teacher's beliefs have been found to influence learning outcomes (Cockcroft, 1982). Beliefs have been considered as an important construct in studies related to teacher professional development, since they are thought to influence perception, goal formation and actions taken by teachers in various contexts and thus may serve as a predictive tool for teacher decisions (Dewey, 1983; Schoenfeld 2005; Törner, Rolka, Rösken, & Sriraman 2008).

Beliefs research has shown that it is important to know what teachers think and why, so as to support them in bringing instructional change or educational reform to their classroom (Battista, 1994; Cooney, Shealy & Arvold, 1998). Thompson (1984) in her seminal paper had discussed the importance of the role that beliefs play in influencing teachers' behavior.

> Failure to recognize the role that the teachers' conceptions might play in shaping their behavior is likely to result in misguided efforts to improve the quality of instruction in the schools. (p. 106)

Thompson's (1984) classical study revealed how an instrumentalist teacher Lynn demonstrated rules and procedures while another teacher Kay engaged students in activities emphasizing process aspects of mathematics. Stigler and Hiebert (1999) explained the difference in practices of US and Japanese teachers based on their beliefs about mathematics as procedural or as understanding the relationship between concepts, facts and procedures. Belief about mathematics as procedures has been taken up one of the dimensions for analysing teacher beliefs in Chapter 4 as discussed in section 4.5. The importance of understanding and using student thinking for teaching has found place in many reform documents and curricula across the world including India (Australian Education Council, & Curriculum Corporation (Australia),1990; National council of Educational Research and Training (India), 2005; National council of teachers of mathematics, 2000).

Structure of beliefs

There appears to be general agreement in the mathematics education community that mathematical beliefs are "personal philosophies and conception about the nature of mathematics and its teaching and learning" (Thompson, 1992). Philipp (2007) proposed a working definition of beliefs considering them as "lenses that affect one's view of some aspect of the world or as disposition towards action" (p. 259) borrowing from the notions of Pajares (1992) and Cooney, Shealy and Arvold (1998). In another definition, beliefs are regarded as conceptions which an individual believes to be true (Ajzen & Fishbein, 1977) and are inferred from explicit articulations or individual actions (Pajares, 1992).

Beliefs have been considered as a messy construct (Pajares 1992) and there are number of terms and notions similar to beliefs which have been used by researchers in mathematics education including knowledge, conceptions, attitudes, values, affect, orientations and identity (Philipp, 2007). At one end of the spectrum beliefs are seen as connected to cognitive aspects and used interchangeably with knowledge and conceptions. At the other end of the spectrum, the relationship between beliefs and affect have been explored through constructs like values, identity and orientations. Wilson and Cooney (2002) argue that in spite of multiple terms associated with beliefs and failure to achieve consensus among researchers for defining beliefs, teacher thinking does have an impact on teaching and learning in classroom. There have been various efforts to bring theoretical consistency in the use of the term "belief". In beliefs research, one line of effort has been to arrive at an agreeable definition of beliefs. Still others, have tried to define the characteristics and structure of beliefs with respect to connections among beliefs and their connections with teachers' practice. Efforts to distinguish beliefs from other notions like knowledge, values, goals and attitudes have also contributed towards clarity over what beliefs are. Still others have identified different dimensions over which beliefs are analyzed. For e.g., the cultural dimension of beliefs suggests that depending on the community that one is member of, the beliefs held might be considered as true and thus be held with the status of knowledge rather than being beliefs. However, the mathematics education community is still to reach a consensus over the use of this term.

Distinguishing beliefs from other constructs

Thompson (1992) distinguished belief from knowledge by noting that beliefs can be held with varying degrees of conviction and may not be consensual as compared to knowledge, which is validated by processes arrived at by consensus in a community. This explains why some beliefs may be resistant to change since they are held with stronger conviction. Thompson distinguished beliefs from affect in terms of the former being more cognitive in nature. However, Maass and Schloglmann (2009) argued that beliefs serve both cognitive and affective functions and thus are part of both cognitive as well as affective structure. They are different from values which are described in terms being desirable or not, while beliefs are classified as being true or false. Thus, Bishop, Seah and Chin (2003) consider beliefs to be more context dependent than values. Clarkson and Bishop (1999) view values as beliefs in action and one may have to prioritize among competing values in one's teaching (Bishop et al., 2003). Törner, Rolka, Rösken and Sriraman (2008) consider beliefs and goals as mutually dependent. In articulating a goal, beliefs are implicitly applied and conversely one can predict teachers' decisions based on knowing their beliefs. For e.g. developing mathematical proficiency can be a value held by a teacher which might be supported by beliefs like developing conceptual understanding, understanding connections across concepts and problem solving. On the other hand mathematical fluency can be a value which may be supported by beliefs that knowing and performing procedures quickly is important for doing mathematics. It is possible for a person to hold incompatible values or incompatible beliefs which may function according to the teacher's priorities or the context in which she is working. For e.g., a teacher may believe that conceptual understanding is important but just before an exam the teacher may focus her attention on making students remember rules and typical problems which may come in the exam as she wants her students to pass the exam and get good grades.

Beliefs as a construct to explain teachers' behavior have also come under criticism from various researchers. One of the reasons for criticism is the messiness of various terms and notions associated with beliefs which are directly or indirectly implicated in a number of research reports. The definitions quoted above are an attempt towards arriving at a common ground with regard to consensus for what are considered as beliefs in the mathematics education community. However, the definitions have been found to be inadequate in describing a complex construct like beliefs and there have been attempts to describe the structure and characteristic of beliefs in order to clarify what beliefs are, how beliefs are connected, how they are held and what their relation with practice is. Another strong line of criticism claims that attempts to explain practice on the basis of beliefs held by teachers imply circularity in definition as beliefs are inferred from the very actions which they are used to explain (Lester, 2002). The distinction proposed by researchers, between articulated and attributed beliefs on the one hand and espoused beliefs and enacted beliefs on the other, is an attempt towards differentiating beliefs held by teachers and the actual practice. Research on various factors ranging from cognitive to sociocultural, which influence whether and how beliefs held by teachers are enacted in practice, further illustrates the complexity of the relationship between beliefs and practice as well as their distinction.

There have been also several methodological critiques of beliefs research such as that the analytical framework is not empirically grounded (Lester, 2002; Wedege, Skott, Henningsen, & Waege, 2006). A critique of the distinction between articulated and attributed belief has been made by Speer (2005), arguing that all reported beliefs in research are attributed since researchers interpret and select the articulations of the teachers when reporting. Also, the terminology used in questionnaires and interviews may not have unequivocal meanings (Speer, 2005) between the researchers and teachers. However, one can argue that the context plays a role in determining whether the practices which are preferred by teachers are actually implemented. This creates the need for distinguishing beliefs that are only articulated in teachers' talk about practices they prefer and those which get reflected in practice. Further complexity is added if beliefs and practices preferred or enacted by teachers are considered as dynamic entities as there have been evidences of change in beliefs and practice as a result of professional development, although beliefs have been described as being resistant to change. Analysis of articulated beliefs, preferred practice and enacted beliefs and practice can contribute towards understanding which beliefs and practices are stable or central to teachers' identity and which are in a state of flux.

Several researchers have tried to establish characteristics of the beliefs as well as belief structure or system which are listed below. Green (1971) and Rokeach (1968) have discussed the nature and organization of the belief system which are discussed below.

1. **Integratedness/ connectedness**: Green (1971) proposed that beliefs exist in relation to other beliefs and thus may be organized in clusters rather than as isolated from each other. Thompson (1992) also proposed that belief system of some teachers are more integrated when there is coherence in the belief structure. I agree with Aguirre and Speer (2000) that beliefs occur in the form of belief bundles that "con-

nect particular beliefs from various aspects of entire belief system" (p.333). Therefore, it is possible that multiple conflicting beliefs are activated during teaching and the decision for action is taken based on the strength of beliefs held, as well as long or short term goals (Schoenfeld, 2003), the context of the teaching situation, unexpected occurrences during teaching and the knowledge held by teacher to deal with the situations arising during teaching.

2. **Primary/ derivative structure**: Some beliefs can be more primary and others derivative in nature. Thompson (1992) described how presenting mathematics clearly to students can be a primary belief which can become the basis of derivative beliefs held, for e.g., that teachers should be able to answer all the questions asked by the students since teachers feel that they should know all the content thoroughly to be able to play the role of the teacher. Rokeach (1968) considered primary beliefs as developing from direct experience and having more influence on behavior while derivative beliefs are learnt indirectly from others.

3. **Central/ periphery structure**: Another feature proposed by Green (1971) was that some beliefs may be more central in nature while others lie at the periphery, with a caveat that primary beliefs may not necessarily be more central. Beliefs that are consistent across time and context in a person's discourse as well as actions have been described as more centrally held as compared to others (Rokeach, 1968). Peripheral beliefs may be more susceptible to change and therefore when beliefs get re-organized as a result of some experience or reflection then it is possible that some primary beliefs which are peripherally held may change along with the derivative beliefs. For e.g., realizing that students can come up with the methods of their own may change the primary belief that all the material must be presented and the teacher may allow opportunities for students to share their strategies.

4. **Stable vs. dynamic beliefs**: Green (1971) distinguished between evidentially held beliefs and non evidentially held beliefs with the latter being more resistant to change. Thompson (1992) proposed that beliefs should be considered as dynamic structures instead of fixed entities which can change in the light of experience and that research should focus on the dialectic between belief and practice and the interaction between the teacher's and students' conceptions during teaching.

5. **Confidence**: Confidence has been interpreted as self belief and as impacting learning (Wesson & Derrer-Rendall, 2011) as well as teaching as it influences access and

use of prior understanding (Hailikari, Nevgi, & Komulainen, 2008).

6. **Conscious/ unconscious**: An individual may hold beliefs consciously or unconsciously and thus may not be aware of the inconsistency between the beliefs. Ambrose (2004) discusses how becoming conscious of one's beliefs can be a stepping stone towards change in beliefs. Studies have used this distinction to explain the inconsistency in beliefs held as teachers may indicate inconsistent beliefs in their talk and practice.
7. **Role of affect**: McLeod (1992) distinguished attitudes from beliefs by considering beliefs as more cognitive than attitudes while attitudes might indicate more or less stable feelings. He explained how belief interacts with affect by giving an example of a boy who believed that mathematics problems should be solved in a few minutes but repeated experience of not being able to solve story problems quickly led to a negative attitude towards them.
8. **Truth value:** Beliefs have been distinguished by knowledge which is certified to be true and justified. However, a person holding certain belief may believe it to be true even when contrary evidences are available. Goldin (2002) has characterized beliefs as having a "truth value" of some kind for the holder (p. 59).

2.4.1.2 Research on beliefs about mathematics, teaching and learning and students

Most of the studies on teachers' beliefs about mathematics, its teaching and learning and about students are with preservice teachers and relatively few studies with in-service teachers are reported. Research is needed on similar topics about in-service teachers' beliefs as a large majority of the teachers employed have not had adequate in-service professional development opportunities to address the issues of teaching mathematics at various levels. Research on in-service teachers' beliefs can help in understanding the teachers' views about efforts related to curricular reform as well as in bringing about improvement and innovation in teaching of mathematics for understanding in general. This understanding can also aid in designing effective professional development programs for in-service teachers that address the concerns and needs of the teachers to support development of conducive beliefs rather than addressing teachers' beliefs directly.

Forgasz and Leder (2008) report in their review of research on beliefs that comparatively fewer studies have been reported for middle school and secondary teachers' beliefs as compared to primary teachers. Also that there is more research on beliefs about pedagogy and learning as compared to teachers' views about mathematics and equity related beliefs except those on gender. Most of the analytical frameworks used by researchers for assessing beliefs were theoretically driven. Teachers' beliefs about teaching mathematics and about students is further discussed in Chapter 4 in section 4.4 and 4.6.

2.4.1.3 Relation between belief and practice

Several studies have described how beliefs serve as a filter for interpreting directives and suggestions for teaching and noticing aspects of others' practice. Grant, Hiebert and Wearne (1998) found that teachers' beliefs about mathematics influenced what they noticed and implemented in their classroom after observing reform minded teaching, with teachers viewing concepts and processes as part of mathematics being more open to implementing changes. Many studies have reported the connection between beliefs about mathematics and those about its teaching and learning or about students' ability.

Researchers have categorized teachers' beliefs about mathematics into different categories. Ernest (1989) categorized conceptions of mathematics as (I) Problem solving view considering mathematics as a dynamic subject that is a result of human invention, (ii) Platonist view where mathematics is considered a structured unchanging body of knowledge and (iii) Instrumentalist view when mathematics is considered as collection of procedures, facts and skills. Researchers have discussed how beliefs held about mathematics and mathematics teaching is shaped by what teachers' had experienced in their own schooling (Seaman, Szydlik, Szydlik & Beam, 2005).

Beswick (2005) described how certain beliefs about mathematics are connected with particular beliefs about its teaching and learning. For e.g., he considered the instrumentalist view of mathematics as theoretically consistent with a view of teaching focused on performance and that of learning as being focused on mastery of skills and passive reception of knowledge. On the other hand, a Platonist or problem solving view of mathematics was considered to be consistent with an emphasis on understanding or being learner focused with opportunities for developing understanding or autonomous exploration. Archer (2000) reported marked differences in the views of primary and secondary school teachers with respect to the nature of mathematics. Primary teachers believed that mathematics is more connected to students' everyday lives as compared to secondary teachers while noting how the teaching of mathematics at primary levels has changed since they studied maths in school.

However, there have also been contradictory findings about the connection between beliefs about mathematics and teaching since other beliefs and contextual factors have been found to play an important role. For e.g., a study by Sztajn (2003) collected rich data from two teachers having the same beliefs but teaching in different contexts in terms of having students from different socio-economic backgrounds. The data included interviews of parents and principals along with teachers' observation, interviews and artifacts like lesson plans and students' work in the classroom. She concluded that teachers' notions of student needs, influenced by teachers' beliefs about children, society and education were important factors in explaining differences in their instruction rather than their beliefs about mathematics which contrasts with the finding that there is a relation between beliefs about mathematics and teaching. Forgasz and Leder (2008) concluded in a review of studies on beliefs from 1997 to 2006 that beliefs held by teachers about teaching are related to what they think about how students learn and how they perceive their students' ability. Beswick (2004) reported through a case study that even when a teacher may hold a problem solving view of mathematics and constructivist approach to teaching, her views about students' ability might affect teaching practices. This is relevant in the context of the calls for change in teaching practice towards providing equity to disadvantaged learners in various research reports and curriculum documents in countries having diverse learners (Trentacosta & Kenny, 1997; Skovsmose & Valero, 2002; Badat, 2009).

Teachers' beliefs about assessment have been found to influence their beliefs about teaching and vice versa. Anderson, White and Sullivan (2005) surveyed primary teachers who acknowledged the importance of problem solving and considered open ended and unfamiliar questions as more appropriate for high achieving students. Countries like England, Japan, Korea and India are cases where teachers' classroom practices have been driven by concerns over student performances in certain examinations (Miyake & Nagasaki, 1997; Tirosh & Graeber, 2003; Shirali & Ghosh, 2012).

Beliefs about gender interact with the beliefs and practice for teaching mathematics. Tiedemann (2000) found from a survey of 52 primary teachers that their views about the ease of learning mathematics and performance were biased towards boys and believed that girls need to put in more effort. Results from a large scale survey of prospective secondary teachers by Forgasz (2001) indicated that teachers from Australia as well as the USA considered that students view mathematics to be a male domain.

Another important relation between beliefs and practice that has been reported in many studies is the apparent inconsistency between the articulated beliefs and the practice observed. Several research studies report dissonances between teachers' beliefs about mathematics, its teaching and learning and actual classroom practices. These discrepancies have been termed as inconsistency between espoused beliefs and practices intended or used by teachers (Quinn & Wilson, 1997; Timmerman, 2004) or between "articulated beliefs" and "enacted beliefs" (Even & Ball, 2009). As described earlier, the explanations proposed for these inconsistencies are based on the proposed structure of bundle of beliefs, that some beliefs are held more centrally than others (Pajares, 1992), or that the constraints and supports available in the teachers' context allow teachers to enact some beliefs in consonance with their present purpose while assigning lower priority to others. Barkatsas and Malone (2005) reported a case study in which espoused beliefs were less traditional than observed practices. Argyris and Schon (1978) distinguished between "espoused theories" and "theories in action" categorizing teachers' report of practices through questionnaire or interview responses as "espoused theories". Raymond (1997) in her study found inconsistencies between beliefs held by a teacher and her practice. Beliefs about mathematics were more closely related to practice in her study than beliefs about teaching and learning. She explained these inconsistencies based on the teachers' concerns about time constraints, scarcity of resources, standardized tests and students' behavior. Lerman (2001) on the other hand, argued that beliefs depend on contexts and thus one to one correspondence can not be guaranteed between beliefs expressed in interviews and practice. The resemblance in beliefs across contexts is, however, possible. Hoyles (1992) concluded on the basis of her work with teachers that beliefs are co-produced in the context and culture in which teachers are situated; for e.g., grade level, textbooks used, teachers' objective and motivation and students' expectation in the classroom may lead to construction of beliefs-in-practice. Laffitte (1993) discussed the values embedded within the school culture that influence the roles and responsibilities that teachers perceive as their own, thereby, influencing teaching, thinking and beliefs.

Skott (2001) used the term *school mathematics images* to describe the priorities that arise in teaching, which can range from developing understanding of mathematics to broader issues of classroom management. He found that while considering multiple motives at the time of decision making teachers experience conflict between belief and practice and explained inconsistency by dominant motives that overpower others. He described how a teacher Christopher focused on mathematical learning and asked students to do problems on their own in one lesson while in another lesson concerns for students developing self confidence in mathematics led him to guide students through solving a problem to arrive at the answer. Thus, although beliefs may be situated, teachers' priorities might change from situation to situation as teachers need to decide between competing "objects" and "motives" (p.25). One needs to be cautious in making general conclusions about teachers' beliefs without rich data. Skott proposed that practices emerge "in and through interaction" in various contexts (Skott, 2009, p.29). Teachers' actions can be explained by their engagement in multiple actual and virtual communities of practice and thus teachers' belief may not be the sole determinant of practices.

Thus studies have indicated that teachers' practice is influenced by beliefs but in complex ways since there are other contextual factors at play and different beliefs might be accorded priority at different times. This indicates that more research is needed to clarify the issue of relation between belief and practice (Da Ponte & Chapman, 2006).

2.4.1.4 Study of beliefs in non-Western cultures

Most of the studies on teacher beliefs have been done in Western cultures and their findings have been used to make global assumptions. However, studies in non-Western cultures have revealed the role of culture and context in the formation of teachers' beliefs making us aware that beliefs can be socially constituted instead of being considered as only a cognitive entity residing in the individual's mind. Cai (2004) is an example of a study where beliefs about effective teaching were considered in both Eastern and Western cultures using semistructured interviews. Wang and Cai (2007) found that Chinese mainland teachers believe that teachers should be able to "craft the knowledge from the textbook for teaching by predicting possible students' difficulties" (p.287) and that the lesson should be coherent. These beliefs supported teacher directed and content oriented teaching in China. Bryan, Wang, Perry, Wong and Cai (2007) found several similarities between teachers from four different regions comprising China, Hongkong, Australia and US. However, some characteristics of effective teachers can be ascribed to their cultures like teachers from Hong kong and China had a more Platonist view of mathematics while others had a more functional view. Eastern teachers considered that memorization can be an intermediate step towards understanding and is helpful in automation and fluency in solving problems thus valuing the integration of memorization and understanding. Western teachers considered only rote memorization which they did not value over understanding. US teachers had concerns for classroom management while Chinese and Hong kong teachers considered teachers' ability to provide clear explanations and stimulating thinking as necessary. Western teachers believed in the use of physical manipulative while Eastern teachers emphasized verbal engagement. Bryan et al. (2007, p.339) comment that "it is impractical to look for a 'national/ regional script' of mathematics teaching. Yet classroom practices are often shaped by cultural, environmental and societal assumptions". It was also noted that most papers published in key journals are from English speaking countries reflecting the issues faced by these countries while issues of addressing diverse learners and teachers in classrooms in low resource systems are not discussed.

In the Indian context, commonly held views include the belief that mathematics is a body of knowledge consisting of known solutions to a well defined set of problems and that not all children are capable of learning mathematics. A study by Dewan (2009) indicates that such beliefs, which are far removed from the ones envisioned in NCF 2005, are held by not only teachers but even administrators, faculty members and directors of teacher education institutions, thereby indicating the extent of challenge to implement the curricular reform. This points to the need to create spaces where teachers articulate and reflect on the beliefs that they hold while respecting the identity of the teacher. Such opportunities may include not only experiences of alternative ways of doing mathematics, but also building an awareness of and sensitivity to students' thinking.

2.4.1.5 Instruments to measure beliefs

The literature reveals that beliefs have a complex structure and are influenced by social, contextual and even ideological aspects. A major methodological weakness identified among studies researching beliefs was that very few studies engaged in theoretical discussion on the construct of beliefs or include a definition of beliefs. Da ponte and Chapman (2006) were critical of exploring teachers' beliefs without simultaneous exploration of practice. The realization of complexity in the nature of beliefs has led to change in the nature of instruments to assess beliefs from quantitative to qualitative assessments. Two most common ways in which beliefs are assessed are the case study methodology incorporating data triangulation from various sources (interview, observations, linguistic analyzes) over a period of time or using belief assessment instruments like surveys or questionnaires (Philipp, 2007). Forgasz and Leder (2008) found that most studies on beliefs had large scale likert type surveys with descriptive statistics along with studies having small scale qualitative studies on teachers' beliefs.

Fennema, Carpenter, Franke, Levi, Jacobs and Empson (1996) developed a likert scale survey of 48 items and 4 sub-scales to assess teachers' beliefs and changes in them. However, most survey based studies on beliefs use previously constructed questionnaires (Ambrose, 2004; Seaman, Szydlik, Szydlik & Beam, 2005; Vacc & Bright, 1999; Wilkins & Brand, 2004). One of the findings related to surveys is that teachers may focus on different aspects of the statement which may not have been intended as the main focus. Therefore, Zollman and Mason (1992) after a pilot with teachers, decided that capitalizing select words to focus teachers' attention might help. For e.g. "students should share their problem solving thinking and approaches WITH OTHER STUDENTS" (Zollman & Mason 1992 as cited in Philipp 2007, p.270). However, concerns have been raised over the accuracy of teacher self report surveys and susceptibility to differences in interpretation, hence pointing to issues of validity for likert type surveys. Also teachers may decide a position on reading the survey item for an issue about which teachers may not have consciously thought during their practice. Ambrose, Clement, Phillip and Chauvot, (2004) designed a web based survey asking respondents to interpret a complex situation by taking a teaching decision and decided whether it gave weak, strong or no evidence for the teacher holding certain beliefs.

Beliefs assessed using interpretations of video cases of teaching, narratives of teachers about their practice (Ambrose, Philipp, Chauvot & Clement, 2003) have been found to be useful. Some have questioned the reliability of ascribing beliefs, since they may depend on the context in which data is gathered (Perrin-Glorian, Deblois, L. & Robert, A. 2008). Some have questioned the methodologies adopted for belief attribution suggesting that researchers and teachers may have different interpretations and meanings (Speer, 2005). This points to the inherent difficulty of describing teachers' beliefs despite their centrality in influencing teachers' thinking and practice, and the need to draw on multiple sources and use mixed methodologies while ascertaining the beliefs of specific teacher groups.

Gresalfi and Cobb (2011) argue that "collective discussions" are just as valid as a source of data about teachers' views and identities as individual interviews since the group context has the same issue of social interaction and positioning that an interview might have.

The above discussion indicates that a mixed method approach might be more appropriate to assess teachers' beliefs. Given the fact that teachers may have their own interpretations of the words used in a questionnaire, it might be better to know how teachers' interpret these words at small scale with diverse samples before implementing large scale surveys. Moreover, to have an in-depth understanding of teachers' beliefs close interactions in several contexts might be helpful to assess them and to understand how contexts affect the beliefs articulated or enacted by the teachers.

2.4.2 Relation between teachers' knowledge and practice

Research studies of teachers' knowledge have pointed to the importance of knowledge that integrates subject matter and pedagogy for effective teaching. Although pedagogical content knowledge and subject matter knowledge have been considered as useful constructs to describe essential knowledge for teaching (Shulman, 1986; Ma, 1999), it is rarely the central focus of any phase of teacher education in India (Kumar, Dewan & Subramaniam, 2012).

Knowledge held by teachers may play a role in whether or not a teacher is motivated to change her practice. Nathan and Koedinger (2000) explained that a teacher with high content knowledge may have a low level understanding of students' thinking as they may have an *expert blind spot*. Thus content knowledge may even act as a deterrent in developing teachers' pedagogical content knowledge. Lack of knowledge of student thinking and how to support development of understanding without resorting to 'telling' may serve as a challenge for teachers in aligning their practice towards reform agendas (Romagnano, 1994; Chazan & Ball, 1999). Other studies have indicated that poor math content knowledge also prevented change in beliefs about practice (Britt, Irwin & Ritchie, 2001 as cited in Forgasz & Leder, 2008).

One of the aspects of a teacher's growth, is the development of knowledge of mathematics in terms of its principles or concepts which is distinguished from mere knowledge of facts and procedures of mathematics. Spillane, Reiser and Gomez (2006) described principled knowledge as signifying the mathematical ideas and concepts that undergird the mathematical procedures, distinguishing it from procedural knowledge. Procedural knowledge has been contrasted with conceptual knowledge in the literature (Scheffler, 1965; Hiebert & Lefevre, 1986; Long, 2005). Skemp's (1976) distinction between instrumental and relational understanding emphasizes the importance of understanding the underlying ideas in procedures and connection with other mathematical concepts over knowledge of the procedure for solving a problem. Some good examples of the distinction between these two different types of knowledge are illustrated in work of Ma (1999), where most of the US teachers displayed knowledge of the procedure but many Chinese teachers displayed knowledge of not only alternative procedures, but also explanations of procedures with relevant conceptual details as well as knowledge of why certain procedures work for all numbers. Here the contrast of procedural view of mathematics is with 'profound knowledge of mathematics', which includes connections between and among procedures as well as related concepts in the form of a network.

2.4.2.1 Frameworks to describe teachers' knowledge

Shulman's notion of pedagogical content knowledge is based on the notion that mere knowledge of pedagogical techniques is inadequate and also that content knowledge in itself is not sufficient for teaching to be successful. This notion has been further extended through the term *mathematical knowledge for teaching* (Ball, 1990b; Ball & Bass, 2000; Thompson & Thompson, 1996) to designate the special way in which teachers need to know mathematics in order to engage in the *work* of teaching.

Teachers' knowledge of student learning and instructional practices has been considered as central to teachers' professional development especially the realization that students are capable of solving mathematical problems without explicit direct instruction in procedures (Jaworski, 2006; Even & Tirosh, 2002; National Research Council, 2001; Fennema et al., 1996; Simon & Schifter, 1991; Cobb, Wood, Yackel & McNeal, 1993; Becker & Pence, 1996; Klein, Barkai, Tirosh & Tsamir, 1998). However, in India these ideas are yet to be explored in classrooms and integrated into teacher education. There have been attempts towards "joyful learning" in the past and making teaching more student-centered (NCERT, 2005) in the classrooms by using manipulative and activities but the role of students in using these resources and opportunities to make their ideas explicit is still very limited. In several studies, the knowledge of mathematics is focused by encouraging teachers to deepen learning of school mathematics in integration with pedagogy or knowledge of students (understanding misconceptions, errors and invented methods). The assumption behind the focus on students' thinking is that it will lead to change in practice by making teachers focus on how students make sense of the content (Swafford, Jones & Thornton, 1997).

Cochran-Smith and Lytle (1999) critiqued both process-product research and interpretive research on teaching for not viewing teachers as possessors of the knowledge of teaching, since the enterprise of research on teaching was almost solely engaged by university researchers. Making a call for practitioners (including teachers, administrators and teacher educators) to adopt an inquiry stance in teaching, they advocate providing opportunities to teachers to build and share their knowledge of teaching by providing "compelling insider accounts of the complexities of teaching" (Cochran-Smith & Lytle, 2009, p. x). I agree with their thesis that practitioner research in which the researcher is involved in both practice and research generates local knowledge of teaching which is also available to other practitioners outside the context. The generation of this knowledge is through a dialectic between theorizing and doing which involves experimentation and looking at evidences critically for indications of student learning.

There is widespread recognition that specialized forms of mathematical content knowledge are important for effective mathematics teaching (Hill, Ball & Schilling, 2008; Ernest, 1999). Researchers have also shown the relevance of mathematical knowledge for teaching in determining mathematical quality of instruction (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008). Several researchers have proposed frameworks aimed at capturing the outlines of such knowledge as well as explaining its close connection with the practice of teaching (Ball, Thames & Phelps, 2008; Petrou & Goulding, 2011). It has been argued that much of this knowledge is acquired in and through the practice of teaching (Ma, 1999) and through opportunities created for professional development (Putnam & Borko, 2000). There is a need to understand in detail and topic-wise, what constitutes mathematical knowledge needed for teaching and how professional development programs can help in building and strengthening such knowledge.

A widely used framework for understanding the specialized knowledge of mathematics needed for teaching has been proposed by Deborah Ball and her colleagues (Ball, Thames & Phelps, 2008). In this "practice-based theory of mathematical knowledge for teaching (MKT)", the authors distinguish between pedagogical content knowledge (PCK), which is closer to students and teaching (such as the underlying thinking behind common errors that students make or knowledge of how to sequence instruction), and specialized content knowledge (SCK), which is not necessarily connected to students or to teaching, and is distinct from both common content knowledge and PCK. Under SCK, Ball et al., include such elements as knowing why algorithms work, having a repertoire of representations of a mathematical concept, and knowing the affordances and limits of particular representations (Ball, Thames & Phelps, 2008). Consistent with their practice-based approach, they identify a set of tasks that are an essential part of the work of teaching, where SCK is implicated. Such tasks include finding an example to make a specific mathematical point, recognizing what is involved in using a particular representation, linking representations to underlying ideas and to other representations, and selecting representations for particular purposes.

Studies on teachers' conceptions of representation are few and have revealed that teachers generally view representations as devices to be used in classroom for purpose of communication (Stylianou, 2010). Teachers acknowledged both cognitive and social aspects of the use of representations. Teachers discussed how they used representations as a tool for communication and explanation, recording, exploration of problem, monitoring and evaluating and ensuring equity in classroom by catering to different learning styles. The study highlighted how it is challenging for teachers to integrate multifaceted representation with instruction. It advocated that teachers should be made aware of multiple roles that representation can play in learning and doing mathematics. Cai (2006) found that teachers preferred symbolic representations over visual representations. Moyer (2001) found that manipulative were perceived by teachers as useful for introducing "fun" rather than learning mathematical ideas and were used as diversions when teachers themselves were not able to generate representations.

Knowledge of the textbook has been recognized as playing an important role in teaching by researchers like Shulman (1986) who proposed that curricular knowledge helps teachers to connect different topics laterally that are covered in the same grade as well as vertically by building upon students' previous knowledge and recognizing important mathematical ideas useful for learning mathematics in later grades. Curricular knowledge has also been considered as important component of the mathematical knowledge needed for teaching. However An, Kulm and Wu (2004) argue that the knowledge of content, curriculum and teaching interact and become transformed in the task of teaching to contribute to pedagogical content knowledge to address the goal of enhancing students' learning. This process is also impacted by beliefs held by teachers. Thus there is a need to study interactions between knowledge of textbooks and other beliefs and knowledge held by teachers.

Rowland, Huckstep and Thwaites (2005) identified four dimensions of teachers' mathematics subject knowledge from a grounded analysis of videos of preservice teachers' enactment, namely, i.e., foundation, transformation, connection and contingency. This framework links teacher knowledge to its enactment. This has implications for how knowledge developed in professional development space gets used in teaching and the various aspects of teaching in which the knowledge gets reflected. Rowland et al. (2005) contend that "Foundation" refers to the propositional knowledge of mathematics along with knowledge of "why" mathematics works in specific contexts and appropriate use of vocabulary. It also includes beliefs about mathematics, purpose of math education and about appropriate conditions for learning mathematics. The transformation component refers to how a teacher transforms the content knowledge in pedagogically useful ways including the choice and use of examples for teaching. Connection refers to the coherence in mathematics teaching by illustrating connections across mathematics and exhibited in the planning of the sequence of instruction and tasks in teaching. The teacher's decision making and response to unexpected events in the classroom like deviating from the plan to address students' ideas or misconceptions.

The frameworks described above for teachers' knowledge of teaching mathematics discuss the different types of knowledge that a teacher needs to have to teach mathematics and how this knowledge is used and reflected in the teaching of mathematics. However, one needs a framework which provides a coherent idea of how knowledge held by teachers plays a role in what tasks and representations get selected for teaching in the classroom and how the teacher manages the classroom interaction to develop understanding of a particular concept. The framework of meaning of integers proposed in Chapter 7 connects the underlying meanings of integers and operation with the procedures performed using contexts, models and symbolic representations.

2.4.2.2 Assessing teacher's knowledge

Researchers have tried to assess teachers' knowledge for teaching using paper and pencil tests, observational methods, interviews, tasks, card sorts and response to video clips. Leinhardt and

Smith (1985) classified teachers' knowledge based on in-class discussions (use of mathematics and their errors) by giving extensive descriptions of the lesson taught. Borko, Eisenhart, Brown, Underhill, Jones and Agard, (1992) also used observation of explanations in the classroom along with teachers' responses (in an interview) to open ended tasks to talk about the content knowledge used in teaching. These studies indicated a move away from considering just subject matter knowledge of mathematics to analyzing the mathematical knowledge demands in teaching.

Recently more standardized instruments have been developed for judging the quality of mathematics in instruction aside from pedagogical aspects. These include the Reformed teaching observation protocol (Pilburn, Sawada, Falconer, Turley, Benford & Bloom, 2000) and the Learning mathematics for teaching: Quality of mathematics in Instruction (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep & Ball, 2008). However, using these instruments requires the knowledge of specific mathematical topic and consistent interpretation based on protocol through training. Also a substantial number and variety of lessons need to be observed before making a generalization about teachers' mathematical knowledge for teaching (Rowan, Harrison & Hayes, 2004). As an alternative to tedious observations, tasks have been developed to assess knowledge of mathematics for teaching based on specific content areas (Borko et al., 1992; Ma, 1999; Tirosh & Graeber, 1990). Researchers have used knowledge derived from studies of children and based on situations that might arise in teaching like addressing misconceptions or giving conceptual justification.

Barwell (2013) has proposed an alternative way to talk about teachers' knowledge by using discursive psychology to analyze classroom interaction. He states that the framework of Ball et al. (2008) assumes a "representational model of knowledge" (p. 605) and involves making assumptions about teachers' and students' knowledge as representations which are not accessible but are inferred through peoples' action. He offers that focusing on discourse in classrooms and trying to understand how teachers and student "construct each other as knowledgable" is more consistent with sociocultural framework of assuming knowledge as situated and distributed and can contribute to fine grain analysis of teaching and learning in classrooms. I agree with this proposal that analysis of classroom discourse as well as discourse in the professional development space can reveal much about the knowledge that teachers hold and about how they use it in different contexts.

2.5 Research studies on inservice teacher professional development

Professional development interventions have been claimed to be effective on the basis of their success in bringing about change in teachers' characteristics like practices, knowledge, beliefs, and identity. Further, curricular reform and student learning have also been claimed as results of effective professional development. These aspects define what changes as a result of professional development and thus get treated as a measure of the professional growth of teacher.

2.5.1 Change in beliefs through professional development

Beliefs have been considered as the clearest measure of teachers' professional growth (Kagan, 1992). How professional development influences the relationship between belief and practice has been studied by many and change in beliefs has been recognized as a complex, difficult and long term process (Clarke, 1994; Swan, 2006). Many studies have a linear view of change in beliefs as a result of engagement in professional development and look for evidences of change in practice as an indication of change in beliefs. However, some studies have claimed that changes in beliefs may not be reflected in practice (Fernandes & Vale, 1994, as cited in Forgasz & Leder, 2008). Guskey (1986) on the other hand posited that belief change occurs as a result of getting evidences of students learning after exploring changed practices in teaching. His model of change though linear, was driven by instructional change followed by change in beliefs.

Leder, Pehkonnen and Torner (2002) noted that although research on role of beliefs in teacher change has been piling up, no consistent pattern has yet merged to inform the process of teacher change. How certain beliefs are activated during teaching might depend on long and short term goals (Schoenfeld, 2003), the context of the teaching situation, unexpected occurrences during teaching and the knowledge held by teacher to deal with the situations arising during teaching.

Several researchers have discussed the process of change of beliefs borrowing from the conceptual change literature which uses the idea of assimilation and accommodation for describing change in belief structure (Piaget & Cook, 1952). Posner, Strike, Hewson and Gertzog (1982) argued that for teachers to change their beliefs, they need to be dissatisfied with their held beliefs while the new beliefs seem intelligible and plausible. It is possible that teachers might add unconnected beliefs to their belief structure rather than making a revision in the way beliefs are held by them leading to inconsistent beliefs being held by same individual. It has been argued that change in beliefs is a process which involves a 'gestalt shift' rather than being a result of argument and reason. Chapman (2002) proposed that change in teachers' beliefs could occur as a result of deconstructing the belief structure including the primary or central beliefs or developing additional beliefs compatible with a reform perspective. However, teachers may not be willing to bring change easily in central beliefs as they are part of teachers' identity. Ambrose (2004) found that as a result of working closely with students pre-service teachers' new beliefs were added to belief structure rather than replacing previously held beliefs. This has implications for even in-service teachers since it indicates that experimentation focusing on student thinking and reflections are important factors contributing to teachers' growth in terms of beliefs and practice.

Franke, Fennema and Carpenter (1997) found no consistent relationship between change in beliefs of participating teachers with their change in practice. In consonance, Knapp and Peterson (1995) found that change in beliefs and practice is interactive rather than one following other. Others have found that beliefs have an 'embedded' nature and teachers 'reconstruct' their beliefs when engaged in innovative practice (Hoyles, 1992). This suggests that a two way relation exists between beliefs and practice and implies that a teacher development program engaging teachers in new kinds of practice and experience might help them develop beliefs conducive to it.

In reviewing the research on teacher change and development, Wilson and Cooney (2002) identified that most studies have taken individual teachers as the units of analysis and studies are needed where a group of teachers are taken as a unit of analysis while undergoing professional development. Most studies about change in teachers' beliefs have been reported as case studies and some have identified teachers' personality characteristics also as a determining factor for whether a change in beliefs structure occurs as a result of professional development or not. Cooney et al., (1998) found that the teacher Greg having a more reflective stance towards his teaching was more open to problematizing teaching, exchanging views and reflecting on beliefs held by others and showed change in beliefs over the course of the study. On the other hand, Henry, who had a more authoritarian view, was frustrated when challenged, did not engage with other views and showed little change. The authors conclude that rather than what beliefs teachers hold, the way teachers hold beliefs has an impact on professional growth. The way a teacher holds a belief refers to which beliefs are held as central or peripheral, isolated or connected, consistent or inconsistent, stable or dynamic and other characteristics of beliefs discussed above. They identified teachers' notion of caring for students and belief about teaching as telling as the main obstacles in changing beliefs and practice as they contribute to teachers' sense of effectiveness.

2.5.2 Teachers' growth as change in practices in classroom

Research on teaching practice has been carried out with the intent of understanding the complexity of teaching and to determine the impact of professional development on teaching practice. Researchers have recognized various dimensions across which change in teachers' practice could be characterized while some have tried to develop frameworks to analyze practice and identify the practices which are preferred amongst teachers. Some have focused on the kind of mathematics that gets discussed in the classroom like procedures or concepts while others have focused more on pedagogy. Analysis of teachers' practice have led to conjectures about their beliefs and orientation that teachers hold. In this thesis, teachers' growth through changes in practice is explored in Chapter 6 and Chapter 7. In the next subsection, research literature has been reviewed regarding the frameworks that have been used to analyze teaching and knowledge held by teachers.

2.5.2.1 Analyzing classroom interaction and practices

Students' learning of mathematics in the classroom is shaped by the opportunities provided through classroom interaction. The most common pattern of interaction discovered in the classroom has been the IRE pattern in which the teacher dominates the interaction by initiating a question, followed by response from students and evaluation by the teacher (Porter, 1989; Stigler & Hiebert, 1999; Cazden, 2001; Mehan, 1985; Hiebert, & Grouws, 2007). Silver, Smith and Nelson (1995) found this type of interaction was more common in schools with a majority of students from economically disadvantaged backgrounds. Practices that are very different from the IRE type of interaction have been identified and studied. For e.g., Kazemi and Stipek (2001) identified the norms established through the patterns of interaction that helped focus on conceptual understanding. These included explanations backed by mathematical reasons, safe space to make mistakes and learn from them, identifying mathematics embedded in strategies, and arriving at consensus through mathematical argumentation.

Researchers have probed deeper into interactions in classrooms by focusing on the teacher's questioning practices. Wood (1998) found that teachers tend to use *funneling* type questions wherein they do most of the thinking for the students. Other studies have tried making a distinction between the lower order questions asked by teachers like factual questions (Brualdi, 1998; Cotton 1989; Vacc, 1993; Myhil & Dunkin, 2002) and higher order questions that engage students in analysis, synthesis and evaluation (Cotton, 1989; Kawanaka & Stigler, 1999; Newmann, 1988). Sahin and Kulm (2008) studied questioning practices of two middle school teachers (a novice and an experienced teacher) categorizing the questions into factual, probing and guiding questions. They found that teachers used more factual questions in the classroom than any other category. Although relatively few higher order questions were asked by both teachers, they recognized that asking 'why' questions helps in assessing students' understanding.

Others have also studied teachers' evaluation practices and how teachers respond to students. Sherin (2002) identified the *filtering approach* which was based not on questioning but on how the teacher reacted to the students' responses to the question by selecting (filtering) from the approaches shared in the classroom, the ones which should be pursued for mathematical discussion. McClain and Cobb (2001) discussed how the teacher's responses to students' strategies help in establishing the norms about which representations are elegant or efficient, since students had to make judgments about it. Empson and Jacobs (2008) use the notion of directive listening, observational listening and responsive listening to differentiate the teachers' efforts to listen and respond effectively to student responses. To develop responsive listening teachers were engaged during professional development in discussion of children's written work and analyzes of videos along with interaction with children and reflecting on it. There are studies which have analyzed change in teachers' response over a period. Crespo (2000) found that the practice of prospective teachers changed along two dimensions, i.e., from the focus on correctness of answers to 'meaning making' and secondly moving from focus on quick and conclusive answers to thoughtful and tentative ones.

Many studies have looked at what mathematics is focused in the interaction between teachers and students and have distinguished focus on facts and procedures from to conceptual approaches to doing mathematics. Thompson, Philipp, Thompson and Boyd (1994) distinguished teachers having a *calculational orientation* from those having a *conceptual orientation* by looking at their focus on procedures or conceptual connections in classroom talk. Schifter (1995) identified broad stages that depicted teachers' understanding of "doing mathematics" and the role of authority in determining correct answers. She proposed that in the first stage, mathematics is thought of as "ad hoc accumulation of facts, definitions and computational routines" (p. 18). The second and third stages are characterized by student-centered activity in classroom with the difference of focusing on "issues of mathematical structure and validity" in the third stage. In the fourth stage, practice is marked by "systematic mathematical inquiry" around the *big* ideas of mathematics (p.18).

The mathematical explanations given by the teachers and students have also been the focus of study. Explanation has been described as "the discourse of an individual intending to establish for somebody else the validity of the mathematical statement" (Balacheff, 1988). Some researchers like van Fraassen (1980) opined that it must answer a 'why' question while Achinstein (1983) contended that explanation may be a response to any type of question seeking understanding. Toulmin (1969) identified the warrants and backing as essential features of a legitimate explanation. Ball et al. (2008) considered giving and evaluating mathematical explanations as part of specialized content knowledge. A study by Ma (1999) distinguished between procedural and conceptual explanations given by teachers and discussed the connection between the two. She found that experienced teachers in China could give conceptual explanations as a result of extensive opportunities for professional development in a community of teachers wherein they analyze the curriculum materials deeply. Among the conceptually based explanations some studies have distinguished between mathematically based explanations which draw from ideas of mathematics and use of symbols and patterns in contrast to practically based explanations which are based on experienced phenomena. Levenson, Tsamir and Tirosh (2010) found that teachers generated more mathematically based explanations in the workshop setting but ended up using more practically based explanations in the classroom since they felt it would be more convincing for students.

Other studies have focused on how teachers developed understanding of how students can give conceptual based explanations. Bowers and Doerr (2001) discussed how prospective teachers moved from the understanding of conceptual explanations as useful for giving clear explanations to students to the idea that students themselves can arrive at these conceptual explanations through engagement in context based tasks due to affordances that are inbuilt in the contexts. The realization that students are capable of solving mathematical problems without explicit direct instruction in procedures has been considered an important moment in teachers' professional development (Fennema et al. 1996; National research council, 2001; Tirosh & Graeber 2003). Goldsmith and Schifter (1997) described change in teachers' practices as organized around transfer of facts and procedures to focusing on building on students' understanding to finally using knowledge of what students can do to plan their teaching. They concluded that development of pedagogical content knowledge is not as orderly as depicted in most research and indicated the need for better understanding of issues faced by teachers when acting on new beliefs and understanding gained in professional development.

Another dimension of teacher's growth with respect to explanations is the realization that interpretations of students can vary from what was intended and depend on their prior experiences. Tzur, Simon, Heinz and Kinzel (2001) illustrated teachers' growth by movement from a *perception based perspective to conception based perspective,* which involves moving from considering that everybody sees the same mathematics in a representation to recognizing that mathematics is a product of human activity and possibilities of variations in the interpretations of representations.

The studies discussed in this section analyze explanations given by the teachers and focus their attention of kinds of knowledge that teachers may have. Researchers have made inferences about beliefs held by teachers about mathematics and its teaching based on the analysis of classroom interaction or report of classroom practices and explicit acknowledgements of beliefs. Askew, Brown, Rhodes, William and Johnson (1997) proposed three types of belief orientations that are relevant to practice, viz., connectionist, transmissionist and discovery. While the transmissionist orientation is based on beliefs about mathematics as procedures and teaching through telling, connectionist orientation is based on belief about mathematics as a discipline which is interconnected and gives importance to students' strategies while teaching mathematics. The discovery orientation involves believing that mathematics is discoverable by students and allowing opportunities to do that. Askew et al. (1997) posit that no teacher may exactly fit into one orientation and may combine characteristics of two or more orientations. I feel that the practices in Indian classrooms and beliefs held by teachers could be characterized along the continuum of transmission oriented and student-centered teaching or teaching responsive to students since connectionist and discovery ideas may not be widely prevalent among teachers. Rather than considering transmission oriented and student-centered practice as binary opposites, they may be construed as opposite ends of continuum.

2.5.3 Study of interaction between different factors that govern professional development

Schoenfeld (2010) posited that teachers' behavior in teaching is a function of their knowledge and resources, goals, beliefs and orientations. While these factors have been found important in the research literature, there are a few studies which analyze the interactions between these factors governing practice. Schoenfeld (2011) describes how these different factors interact during in-the-moment decision making by teachers. Goals at multiple levels that teachers set out to achieve drives them towards selecting some resources. "Orientation" is a more general term than beliefs, which refers to the way an individual perceives and acts in a social situation depending on what aspects of situation are salient and the number of beliefs held which are connected to each other.

There are certain other broad frameworks that integrate perspectives of mathematics, student thinking as well as pedagogy to identify what aspects of teaching support or constrain the development of students' understanding. Fennema et al. (1996) used a framework that depicts how beliefs, knowledge and practice interact with each other. They have described the levels at which teachers progressed in their practice through focusing on students' thinking during professional development using a research based model of children's thinking. Instruction at Level 1 focused on procedures and was textbook directed while at Level 2 teaching still had fixed routines although enriched by tasks discussed in the teacher workshops. Level 3 was characterized by teachers giving opportunities to children to report their solution of non textbook based problems. Level 4 indicated substantial change in teachers' beliefs as teachers based their decisions on students' capabilities and solution strategies.

The realization that teaching is a complex activity has led to the development and use of integrated frameworks that focus on the aspect of content, pedagogy as well as student thinking. Based on the notion of mathematical knowledge for teaching Deborah Ball and her colleagues have proposed a framework for analyzing classroom instruction and establishing relation with mathematical knowledge for teaching (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep & Ball, 2008) called the 'Mathematical quality of Instruction'. The themes in the framework are based on prior research on classroom instruction, some of which reported deficits while others reported affordances of mathematical knowledge held by teachers to develop understanding of mathematics. Thus it represents both the problematic aspects of mathematics teaching like errors, responding inappropriately to students and lack of connection of classroom work to mathematics as well as affordances available like richness of mathematics, mathematical language and appropriate response to students. By relating teachers' performance on paper and pencil tests for mathematical knowledge for teaching and analyzing videotaped lessons of teachers, they have tried to establish that there is a strong relation between mathematical knowledge for teaching possessed by teachers and the quality of mathematics instruction. Teachers having high MKT exhibited many examples of rich mathematics and skill in responding to students while ensuring equitable opportunities to learn over classroom observations spread throughout the year on different topics. On the other hand teachers with lower knowledge scores exhibited more variability across lessons and good lessons were generally supported by textbook or professional development.

Another framework, which analyzes how knowledge of mathematics impacts classroom teaching has been developed by Adler and Ronda (2015) called "Mathematical discourse in instruction". The authors use a sociocultural framework to analyze the object of learning in lesson episodes by analyzing the kind of exemplification used, explanatory talk of teachers and learner participation. Exemplification involves both examples as well as tasks which the teacher uses in the classroom. Similar examples may help in generalizations while a variety of examples may help in identifying different classes and more powerful generalizations. Tasks can be framed having low mathematical demand by using known procedures or higher demand through application of a concept, establishing connections and problem solving. During the explanatory talk, the teacher may use criteria which are non-mathematical in nature or rely on authority. Alternatively she may rely on generalizing and proof as mathematical criteria to establish an assertion. In giving explanations, the teacher may rely on colloquial names or may use mathematically precise and formal terms. Learner participation captures students' responses to the tasks as well as other instances where learners are invited by the teacher to contribute to classroom discussion. Researchers were able to use these analytical ideas to describe the different mediational means for mathematical learning that were at work in a teachers' classroom through her use of examples and the explanatory talk.

Bray (2011) depicts interactions between beliefs, knowledge and practice while considering beliefs and knowledge as "evolving cognitive constructs that are constantly reinforced or revised by the influence of experiences" (p. 4). She studied teachers' error handling practices during class discussion and found that practice varied across three dimensions where the focus of the teacher can be on flawed solutions or development of conceptual understanding or encouraging the class as a community to engage with error. She found that some dimensions of practice are mediated by beliefs held by teachers (e.g., sharing wrong solutions might confuse or embarrass students versus mistakes are resources for student learning). Other dimensions like discussing concepts to address errors might depend on teachers' knowledge of the key mathematical idea underlying the error which help teachers in developing lesson images and premeditated scripts with conceptual questions and explanations for addressing anticipated errors. She recognized that teachers need "time, intellectual space and human support" (p. 35) to reflect on their beliefs and practice, deepening their knowledge and feeling the need to adopt reform based teaching.

Besides interaction between beliefs, knowledge and practice, interaction between beliefs and knowledge has been studied relatively to a lesser extent. Ernest (1989) reported how two teachers who have similar knowledge may still teach differently because of variation in beliefs. Knowledge held may not be necessarily used by the teachers (Brown & Cooney, 1982) or may lead to teachers being 'blind' in noticing the difficulties faced by students. Ma (1999) found that even teachers who hold student-centered beliefs about teaching mathematics, may not be able to give explanations that are conceptual in nature in order to develop students' understanding. Thus knowledge of concepts underlying procedures and their interconnections is important in bringing about any substantive change in pedagogy.

2.5.4 Frameworks and principles of design of professional development for teachers

There exists extensive research about different aspects of professional development which have been reviewed by experts in the field to identify the key factors that contribute to effective professional development. Guskey (1995, p.117) identified that the "enormous variability of the educational contexts" makes it imperative to identify the unique features of the setting and the assortment of elements of professional development which makes it work. Several researchers have tried to identify 'principles of design' and frameworks essential to make professional development initiative successful either through review of literature or through reflecting on their own experience.

In a review of literature on teacher education, Loucks-Horsley and Matsumoto (1999) have identified four categories for articulating the principles of design, such as content, process, strategies and structures, and context.

1. The principle related to **content** define what gets focused in the professional development initiative which Loucks-Horsley et al. (1999) have identified as "subject matter, learners and learning and teaching methods" (p.262).
2. The **process** principle relates to how tasks are enacted and the processes in which the teachers are engaged. They have identified four processes that contribute to learning i.e.
    1. Learner-centered: by building on what learners know,
    2. Assessment-centered: by providing opportunity for feedback and revision to teachers, and
    3. Community-centered: by giving teachers' time to work together and develop a community.
    4. Knowledge-centered: by getting opportunities to develop knowledge of planning and strategies for teaching,
3. They have identified five different types of **strategies** which are used to design professional development.
    1. The first category of strategies is "immersion" which involves engaging in doing mathematics through problem solving, investigations, etc.
    2. The Second strategy is to use curriculum materials to develop teachers practice to

implement the materials.

3. The third is examining practice through records or artifacts of practice.
4. The fourth is the collaborative strategies like mentoring, working together, etc.
5. Lastly, vehicles and mechanisms like workshops, topic study groups are used to engage teachers.

4. **Context** for professional development refers to the extent of support and nurturing that is provided to the teacher to try out ideas learnt through in-service professional development in the classroom. This organizational context ranges from schools to school system to state systems to national systems.

Tirosh and Graeber (2003) identified content and process as the two main aspects of effective professional development which are interactively connected. On the contrary, Some researchers have argued that a strong focus on process is needed in in-service professional development. However, others have reported that when professional development activities are focused on how and what is to be done in the classroom without discussion about why it should be done or taking teachers' ideas into consideration, the superficial features of instruction may change, but the fundamental character of teaching and learning is unlikely to be altered (Baker & Smith, 1999). However, an earlier meta analysis of research showed that programs that were too theoretical and lacked modeling of techniques were least effective in motivating teachers to use them, while including the conceptual aspect greatly improved impact as compared to mere demonstration of techniques (Showers, 1987). This indicates a need for balance between philosophical and practical considerations.

The multiple ways that professional development efforts influence classroom practice indicate the need for frameworks that acknowledge the complexity of the process. In many studies professional development is characterized as the "change" (Clarke & Hollingsworth, 2002) that is exhibited by the teacher in the contexts of the beliefs or knowledge held and practices executed in the classroom. Clarke et. al. (2002) proposed an empirically grounded framework for teacher 'change' which is non-linear and describes change as a process of reflection and enactment in "the personal domain (teacher knowledge, beliefs and attitudes), the domain of practice (professional experimentation), the domain of consequence (salient outcomes), and the external domain (sources of information, stimulus or support)" (p. 950). While this framework explains the process of change that the individual teacher undergoes, one needs to also consider that the teacher is situated in a social context. Professional development occurs through the process of peer interaction in a professional development context within schools and outside school settings.

Some of the principles found common across various studies and reviews of research on teacher professional development are listed below:

1. The design should be based on sound theoretical ideas related to learning of students as well as of adults (Knapp, 2003; Mewborn, 2003).
2. The focus of the initiative should be on critical activities related to teaching (Borasi & Fonzi, 2002) and providing a diverse set of experiences to teachers. Opportunities provided for learning collaboratively, both during teaching and through reflection on teaching have to be considered (Clarke, 1994; Guskey, 1995; Hawley & Valli, 1999; Elmore, 2002).
3. A concrete image of quality and alternative practices for teaching should be provided through modeling or videos to challenge teachers and engage in reflection (Knapp, 2003; Corcoran 1995, Borko & Putnam, 1995).
4. Many researchers have recommended a strong focus on development of content knowledge along with knowledge of students, culture and of teaching and learning (Ball & Cohen 1999, Wilson & Berne, 1999, Borko & Putnam, 1995).
5. Teachers have been viewed as both objects and agents of change (Sowder, 2007). Researchers have recognized over time that teachers should play an important role in deciding the foci of the initiative. Recognition of their needs and respecting them as professionals by considering them as partners helps develop ownership of the initiative by teachers. Hargreaves (1995) observed that teachers may not take up ideas from professional development efforts if they are imposed or if teachers are not involved in the developmental cycle or when other teachers in the school are not part of the initiative.
6. Continuous on-going engagement is needed based on understanding that change is a gradual and difficult process which requires on-site follow up and support and understanding of the unique contexts and cultures in which teachers work (Clarke, 1994; Guskey 1995).
7. The role and participation of other school staff and leaders has been considered important (Fullan, 1987; Clarke, 1994; Elmore, 2002) to develop a "culture of support" (Villegas-Reimers, 2003), which can contribute towards integration of the program with the school life (Guskey, 1995; Garet, Porter, Desimone, Birman &

Yoon, 2001).

8. The context also plays an important role in determining the success of a professional development initiative, which includes historical, social, economic and cultural contexts which is especially true for third world countries (Supovitz and Turner, 2000; Johnson, Monk & Hodges, 2000). Provision of time for teachers to engage in professional development activities on a regular basis is a challenge for implementation as time for planning and knowledge development is not recognized and valued in many countries, China, Germany and Japan being among the exceptions.

9. Most studies on teacher professional development have elaborated on the connection between teacher learning and student learning. However, due to reform efforts often not being in line with the systemic change, it was difficult to show student learning as an outcome (Darling-Hammond & Sykes, 1999). Darling-Hammond and Sykes (1999) recommended that changes in instructional materials, tests, curriculum framework should match the efforts done in teacher professional development, while teachers should get opportunity to focus on student learning using the curriculum materials.

10. Materials for teachers, description of facilitator roles and teacher outcome measures (Cohen, Raudenbush & Ball, 2003; Borko, 2004) need to be well defined for implementation. Research is needed for understanding professional development of teacher educators. However, aspects like pedagogy for students and teachers, knowledge of content, knowledge of classroom as well as schools, institutions and also national education system is necessary.

While the above framework given by Loucks-Horsley et al. (1999) and other principles provide the broad categories to describe professional development, I believe that one needs to have a specific framework based on the underpinning philosophical or mathematical ideas inherent in the design as well as enactment of the professional development activity. The frameworks used in the design of workshop and the engagement in the activities in topic study group have been described in Sub-study 2, 3 and 4 in Chapter 5, 6 and 7 respectively. The framework in Chapter 5 focuses on the process aspects by illustrating the philosophical and pedagogical ideas used in design and enactment of tasks during the professional development workshop. I argue that mathematical content focused in the professional development initiatives need to be unpacked and ideas that exist in research literature need to be integrated in the design of PD activities. In

Sub-study 3, the sub-construct theory of fractions has been applied to analyze teaching and change in teaching over the two years. In Sub-study 4, a framework was developed for specialized content knowledge for teaching integers through a review of literature around meanings of integers and integer operations used in various contexts and models. These topic specific frameworks facilitated connecting teacher learning of mathematical ideas to how they were used in class by the teacher as well as students. These process and content frameworks focus on different aspects of professional development initiatives and thus they are considered as complementary rather than any of them having giving primacy over the others. Integration of topic specific frameworks of teacher knowledge with the professional development design is rarely found in the literature. One such example is the CGI study in which researchers have used a research based framework to categorize the type of problems for different operations and the strategies used by students to solve these problems (Carpenter et al., 1999).

2.6 Research studies on key features contributing to effective professional development

Teachers' change in belief as a result of professional development efforts is a well researched topic. Researchers have tried to identify different aspects of professional development that contributed to change in beliefs towards educational reform oriented teaching. These aspects include engagement in continuous professional development, socialization and experience (Richardson, 1998), opportunities to consider and challenge held beliefs (Wood & Sellers, 1997: Borko, Mayfield, Marion, Flexer & Combo, 1997), reflection (Clarke, 1997; Raymond, 1997; Senger, 1998; Artzt & Armour-Thomas, 1999), collegial discussion, providing images of alternative teaching practices, using student work to understand student thinking (Sowder, 1998), action research collaboration with peers or experts like teacher educators or researchers (Edwards & Hensein, 1999) and engagement in a community (Ambrose, 2004).

Factors that have constrained teachers' change of practice include beliefs about student ability for engaging in mathematics and especially for students from low socio economic background (Sztajn 2003, Arbaugh, Lannin, Jones & Park-Rogers, 2006), school micro-culture, lack of support in classroom, practical constraints in translation of beliefs into practice (Quinn & Wilson, 1997), limited content knowledge of mathematics (Halai, 1998; Raymond, 1997; Steele, 2001; Britt, Irwin & Ritchie, 2001) or even lack of confidence in teaching (Beswick, Watson & Brown, 2006). Grant, Hiebert and Wearne (1998) tried *showing* teachers alternative instruction based on student's thinking and evaluated how those teachers implemented the features of the alternative instruction in their own teaching. They found that teachers interpreted the demonstration in accordance with their held beliefs failing to understand the intended purpose of using manipulative and tasks to provide students with opportunities to share their strategies. Instead they implemented lesson plans without substantial change in their teaching goals of developing skills of students for learning procedures and the role of teacher being to present solutions or explain rules.

Analysis and evaluation of studies on teacher professional development has revealed several factors or elements that contribute towards reshaping teachers' practice through professional development. In the following list, several themes have been identified relevant to this study which have been used for designing opportunities and tasks for professional developmentwhich are discussed in the subsections below.

1. Engaging with mathematics content
2. Using artifacts from practice for teacher learning
3. Learning from designing instruction: Lesson Study
4. Role of reflection in teacher learning
5. Participation in communities and collaboration with others
6. Role of the teacher educator/facilitator/researcher

2.6.1 Engaging with mathematics

Content knowledge of the subject is an important aspect of professional development of mathematics teachers that needs to be focused. In India and elsewhere, knowledge of mathematics is generally assumed during preservice teacher education. Thus teachers replicate the same approach to mathematics that they had experienced during their schooling in their classrooms. They would typically not have had an opportunity to develop understanding about processes of mathematics like making representations, problem posing, problem solving and reasoning. Inservice teacher professional development interventions have therefore used various ways to engage teachers in "doing mathematics" through problem solving or solving non-standard or real world problems, engaging in mathematical processes for mathematical abstraction and generalization. Thus teachers take up the role of learners, which they are expected to subsequently reenact with students. The knowledge of mathematics focused in most professional development is deeper learning of school mathematics in integration with pedagogy or knowledge of students (understanding misconceptions, errors and invented methods) with an assumption that it will lead to change in practice (National research council, 2001; Swafford, Jones & Thornton, 1997).

A review of literature indicates that such activities are taken up during workshops or summer institutes in which teachers are expected to take up the role of a learner. Amit and Hillman (1999), in a workshop as part of a three week summer institute, engaged teachers in performance based assessment through tasks based on real world contexts which were open ended and had multiple solutions. Some studies have focused on teachers' knowledge of processes of mathematics in professional development context through problem solving (Ball, 1990a; Schiftor & Simon, 1992; Borasi, Fonzi, Smith & Rose 1999) followed by adopting the role of a teacher. These studies have reported how teachers developed knowledge of alternative ways of doing mathematics. Teachers reported that these experiences motivated them to to rethink their pedagogical beliefs and practice in not only the instructional unit that was the focus of professional development but for other topics of mathematics also. However, studies have also reported how it is challenging to bring these ideas into practice as teachers face number of dilemmas and conflicts while implementing them in the classroom (Brown, Carter & Richards, 1999).

Sowder Philipp, Armstrong, & Schappelle (1998) engaged in-service middle grade teachers in conceptual discussions on a specific topic (topic study groups) by having regular meetings over the course of two years. The study showed how conversations changed during the initial and later years along with change in understanding of the content and teachers' comfort level with the content. The changes during meetings were reflected in changes in classroom discourse, teachers' beliefs about student capabilities and the curriculum and increased students' learning. They argued for extended support to teachers to bring about 'meaningful' change in teaching.

Weinzweig (1999) described the use of 'problem situations' to engage teachers in experiencing ways of thinking and doing mathematics in which there can be multiple pathways and solutions and problem solving becomes an important component of teaching. The workshops were held over the period of a year with time in between workshops for teachers to try out ideas. The impact was visible in the form of demands by participating elementary teachers for more such mathematics focused courses and participation in education research and communities.

Irwin and Britt (1999), who engaged with teachers through regular workshops over two years, found that teachers who had deeper and more integrated knowledge of mathematics, had a view of mathematics as 'fallible' and a 'product of human invention' and were more likely to use a problem solving approach in the classroom. They found that a teacher who was confident and had in-depth conceptual and relational knowledge was able to lead the class to construct problems on their own and was able to relate them for students. However, another teacher Emily, who had anxiety issues with mathematics was not able to overcome it to engage students in mathematics embedded in tasks. The authors suggested providing space for such teachers to voice their anxiety and working with the curriculum problems starting from their own context rather than using challenging investigations, which Emily was not confident of mathematically.

2.6.2 Using artifacts from practice for teacher learning

Practice based professional development approaches based on situative philosophy consider using artifacts from practice as a promising resource for engaging teachers in analysis and reflection on teaching, besides their being sources for teachers' learning of mathematics, pedagogy, curriculum as well as student thinking. These artifacts can be in the form of curriculum materials, classroom situations in form of written cases or video records, student work, blackboard work, tasks used for teaching and even teachers' reflection on teaching. They can be sourced from traditional as well as non-traditional classrooms, since both can be used to engage teachers in thinking about practices used for teaching mathematics. Stein, Smith, Henningsen and Silver (2000) classified the 'cases' used for professional development of teachers into 'dilemma driven' (which encapsulate dilemmas of teaching) or 'paradigm' cases which "embody certain principles or ideas related to the teaching and learning of mathematics" (p. 33). Similar categorization can be done for the artifacts used as some illustrate the traditional approaches but can be used to engage teachers in critical analysis or see teaching from different perspectives, for e.g., contrasting teacher perspective to that of the student. On the other hand, artifacts from teaching can also be illustrative of exemplary work or reform oriented teaching, which can be used for analysis of elements that contribute to teaching in this manner and the type or evidences of student learning as a result of such teaching. Efforts to bring about professional growth by bringing about change in traditional mathematics teaching, requires the development of an alternative image of teaching mathematics where the focus is on understanding rather than memorizing procedures. Studies indicate that it is possible to raise teachers' awareness of alternative approaches to teaching through using artifacts that have examples of modeling new approaches, but this may not translate into actual change in teaching practice as teachers need to analyze and critically reflect on these artifacts with colleagues to understand embodied principles and may require extra assistance from teacher educators, support from administrators and time to translate ideas into practice (Goldstein, Mnisi & Rodwell, 1999; Markovits & Even, 1999).

In a 3 year longitudinal study, Sowder and Schapelle (1995) noted the importance of providing support to teachers for implementing meaningful change in practice through conversations and questioning in a professional development setting. Teachers and researchers met periodically over a two year period and interacted in three hour long seminars focused on understanding the concepts involved based on the content needs of the teachers. The impact was visible through change in teachers' conversations, understanding of content, student expectations, role of curriculum and questioning pattern in the class room, which led to improved student learning outcomes. These studies indicate that it is possible to initiate reflection among teachers and motivation for change in practice through engagement in understanding mathematics in the curriculum deeply, perhaps in the role of learners first and then by providing extended support to teachers in their classroom explorations.

2.6.2.1 Using classroom situations for analysis and professional development

Some studies have also used classroom situations as a tool to engage teachers in analyzing teaching and think about similar situations that may arise in their teaching. This is done by using written cases of classroom events or by using multimedia or videos to show these events in a professional development setting to a group of teachers and then engaging them in critical analysis and reflection on these events. These types of resources are extensively used in practice based professional development since it help in presenting the complex knowledge of teaching embedded in these situations to teachers to be analyzed and discover similarities and differences with their own practices. Teachers can relate to video cases, which are believed to be excellent tools to engage teachers in discussing mathematical as well as pedagogical ideas, generalize these ideas and develop a precise language to discuss practice while also providing a "safe place" (p.5) to analyze practice carefully (Mumme & Seago, 2002).

Studies have found that case-based resources helps teachers in engaging in critical analysis, developing awareness of student learning and deep understanding of mathematics, reflection on teaching, comparing their ideas with peers while attempting to justify their own ideas, developing pedagogical content knowledge and revising beliefs about teaching (Barnett 1998; Barnett &Friedman, 1997). Barnett (1998) argues that it is the "collective inquiry and critical reflection" (p.92) that really helps teachers in arriving at in-depth understanding of teaching practice and anticipates that a teacher engaged in such practice will subject her ideas to critical examination and will thereby make better informed classroom decisions. Barnett and Friedman (1997) argue that these cases allow teachers to look at classrooms from students' perspective and allow reflection on student thinking. Reflection on teaching practice in a group setting has also been endorsed for bringing about change in teacher by others (Walen & Williams, 2000).

Biza, Nardi and Zachariades (2007) used hypothetical classroom situations that are likely to occur in practice, where teachers solved problems and analyzed them to identify underlying learning objectives, examined possible student errors and wrote feedback. It helped in identification of mathematical, didactical and pedagogical issues that increased teachers' awareness.

Using students' work and thinking as a resource for professional development

Analyzing students' work has been identified as promising for developing teachers' knowledge about students' thinking in general and in relation to specific content. It helps to highlight how students think in qualitatively different ways from adults and also individual differences in interpreting contexts and sense making. Students can exhibit different levels of understanding as they continuously try to make sense of their experiences within the classroom and outside school. In the process they may develop understanding very different from what was intended by the teacher. Awareness of children's conceptions helps the teacher in addressing learning difficulties of students by being able to identify the conceptual gap, while also making them aware of the potential that a student's meaning or strategy has for encouraging discussion and learning in the class.

Borko (2004) has pointed out that studies taking individual teachers as units of analysis show how teacher learning is a "slow and uncertain process" as teachers find some practices easier to adopt than others (p.6). She reports how teachers in the Cognitively Guided Instruction (CGI) program found eliciting student thinking easier compared to using students' thinking to make decisions. The CGI program (Carpenter & Fennema, 1988) used students' work to show how students can solve problems using different resources without being taught the procedure. They found that as a result of being aware of different solution strategies that children adopt, teachers encouraged problem solving and listened to them more while exhibiting changes in their beliefs about children's abilities and using knowledge developed in workshops. As an extension of the CGI project, Franke and Kazemi (2001) developed a community of practice of teachers in school by providing on site continuous professional development. Teachers shared, explored and discussed student work to develop their own mathematical thinking, which lead to experimentation and change in classroom practice to support students' thinking. There is strong evidence from the CGI study that when teachers are engaged in "sense making around children's thinking" (p.103), it can motivate teachers to listen more closely to their students and encourage them to give explanations of their answers and probe their thinking (Franke & Kazemi, 2001). Franke et al (2001) found that it was easier for teachers to elicit students' thinking than to respond to students' thinking in appropriate ways to develop understanding. They explain that "as teachers engage in listening to their students' thinking, they learn more about possible problems to pose, strategies to expect, and relationships that exist between problems and strategies". Another important gain in teacher's learning was the role teachers adopted as "learners" to analyze student thinking and seeing classrooms as a site for learning about teaching such that their growth was "generative" in nature by applying the learnt knowledge to newer areas. Classrooms became the site for teachers' learning and workgroups, places for reflection on experimentation.

Similarly, in Teaching to the Big Ideas project Schifter and colleagues (Schifter, Russel & Bastable, 1999) used students' work and videos of interviews and classroom discourse to develop practices to engage students in mathematical discourse. Teacher learnt to recognize mathematical ideas embedded in students' work.

Cohen (2004) while using students' work in the Developing Mathematical Ideas project noted that rigor and supportiveness of the discussion in the professional community supported teacher learning in different topics. In the Integrating Mathematics Assessment (IMA) project, Gearhart and Saxe (2004) went a step further by recognizing that ongoing assessment is critical to support development of understanding. They involved teachers as both learners of mathematics and researchers of children's mathematics before they implemented the curriculum in their classroom with integrated assessment.

2.6.3 Learning from designing instruction: Lesson study

Lesson study is an established cultural practice in Japan for teacher professional development. The lesson study cycle includes stages of goal setting, curriculum analysis, lesson planning, teaching the lesson and its observation followed by debriefing and reflection on the lesson. Lesson study has been used in studies with various purposes in mind and have yielded promising results for building communities and producing knowledge and resources for teaching. Ball and Cohen (1999) have suggested that producing knowledge for teaching through collective reflection is a more worthwhile goal than producing lessons that can be copied. This knowledge can be produced in a community comprising both teachers and teacher educators as it allows making shared knowledge among teachers explicit while "observations and replications across multiple trials can produce dependable knowledge" (Hiebert, Gallimore & Stigler, 2002). Lesson study has been found to be promising for promoting shift in practices (Lewis & Tsuchida, 1998; Murata and Takahashi, 2002), exploring effective teaching practices (Chokshi & Fernandez, 2004; Lewis, Perry & Hurd, 2004; Lewis, Perry & Murata, 2006), developing knowledge of mathematical content and teaching (Fernandez, 2005; Fernandez, Cannon & Chokshi, 2003; Yoshida, 2008) and improvement in everyday practice over time (Murata, 2010, 2011).

Results from studies using lesson study have elaborated on how different phases contribute towards teacher learning. Some studies indicate that the phase of collaborative discussion on topics is as if not more important than the collective observation of lessons. Lesson plans can serve as "concrete scaffolds for teachers to focus their attention and learn about specific content areas under discussion" (Hart, Alston & Murata, 2011, p.7). It has been found that collaboratively anticipating and discussing student thinking while making lesson plans leads to teacher learning and that lack of subject matter knowledge and reasoning skills constrain the opportunities for learning during lesson study (Fernandez, 2005). Meyer and Wilkerson (2011) elaborate on how opportunities to develop teachers' knowledge arise through the discussion of concepts and instructional strategies prior to making a lesson plan rather than through the use of an existing lesson plan and focusing on its implementation. Eley (2006) found that teachers discussing their reasoning behind planning use "contextually localized models" of what students do instead of ascribing to general conceptions of teaching focused in teacher education courses. This points to the potential of using collaborative lesson planning as a tool for professional development as it creates space for articulation and negotiation of beliefs and building on teachers' situated knowledge of students.

Analysis of teachers' discourse within interventions has shed light on both the individuals' unique beliefs, knowledge and dispositions that they bring to the participation in the professional development context but also on how interactions with others shape individuals' trajectory of learning within the group interactions while contributing to development of shared understanding and knowledge of the group as a whole.

Scholars have found that several challenges exist in adopting lesson study into a new culture as Japanese lesson study is a part of a system where teachers take up control of their own professional development rather than an exercise planned by others for teachers. Sannino (2010) describes the case of a teacher whose discourse underwent a shift from showing resistance to externalizing the tensions and conflicts experienced by her and facing them. On confronting her tensions she experimented in her assessment practice by "breaking out of the traditional mold of individual evaluation" (p.843) thereby exercising her agency by finding her own meaningful way to address the issue of assessment in her class. Murata, Bofferding, Pothen, Taylor and Wischnia (2012) illustrated how connections were naturally made between craft knowledge (Ruthven & Goodchild, 2008) and scholarly knowledge while participating in lesson study while individual teacher talk paths varied within the group path. The craft knowledge was related to knowledge that teachers have about their practice while scholarly knowledge is the knowledge obtained through researching teaching. Connections between two kinds of knowledge occurred as a result of teachers sharing the materials, representations, content and knowledge about students in a manner accessible to others and feeling the need to access scholarly knowledge to address questions arising in the process of lesson study like why some methods are used in lower grades but not in higher grades.

2.6.4 Role of reflection in teacher learning

The role that reflection plays in teachers' learning and professional development has been well recognized over the years in many theoretical writings as well as empirical studies. Dewey (1910) can be considered as one of the earliest pioneers to talk about the importance of reflective thinking in teaching. Dewey defined reflection as "active, persistent and careful consideration of any belief or supposed form of knowledge in the light of grounds that support it and the further conclusions to which it tends" (italics in original, p.9). To him reflection mirrored the scientific method which he considered as the phase in which an experience is spontaneously interpreted, followed by naming or identifying the problem. Possible explanations are generated and then converted to hypotheses, which are established by experimenting or testing.

In the literature there are varied interpretations and synonymous terms used (like metacognition), which has made it difficult to clarify how reflection is different from other types of thought and thus to assess and relate it with teacher professional development. However, researchers like Schon (1983) have argued for reflection as not just a cognitive act but as a practice, described through his notion of 'reflective practice'. Here the distinction between thought and action gets erased when he argues that a practitioner can engage in 'reflection-in-action' through intuitive ways of responding to the situation besides engaging in 'reflection-on-action'. Schon (1983) introduced the idea of the reflective practitioner, while criticising the idea of 'technical rationality' in which one assumes that the practitioner is a technician who uses academic knowledge for problem solving in practice. He proposed that greater importance needs to be given to the problem setting process of identifying the problem in a certain context, in contrast to instrumental problem solving. He notes that reflection is mostly a result of coming across an unexpected outcome that makes us think. Below is an excerpt about reflective conversation (Schon, 1983), in which Schon highlights the agency of the practitioner in selecting the problem, constructing the situation in his mind and deciding how to address it.

> In a practitioner's reflective conversation with a situation that he [or she] treats as unique and uncertain, he functions as an agent. Through his transaction with the situation, he shapes it and makes himself a part of it. Hence, the sense he makes of the situation must include his own contribution to it. Yet he recognizes that the situation, having a life of its own distinct from his intentions, may foil his projects and reveal new meanings. (p. 163)

Mewborn (1999) identified the characteristics of reflective thinking in her study as being different from recollection or rationalization and requiring introspection as well as external probing. She considered action as an "integral part of the reflective process" (p.317). Reflection has also been associated with the Inquiry approach and pedagogical theorizing (Barnett, 1998).

In empirical studies, reflection has been considered as both the means to bringing about change in teachers' practices as well as an indicator of the effectiveness of professional development in making teachers think. Sowder (2007) noted that the "degree and kind of reflection is often used to describe teacher change" (p.198). Avalos (2011) pointed out that reflection is used as an instrument for change in studies characterizing professional development. Ball and Cohen (1990) contend that teachers learn by analyzing and reflecting on practice rather than learning new strategies and activities. Mason (2002) also contends that systematic reflection on students' learning and teachers' interaction in the classroom contributes to teacher learning. However, Farah-Sarkis (1999) points out that designing in-service program for engaging teachers in reflection is not easy, since teachers' beliefs and conceptions depend on many factors including educational background, pedagogical culture in school as well as educational system. Chapman (1999) reports how focusing on teachers' personal experience and meaning to develop self understanding through the process of narrative reflection had a positive impact on their beliefs about problem solving and teaching.

As Mewborn (1999) indicated the need for a social situation for engaging teachers in reflection, and some researchers have engaged teachers in reflection in collaboration with researchers or peer teachers. Jaworski (2003) elaborates how teacher and researcher collaborated together to analyze interaction in the classroom. Scherer and Steinbring (2006) discuss how "professional joint reflection" (p.157) for planning and analyzing teaching between teacher and researcher in a collaborative project could lead to changes in the teacher's interaction in the classroom in the long term. Here the teacher and the researcher played the role of both external reflective observers of practice and of actors involved in the process of interaction. Teachers gained a better understanding of student learning and engaged with the challenge of disrupting patterns of established teacher centered practice to a more student centered one.

2.6.5 Participation in collaboration and communities

Several studies adopting the sociocultural framework have established that professional communities contribute towards improvement in instruction and student learning (Little, 2002; Wineburg & Grossman, 1998; Grossman, Wineburg & Woolworth, 2001; Stein, Silver &Smith, 1998). In a similar vein, collaboration between teachers and between teachers and researchers/ teacher educators have also been considered promising to bring about change in practice in classrooms towards developing mathematical understanding.

Professional learning communities can comprise teachers, teacher educators and researchers, who are engaged in the enterprise of mathematics education. They allow bringing varied experience and situated knowledge of students and contexts of teachers into the discourse of the community for reflection and developing insights, while the presence of university educators helps in bringing critical and reflective stance in conversations and bringing research based ideas of teaching and learning into the discourse (Goldenberg & Gallimore, 1991; Saunders, Goldenberg, & Hamann, 1992; Richardson & Anders, 1994). Vescio, Ross and Adams' (2006) review of professional learning communities indicates that participation in these communities did lead to change in teaching practice in terms of being more student-centered. However, several researchers (Brodie, 2012; Kazemi & Hubbard, 2008) have argued for developing deeper understanding of the interactions within such communities and connecting it to change in practices in the classroom.

Several researchers have tried to articulate the features that define a true community as different from a group of people working together. Borko (2004) notes that "norms that promote supportive yet challenging conversations about teaching are one of the most important features of successful learning communities" (p.7). Features of a true community include sharing a sense of purpose (Secada & Adajian, 1997), accountability to each other (Brodie, 2013), ownership, intensive and long term professional relationships, and critical review of teachers' practices among others. Westheimer (1999, p. 75) identified "shared beliefs and understandings, interaction and participation, interdependence, concern for individual and minority views" as important features for defining a community. Stoll, Bolam, McMahon, Wallace and Thomas (2006) identified the characteristics of professional communities that make them effective – shared values and vision, collective responsibility, reflective professional inquiry, collaboration as well as promotion of group and individual learning.

Although the communities of practice approach has been used in many studies to engage teachers, Adler (1998) has criticized the use of Wenger's theory of communities of practice (1998) since the theory is based on anthropological data of apprenticeship relationships which is not the kind of relationships which occur in education institutions. Graven (2003) criticizes Wenger's assumption of "minimizing teaching to maximize learning" which questions the role of the teacher in the process of learning. Graven presented evidence for the emergence of teachers' "confidence" as a result of engaging in a community of practice. The teachers developed the confidence to admit what they did not know and needed to learn, which helped in sustaining ongoing learning of teachers. However, several research projects have made the use of communities of practice in the form of professional communities, collaboratives, communities for Inquiry and teacher networks (Grossman, Wineburg & Woolworth, 2001).

A growing number of research studies provide evidence that teacher's participation and collaboration with others in professional development contexts have an impact on practice. Most have found student work, tasks and lesson design as a useful focus for discussion in the community. Kazemi and Franke (2004) reported shifts in teachers' participation in work-groups over the years as they shared student work generated from the problems decided in the work-group. The shifts involved at first attending to the details of student thinking followed by developing possible learning trajectories for students. Brodie (2012) uses the term "professional learning communities" for communities organized to support teacher professional development and argues that it is a "sustainable and generative method of professional development" since communities sustained themselves when teacher's participation in the community and the classroom practice informed each other as the teacher tried to engage with learner errors in her teaching. By analyzing sessions where teachers discuss lesson design and reflections on their teaching they arrived at the notions of challenge and solidarity as important in teachers developing accountability for each other and their professions. Jaworski and colleagues (Jaworski, Goodchild, Eriksen & Daland, 2011; Goodchild & Jaworski, 2005; Jaworski & Goodchild, 2006) analyzed how discussion on tasks designed by the team of didacticians enabled building a community to engage in mathematics together, providing a basis for raising pedagogical issues and generating examples of the tasks that teachers could use in their classrooms. Though the task were adapted differently by the teachers in their classrooms, tensions were experienced for developing teachers' mathematical awareness and providing opportunity for self- direction.

Another form of social engagement for teacher professional development has been the collaborative partnerships between teacher educators and teachers and among teachers. It has taken the form of collaborative inquiry (Krainer, Goffree & Berger, 1999; Lin & Cooney, 2001; Wood Scott, Nelson & Warfield, 2001; Jaworski, 2010), classroom coaching as collaboration (Becker & Pence, 2003), collaborative action research (Raymond & Leinenbach, 2000), and even just dialogue between researchers and teachers in a collaborative context. The role of teachers as researchers has been presented as a model of professional development in various countries including UK and USA (Hollingsworth, 1995) as it portrays the vision of teacher as a professional engaging in action research to improve ones' own practice and context or work while contributing to developing knowledge about teaching.

Erickson, Brandes, Mitchell and Mitchell (2005) identified the features that promoted success of the collaboration between classroom teachers and teacher educators as "a mutually held understanding of what types of classroom practices nurture good teaching and learning, a setting where teachers have a strong commitment and control over the project and decide on its direction, and a structure that allows teachers and teacher educators to meet regularly in an atmosphere of trust and mutual understanding."

2.6.6 Role of the facilitator in professional development

There are few formal programs for professional development of teacher educators along with dearth of studies which identify strategies used by effective teacher educator or for their own professional development.

Schifter and Lester (2002) have argued that teacher professional development requires active facilitation for pushing teachers to think beyond the tasks that they engage with in professional development to the actual issues related to teaching like strategies used in classrooms. They noted that facilitators need to have deep understanding of subject matter content and students' thinking, understanding of learning goals of teachers and participants' perspective so as to make choices about appropriate representations that can make teachers rethink their deeply held ideas. Remillard and Kaye (2002) found that challenges arose for facilitation when the goals of facilitator, expectation of the participations and curriculum agenda do not match. They classified such instances as "openings in the curriculum", as these moments can lead to deeper inquiry and learning of teachers. They posit that the facilitator needs to be able to identify such openings, understand the underlying tensions, anticipate consequences and take action. The role of the facilitator in guiding a lesson study has also been identified as important in providing guiding questions and eliciting teachers' pedagogical content knowledge which had not been formally shared before (Hart & Carriere, 2011).

Zaslavsky and Leiken (2004) have discussed how engaging in the community of practice can help in professional development of mathematics teacher educators by engaging in challenging mathematical tasks to develop deeper understanding of mathematics, developing sensitivity to students thinking while also becoming sensitive to teachers' learning. The argue that engagement within the communities help in understanding the complexities of management of learning of both students as well as teachers.

2.7 Conclusion

The foregoing discussion points to the complexity and challenge of designing and implementing effective PD for mathematics teachers. The discussion of the Indian context reveals the critical need to design sound in-service TPD programs. The review of frameworks and principles for TPD points to the importance of teachers' role and agency in their professional development. There is a need to understand the forms of expertise that Indian teachers of mathematics bring to PD interactions and the kind of agency that they exercise, when a space for it exists. Efforts to "train" teachers without addressing their core beliefs about mathematics, and about the teaching and learning of mathematics are only likely to produce superficial change. Thus it is essential to obtain an understanding of the kinds of beliefs that are prevalent around Indian mathematics teachers, the ways in which they hold, and how these beliefs shape their teaching practice. The context of reform, vigorously promoted by the NCF 2005, introduces perturbations in teachers' belief systems, but this may not result in genuine movement towards realizing the new vision of education. There is need therefore to understand the interaction between core, entrenched beliefs and the "new" beliefs engendered in the reform context.

Sowder (2007, p.97) emphasized that change is a "process rather than an event, it must be considered in terms of continuous growth over time". This calls for the need to study the relationship between beliefs, knowledge and practices when teachers are undergoing a transition in the sense that teachers explore new practices after intending to change following a professional development experience or explore new curriculum material after curriculum change. This will help understand the process by which teachers develop or can be supported to develop the beliefs and associated pedagogical content knowledge to sustain the use of practices and helping students learn. Studies on teachers' learning-in-practice can bridge the gap between understanding how change in beliefs and practice is supported or constrained by knowledge on one hand and on the other hand to bridge the gap between theory and practice in teacher education (Adler et al., 2005; Even & Ball, 2009).

Further, the literature reviewed indicates how the mathematics education research community has gained increasing awareness of the specialized knowledge of mathematics to teach effectively. There is need to investigate how the goal of strengthening teachers' specialized mathematical knowledge can be foregrounded in PD interventions. What role does knowledge play in constraining or facilitating change in practice, and how does it interact with the changing beliefs of a teacher. There is thus a need to investigate teachers' professional development in a wholistic manner that takes context, beliefs and knowledge into account. The present research study takes a step in this direction.

3

The Research Study

In this chapter, the design of the research study as a whole as well as of the four sub-studies is discussed. The methodology used in the four sub-studies to collect data and the decisions taken for data reduction and data analysis are described. In section 3.1,the purpose of the larger study under which the four sub-studies were constituted is discussed. In section 3.2, the overarching research question as well as the specific research questions dealt in each of the sub-studies are presented. Sections 3.3 has details of the research design followed by section 3.4 with details of the participants in the study. In section 3.5, the rationale for the suitability of the methods adopted for data collection and analysis is presented followed by the description of data collection and data analysis methods used for each of the Sub-study. Finally, in section 3.7, the roles that the researcher has played in the study and how it bears on data collection and data analysis has been discussed.

There have been several debates in the past about the design of educational research. The design of the research study and the methods chosen for data collection and analysis determine the robustness and the usefulness of the findings from a study. The selection of methods should be guided by the research questions such that they are answered to satisfactory depth and detail within the scope of the study. The design of educational research itself has undergone several developments. While, in the past, the experimental designs borrowed from science research programs were used in education research, based on positivist assumptions, there is increasing awareness among educational researchers about the limitations of these designs for use in educational research. These limitations stem from the fact that educational research participants are "human" subjects, who have their own volition and agency and display actions based on their beliefs and assumptions as well as the norms accepted in different social settings.

Another important factor that influences the findings of the educational research is the interaction of the researcher with the participants. While earlier research designs attempted to limit the interactions of the researcher with the participants, trying to make observations as objective as possible, now participant observation by the researcher is considered as valuable in developing in-depth understanding of the situation through immersion in the research context under study. The recognition of theory ladenness of the data means that even when the researcher is doing observations as a non-participant, the observations recorded will be determined by what he/she considers important and worthy to notice. Immersion in a research context thus allows one to experience the field from the eyes of the insider and to understand participants' perspectives more closely by interacting with them. In fact, collaboration with the teachers in the classroom for educational research allows teachers to take ownership and participate in the research on an equal footing with the researcher. Educational research has also moved from having a fixed research design to having flexible and emergent research design depending on the circumstances and factors considered important in different phases of the study. This has led to emergence of the design based research as a very promising field to study the effect of interventions in the field on the on hand and on the other hand to develop a theoretical understanding based on the empirical findings thus developing theories grounded in the reality (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003).

3.1 Purpose of the Study

The main purpose of the study was to analyze how beliefs, knowledge and practices of the teachers participating in the study are influenced and modified as the result of the participation in the professional development initiatives across the two years of the study. The PD interventions were aimed at collaborating with teachers so as to promote changes in teachers' practice towards teaching that is more responsive to the development of student understanding. Thus, it was first important to understand the participating teachers' beliefs, knowledge and preferred practices. Such an understanding was developed through the analysis of the teachers' questionnaire and interview responses during the teacher professional development workshop in the first year at the start of the study. This has been discussed in Chapter 4, which reports Sub-study 1. Secondly, analyses of the process of professional development within the setting of the workshop as well as during the follow up in the classroom in collaboration with the researcher was done to understand how the process influenced teachers' beliefs, knowledge and practice. This has been reported in Sub-studies 2, 3 and 4 in Chapters 5, 6 and 7 respectively.

Thus the thesis consists of the four Sub-studies listed below:

Sub-study 1: Beliefs and Practices of teachers in context of curriculum reform

Sub-study 2: Professional development Workshop – design and enactment

Sub-study 3: Role of beliefs and knowledge in teaching of fractions: Case study of Nupur

Sub-study 4: Topics focused professional development on the teaching of integers

3.2 Research Questions

The larger question being investigated in the study is: *In the context of the classrooms chosen for the study, what factors support teachers in adopting learner centered practices and what factors inhibit or constrain them in doing so?*

The factors that will be focused upon are those that will be relevant to in-service programs for teacher professional development and continuing teacher support.

Following are the sets of research questions addressed in each of the four sub-studies:

Sub-study 1:

1. What are the core and peripheral practices of the teachers in the sample with respect to the teaching of mathematics?
2. What beliefs are core or peripheral as indicated by the teachers' articulation and the practices preferred by the teachers?
3. What is the relation between beliefs expressed and the practices preferred by the teachers?
4. What is indicated about teachers' knowledge from their explanations? What is the relation between preferred practices and the knowledge held by teachers?

Sub-study 2:

1. What aspects of the workshop design and enactment are important from a TPD perspective?
2. How did the workshop tasks encode the design principles?
3. How was teachers' agency enabled in the course of the enactment of the workshop?
4. What aspects of the teacher educators' enactment of the task and interaction facilitated engagement by the teachers?
5. What were the learning gains from the PD workshop as perceived by the teachers?

Sub-study 3:

1. What changes did Nupur try to bring in her classroom practice specifically with regard to the way in which tasks for students were framed and implemented? How did her beliefs and knowledge support and constrain the changes that she tried to implement?
2. What textbook resources were available to her to support her teaching and how did she make use of these resources?
3. What was the role of the researcher as a collaborator in Nupur's teaching?

Sub-study 4:

1. What were the teachers' concerns about the teaching of integers and how are they related to issues of meaning of integers?
2. How did teachers construct Specialized Content knowledge (SCK) for teaching integers using the framework of integer meanings through the exploration of contexts?
3. How did the criteria used by teachers for judging adequacy of representations evolve in the course of the topic study workshops?
4. What was the impact of the learnings from the topic study workshops on teaching of integers as reported and as observed?

3.3 Research Design

The study covered a span of two academic years. Sub-studies 1 and 2 took place in the first academic year 2009-2010 (June to April), while Sub-studies 3 and 4 took place in the second academic year 2010-2011. These sub-studies collectively build a comprehensive picture of professional development interventions in the study in various settings and forms. The details of the time line, research tools used, number of participants, and the role of the teacher in different sub-studies is given in Table 3.1.

Table 3.1: Professional development interventions during the two years of the study

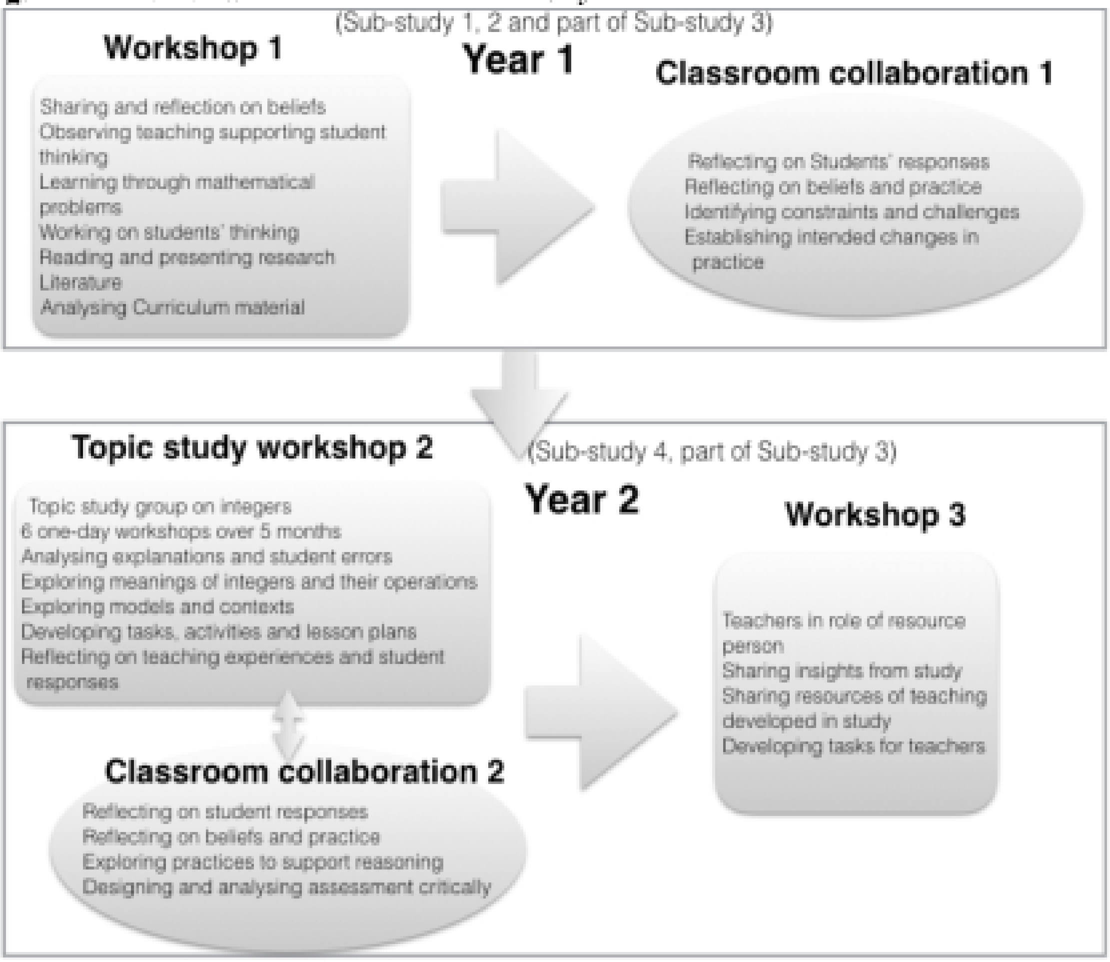
Study	Time	No. of teacher participants	Research tools used	Data analysis	Professional development setting	Teachers' role
Sub-study 1	May-June 2009	26: 18 primary + 8 middle	Questionnaire	Quantitative descriptive analysis	Workshop	Respondent
		11: 5 primary + 6 middle	Interviews	Qualitative thematic analysis	Workshop	Respondent of questionnaire

Sub-study 2	May-June 2009	13 (5 primary + 8 middle)	Audio and Video data, researcher notes	Qualitative coding/ grounded theory approach	Teacher educator led workshop	Learner and community member
Sub-study 3	July-Sept. 2009-2010	1 Primary and 1 middle school teacher (not reported)	Audio data, Researcher notes	Qualitative coding/ grounded theory approach	Collaborative school based setting	Teacher-collaborator
Sub study 4	August to November 2010	4 middle school teachers	Audio and Video data, researcher notes, lesson plans, presentations	Qualitative coding/ grounded theory approach	Collaborative workshops/ meetings	Learner, designer and teacher educator
		4 middle school teachers (3 reported)	Audio data, researcher notes, student work	Qualitative coding/ grounded theory approach	Collaborative school based setting	Teacher and researcher

Sub-study 1 provides the overview of the beliefs held by the participant teachers in the beginning of the study and the preferred practices for the teaching of mathematics. It also illustrates how teachers' thinking and practice have been informed by the current scenario of curriculum reform in the education system and the kind of inputs required to engage teachers in making meaning of the new curriculum. Semi-structured interviews and questionnaire about teachers' beliefs and practices using likert type items were used. 26 teachers responded to the questionnaire and 11 teachers were interviewed about the practices engaged by the teachers and for their beliefs about mathematics, its teaching and learning and about students.

Sub-study 2 took place in Year 1 in the setting of a ten day long workshop (including a non working Sunday) during the summer vacation. 19 teachers participated in the workshop, 13 of whom belonged to a nationwide school system. The goals of the workshop were strengthening teachers' knowledge relevant to teaching, providing opportunities to articulate and reflect on beliefs and practices, developing awareness of alternative practices and resources and developing a sense of community among teachers, teacher educators and researchers participating in the study. The workshop tasks included observing and reflecting on non-traditional teaching, learning through solving and analyzing problems, anticipating and reflecting on student responses, discussing mathematics education research literature, analyzing textbooks, and articulating beliefs about teaching, students and mathematics. More details of the workshop can be found in Chapter 5. The workshop tasks included a range of topics and concepts in school mathematics such as whole numbers and operations, fractions, ratio and proportion, and algebra The teachers who took part in ten day professional development workshop responded to the full questionnaire before the workshop and once again, on the last day of the workshop, wrote their responses to parts of the questionnaire dealing with their beliefs about the teaching-learning of mathematics and the nature of mathematics and their preferred teaching practices.

Figure 3.1: Overview of Year 1 and 2 of the study



Sub study 3 is a case study of a primary teacher Nupur, who was a participant in Sub-study 1 and Sub-study 2. Data from the classroom observations following the orientation workshop in the first and the second year have been analyzed. The researcher visited the classroom of the teacher after the workshop. The purpose was to establish collaboration with the teacher in the classroom, to identify the take up from the workshop and the challenges faced by teacher in implementing intended changes. During these visits, the researcher frequently reflected on the lessons together with the teacher and discussed plans for future lessons. The visits revealed that teachers needed to develop knowledge and resources for specific topics which would facilitate the change in teachers' practice towards developing student understanding. This was addressed through the Topic Study Group workshops held in the second year.

In the second year, Sub-study 4 involved six one-day meetings of a Topic study group spread over a period of five months (July-November 2010) while the teachers were teaching in their schools. The meetings were held at HBCSE and thus provided an uninterrupted professional development space to teachers. The specific topic of integers was chosen as the focus of the workshops by the four middle school teachers, all of whom had attended the professional development workshop in the first year of the study. Four middle school in-service teachers (3 female: Swati, Anita and Rajni; 1 Male: Ajay, all pseudonyms) engaged in collaborative investigation on the sixth Grade topic of *integers*.

The primary teachers group consisting of 4 primary teachers from Mumbai decided to study the topic of Multiples and factors. They analyzed. the textbook problems and studied problems from other textbooks and resource books. Discussion and sessions to develop content knowledge for underlying concepts like types of multiplication and division problems were held. However, due to administrative constraints, the primary teachers were not able to fully use the problems developed in the topic study groups in the classroom since the topic was included in the September term exam while they were planning for December term. It was not feasible to transcribe and analyze the data of both the topic study groups. The middle school teachers were able to implement some of the ideas discussed in the workshops in the classroom and thus were selected for detailed analysis which is presented in Chapter 7.

In the months when the groups held meetings, the researcher observed classroom teaching of three teachers Swati, Rajni and Anita, and held post lesson discussion on classroom interactions. The two teachers Swati and Anita had expressed an intent to change their classroom practice and had shown more active engagement in the meeting as compared to the other two teachers and were provided support for classroom teaching. A resource team of five members consisting of researchers (who also played the role of teacher educators) and graduate students planned and facilitated the meetings of the topic study workshop. Usually, two to three resource team members were present along with the four teachers for each workshop meeting. The aim of the topic study meetings was to focus on a topic that was challenging to teach, to develop a deeper understanding of the topic, and to plan for teaching. The teacher educators, who were also researchers, refrained from conveying to the teachers that they needed to make specific shifts in their teaching practice, for e.g., from teaching rules to teaching for understanding and reasoning. The focus was more on collaboratively developing mathematical knowledge for teaching and resources, which was expected to facilitate teacher learning and influence teachers' decisions in selecting and designing tasks.

In parallel to the workshop, classroom visits were made to observe lessons of 3 teachers during the period in which the Topic study group meetings took place in HBCSE. The purpose of this Sub-study was to identify the take up from the topic study meetings in terms of resources used like tasks, activities and representations, challenges faced by teachers in implementing intended changes in practice as well as observing the interplay between teachers' knowledge, resources used and the classroom interaction. Thus in this Sub-study the professional development setting included both the school and classroom as well as HBCSE as a distinct out of school space for professional development.

There were major differences in the approach to professional development in the Year 1 and 2. In the Year 1 the design considerations took on the objective of building awareness of alternative pedagogies through observing non-traditional teaching and reading research texts which had evidences of student arriving at interesting insights about mathematics on their own. The focus was not on a particular topic and the objective of the school based collaborative work was to see what alternative pedagogies teachers explored in their teaching and the challenges faced by them in doing that. The findings from Year 1 informed the design considerations for Year 2. The major shifts in Year 2 of professional development included focus on a specific topic of integers selected by teachers themselves, collaborative nature of workshops, stronger connection between workshop and school based teaching of the topic and opportunity for teachers to adopt a more central role of being a resource person in a workshop.

As a result of change in design, the activities that teachers engaged in both the phases were also different in some aspects as shown in Figure 3.1. More details of these activities are given in the respective Sub-study chapters. Teachers had varying roles in the four sub-studies. In Sub-study 1, the teachers had the role of respondents to the interview and questionnaire, which provided opportunities to articulate their beliefs and talk about their preferred practice. The teacher educator led workshop tasks in Sub-study 2 prompted teachers to articulate their knowledge of subject, teaching as well as students. In doing so the in-service teachers played the role of a knowledgeable learner who is extending her knowledge through engaging in tasks and social interaction. In Sub-study 3, the teacher Nupur adopted the role of being a teacher who collaborated with the researcher to engage with students' thinking by discussing students' response in the classroom. In Sub-study 4, during the meetings, the teachers' role gradually became more central to their own learning by engaging in designing, evaluating and using the tasks using the theoretical framework provided by researchers playing the role of teacher educators. The participating teachers in Sub-study 4 also had the role of teacher collaborator by sharing and analyzing their experiences of using tasks constructed by them in topic study meetings. Thus, these four sub studies depict the professional growth of participant teachers over these two years as they moved from the role of a knowledgeable learner to that of a reflective teacher who is conscious of his/her own beliefs, responsive to students and develops knowledge through exploration and reflection on use of tasks beyond the textbooks.

3.4 Participants

Participants in the study were mathematics teachers teaching primary and middle grades in a nation-wide Government school system and were nominated by their principals to participate in the study. These teachers had many years of experience of teaching ranging from 17 to 23 years and were between the age range of 42 to 54 years. Generally all the teachers had bachelor's degrees in mathematics or science and in education, while some also had a master's degree in mathematics. The details of the participants in the various sub-studies are presented later.

Purposive sampling process was adopted to select the participants in each of the sub-studies. The teachers as participants were selected based on the research questions identified for the Sub-study and the conjectures about how studying particular teacher will help in understanding the process of professional development and knowing how it contributes to teacher learning. How participants were selected and their characteristics are described for each of the sub-studies in the paras that follow.

In Sub-study 1, the questionnaire data analyzed in the study is from 26 teachers which includes two groups of teachers from the same nationwide system of schools. A group of 13 primary teachers responded to the questionnaire before attending a one day workshop. Another group of

13 teachers attended a ten day long PD workshop also responded to the questionnaire before the workshop. This group had 5 primary teachers and 8 middle school teachers. Of the 8 middle school teachers 3 were male and there was no male primary teacher.[1] Thus of the 26 respondents, 3 were male and 23 were female. Of the 13 teachers from the system who participated in the professional development workshop, 4 primary and 4 middle school teachers were local, i.e. from Mumbai, and participated further in the classroom collaboration and Topic Study Group phases. Of the 13 teachers who attended the 10 day long PD workshop, interviews of 11 teachers was taken. All of them had a bachelor's degree along with B.Ed. as professional qualification. The age range of these 23 teachers was from 28 to 50 years.

In Sub-study 2, the interactions of the group of 13 teachers who participated in the ten day long workshop has been analyzed during the different sessions of the workshop along with the task features and articulations of the teacher educator. There were 8 middle school teachers (5 female + 3 male) and 5 primary teachers (all female). Four of the middle school teachers lived out of Mumbai. All the teachers who participated had more than 15 years experience of teaching and were between the ages of 39 and 50 years.

In Sub-study 3, the case study of one primary teacher has been reported. Although observations were done for one middle school teacher, it has not been reported as classes were not transcribed. The primary teacher Nupur was selected for case study as she was a highly motivated teacher during the PD workshop and had expressed intent to adopt learner-centered practices in the classroom. Analysis of the teaching of fractions by her has been presented from observations done during the two years of the study. Nupur was a resident of Mumbai and taught in a Mumbai school. More details about her are available in Chapter 6.

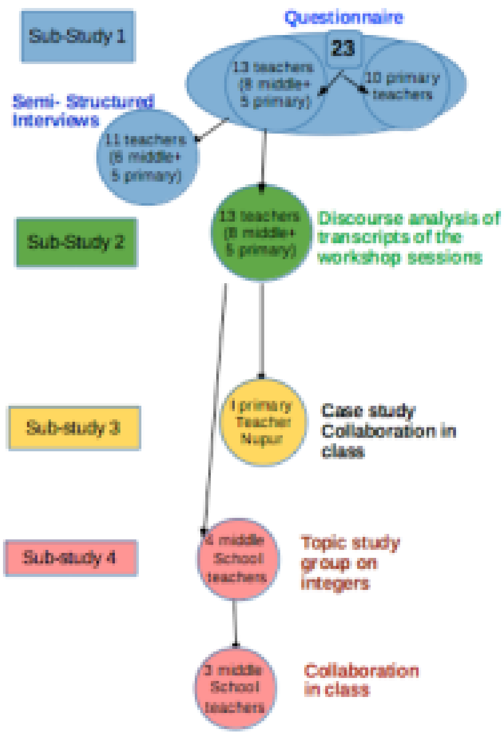
In Sub-study 4 in Chapter 7, analysis of interactions of 4 middle school teachers in the 6 one day topic study workshops on integers and of three middle school teachers (female: Swati, Anita and Rajni) teaching integers in their class is presented. All the 8 local teachers (4 primary and 4 middle school teachers) participated in the 6 one day topic study workshops in the second year (2010) after the first workshop. The primary teachers group was not able to implement their plans properly in classrooms due to constraints of the timetable and upcoming examinations.

Table 3.2: Background information about participant teachers in Sub-study 4

1 The number of male teachers at the primary level is much smaller than female teachers in this school system.

Teacher (Pseudonyms)	Age and Gender	Qualification	Teaching experience (Primary + Middle/ Secondary)	Average number of students in class from 2007-09 (self reported)
Swati	42, F	M.Sc, Maths, B.Ed	10+7	45
Anita	47, F	B.Sc, Maths, B.Ed	20+3	40
Rajni	53, F	M.Sc, Maths, B.Ed	0+23	45
Ajay	54, M	B.Sc, Maths, B.Ed	0+22	40

Figure 2: Flowchart of Participants in Sub-studies 1, 2, 3 and 4



3.5 Suitability of the Research Design and Methods

Kelly and Lesh (2000) have advocated for the diversity of research methods to be used in research to respond to the need of analyzing and describing teaching and learning. The methods used in the present study were chosen based on the research questions. The multiple methods used in the various sub-studies include questionnaire, interviews, discourse analysis, design experiment and case study. The design of the larger study is based on design experiment methodology, where the first phase of the study informed the second phase. The methods of data collection and analysis in the sub-studies have largely been drawn from the qualitative research method paradigm.

Qualitative research is a broad term for interpretive methods based on a phenomenological paradigm like interviews, participant observations, case studies (Best and Kahn, 2003). This study has used qualitative research methods with the exception of use of quantitative methods in the form of a belief survey in Sub-study 1. Miles and Huberman (1994) advocate the use of qualitative research methods to provide in-depth detail about an area or to gain a new perspective. Since the phenomena of change in teachers' beliefs, knowledge and practice and their relation to each other has not been extensively researched in the Indian context, qualitative research methods are appropriate to explore these phenomena.

The design of the workshops and its follow up are based broadly on the design experiment methodology where-in researchers design professional development based on their assumptions and stated hypothesis. Teachers' engagement within the workshop and in the classroom is analyzed. to inform the design of the subsequent professional development efforts. Cobb et al. (2003) argue that design experiments have "both a pragmatic bent – 'engineering' particular forms of learning – and a theoretical orientation- developing domain specific theories by systematically studying those forms of learning and the means of supporting them" (p. 9). Design experiments have been used for researching teacher education both to design interventions as well as to develop theories related to teacher education. Cobb et al. (2003) argues that the relation between theories and work in practical education contexts is important and worthy of study through design experiments as it allows one to trace the how learning occurs in different contexts and the role of the interventions and tools used in that context. The important steps in the design experiment are designing, putting into practice, observing and analyzing and then revising the design to be tested again.

Cobb et al. (2003) have identified the cross cutting common features of design experiment that vary from being classroom experiment with the teacher as collaborator or design of pre or in-service education. The first common feature is that design experiment is about developing theory about the process of learning and the means designed to support that learning, which in the case of this study is the 'learning of teachers'. The second common feature is the highly interventionist nature of methodology. The third feature is that all design experiments have prospective and reflective phases and thus one should implement the intervention based on a hypothesized learning process but also capitalize on contingencies as the design unfolds. The fourth common feature is the iterative design of making and testing conjectures which when refuted leads to design of new conjectures. However, in this study, particular forms of intervention only had a single iteration. The overall intervention involved successively trying different forms based on the feedback from the prior interventions. The fifth feature is the generation of "humble theories" which are domain specific and are specific to the activities designed. In this study, we (researcher and thesis supervisor) have collaboratively arrived at frameworks that are suitable for designing topic specific professional development interventions for the topic of integers, and to a limited extent, for the topic of fractions. A local theoretical framework for analyzing teachers' beliefs and practices has been proposed in the thesis that takes into account their interaction, as well as a framework for design and interaction aspects of a PD workshop.

In this study, the design experiment methodology is used in combination with qualitative inquiry. The workshops and the classroom are the sites of intervention for teachers' professional learning and the effort is to obtain both empirical account of interventions that supported teachers' learning along with theoretical understanding about what and how the aspects within the intervention influenced teacher learning. The design of the study consisted of two phases in which the results of the first phase informed the design in the second phase. The design of these phases were based on conjectures about what can inform teacher professional development while there was leeway for contingencies which arose in the field. The first phase was designed as a ten day long teacher professional development workshop along with the follow up based on the assumption that teachers will be able to use the examples and artifacts of teaching shared in the workshop to adopt student-centered practices in their teaching. The demands and expectations from the teachers to request researcher's participation in the classroom led to the adoption of the role as a participant in teaching. Identification of challenges (as a case study in Chapter 6) faced by the teacher Nupur (pseudonym) led to the design of topic-specific workshops in the second year which occurred in parallel with the teaching of the particular topic.

Cobb et al. (2003) suggested documenting the learning ecologies at multiple levels which includes the tasks, discourse used, norms established and tools and material provided (to name a few). The analysis leads to development of an interpretive framework to understand the complexity of the way design supports participants' learning. In this study, the goal is to develop an interpretive framework to understand how teachers' beliefs, knowledge and practice interacts and influences their participation and adoption of new ideas and practices in the classroom. As compared to naturalistic inquiry, the design experiment draws on previous empirical research to make design decisions. It is therefore important to distinguish between the elements of intervention and conditions that were present or assumed as the background conditions. Teachers' beliefs and preference for practice has been studied in Sub-study 1 and are assumed as background conditions on which the effect of the interventions has been studied.

The phenomena of teacher professional development and growth is a continuous process which can occur at various sites including the professional development site during the workshops, while teaching in the classroom, while talking to researcher or colleagues and even while having informal conversations with other teachers or teacher educators. The settings where teachers' interaction have been analyzed. can be termed as sites of professional development. The Sub-study 2 explores the site of workshop and the interactions between various elements designed in the workshops as well as the participants. The Sub-study 3 explores the classroom teaching and discussions with the researcher in the role of collaborator as the site of professional development. Sub-study 4 explores both the workshop as well as the classroom teaching as the sites for professional development and tries to explicitly draw connections between the two.

3.6 Data Collection and Data Analysis Procedures

Data collection across the two phases was both in the form of researchers' notes of participant observation and audio records which were later converted into transcripts, excerpts or notes. Additionally, in Sub-study 1, data was collected in the form of responses to questionnaires and interviews for assessing teachers' beliefs and preferred practices. Written consent (Appendix 8) was taken for observation and audio recording from all participants in the study. The rationale for selecting these data collection methods and theoretical considerations informing the data selection as well as analysis in the four sub-studies are described in the sub-sections below.

The data analysis procedures adopted in this study are largely based on those explicated by Miles and Huberman (1994). They suggest going through the data and performing data reduction, data displays and then drawing conclusions and verifying them through comparison. Data reduction involves working on field notes and transcripts and then identifying the underlying themes and patterns by writing memos about the notes. The themes are identified such that they reflect participants' perspectives, ideas and thoughts about the issue or phenomena under study. Themes are given codes and transcripts are read and re-read again to identify similar themes as well as the variations. Data display is also a form of data reduction in which the similar codes and variations are represented in the form of tables or concept maps and relations are identified between them to subsume a number of categories under a super category or form two different categories. These lead to identification of emergent codes and themes which can guide further data analysis. The final phase of data analysis involves framing conclusions which are arrived at through studying and working on the data displays to verify the themes in similar events and identifying their trustworthiness. While quantitative results are evaluated on the standards of validity and reliability, Lincoln and Guba (1985) identify the trustworthiness of the results as the criteria for evaluating qualitative data analysis. This includes several constructs like credibility, transferability, dependability and confirmability.

1. **Credibility**: Credibility is similar to the concept of internal validity for quantitative data analysis where the researcher has to critically analyze the conclusions to see if they make sense, describe the participants' perspectives adequately and represent the true picture of the phenomena (Miles and Huberman, 1994). In this study, credibility has been ensured through deep immersion of the researcher in the field as a collaborator with the participants, keeping thick records of interaction in the field and discussing the observations and conclusion with the supervisor and other research colleagues to exclude biases in the conclusions. The two year long association of the researcher with the teachers helped in establishing rapport with them so that they were able to frankly express their views and tensions felt by them during teaching. Use of both classroom observation as well as meetings with the teachers before or after the lesson ensured that the researcher was able to document teachers' beliefs and knowledge as expressed in these meetings and served as a method of triangulation. Member checks with the participants was not possible due to huge gap in the time of data collection and writing of the results. Also, the sensitive nature of claims about the beliefs and knowledge are difficult to ascertain through member checks.

2. **Transferability**: This construct is similar to the concept of external validity for quantitative analysis where a researcher seeks to determine if the results attained in one context are transferable to other contexts. By providing rich descriptions of the interactions,

participants' perspectives and details of the contexts and settings in which the teachers engaged in different professional development initiatives, readers are expected to make the decision about what conclusions and aspects are transferable to the contexts they chose to study or analyze.

3. **Dependability**: Similar to reliability, dependability is about whether the results of the study are consistent over time or across the researchers (Lincoln and Guba, 1985; Miles and Huberman, 1994). To facilitate other researchers to replicate the study, it is important to give the details of the processes and tools used in the study. This has been done by giving the appropriate description of the tasks and frameworks used as well as the processes used with the teachers during the workshops and classroom teaching along with the efforts done by teacher educators to engage the teachers.

4. **Confirmability**: In place of objectivity, a qualitative researcher aims for confirmability. The qualitative research paradigm acknowledges that it is difficult to ensure objectivity since tools like questionnaire and interviews are designed by humans and thus researcher bias is inevitable (Shenton, 2004). However, to achieve confirmability the researcher has the responsibility to demonstrate that the findings are not based on her own assumptions and predispositions but emerged from systematic analysis of the data. One of the ways to ensure this is by the researcher explicating her own assumptions, biases and beliefs that guided the decision making during the data collection and data analysis. The reflective commentary in the chapter along with presentation of evidences from data in form of excerpts in the chapter will hopefully ensure the confirmability of the findings of the thesis.

3.6.1 Sub-study 1: Beliefs and practices of teachers

Sub-study 1 involved research tools in the form of semi-structured interviews and questionnaires. Some of the interview questions and items in the questionnaire were adapted from previously published questionnaires from studies done elsewhere.

3.6.1.1 Tool construction: Questionnaire and interview

The purpose of the research tools, namely, interview and questionnaire in Sub-study 1 was to know the beliefs and preferred practices of teachers. The intention was to develop an understanding of the complex interaction between beliefs and knowledge held, practices preferred as well as cultural and political aspects influencing teachers' thinking and decisions. Since the research is exploratory in nature, the focus was not on the validation of the tools but on ascertaining which tools allow better glimpse of the complexity of the beliefs held by teachers, prevalent practices and constraints imposed on beliefs held as well as preferred practices. Since there was a paucity of studies related to assessing teachers' beliefs about mathematics, its teaching-learning and students in India, it was essential to explore how beliefs are expressed by teachers using exploratory tools as well as tools used in other studies done elsewhere in the world. Therefore, it was decided to use both questionnaire as well as interviews to assess teachers' beliefs and practices.

To assess teachers' beliefs about mathematics, teaching-learning process, students and self efficacy as well as to know the preferences for practices in teaching of mathematics, a framework was developed to identify themes that need to be focused in both interview questions as well as the questionnaire. The framework is given in Table 3.2 below.

Table 3.2: Framework for selecting and constructing questions for interview and questionnaire

1. Beliefs
   1.1. About mathematics:
      1.1.1. Fixed body of knowledge vs. discoverable
      1.1.2. Mainly isolated pieces of procedural knowledge which is handed down vs. Richly interconnected conceptual knowledge based on reasoning and justification
      1.1.3. Abstract, esoteric vs. connected with real life
   1.2. Teaching and learning of mathematics: Transmissionist view vs. Constructivist view.
2. Attitudes
   2.1. Attitudes to and expectations about students, sensitivity to students
   2.2. About oneself in regard to mathematics: Confidence, Anxiety, Liking for maths, Autonomy vs. Reliance on authority
3. Practices: It is theorized that practices essentially fall along one dimension represented by the transmissionist vs constructivist continuum. (Other equivalent pairs are teacher-centered vs. Student-centered.) Practices that can be considered near the transmission/ teacher-centered continuum are
      1. Showing and teaching procedures before solving
      2. Asking students to practice repeatedly

3. Avoiding errors 4. Focusing on standard algorithms only 5. Teaching shortcuts 6. Following the textbook closely 7. Focusing on speed of solving problems
Practices that can be considered near the Student-Centered continuum are: 1. Encouraging students to express their ideas, explanations and justifications 2. Use of multiple representations 3. Connecting school mathematics with students' daily lives 4. Encouraging alternative algorithms 5. Encouraging multiple languages 6. Using student errors to understand and respond to students' thinking 7. Encouraging reasoning and understanding why procedures work

The above framework was used for designing the questionnaire. However, the analysis of the questionnaire has been done by comparing the results from the questionnaire items with the categories that emerged from the analysis of the interview through open coding. This is described in more detail in a later sub-section on data analysis and also in Chapter 4.

3.6.1.2 Questionnaire

Data about teachers' beliefs and practices were collected through Likert type written questionnaires (having both positive and negative statements). The questionnaire was developed on the basis of the framework in Table 3.2. Items were adapted from the existing literature (Swan, 2006; Fenemma and Sherman, 1976) since they corresponded to various aspects of the framework. Many items in the first draft of the questionnaire were adapted from Swan (2006). These questions and items were discussed in the research group comprising supervisor, research scholar and a research assistant. The selected items were reviewed and adapted according to Indian context. Further additional questions were added based on the framework and prior experience in teacher education and Indian school system. This draft was shared with experts comprising 3 faculty members who had prior experience in teacher education and research tools construction. The modifications were made in both the tools based on experts' suggestions. The questionnaire had six parts focusing on

(1) Background personal data,

(2) Frequency of practices adopted,

(3) Teachers' beliefs about mathematics,

(4) Beliefs about its teaching and learning,

(5) Beliefs about self and

(6) Beliefs about students.

Pilot survey

The questionnaire tool was piloted, first with 20 members of the institute and then with 50 teachers belonging to the same nationwide government school system as the participants. Participants attempting the questionnaire were asked to indicate items which seemed unclear to them. Items were modified based on feedback received by participants. The objective of the pilot was to see how participants respond to questions and not to standardize the questionnaire since it probed teachers' beliefs in an area where little research has been reported in India. Therefore the purpose and use of these research tools was more for exploration of different ideas that get captured through these tools to help get a picture of the variability in teachers' views related to the themes selected.

Validity and reliability

Content validity of both questionnaire and interview was done by experts (researchers, teacher educators) and changes were incorporated as per suggestions. Data were coded and reverse scores were given to the negatively worded items. Reliability analysis was carried out using the R statistical package. Cronbach alpha values were obtained for each part of the questionnaire and were found to be in the range of 0.75-0.9, except for Part 3 (alpha = 0.53) dealing with teachers' ideas of mathematics. It was felt that the questionnaire items as perceived by the teachers did not strongly distinguish their ideas of mathematics from effective ways of teaching and learning mathematics (Part 4). Hence Parts 3 and 4 of the questionnaire were combined into a single scale, for which the Cronbach alpha was found to be 0.81, which was higher than the value for the parts taken separately. Efforts were made to describe the scales obtained and to extract sub-scales. In general, the sub-scales did not yield high alpha values, except for the gender preference sub-scale consisting of 4 items of Part 6 (cronbach alpha: 0.84).

Participants in the workshop including the 13 teachers from the government school system completed the questionnaire in about an hour. Another group of 10 teachers who participated in a one day workshop belonging to the same system also responded to the questionnaire and their responses were analyzed. along with the 13 teachers from the earlier workshop.

The final versions of the questionnaire are given in Appendix 1.

3.6.1.3 Interviews

The interview questions were prepared based on the themes of the framework for assessing beliefs of teachers in Table 3.3 (See Appendix 2). Additionally, a question (Q. 11) was incorporated through which one can get an idea about teachers' knowledge for teaching mathematics. Some of the questions in the interview were influenced by the interview used by Ball (1988). The draft of the interview schedule was prepared and given to 3 faculty members to obtain feedback and modifications were made based on it. The prompts prepared for the interview are given in Appendix 2.

Teachers were explained the purpose of interview so as to understand teachers' views about different practices prevalent in the education system for teaching mathematics and reforms in the education system. The researcher started by introducing herself and asking the teachers to talk about their personal bio and the reason they chose to become a teacher. Each interview lasted from about half hour to an hour depending on how much the participants shared. Teachers were probed to elaborate on the points and give examples from their classrooms if the responses were not clear. Since the setting of the interview was an institute, a professional development space away from school, it allowed for teachers to speak at length without facing the issues of taking classes in school or being overheard by a colleague or a superior.

The researcher's prior experience in another study of interviewing a teacher about the beliefs and interacting with her helped in conducting the interview with teachers. The general practice was to ask about preferred practices first and then to probe deeper to elicit teachers' beliefs related to that particular practice. Semistructured interviews were used in this study as this was the first interaction of the researcher with the participants and to allow exploration, one needed to leave scope for further probes when something interesting was shared by the participants. In trying to keep the interactions in the interview spontaneous, a particular question was asked at the point when the participant spontaneously mentioned something related to it.

In this study, the interview was chosen as a research instrument to assess beliefs as well as preferred practices as they provide access to the person's values, beliefs and knowledge (Tuckman, 1972). Further, interviews could also lead to identification of variables and relationships which the closed responses of a questionnaire may not elucidate. Thus, the interview data in this study serves to complement the questionnaire data providing alternative and fine grained data to contrast and compare with the questionnaire responses. The responses from both instruments have been used in the chapter to gain an understanding of teachers' views and beliefs. However, the assumption behind using interviews as a source of data is that the knowledge is not only explicated through interview but is also generated through conversations (Kvale, 1996). Thus in myview, the interview data was generated as a result of interaction between researcher and the participant. I agree with the Laing's (1967) position that rather than data being objective, data from interview is intersubjectively created as the participants' viewpoint interacts with the viewpoint of the researcher both during the interview through verbal and non verbal interactions as well as during the analysis. Therefore, the researcher tried her best to adopt both the teacher as well as the researcher's point of view during the collection and analysis of the data. The researcher also shared her own background of being a teacher to put teachers at ease and persuade them that the researcher understands the issues faced by the teachers. The interview data elicited from the interviewee is thus the result of the co-construction that occurs as result of interaction with the interviewer.

During the interview about beliefs and practice, respondents had to think and respond about issues that they may have not reflected on or thought about in their course of their teaching career. In trying to respond to the interview questions, respondents constructed their own narratives about their teaching and their teaching career. Their responses indicated not only their own choices and beliefs but also the factors that influenced their practice such as the role of experience, schooling, reform context and administrators. Since the participants had responded to the questionnaire before they were interviewed, the questionnaire items and teachers' responses to it did influence the way teachers responded to the interview questions. The questions also probed for meanings that participants hold for the key words like "explanation" and "reasoning" that were used in the questionnaire and thus provided important data to analyze findings from the questionnaire. However, due to the teachers' own personality characteristics and the interviewer's allegiance to the institute, it might have caused some teachers to not divulge their beliefs and practices honestly. The data has been analyzed. by contrasting the questionnaire and interview responses and considering the various issues of power associated with the position of being a teacher in a reform context. Thus inconsistencies between the data from the questionnaire and interviews have been treated as an indication of lack of strength in beliefs.

3.6.1.4 Analysis of interviews and questionnaire responses

Of the 13 teachers participating in the ten day long professional development workshop, 11 were interviewed, which included 6 middle school teachers (Teachers M1, M2, M3, M4, M5 and M6) and 5 primary school teachers (Teachers P1, P2, P3, P4 and P5). Semi structured interviews were done with participant teachers during the first ten days of the workshop in 2009. The interviews were fully transcribed. Themes were arrived at by first open coding the interview data to get emergent codes which were subsumed into themes. The transcripts were open coded (Miles and Huberman, 1994) for descriptive categories like "good student- accuracy", "good student- thinking", etc. This was followed by comparing transcripts of different teachers for the related descriptive categories like "mathematics- as calculation" and "mathematics- as reasoning". These related categories were then organized and subsumed into a theme. The questionnaire items were categorized into the themes that emerged from the interviews and teachers' response to the questionnaire item was compared with that during interviews. Triangulation was thus done considering questionnaire responses and interview data and teachers' response to the relevant questionnaire items related to the themes were considered. Considering teachers' interview excerpts related to a descriptive category revealed the variations among the views held by the teachers as well as the strength of a view among the group of teachers. The analysis led to the emergence of the categories of core and peripheral practices and beliefs to explain the view held strongly or weakly among the teachers across the interview and questionnaire. These categories and the important themes that emerged have been described in detail in Chapter 4.

3.6.2 Sub-study 2: Design and enactment of PD workshop

In the first year, a ten day professional development workshop during the summer vacation was held for 13 teachers belonging to a nationwide government system of schools. The goals of the workshop were strengthening teachers' knowledge relevant to teaching, providing opportunities to articulate and reflect on beliefs and developing a sense of community among teachers, teacher educators and researchers participating in the study. The workshop tasks included observing and reflecting on non-traditional teaching, learning through solving and analyzing problems, anticipating and reflecting on student responses, discussing math education research literature, analyzing textbooks, and articulating beliefs about teaching, students and mathematics.

Teachers were engaged in a collective experience of reflecting on their beliefs and practices that are common place in teaching of mathematics using the artifacts derived from practice. The details of the principles of workshop design as well as details of the interaction in the sessions are provided in Chapter 5.

3.6.2.1 Data collection and analysis

Cobb et al. (2003) discuss design experiments for the purpose of "engineering particular forms of learning and systematically studying these different forms of learning within the context by the means of supporting them" (p.9). The design of the workshop was undertaken with the purpose of making teachers aware about their own beliefs, making their knowledge of mathematics teaching explicit through use of situated tasks and make them aware of alternative practices that can be used for teaching mathematics. The plan of sessions and tasks thus embedded the principle of situatedness, challenge and developing the sense of community. Data from the workshop was collected in the form of video recordings of the sessions which were later transcribed. Additional data in the form of teachers' work and responses to tasks were also collected. Transcripts and descriptions of the sessions in the workshop were prepared from the video records of the sessions. The coding process was adapted from Miles and Huberman (1994) as well as Corbin and Strauss (2008) to develop emergent codes from the data. Corbin and Strauss have discussed how analysis of transcription involves interpretation by the researcher and thus the researchers are "the translators of other persons' words and actions" (p.49) who reinterpret and revise their interpretations in the process of analysis. This process was facilitated through brainstorming and the discussion of the insights from the data along with evidences from transcription in a group comprising the researcher, supervisor and another researcher colleague. The discussions helped in arriving at the principles which seemed to be reflected across sessions and in the actions of the teacher educator.

The coding was done broadly in three categories: the design features, the facilitation features and teachers' explorations and reflections. The descriptive codes identified for the design features included coding for the task as asking teachers to articulate student thinking, analyze student error, predict student responses, propose teaching approach, identify conceptual gap, explanation of the procedure, identify key concepts, etc. The descriptive codes for facilitation features included soliciting teachers' responses, revoicing, comparing responses, proposing teaching approach, making counter-argument, probing meaning, reviving discussion, making inferences, making connections, sharing protocol, etc. The descriptive codes for teachers' articulations included making conjectures, identifying student errors, inferences, assertions, challenging assertion, resisting, initiation, supporting, giving example and giving explanation. Thus, initially the coding was at the descriptive level. However, through analysis of description of different sessions, researchers were able to identify the conceptual themes like the role of tasks situated in practice, teachers agency in engaging with the tasks and interactions in the sessions as well as the belief goals of the teacher educator in the sessions. The consensus about the coding was established by discussion among the coders. After the initial coding for two different sessions, the rest of the coding for other sessions was done by the researcher alone.

3.6.2.2 Analysis of questionnaire responses

The 13 teachers who had participated in the professional development workshop, responded to the questionnaire again on the last day, keeping in mind their views and preferences for their practice in future. Paired t-test was used to find if the difference between the means were significant for the responses before the workshop and after the workshop. The online tool to calculate t-test value and significance was used (https://www.graphpad.com/quick). The items for which the difference between the means was significant have been presented in Chapter 5.

3.6.3 Sub-study 3: Case study of Nupur

The workshop was followed by visits by the researcher to the classrooms of two teachers – one primary and one middle school teacher for about one month each. During these visits, the researcher frequently reflected on the lessons together with the teacher and discussed plans for future lessons. Chapter 6 in this thesis reports the Sub-study 3 in form of a case study of the primary teacher Nupur (pseudonym) who held positive beliefs for student-centered teaching and intended to change her practices towards the same but faced challenges in adopting these practices. The beliefs held by the teacher, the tensions experienced among beliefs while exploring new practices in the classroom have been discussed in the chapter. The lessons observed in the first year were on the topic of fractions, and in the second year on the topic of Area and fractions. Since the preliminary analysis showed the challenges as being content specific, the lessons on fractions in both years have been chosen for analysis. The observation of the classrooms have been analyzed. for the tasks that were designed or used from the textbook as well as the way the teacher managed the classroom interaction after posing the task. The aspects coded for the task framing and task enactment have been described in Chapter 6.

3.6.3.1 Suitability of the Case study method

Yin (2009) acknowledged that the case study method is determined by the nature of the research questions where the researcher attempts to gain understanding of "how" and "why" questions. In Sub-study 3, the attempt is to understand how the beliefs and knowledge held by the teacher influenced the ways she selected and implemented the tasks in the classroom and how her participation in the study influenced her practices, beliefs and knowledge. Since she had expressed positive beliefs towards adopting student centered practice, she was a suitable case to study the kind of challenges that arise when the beliefs are conducive for adopting a practice. The case study delves both on her interactions in the classrooms with the students as well as on her discussions with the researcher outside the classroom. Since the relationship between the researcher and the teacher is that of collaboration and the researcher herself was interacting with students in the classroom, the data related to researchers' interactions with the students and the teacher also become the part of the case study. Thus the case of Nupur is of a teacher located in the physical space of the classroom and school and in a social relationship with the students and the researcher.

The context of the case study, the description of the school, Nupur's characteristics and the socioeconomic status of the students in the Nupur's classroom have been described in detail in Chapter 6. The chapter also includes a summary of the lessons taught by Nupur which have been analyzed. in detail in terms of task framing and task implementation.

3.6.3.2 Data collection

For the case study, data was collected from different sources which can be categorized into background data for the study, data of teaching from the classroom and data of teacher and researchers' meetings.

Background data was collected by ethnographic notes made by researcher during visits to the school where Nupur worked for around 2 months each in two consecutive years. The researcher visited the school almost daily and spent time in the staffroom and interacted with other teachers along with Nupur. She also interacted with the headmistress and the principal and attended a few official meetings taken by them with the teacher to get a wholistic sense of the working ambience in the school. Permission was taken from both the principal and the headmistress for classroom observation along with consent from the teacher. A note was sent home with the students of the class explaining the purpose of the study and that the students' real names will not be used in the study to ensure that the parents did not have any objections to the study. The circulars given to the teachers to direct their teaching were also studied, although not reported in the thesis. Interviews were taken with students of Nupur's class to get an idea about their socioeconomic status along with the interview of the other two sections belong to the same grade. This data is reported in Chapter 6.

Data about teaching was obtained through audio recording through an audio recorder on teachers' table kept in the front of the class. This made it difficult to clearly audio record students' responses in the classroom. The researcher took notes in the classroom to record teachers' questions and students' responses. Although the names of the students were not noted in the records, the gender of the students was recorded using the symbol "Sg" for girl and "Sb" for boy along with their responses. The researcher also completed the notes after the class based on her memory to make the notes comprehensive along with a brief reflection of her own about what aspects of teaching were salient to her. Tests were also given by the teacher to students and some tests were made by researcher and teacher together. This data has not been included as due permission was not taken from the teacher to share this. The teacher and researcher jointly took a one day workshop with the other primary teachers of the school at the end of the first year observation, which was audio recorded and has been used in the chapter to substantiate the arguments about what Nupur learnt from the collaboration and what challenges she had faced. The analytical framework used to analyze the data from the classroom has been discussed in detail in Chapter 6.

Data about teacher and researcher's meeting was obtained through a voice recorder during the meetings before and after the lesson to discuss students' responses and planning the tasks in the subsequent lessons. The researcher also made notes of the remarks made by the teacher during the conversation in the staffroom which were relevant to what was being taught in the classroom.

3.6.4 Sub-study 4: Topic focused professional development

In the second year, six one-day topic study workshops on integers were held for the four middle school teachers spread over a period of five months while the teachers were teaching in their schools. All the teachers had attended the professional development workshop in the first year of the study. Out of the four teachers, the researcher visited the classrooms of the three teachers (Rajni, Swati and Anita) while they were teaching integers. The fourth teacher did not teach integers in the sixth grade and thus classroom observation was not possible. At the end of the second year, the teachers' group conducted an extended workshop session of three hours on teaching integers for peer teachers from same school system.

The aim of the topic study workshops was to focus on a topic that was challenging to teach, to develop a deeper understanding of the topic, and to plan for teaching. The teacher educators, who were also researchers, refrained from conveying to the teachers that they needed to make specific shifts in their teaching practice, for e.g., from teaching rules to teaching for understanding and reasoning. The focus was more on collaboratively developing mathematical knowledge for teaching and resources, which was expected to facilitate teacher learning and influence teachers' decisions in selecting and designing tasks. The middle school teachers' group chose the topic of integers to be taught in Grade 6 as the topic for the collaborative workshop. Meetings were held over six days spread over a period of 18 weeks. The discussion in the workshop can be broadly divided into four stages: (i) initial discussion of issues related to the teaching of integers (Day 1 and Day 2) (ii) engagement with contexts in which integers can be applied meaningfully (Day 2 and Day 3) (iii) planning for teaching (Day 4) and (iv) reflection on teaching and preparing a workshop session for other teachers (Day 5 and Day 6). The stages are convenient divisions with overlaps and elements of each phase present in the other phases.

The teacher educators' role was to initially elicit from the teachers the approaches that they used in the classroom and the challenges that they faced. On the second day, a worksheet (see Appendix 5) on integer meanings designed by the teacher educators based on research on the teaching and learning of integers was the focus of an extended discussion on contexts where integers were used and the associated integer meanings. On subsequent days, the teacher educators supported teachers in examining the learning outcomes addressed by the textbook chapter, in designing instruction and in preparing for a workshop session for peer teachers. Such support consisted in helping individual teachers in identifying and preparing learning resources that they wished to use in their classrooms, and sometimes in designing student tasks around a context chosen by the teachers.

The data that is analyzed in Chapter 7 consists mainly of transcripts of audio recordings of the workshops. The audio-recording included interactions between teachers and teacher educators in the workshop as well as teachers' reports and reflections on using resources developed in the workshop for teaching. Additional data was used from teachers' individual lesson plans and presentations made by teachers to their peers in the last meeting of the workshop. The discussion in all the four stages of the workshop was fully transcribed. The codes used with the transcripts are described in detail in Chapter 7 along with the framework of integer meanings used for the analysis.

The researcher and her supervisor independently coded the transcripts. After initial coding, codes were merged and simplified to remove ambiguities. Differences between the two coders in coding were resolved through discussion; when they could not be resolved, both codes were marked together for the particular turn. The codes were used to collect together utterances that were related to a common theme, that indicated the broad features of the discussion. The codes were also used as filters to focus on specific aspects of interest, and to validate the claims made. The discussion in Chapter 7 uses extracts from the transcript guided by the coding scheme to support the arguments made.

3.6.5 Role of the researcher

According to Burton (2002) it is important to make clear the grounds on which the decisions were made to regarding data collection and analysis. Since this study adopted qualitative methods for informing data collections and analysis, the role of the researcher becomes important as the researcher is both an instrument for data collection as well as the instrument for data analysis. Thus writing is informed by the researcher's past experiences and views which need to be made explicit to the audience. The role of the researcher in the form of a collaborator and a participant during the classroom observation rather than as an outsider has thus been described. The choices made during data collection and data analysis are influenced by both the ideas and predispositions with which the researcher enters the field as well as the increasingly nuanced understanding of the situation that the researcher develops through the interactions in the field. To ensure that authentic data is collected a researcher needs to constantly reflect and be aware of his/her own assumptions and constantly be critical of them in the light of evidences gained in the field. Thus, a researcher has to keep his/her mind open to unanticipated leads and relationships which are discovered in the field and has to continuously adapt the design towards finding the answers to the questions he/she was seeking. Thus researcher plays an important role in eliciting, noticing and interpreting the behaviors/articulations occurring in the settings. The 'thick' description of the setting having details and specific helps the researcher to unearth important themes, categories and inter-relationships providing deeper insights into the situation. The assumption behind these methods is that complete objectivity is impossible and thus it is important for the researcher to make his/her own assumptions and theoretical understanding explicit to allow reader to make sense of the way data is presented and interpreted thereby establishing credibility of the findings. In other words, the researcher is the instrument for data collection and analysis and she needs to be aware of the role played by her in this process.

As a researcher who utilized qualitative research methodology in the design experiment process for teacher professional development, my role was very complex. I played the role of participant observer in all the settings while the role of teacher educators was played by faculty members and other research team members of HBCSE. However, in the classroom I had to take a more active role due to the teachers' demand that I engage in classroom interactions. My role as a participant observer enabled me to notice and analyze the interactions that contributed to teachers' understanding and understand the process of professional development as a continuous process.

My previous experiences in my life contributed to the way the research study was designed and data was collected and analyzed. My own experience of being a government school teacher and teaching maths at primary level contributed to my understanding of what students are learning in the classroom as well as the challenges and feeling experienced by the participating teachers in the study. My experience as a government school teacher had made me aware of the isolation faced by the teacher in not having academic support to manage the challenges of student learning. However, my own efforts in the school made me believe that it is possible for a teacher to work in this setting and use resources to ensure conceptual understanding of mathematics amongst the students. My efforts to introduce student-centered pedagogy in my lessons while doing the study for fulfillment of M.Ed. part-time degree also contributed to the understanding about how students react to the change in pedagogy and ways to engage them as well as the understanding that these changes take time to be adopted in the classroom as a norm. Thus, it helped me in continuing to be patient and observant of the learning exhibited by the teachers as well as students.

My participation in teaching in the summer camps organized at HBCSE using the ideas from research for teaching fractions had made me aware how difficult it is to make a switch from relying on the textbook to making decisions about the tasks and to plan teaching based on students' responses and their evolving understanding. These summer camps also taught me classroom strategies to engage students in explaining their methods and reasoning after working on problems for which they were not taught or told the procedure to solve. In another pilot study before this research study, a collaborative research design was adopted instead of an observation based study with a practicing teacher to study the learning of fractions. She had discussed how she felt uncomfortable with me observing her lessons as she felt that her teaching might be evaluated. She felt that my position as the researcher has the power as I was not obliged to take part in the lessons even when I know of strategies that can help students learn fractions. Following this conversation, a collaborative research design was adopted in the study to study students' responses and their learning which she used to provide feedback to the students. In this pilot study, I had participated as a participant observer and discussed with the teacher what students are learning about fractions based on their responses and solutions in the classroom and had planned the future lessons together. Similar expectations were experienced by the researcher from the participating teachers in the reported research study. Thus the researcher adopted the role of the participant observer to work with the teachers in the various sub-studies reported in this thesis. As in the pilot study, I decided to collaborate with the teachers and intervene in the classroom on the invitation of the teachers.

4
Teachers' Beliefs and Practices in the Context of Curriculum Reform

4.1 Introduction

The position paper on the teaching of mathematics, an important paper for the National Curriculum Framework 2005 recognizes that teaching of mathematics has been textbook centered with the focus on learning mechanical procedures rather than developing students' power of mathematization and reasoning (NCERT, 2006c). The document recognizes inadequate teacher preparation as one of the reasons for the prevalent approach to mathematics teaching. As described in Chapter 2, studies elsewhere in the world have indicated that focus on change in teaching strategies without taking *teacher thinking* into consideration results in teachers making superficial changes which do not lead to any significant change in student learning opportunities (see Chapter 2). It is, therefore, important to first understand the beliefs and practices that are prevalent among teachers in order to support reform in teaching that is not superficial.

As described in the previous chapter, this study is situated in the context of a project aimed at collaborating with teachers to develop teaching practices that support the development of student understanding. In order to support teachers in initiating change in classroom practice, it was important to let teachers articulate and become aware of the beliefs that they held and the practices that they preferred. It is also important to understand the beliefs held and practices preferred by teachers, as these throw critical light on the teachers' reflections and actions in subsequent phases of the study. Hence the relevance of the Sub-study being reported in this chapter.

In this chapter, an analysis of the questionnaire and interview responses of teachers who participated in the professional development workshop in Year 1 of the study is presented. This was the first interaction that the researcher and the workshop design group had with teachers and the analysis of questionnaire and interview illustrated the beliefs and practices considered important to teachers at that point of time. The responses to the questionnaire are from 26 teachers belonging to the same school system from two different events. The first was a workshop conducted in Year 1 for a period of ten days in which 13 teachers from a nationwide government school system participated. The second event was a short, half a day workshop also held in Year 1 in a school belonging to the same school system, in which another set of 13 teachers participated. Interview data of 11 teachers who participated in the first ten day workshop has been analyzed for this chapter. Out of these 11 teachers, 5 are primary school teachers, while the other 6 are middle school teachers. The primary teachers are designated as P1 to P5 and the middle school teachers as M1 to M6. Chapter 6 presents a case study of Nupur, who is **P1**. Chapter 7 discusses the involvement of Swati (**M1**), Anita (**M2**), Rajni (**M3**) and Ajay (**M4**) in topic focused workshops. Since these teachers will be discussed in subsequent chapters under different names (all pseudonyms), reference to them in this chapter is made in bold font. Since the first point of contact for teachers was during the first workshop held in summer vacation of Year 1, it was not possible to obtain data about teachers' practices from classroom observations to corroborate the teachers' responses in questionnaire and interviews. In later chapters, it will be discussed how the beliefs and practices reported here connect to discussions in teacher development workshops and to actual classroom practices observed.

4.2 Beliefs and practices of teachers: The underlying interconnected themes

The most common practice associated with teaching mathematics in Indian classrooms is probably showing procedures and examples to solve a problem, followed by students solving similar problems from the textbook. Through repeated practice students are expected to identify the type of problem and use a suitable memorized procedure for solving. From experiences shared during discussions with teachers and teacher educators, it appears that several generations have gone through similar teaching, indicating that it is a common cultural norm for teaching mathematics. Thus the teaching of mathematics is mechanical and is essentially about teaching and memorizing procedures. In the mathematics education research literature, this type of teaching has been recognized as a "transmissionist" view of teaching, which has been contrasted with a "student-centered" view of teaching. The student-centered view describes teaching that involves giving opportunity to students to share their ideas and teaching that is responsive to student understanding. The use of the word "constructivist" is avoided here because in the Indian context (and in the context of some other countries), the word constructivist has acquired the connotation of a limited role for the teacher. The teacher is supposed to function merely as a "facilitator" who supports students in their active construction of knowledge. In contrast, I believe that the teacher has an active and important role in developing students' understanding and hence I prefer to use the phrase "student-centered teaching" or "student responsive teaching". However, "transmission" oriented teaching or "transmissionist teaching" captures the practice prevalent in many Indian school settings well, where the teacher attempts to transmit a "packaged" knowledge of mathematics to students.

Another distinction that is prevalent in the literature is based on the type of mathematical knowledge that is focused in teaching which can be purely procedural or allow for deeper connections between procedural and conceptual knowledge to be explored. The first distinction between transmissionist and student-centered teaching points to the beliefs that teachers may hold about teaching, whereas the second distinction between procedural and connected nature of mathematical knowledge indicates the beliefs teachers may hold about the nature of mathematics. These distinctions are indicated by the fact that some researchers Some have described these latter distinction as the contrast between "transmissionist" and "connectionist" teaching (Swan, 2006). Other beliefs like the capability of doing mathematics being innate or acquired might also influence the choice of practice as well as the self efficacy beliefs of the teacher.

These distinctions present in the research literature frame the discussion in this chapter on preferred practices and beliefs held by the teachers in the study and have helped in arriving at the framework discussed in the next section on core and peripheral practices and beliefs.

It is acknowledged that beliefs and practices reported during the interview are "articulated beliefs" and "preferred practices". Teachers' articulations in the interview about what they prefer and think as important while teaching has been considered as "beliefs". These articulated beliefs may be different from actual practice as discussed in the Chapter 2.

In this chapter, the discussion focuses on how teaching practices described and preferred by teachers align largely with the "transmission" view of teaching and procedural view of mathematics although they also appear to be related to "student-centered" teaching and a focus on reasoning. The transmission view of teaching, leads to a preference for practices like showing procedures or solutions on the black-board to students before asking them to copy the solutions in their notebook and then perhaps asking them to solve a similar problem for practice and memorizing procedures. Teachers aligning closely to a transmission view, generally try to avoid students making errors by helping them identify the operation/ formula to be used and in the case of error repeat the procedure again for the students.

Practices aligning with the student-centered view may involve providing opportunities for students to exercise their agency while engaging them in tasks and activities. The teachers may encourage students' solutions and ideas for solving a problem, asking students to give reasons and explanations of their answers and make and explore conjectures. Such an approach to teaching may involve connecting maths to daily life and contexts and activities where students make sense of their experiences while making connections with formal mathematics. Even in student-centered teaching, the teachers' role is important in promoting learning by making choices and selections of the contexts and activities; building on students' explanations to identify and make connections with key mathematical ideas and helping them in adopting more efficient strategies for solutions.

The work of Ball, Hill and Bass (2005) and Ma (1999) indicates how knowledge of procedures and concepts are intricately connected and thus one needs to focus on the connections between procedures and concepts. For e.g., knowledge of standard procedures of say subtraction, division, etc., involves understanding of certain key concepts like place value. However, when teachers' give simplified or mathematically inadequate explanations of procedures which do not take into consideration the key ideas, teaching can be called as essentially procedure focused. Procedure focused teaching also focuses on only the standard algorithm or solution while discussing a problem, emphasizes the steps of the procedures for memorization, and considers speed and correctness of a calculation as an indicator of learning. When teachers give conceptually based explanation of procedures, they use contexts, representations or ideas to develop the meaning of mathematical concepts, establish connections between procedures and concepts, and use their understanding of how and why procedures work to identify and discuss a mathematical idea in a students' solution. Procedural and conceptually focused teaching can also be considered as two ends of a continuum.

The questions addressed in this chapter are:

1. What are the core and peripheral practices of the teachers in the sample with respect to the teaching of mathematics?
2. What is the relation between beliefs expressed and the practices preferred by the teachers?
3. What beliefs are core or peripheral as indicated by the teachers' articulation and the practices preferred by the teachers?
4. What is indicated about teachers' knowledge from their explanations? What is the relation between preferred practices and the knowledge held by teachers?

In the following sections, I discuss findings from the interview (Appendix 2) and questionnaire about practices and beliefs (Appendix 1). In section 4.2, I discuss the core and peripheral practices which teachers reported in the interviews and the questionnaire. In section 4.3, 4.4, 4.5 and 4.6 I discuss beliefs held by teachers about the teaching and learning of mathematics, the nature of mathematics, beliefs about students and beliefs about self as a maths teacher respectively. In section 4.8, I discuss the role of knowledge of concepts and procedures in influencing the choice of practice through analyzing teachers' mathematical explanations. The last section discusses the framework of how core and peripheral practices are related to the core and peripheral beliefs as well as knowledge.

4.3 Core and peripheral practices for the teaching of mathematics

The research literature on beliefs indicates that not all practices are used consistently by teachers; teachers consider some practices as more important than others and hence adopt them more frequently. In the context of educational reform, teachers have been found to change their practice only superficially or just include new practices in their repertoire, without actually revising their teaching towards fulfilling the reform goals which are generally directed towards changing a traditional way of teaching (Cohen & Ball, 1990). Research has also indicated that it is teachers' beliefs and values that guide the choice of practice and decision making in the classroom and thus it is important to study beliefs in relation to the practices used in the classroom. Green (1971) identified that beliefs can have varying significance and thus can be classified as core and peripheral beliefs, explaining teachers' choice of practice. This framework has been extended to categorize the practices as core and peripheral practices which are guided by core and peripheral beliefs respectively. It is proposed that the relationship between practices and beliefs is a complex one, influenced by other factors including the teachers' knowledge and sociocultural factors like systemic expectations in times of educational reform.

"Core" practices are those that teachers rely on and adopt frequently. Thus they are practices to which teachers ascribe importance and usefulness, of which many instances of classroom use are cited, and where teachers possess the knowledge to support the practice. These core practices are supported by component practices that help to maintain their use and frequency and are consistent with them. The core practices form a coherent cluster and one expects consistency in the teachers' articulation of them across tools like the questionnaire and interview. Teachers display and hold knowledge needed to use these core practices. For e.g., teachers will have knowledge of steps of the procedure to support the practice of showing procedures but not necessarily of how these procedures work. The core practices are guided by the core beliefs held by the teachers which are either explicitly articulated in response to the questionnaire or interview or can be inferred from core practices and its component practices. These core beliefs indicate a definite view about the nature of mathematics and mathematical learning which in turn explains the use of core and component practices. Core beliefs are also indicated by the consistency of teachers' response to probes about practices as well as beliefs related to teaching and learning, and mathematics.

In this thesis, some practices have been termed as "peripheral" when they are given less priority by teachers during interviews in terms of their importance and usefulness for learning mathematics. Teachers might be using these practices infrequently in the classroom, without clarity about their use and purpose, and might lack knowledge to support using the practice efficiently. A practice may also be considered as peripheral when there is inconsistency between beliefs expressed and reported practice or conflicts and tensions expressed by teachers in implementing a practice. A peripheral practice may be an imposed practice through the education system, which teachers use to fulfill the demands of the system, but do not foresee its usefulness in the teaching-learning process. Peripheral practice may be supported by a peripheral belief which does not directly align with the other core beliefs. On the other hand, a peripheral practice may be supported by core belief if it is consistently expressed in both questionnaire and interviews, is coherent with other core beliefs and teachers discuss the factors which constrain the use of the practice.

The core and peripheral practices have been analyzed for the group of 11 participating teachers, who responded both to the questionnaire and the interview, as well as who took part in the 10-day professional development workshop (see Chapter 5). The data from the questionnaire and the interviews of the teachers participating in the workshop taken together indicate that teachers' views were more aligned to a transmission view of teaching and procedural focus in mathematics. While the questionnaire responses by themselves indicated a more positive alignment towards student-centered teaching and focus on reasoning, teachers' description and examples during the interview focus on showing procedures, repeated practice and getting correct answers and indicated a lack of knowledge of connections between concepts and procedures. Analysis of the interview further indicated that showing procedures, repeated practice of problems and focus on speed and correct answers while avoiding errors are the core practices for teaching maths. Use of activities, connection with daily life, engaging in reasoning, giving an opportunity to students to voice their thinking remain peripheral practices, which teachers use for the purpose of making mathematics interesting and not necessarily to develop an understanding of mathematics.

In the following sections, theevidence from questionnaire and interview is described which illustrate the features of the core and peripheral practices as well as beliefs.

4.3.1 Core practices

There were four core practices that were identified by the analysis of teachers' interview and the questionnaire. They are:

1. Teaching by showing procedures or solved examples
2. Giving students repeated practice of solving problems
3. Focus on speedy solutions
4. Following the textbook

These four practices were supported by several component practices and together these core practices and component practices form a coherent view of teaching mathematics by transmitting knowledge to students. The teacher works as the mediator between the textbook and the students and helps students in remembering what is there in the textbook and solves all the problems of the type given in the textbook. The function of the teacher is to make this package of knowledge in the textbook easier to learn or more interesting.

4.3.1.1 Teaching by showing procedures or solved examples

Teachers' responses to the questionnaire and interview indicated that teachers focus on telling the steps of the procedure without giving an opportunity to students to solve or figure out a problem on their own. Analysis of interview and questionnaire data of teachers revealed that the views expressed by teachers indicating a procedural focus in the interviews are consistent with those expressed in the questionnaire about the frequency of practices used. This indicates that showing procedure may be one of the core practices. Further analysis of interview indicated that some component practices supported this core practice, which are listed below

1. Telling steps of the procedure, considered as an 'explanation', which students are expected to record correctly in their notebooks.
2. Avoiding students' mistakes by giving/ engaging the class in a careful explanation of steps of the procedure.

Showing the procedure as explanation

Teachers' responses to questionnaire items given in Table 4.1 below indicate that almost all teachers (96%) considered "explanation" as telling the steps of the procedure to students which they engaged in most of the time or almost always (2.23). 50% of the teachers gave this explanation in the beginning of the lesson, most of the time (2.1).

Table 4.1: Questionnaire responses for practices related to showing procedures (All numbers shown are percentages)

No.	Item	% Almost Never	%Some of the time	%Half of the time	%Most of the time	%Almost Always
2.1	In the beginning of the class, I show students how to solve a particular problem and then give similar problems to practice from the textbook.	15	19	15	27	23
2.23	To explain the mathematical concept, I show the steps of the procedure to solve the problem.	0	4	0	38	58

Description of a 'typical lesson' in the interview (Appendix 2, Q. 8) indicated that showing procedures is the main part of the lesson. Teachers showed the general procedure or showed how to solve an example problem. Sometimes teachers solicited answers from students by posing a question.

Excerpt 4.1

> My typical mathematics period involves giving explanation by examples, doing activities from the textbook and then solving questions by students. (P2, personal interview, May 25, 2009)
>
> First I explain one or two times then answer comes out orally. If not I solve the question.... I explain on the blackboard and then give some connected problem. Sometimes I solve on the blackboard and then give them. (**M3**, personal interview, May 28, 2009)
>
> First I give a chance to students to think and if they don't get it, then I give the answer. (**M2**, personal interview, May 27, 2009)

Responding to the question to explain what teachers' meant by explanation, (Appendix 2, Q12) teachers considered showing the procedure as equivalent to giving an explanation.

Excerpt 4.2

> Explanation is the explanation of steps of procedure. (**P1**, personal interview, May 25, 2009)
>
> After giving the explanation for 2-3 examples, [I tell them] page number [on which] activity is there, then you can solve the questions. (**M4**, personal interview, May 28, 2009)

Although no direct question was asked about educational reform, teachers talked about the changes they had to make while using new textbooks and implementing reform directives. Some teachers acknowledged that as a result of educational reform, expectations are that teaching should become more interactive. The teachers have responded to this expectation by asking questions to check students' understanding of previously learnt procedures and asking students to solve the problems on the blackboard with the help of the teacher. So, though student participation had increased, the focus was still on memorizing the steps of the procedure.

Excerpt 4.3

> Previously, it was mostly that you solve the problem and give them the method.... Now children have to come out in the introduction part and children have to be involved in the blackboard work.... Firstly, I check if they remember what was taught in previous period.... While introduction, you start with what students know. (**M1**, personal interview, May 27, 2009)

Thus, showing procedures and even asking questions to students resulted in constraining students' agency, since it did not allow an opportunity to students to think of their own ideas for solving a problem. Student agency was further constrained by teachers expecting students to copy the solutions, which were solved on the blackboard, correctly in their notebooks. The teachers justified this practice while describing their typical lessons, as a way of students having proper records in their notebooks, which the students can refer to later when they are going through notebooks. From a teacher's (P3) response it seems that notebook record is also taken as a record of what was done in the class and thus teachers insisted on students copying correctly to avoid students' mistakes. However, P5 insisted that she does not like it when students copy without thinking.

Excerpt 4.4

> Whatever they write in their notebook should be as per the teacher's instruction otherwise teacher gets blamed for teaching – doing wrong on the blackboard. (P3, personal interview, May 26, 2009)
>
> [I tell students] those who have done wrong copy – rub and do it again....[the mistake] It is corrected on board. Then I tell them to copy it correctly. If you make a mistake... if you make a mistake the hand by which you write will get beaten. (P4, personal interview, May 26, 2009)
>
> [response that teacher does not like students to copy without understanding]... children just copy it… I don't like that when they are not thinking. (P5, personal interview, May 30, 2009)

This indicated that teachers considered these solved examples as templates which students should follow step by step to write a solution to a similar problem. The practice of copying solutions from the blackboard was found to be quite common among primary teachers and 60% of the teachers responded to the questionnaire (2.12) that they engage in it most of the time or almost always.

Avoiding student mistakes by sharing procedures

Teachers' responses to the questionnaire reflected a concern with channelling students' thinking in ways that would prevent them from making errors. 53% of the teachers avoided mistakes by carefully explaining before asking students to solve themselves while 96% would, most of the time, tell the steps again if students make an error (Table 4.2, 2.17, 2.2). Although 56% teachers also agreed to "allowing mistakes and discussing them" most of the time, the discussion involves focusing on and repeating the steps as indicated in the interviews (2.29). Teachers' responses to in the questionnaire show that most teachers most of the time ask students to compare their methods (2.18), while sometimes they may tell the correct answer. The response to 2.18 may mean that teachers ask students to compare the steps in the procedures to identify where the mistake occurred as few teachers believed that students can solve a problem on their own without being shown the procedure.

Table 4.2: Practices related to student agency and avoiding mistakes

No.	Item	% Almost never	%Some of the time	%Half of the time	%Most of the time	% Almost Always
2.2	If any student does not understand what was taught, I explain to the student once or twice by repeating the steps in detail slowly.	0	0	4	38	58

2.17	I avoid students making mistakes by explaining things carefully first.	26	11	7	30	23
2.18	When students get different answers then I ask them to compare their methods and decide which one is correct.	8	15	0	42	35
2.25	When students get different answers then I tell them at once which one is correct.	15	35	4	19	27
2.29	I allow students to make mistakes and then discuss them.	7	15	19	30	26
2.12	After students have solved the problem, I give them correct solution so that they can copy it down.	8	23	8	34	27

Table 4.3 compares questionnaire and interview responses and indicates that most teachers focused on the correct answer, avoided mistakes and showed the procedure when students made an error. However, all middle school teachers responded to the questionnaire item 2.29 that they allow students to make mistakes and then discuss them. Two new points emerged during the interview that were not assessed in the questionnaire. The first was that the teachers avoided mistakes by discussing how to identify the operation/formula to solve the problem. Secondly, they discussed typical questions likely to come in exams so that students know the correct solution. The discussion of interview responses below illustrates how these practices are consistent with each other as teachers try to either avoid mistakes by telling procedures or retelling procedures in case of the mistake.

Table 4.3: Comparison of questionnaire and interview responses of primary and middle school teachers for the practices related to avoiding student mistakes

No.	Description of Practice	Question-naire Item No.	Primary teachers' interview response* (N=5)	Primary teachers' questionnaire response** (N=5)	Middle school teachers' Interview response* (N=6)	Middle school teachers' questionnaire response** (N=6)
1	Focusing on correct answer	2.25	5	2	4	2
2	Avoiding mistakes	2.17	4	2	4	4
3	Showing procedure in case of error	2.2	5	5	5	6
4	Allowing and discussing mistakes	2.29	-	2	-	6
45	Identifying the	-	5	-	4	-

	operation or method to solve					
6	Typical questions which come in exams	-	1	-	5	-

* This column gives the number of Primary or Middle school teachers who mentioned the use of this practice in the interview.
** This column gives the number of Primary or Middle school teachers who responded to the questionnaire item with most of the time or almost always.

When teachers were asked in the interview about how they responded to wrong answers by students (Appendix 2, Q. 11), most teachers' responses indicated that they rarely made efforts to understand why the student had made the mistake. The strategies that teachers adopted for addressing student errors was to indicate where the correction had to be done, repeating the steps of the procedure and asking students to do repeated practice. Teachers also asked 'good' students to 'explain' the steps to the students who have made mistakes. This is perhaps what teachers saw as comparing the methods in questionnaire item 2.18. However, many felt that errors had to be rectified immediately by the teacher.

Excerpt 4.5

> The wrong answer gets corrected on the board only. (P4, personal interview, May 26, 2009)
>
> If somebody gets a wrong answer, I ask him/her to do again or I tell that you have done steps till here correctly. (P3, personal interview, May 26, 2009)
>
> If somebody got a wrong answer, I help by first looking at their knowledge. If it is a knowledge problem I tell them what to do, e.g. formula.... If they make calculation mistake I tell them. (**M4**, personal interview, May 28, 2009)
>
> Which step they have gone wrong I will tell.... If wrong in the calculation, then I tell them to calculate again carefully. If there is a mistake in the method, I will repeat the statement. For e.g. For heights and distances figure would be wrong or they would have to take reciprocal for tan theta. Then, I will ask the definition of tan theta and they know by themselves what they have done wrong. (**M3**, personal interview, May 28, 2009)

Thus teachers relied on telling students how to solve the problem and the step that they had done incorrectly when there was a mistake, rather than using student errors to understand the conceptual gaps. However, **M2** shared her limitations in understanding conceptual difficulties when a student did not understand how to solve, even after repeating the explanation. She shared that she generally asks students "Why it is wrong, why did they do wrong?" and later added that her strategy was to "...explain [to] them again whatever is done in class". She tries to change the explanation but if they still don't get it, she said, "I stop there. May be some other concept is involved".

Most teachers described in the interview, how they try to elicit the correct solution from students by explaining the question, sometimes focusing on textual cues and showing or recalling solutions to similar problems. The primary teachers focused on identifying the correct operation to be performed with numbers while middle school teachers focused on methods and specific types of questions and their solutions. P2 described an activity which she designed to develop the understanding of the words for denoting addition like "total", "altogether" for class 1 children by mixing beads from smaller bowl to a bigger bowl and asking students to use these words. She determined the success of lesson by students being able to identify the correct operation while solving problems. While most middle school teachers considered discussion of typical questions important, this practice was not prevalent among the primary teachers perhaps because of no detention[1] policy and greater autonomy in constructing the question paper.

Excerpt 4.6

> I make students repeat the questions if they are important like linear equations, age problem, upstream-downstream questions because I want to make them familiar with the sentence language. I am doing on the blackboard and they are copying so whether they understood or not, I can know only when they do it again. (**M3**, personal interview, May 28, 2009)
>
> Students get scared when the language of the problem is changed e.g. total, altogether, more than…. They don't understand what operation is to be done. Thus reading suggestively helps [in identifying the operation. (P2, personal interview, May 26, 2009)

Many teachers indicated that they are pleased to get correct answers from students and especially from weaker students when they responded to the question asking them about the kinds of responses from the students that makes them happy (Appendix 2, Q 14).

Excerpt 4.7

> What makes me happy – when maximum students are able to answer the question. If only 2-3 answer that means that concept is not clear. Suppose there is very less response only bright ones then I ask a simpler sum and I get a better response. Then I ask little more difficult. (P3, personal interview, May 26, 2009)
>
> If all are giving response then I feel that concept is clear. (P4, personal interview,

1 No detention policy: Section 16 of the Right of Children to Free and Compulsory Education (*RTE*) Act, (GOI, 2009) stipulates that '*No* child admitted in a school shall be held back in any class or expelled from school till the completion of elementary education'.

May 26, 2009)

Inferences about beliefs

The above discussion indicates that showing procedures to students is a core practices, which most teachers engage in, and which is supported by component practices. These component practices reflect how the central idea behind these practices is an assumption that students learn best by showing procedures and thus teachers resort to telling procedures when discussing a problem or an error. Opportunities for students to think about their own solutions decrease as teachers share the procedure to avoid errors and ask students to copy the solutions from black-board.

All these component practices have come into play as teachers considered showing and remembering the procedure as an important part of doing mathematics in school. This indicates that the belief supporting this core practice, namely that the best way to teach mathematics is by showing procedures, is strong and thus a central belief. The other beliefs associated with this core practice like the practice of procedures and focusing on quick calculations are discussed in next subsections.

4.3.1.2 Importance of repeated practice

Repeated practice of mathematical problems has been a common strategy for learning mathematics. The focus on procedures in teaching is consistent with engaging in repeated practice of similar problems assuming that it will help students in memorizing the procedure as well as knowing which procedure is to be adopted for solving a certain kind of problem. Teachers' responses to questionnaire and interview were fairly consistent indicating it to be a core practice among teachers.

Responses to questionnaire items (2.1, 2.8 in Table 4.4) show that 50% of the teachers affirmed that they ask students to practice similar problems after showing the procedure most of the time.

Table 4.4: Questionnaire responses to items related to repeated practice of procedures

No.	Item	% Almost never	%Some of the time	%Half of the time	%Most of the time	% Almost Always
2.1	In the beginning of the class, I show students how to solve a particular problem and then give similar problems	15	19	15	27	23

	to practice from the textbook.					
2.8	I ask students to practice the problems very similar to the one done in class as home work.	8	11	15	31	35

Analysis of the interview responses revealed that teachers considered repeated practice of problems as important for learning of mathematics and adopted several component practices for supporting it, which are listed below. Evidences about this core and its component practices were obtained from teachers description of a typical lesson in response to Q. 8 (Appendix 2) and Q 15 about engaging students in lot of practice. Table 4.5 depicts the frequency of teachers who mentioned the use of these practices in the interview.

1. Giving similar problems after showing the procedure on the blackboard
2. Teachers constructing similar questions for practice
2. Asking students to repeat the problems done in class as homework
3. Giving problems to practice after activity
4. Giving easier problems to practice for weak students

Table 4.5: Comparison of questionnaire and interview responses of primary and middle school teachers for repeated practice of similar problems

No.	Practice description	Questionn -aire item number	Primary teachers' interview response* (N=5)	Primary teachers' questionnaire response* (N=5)	Middle school teachers' Interview response* (N= 6)	Middle school teachers' questionnaire response* (N= 6)
1	Giving similar problems after showing procedure on the blackboard	2.1	4	1	6	4
2	Teachers constructing similar practice questions	2.8	4	1	5	6
3	Asking students to repeat the problems done in class as homework	-	3	-	1	-
4	Giving problems to practice after doing an activity	-	5	-	5	-
5	Give easier problems	-	0	-	5	-

	to practice for weak students					

* See explanatory note for Table 4.3 on page 120.

In Table 4.5 above it is evident that most teachers who were interviewed engaged students in practicing similar problems after showing a procedure and also after doing the activity. More middle school teachers engaged in making practice questions. All the middle school teachers selected lower level questions for weak students to practice so that they can pass the examinations, while this practice was absent in primary classes.

What to practice – Same problems vs. constructing similar problems

During the interview, teachers affirmed the importance of practice. Some teachers while responding to Q. 7 about the textbook (Appendix 2), felt that the textbook did not contain enough practice questions, so they themselves made practice questions for students or took the help of some resource books. This was more common among the middle school teachers. The primary teachers mostly asked students to repeat the work done in class as homework. Responding to Q. 15 (Appendix 2) about engaging students in a lot of practice, most teachers believed that practice of similar problems has a role to play in learning mathematics, which will be discussed further in the section on beliefs (4.3) of teaching and learning mathematics. However, there were differences among teachers in the extent to which practice was emphasized. Some teachers expected students to do 40 similar problems in one go, whereas others felt that practicing 4-5 times is enough.

Excerpt 4.8

> I don't give homework. I tell them, whatever I did in class, you do it again in your rough book. (P4, personal interview, May 26, 2009)
>
> Practice of similar kinds of sums at least needed one or two times. Means if once you have found a logic of something then you should apply, you should see. 30 sums and all are not needed. (P5, personal interview, May 30, 2009)
>
> They do need practice after activity. Minimum 5 sums should be done. If the sums are of different type then we have to increase the number. (italics mine) (P2, personal interview, May 30, 2009)
>
> Homework is similar problems to those done in class. I give 50 problems in 2 days...Because in one day when they solve [many] problems then only they get it. (**M1**, personal interview, May 30, 2009)
>
> They should become thorough means that they have to do a lot of questions even when they know the principle. Along with that whatever is there in the textbook... I

> give similar questions for homework by changing the angles, distance, speed to make them thorough. (**M3**, personal interview, May 30, 2009)

Some teachers like **M2** felt that practice would not help in learning geometry as there is always a possibility to make different types of problems. Due to this, she felt she cannot address students' learning adequately as a teacher.

Excerpt 4.9

> Other things you can practice, but geometry, you can't. Like, even if you do many sums, there can be a different sum to solve. That is where we are facing the problem, children are facing a problem. That is where we face the problem in teaching also. (**M2**, personal interview, May 27, 2009)

Repeated practice for weak students

Teachers used repeated practice not only as a method to teach but also as the strategy to prepare weak students for exams. Their response to Q. 16 (Appendix 2) about students from poor home background indicated that most of these students were weak in mathematics. Teachers also used repeated practice as a way to address poor and weak students' difficulties by selecting 'knowledge-based' and 'typical' questions (requiring memorization) according to the "limit" of the students and engaging in "drilling" to make them pass (M5, M6). In line with this, **M1** proposed that separate question paper of "lower level" should be there for these students.

Excerpt 4.10

> You have to stop at a simple problem. You tell them to practice only this. After they have practiced, then you come to know [that] they can do this. Because at home, there is nobody to help. So if I am giving 50 problems, children with no help at home will be able to do only 20. Only the easier ones. They may not be able to do all. I want to stop at that. Let them practice these 20 again... I have to bring them to passing level. Till 10th they have to study maths no other way. They don't have the aptitude but I have to bring them to the minimum level and stop. (**M2**, personal interview, May 27, 2009)

> First is, we classify the chapters, the ones which are basic. Like, I first know what is the limit [of the student]…. I know these children are not going to take to any mathematics further. So, it is just that they have to pass the exam... questions which are very easy... first is knowledge based... with that, I know the least fundamental requirement they know.... I give them typical problems which come in examinations. My success rate in doing this, so far is good, high. They pass. They get through and most often the parents understand that their children are not up to the mark in mathematics... So, they see to it that they go to other branches and they are more successful, elsewhere. I don't discourage, I tell them that it is better that they avoid maths. (italics mine) (M5, personal interview, May 29, 2009)

> Because they don't have anybody at home to look for them, they don't study at

> home. They only study in school.... Easy sums and easy topics we decide for him, that is what he will understand quickly and with ease... because there is lacuna from primary.... Because, there is lot of gap which is there in secondary, so he is not able to catch the difficulty level. So it is better that we give him easy topics and quickly.... What topic is of his level, that we decide and through drilling we try to give him basic education of that level. (M6, personal interview, May 29, 2009)

Reflected in these excerpts is the expectation that these students will not continue studying mathematics beyond the point where it is compulsory for them to pass in mathematics. Teachers perceived themselves as being unable to help these students, either because of the large gaps in their understanding, or because of their perceived incapability. They felt that they could fulfill their responsibility by helping these students pass the examination.

Inferences about beliefs

The above discussion shows how engaging students in the practice of problems can be considered as a core practice. This indicates a belief among teachers about mathematics that procedures are an important part of school mathematics. Another belief evident about learning mathematics is that, students learn procedures by repeatedly practicing procedures with similar type of problems and thus memorization plays an important role in learning mathematics. The assessment system also contributed to this practice as teachers "teach to the test" and focus on making weak students just pass the exam.

Teachers had low expectations from the students from poor homes, who did not have anyone at home to help them. This was reflected in their practice of selecting 'lower level' questions to work on and giving special attention by engaging them in drilling. This indicated teachers' belief that students from poor homes have difficulty in learning mathematics which can be addressed by memorizing solutions to simpler problems. Thus, belief about learning mathematics by memorizing is held more strongly in the case of students from disadvantaged backgrounds. This also reflects an underlying belief about school mathematics being difficult for students, which will be discussed in the section on "Beliefs about the nature of mathematics" (Section 4.4). Teachers' practice is also influenced by the kind of future careers that the teachers envision for these students.

4.3.1.3 Focus on speedy solutions

In India, the competitive examinations after Grade 12 act as gatekeepers to future careers, and include mathematics as an important component. In order to clear these examinations, one is required to solve many mathematical problems quickly. This has led to a general perception of a good student of mathematics as being quick in doing calculations and getting correct answers. Teachers have been teaching shortcuts to the students, to help them to perform well in these competitive examinations. With the increase of examinations based competitions at the primary and middle school levels, has led to a greater emphasis on knowing shortcuts.

Shortcuts are the strategies through which number of steps can be reduced in a procedure, for e.g., cancellation while dividing whole number or multiplying fractions. Teachers' response to the question about teaching of shortcuts (Appendix 2, Q. 13) in the interview indicated that formulas and procedures which make any lengthy procedure short and convenient are considered as shortcuts. For some teachers like **M4**, teaching shortcuts was part and parcel of teaching mathematics, since he considered that all formulas are shortcuts. For e.g., he considered the formula for simple interest (PRT/100) to be a shortcut which can be derived, but is remembered as a formula. P2 gave the example of 'carrying' in vertical addition as a shortcut.

Excerpt 4.11

> Generally I explain first, then tell the shortcut method.... Formula is a method which we have written in shortcut. Everybody uses it eventually. (P2, personal interview, May 26, 2009)

In the questionnaire, a few teachers seemed to prefer the practice of telling shortcuts and making students work quickly on problems. Almost 26% of the teachers indicated that they ask students to work quickly most of the time. Almost 35% of the teachers never taught only one method to solve a question, indicating that perhaps teachers are open to different methods. However, this does not necessarily mean that teachers allow students to share methods of their own, since many teachers shared in interview that they teach shortcuts.

Table 4.6: Questionnaire items related to practice of focusing on speed of solutions

No.	item	% Almost never	%Some of the time	%Half of the time	%Most of the time	% Almost Always
2.9	I teach only one method for each question.	35	35	12	15	0
2.27	I ask students to work more quickly on their problems.	8	31	19	27	8

The interview data indicates that teachers focus on developing speed in solving problems through component practices like teaching only one method to solve a problem, telling shortcuts, asking students to work quickly (evidence from description of typical lesson) and appreciating students who solve quickly (Response to Q.14, Appendix 2). Almost all the middle school teachers told shortcuts to students, whereas three of the primary teachers discussed them. The other two primary teachers taught only one method. The summary of a number of teachers who mentioned the use of these component practices is given in Table 4.7 below.

Table 4.7: Comparison of questionnaire and interview responses of primary and middle school teachers for the practice of focusing on speed of solutions

No.	Practice description	Questionnaire item number	Primary teachers' interview response* (N=5)	Primary teachers' questionnaire response* (N=5)	Middle school teachers' Interview response* (N= 6)	Middle school teachers' questionnaire response* (N= 6)
1	Teaching only one method	2.9	2	1	2	2
2	Telling shortcuts	-	3	-	5	-
3	Appreciating quick solutions	2.27	3	1	4	4
4	Alternative procedure other than standard method	-	3	-	0	-

* See explanatory note for Table 4.3 on page 120.

There were teachers who believed that students should be allowed the opportunity to think about their own methods or shortcuts. **P1** thought that students should not be restricted to follow a particular method and should be given freedom to follow whatever method they want. P5 agreed that students may arrive at shortcuts on their own while doing mental maths but was not able to give an example.

Teaching Shortcuts

In the interview, teachers were asked if they directly teach shortcuts. If they admitted to telling shortcuts, then they were asked about how shortcuts help in learning mathematics.

Excerpt 4.12

> I teach shortcuts sometimes. I will tell, you can do like this also. Whatever way you understood you write. On the contrary, I don't like reducing the steps and writing.... Writing properly and elaborate... with all the steps only, I like with reasons. (**M3**,

personal interview, May 28, 2009)

Except for M1 and P3, all teachers felt that shortcuts should be taught. Some teachers, including primary teachers, admitted to teaching shortcuts and gave the justification on the basis of the utility of shortcuts in competitive exams. M5 felt teaching shortcuts is useful to students to solve questions in competitive exams where there is a time limit but felt that shortcut is not for the average child as many children "skip the steps as shortcuts, and they are not able to do mental calculation correctly, because of being anxious, and make mistakes". He believed that shortcuts should be taught in higher classes, i.e., from 9th grade onwards since they help students perform in multiple choice questions. He felt that knowing a shortcut is beneficial when students have to respond to one-mark questions requiring lengthy steps.

Appreciating quick solutions

There were five teachers out of eleven who believed that a good student of mathematics should be able to do mathematics problems quickly and accurately. Teachers taught shortcuts depending on their estimate of the capability of students. P5 did not teach shortcuts in her class as most students belonged to poor homes. P4 considered answering the question immediately with the correct answer to be an indicator of being good in mathematics and thus taught shortcuts. She felt that students "pick up" the shortcuts very fast, "they are not confused and they do sums with confidence".

Inferences about beliefs

Focus on speedy solutions came across as a core practice since almost all teachers tried getting students to solve problems quickly. Many taught shortcuts and consider them useful in increasing speed though some were aware that they do not aid in understanding maths. Thus it is a core practice since teachers focused on speed of calculation as it helps students in exams to get better scores. This was consistent with the belief about mathematics as procedures which need to be transmitted to students. As discussed in a later section on beliefs about good students of mathematics (4.3.2) while discussing beliefs about teaching and learning mathematics (section 4.3) students, there were teachers who were critical of the practice of teaching shortcuts and students responding quickly and not allowing others time to think. However, the importance of speed of calculations required in competitive exams motivated these teachers to share shortcuts with students. Students getting good scores in examinations strengthens teachers' belief in their self-efficacy, thus promoting this practice. Teachers' belief about mathematics being an innate ability, made them believe that only bright students will be able to use shortcuts efficiently. Thus, the decision to share shortcuts in class depended on the estimated ability of the students in their class.

4.3.1.4 Following the textbook

Although teachers reported a change in their teaching approaches after the introduction of new textbooks, there was an indication that not much had changed in terms of learning opportunities for students. One of the objectives of the new textbooks was to allow children to "generate new knowledge" through imaginative activities and questions (NCERT, 2006d, Foreword). However, teachers' views were not conducive to meeting this objective. Responding to question 6 and 7 in the interview (Appendix 2), they shared that they followed the textbook, but focused on doing exercises and questions from the textbook, without caring much for the philosophy underlying the textbook questions. In comparison, the old textbooks included presenting a method, followed by exercises with a large number of similar problems. The new textbooks included alternative procedures and use of contexts in a more authentic manner, and the use of activities. Following the textbook is thus a core practice since, teachers followed the sequence given in the textbook, focused on topics and exercises given in the textbook, but did not necessarily subscribe to the purpose for which those exercises were constructed.

In the questionnaire, 64% of the teachers shared that they follow the sequence of the textbook most of the time or almost always(Item 2.19). However, 43% of the teachers also shared that they give problems only from the textbook, some of the time. This indicated that they give problems apart from those given in the textbook which they either construct themselves or take from a resource book.

Table 4.8: Questionnaire responses to practices related to following the textbook

No.	item	% Almost never	%Some of the time	%Half of the time	%Most of the time	% Almost Always
2.19	I follow the sequence given in the textbook in my teaching	8	19	8	38	27
2.31	I give problems only from the textbook to my students	27	43	19	11	0

The analysis of component practices (see Table 4.9) for following the textbook indicate that most teachers used the textbook for planning teaching, determining the sequence of teaching and used the problems given in the textbook. However, the problems were not restricted to those in the textbooks and teachers constructed similar problems.

Table 4.9: Comparison of questionnaire and interview responses of primary and middle school teachers for the practice of following the textbook

No.	Practice description	Questionn -aire item number	Primary teachers' interview response* (N=5)	Primary teachers' questionnaire response* (N=5)	Middle school teachers' Interview response* (N= 6)	Middle school teachers' questionnaire response* (N= 6)
1	Planning on the basis of the textbook	-	5	-	6	-
2	Following the sequence of the textbook	2.19	4	4	6	4
3	Doing the problems and activities from the textbook	2.31	5	0	6	0

* See explanatory note for Table 4.3 on page 120.

Teachers indicated in their interviews the extent to which their teaching is dependent on or influenced by the textbook. Most teachers confirmed that they follow the sequence of the textbook since it is mandated by the system. They have to complete the chapters given in the textbook within the stipulated days, according to the centrally constructed monthly plan made by authorities. The selection of the chapters to be assessed in the term-wise examination was done by central authorities according to the monthly plan. Teachers were held accountable for completing the syllabus according to the plan, and were asked about the number of chapters completed during supervision by administrators like the headmaster, principal or inspector.

Excerpt 4.13

> I teach by the sequence given in the textbook. I follow the textbook completely while teaching. (**M1**, personal interview, May 27, 2009)
>
> I follow the textbook completely while teaching. (But) I am not able to finish the syllabus if I try all the activities of the textbook. (**M2**, personal interview, May 27, 2009)
>
> We have to teach from exam point of view where 2-3 chapters will come in a unit test set by others. (**M1**, personal interview, May 27, 2009)

Most teachers said that they refer to the textbook for planning. While planning they decide on the content and questions using the textbook. Some teachers also considered students' previous knowledge or their 'ability' for planning. Some teachers said that they taught according to the description of the chapter given in the textbook and go page by page, engaging class in all the activities as well as the exercises given in the textbook. However, other teachers felt that it is not possible for them to do each and every activity or task given since they do not have time.

Excerpt 4.14

> I prepare the lesson then I look at the textbook but I don't look at the textbook while teaching. (**M4**, personal interview, May 28, 2009)
>
> We take what is to be taught from the textbook and plan on the basis of the level of the child....Sometimes children are low at grasping. Thus some activities [but] not all can be done. (**M2**, personal interview, May 27, 2009)
>
> I think about how to express maths to children or which particular problem I have to explain – that I plan. (P2, personal interview, May 25, 2009)

In the interviews, teachers described the differences between the old and new textbooks. They also contrasted the old method of teaching with the new method which they felt was influenced by the nature of new textbooks that they followed. Teachers' attitude towards the new textbooks was based on how they interpreted the experiences of mathematics that they themselves had had as school students. Teachers who had had negative experiences of learning mathematics by rote memorization, appreciated the new textbooks. However, those who had experienced success with the old method were critical of the new textbooks for having few practice problems.

Excerpt 4.15

> [The] New textbook has activities and is colorful so all children like it. (P2, personal interview, May 25, 2009)
>
> It is (new textbook) more explicit. In earlier books many things were hidden.… Before exercises one 'Do this' and 'Try this' is there. It is very interesting for children. (**M3**, personal interview, May 28, 2009)
>
> In the earlier textbook of Class 5, there were so many theories and many exercises with numbers. But in the new textbook, not much numbers, only practice sums. It is not given, multiply it, divide it. For the old textbook, we had to teach just the theory, that is, how to multiply and not forced to do the activity. Now questions and the textbook are designed like that. (P3, personal interview, May 26, 2009)
>
> We construct our own problems because there are less questions given in the textbook. (P4, personal interview, May 26, 2009)

Some teachers also talked about the change in their thinking, regarding the teaching of mathematics, after going through the new textbooks. M6 shared that, earlier he used to teach by telling steps for solving problems followed by practice of questions using the old textbook. But now, he felt that doing only exercises is focusing on "how to solve" and "rote memorization" which was supported by earlier textbooks which would have "one example followed by 30-40 questions in exercises". However, for some teachers' the focus was still on covering the syllabus from the textbook by covering topics and doing problems given in the textbook rather than engaging students in making sense of mathematics.

Excerpt 4.16

> Yes I refer textbook because you have to follow the syllabus and complete this much portion before July. (P2, personal interview, May 25, 2009)
>
> When we teach we concentrate more on the concept rather than the book. We clear the concept through examples and then derive formulas. (**M4**, personal interview, May 28, 2009)

Some teachers like **M3** considered the textbook to be a resource for students also.

Excerpt 4.17

> I tell them before teaching tomorrow you study introduction part at least and come and I will ask questions from it. Then only I come to know if they have read and understood. They will bring so many things they have understood. (**M3**, personal interview, May 28, 2009)

Inferences about beliefs

Teachers' decisions about teaching were largely guided by the textbook as indicated by the above excerpts. This indicate teachers' belief that teaching in classroom has to be heavily guided by what is given in the textbook especially the problems and exercises. This practice restricted teachers' agency in using their judgement based on their own knowledge of what to teach and how to teach. On the other hand, it served as a resource for teaching. As discussed earlier, the new textbooks also facilitated reflection on the nature of mathematics and its teaching for some teachers and provided examples of alternative ways of doing and teaching mathematics. Teaching was also influenced by the type of assessment as teachers focused on improving student performance in exams using typical questions. Thus teachers' belief about following the textbook was restricted to the use of problems and not its philosophy. This points to the need to have assessment reforms, aligned with the philosophy of the textbook, to bring about effective educational reform.

4.3.1.5 Conclusion

The analysis of data from interview and questionnaire indicate that showing and practicing procedures are the two core activities which are held with greater strength as compared to other proposed core activities like focusing on speedy solutions and following the textbook. This is indicated as more teachers have consistently agreed to using the first two core practices in their classroom and there is coherence due to overlap in the component practices that support them. For e.g., teachers show procedure and then engage students in practice, telling the procedure and practice are considered as the remedy for errors as well as for avoiding errors. The primacy of correct solutions is also aided by showing and practicing procedures. The focus on speedy solutions is not indicated with similar strength. This could be because teachers perhaps engage with the practice only for students whom they perceive as ‘good students’. Teachers having more students from poor homes refrain from using it. The textbook, being an important tool mandated by the system, has a great influence on teaching. Teachers consider finishing the syllabus as an important goal. However, it still does not ensure that activities and questions given in the textbook are done in the same way as it was intended by the curriculum makers, as teachers adopt a procedurally focused approach with the new textbooks. This could be the reason for the comparatively lower strength of the core practice of following the textbook for teaching, as teachers designed their own practice questions when textbooks lacked them. The lack of practice questions in the textbook made the practice of using the textbook inconsistent with the core practice of focusing procedures and repeated practice of similar problems. On the other hand, a mandatory requirement from the system meant that teachers need to use the textbook for teaching.

4.3.2 Peripheral practices

Some practices were adopted by the teachers due to educational reform measures, which were implemented through circulars, changes in the textbooks and the inputs that teachers received in in-service workshops aimed at helping them adapt to the new curriculum framework. The use of activities is a peripheral practice that was adopted by teachers as a result of educational reform implementation. Other practices that were implemented as part of reform included connecting mathematics to students’ daily lives, use of manipulative and practices aimed at improving equity of learning opportunities which teachers either engage with lesser frequency or their use is not central to their goals of mathematics teaching. This led to terming such practices as peripheral practices. Focus on explanation and justification is another peripheral practice, which teachers said was part of their practice in the questionnaire and interviews but engaged in only infrequently, if at all, while teaching since they felt that students are not capable of understanding and giving reasons.

4.3.2.1 How teachers used activities in the classroom

Teachers had encountered educational reform through the change in the textbooks, workshops focused on reform that they had attended and various circulars issued by the authorities. Teachers had incorporated key slogans into their discourse like "activity based method", "play-way method", "learning to learn", which were used frequently in the school meetings. Teachers felt that the new textbooks, which had activities, advocated such teaching methods.

Table 4.10 illustrates the frequency with which component practices that made up the practice of using activities were reported by the teachers. These component practices include the variety of ways in which activities were used by the teachers in the class ranging from a demonstration, calling a few students to the blackboard, engaging the whole class and, using concrete materials or manipulative. The interview analysis indicated that use of activities is a frequent practice in the primary classes, whereas it is an infrequent practice in the middle school. Primary teachers used the activity in all different forms while middle school teachers mostly demonstrated an activity in the class. Another aspect which determined whether the activity was used frequently or not was whether the teacher felt confident enough to design activities on her own for the class or used the activities from the textbook. The former practice was indicative of student responsive teaching. Again, the primary teachers used activities both from the textbooks and those constructed by them, whereas the middle school teachers used activities mostly from the textbook.

Table 4.10: Comparison of interview responses of primary and middle school teachers for the practice of doing activities

No.	Practice description	Number of primary teachers mentioning use in interview (out of 5)	Number of middle school teachers mentioning use in interview (out of 6)
1	Demonstrating the activity	5	4
2	Engaging students in doing the activity	4	2
3	Use of concrete and pictorial representations	4	2
4	Doing an activity from the textbook	5	4
5	Designing the activity	4	1

For the teachers, "activity" meant the use of a visual or concrete material or a context in the class to illustrate an example or a task. It typically involved a demonstration by the teacher followed by the students' repeating what the teacher has demonstrated. For some teachers, anything that was not related to directly teaching procedures or calculation was termed an activity. In contrast, activity in the textbook was given to engage students in thinking about some aspect of the concept and coming up with their own ideas related to the concept. For e.g., in the Grade 5 textbook (NCERT, 2005, p. 60), there is a description of a game in which students color a circle divided into 12 parts by picking tokens by turn which have fractions as well as equivalent fractions written on them. The textbook footnote has an instruction for teachers "The coloring circle game and many more such activities should be done in the class. The follow-up discussion for such activities will play a major role in developing children's conceptual understanding about fraction" (NCERT, 2005, p.61). This indicates that the game is considered as an activity by the textbook writers and they expect students to use their own knowledge of fractions in playing the game and expect the teacher to take up follow-up discussion after the activity. As the following descriptions of activity given by teachers indicate, the teachers' implementation of activities was very different from what textbook developers had imagined.

The teachers reported that they used activities for different purposes, like an introduction to a concept, creating interest in students, and for developing the understanding of concepts. However, no teacher elaborated on how activities can help in developing understanding or eliciting students' ideas although all described how they use activities in teaching for introducing the concepts. Some teachers, when they used the phrase "introducing the concept" through the use of activity meant showing the procedure using drawings or manipulative.

Excerpt 4.18

> By doing the activity I introduce the concept.... I might show one sum how to do it and the rest they have to do themselves. (P5, personal interview, May 30, 2009)
>
> Whole class should be doing the activity. Not that one is doing and others are watching... They do activity in groups but they don't talk.... After group work, they come to me.... The concept gets clearer by activity even if it takes time.... After activity, drilling practice is needed to be perfect. (P4, personal interview, May 26, 2009)

Most middle school teachers described activities from the textbook (Except **M1**) while primary teachers talked about activities designed by them or taken from some resource book. Since primary teachers have been using activities and manipulative even before the introduction of the new textbooks, they were more used to designing activities on their own.

Excerpt 4.19

> So I tell take page number this and do "do this" (activity) question. (**M3**, personal interview, May 28, 2009)
>
> Doing activity is compulsory now. We use resource books to design activities. (P4, personal interview, May 26, 2009)

Teachers' description of activities indicated that some teachers view demonstration using manipulative or drawings as an activity, the purpose of which remained showing procedure to solve the problem. Some teachers did indicate that they involve students in *doing* the activity after a demonstration. However, a few teachers spoke about the ideas or concepts that students arrived at or shared after doing the activity. P4 described activities that she used to develop the understanding of mathematical concepts using different representations, for e.g., making groups of children of various sizes to understand multiplication tables. While teaching place value she felt happy that students were able to recognize that "2 bundles [of ten] and 2 sticks are 22". She used pictures and illustrations for making problems, for e.g., multiplication based on spiders' legs, butterflies' wings etc.

Excerpt 4.20

> Once, I had ones and tens introduced in the first standard. That time also I had made bundles. I said in this bundle -10 pencils. I took it out and counted . How many bundles? [They say] "1". Then I say, Can we say 10 ones? [They say] "Yes, Ma'am". Then, I showed them 10 ones and one. They said, "eleven". Then, 20 [i.e.] 2 bundles having 20. (P4, personal interview, May 26, 2009)

P5 described an activity in which she encouraged children to come and arrange vegetables in groups in various patterns and then ask them to make patterns in groups using pencils, pens, sharpeners, etc., followed by children standing in groups. She had followed this by representing these patterns on the board using symbols and then engaging students in counting.

Teachers very rarely mentioned the conceptual discussion during or after the activity. However, **P1** and P3 described how they used contexts for doing an activity with students and then discussed the concept with students. P3 described an "activity" which she demonstrated to students, using a context of sharing a Guava, for developing the understanding of fractions. She described a situation in which a mother gave her 3 children, one Guava each, cut into 4 pieces. The elder child gave half of the Guava back to the mother, followed by the other two younger siblings. But then, the mother returns half of what they gave, back to them, so that all of them have ¾ Guava each. Then, she discussed the fractions ¼, ½ and ¾ mentioned in the story with the students. Another teacher, **P1** shared that she uses the activities and the contexts to introduce a new concept. She described how the concept of a fraction can be explained using the context of sharing chapatis (round flat bread) by showing how to make half of chapati and then half of half of a chapati, using pictures and parts of a paper. **P1** elaborated on how activities help students in remembering, understanding and coming up with different methods to solve the problem. She appreciated the new textbooks which had plenty of activities drawing connections with students' daily lives.

Excerpt 4.21

> This is more practical [new textbook]. Examples are given from life.... Child will never forget with this.... In that [old textbook] we used to teach and then student used to make mistakes.... One method will not do. 2-3 methods will definitely come. So it is important to see and understand the method.... We have to move ahead from the previous concepts. So we will take help of some activities. Some problems we will give to students and will ask how it will be done. (**P1**, personal interview, May 25, 2009)

Most teachers felt that practice of problems is still needed after doing the activity. Their description of a typical lesson involved activities for introduction followed by explanation through example problems, followed by students solving similar problems. The use of activities was intended to "clear" the concept for some teachers and for other teachers it served as means of "introducing the concept" and making mathematics interesting. Both the primary and the middle school teachers focused on teaching procedures after doing the activity with students. This could be because the assessment questions are of the type where students have to perform calculations and are not activity based as indicated by **M2** in the excerpt below.

Excerpt 4.22

> With the help of 'Do this' [activity in the textbook] we will tell. Then, they are getting standard questions from the textbook. (**M3**, personal interview, May 28, 2009)
>
> To introduce the chapter, one can use this (activity method)...This method (activity) if I use, I will not be able to do all the textbook exercises. Within one month children will have to sit for exams. In the test, we have to give those (exercise) questions. Only these activity questions are not there. There we are stuck up. (**M2**, personal interview, May 27, 2009)

Primary teachers' belief that mathematics should be simplified for students made them choose concrete materials, contexts and activities for doing mathematics. They indicated their happiness in increased student participation as a result of the use of these resources. Middle school teachers' focus on the other hand was more on developing fluency in solving different type of problems and thus some teachers indicated resistance to activities and use of teaching aids while others who agreed to use it were not able to describe activities beyond mentioning those in the textbook. Some primary teachers also shared that they were happy with the imaginative response that they got from students while doing activities. Almost all the teachers mentioned using activities in the classroom but they also showed solution procedures to students while or after doing the activity rather than using the activity to elicit and work on students' ideas and mathematics emerging from engagement in the context. It is possible that knowledge of only limited ways of representing mathematical concepts and lack of experience of using it with children might be inhibiting teachers from using activities in the way intended by curriculum designers. This explains why the use of activities is a peripheral practice for learning mathematics especially at middle school level since it does not align well with the view of teaching by transmission. Teachers need to understand and appreciate students' ideas as well as contexts that help generate students' ideas.

Almost all the primary teachers said that they use activities, while middle school teachers were not as enthusiastic about using activities, citing various reasons like burden of syllabus, completion, lack of time to complete activities, etc. While discussing activities in interviews, primary teachers expressed various ideas of how they view activities as contributing to learning of mathematics. For e.g., P5 discussed how mathematics has become simpler because of the new textbooks where contexts and games are a part of the chapter. She appreciated how the textbooks allow students to engage in creative activities, although she wished that they had more practice sums.

Excerpt 4.23

> P5: it is creative and it gives you joy.....when we started those days mathematics was complex...now for last 2-3 years they have made it much simpler and still concept oriented...one drawback...they lack practice sums because maths has always been practice.. its not only concept. (P5, personal interview, May 30, 2009)

Teachers were able to recognize and appreciate how when students are given opportunity to exercise their agency, it could lead to imaginative and creative problem solving. P2 described how she was happy because the "children developed their own imagination without my help" when they engaged in the textbook task of arranging different shaped triangles to make other shapes and many made fish in a variety of shapes. P4 also felt that her best lesson was using tangram, where students made different shapes using the tangram shapes like house, gun, etc., on their own.

The description of activities by teachers had the intention of transmitting the ideas or procedures to students using concrete materials, in which some teachers gave leeway to children to arrive at different solutions. However, the main purpose was to transmit the procedures for solving rather than eliciting students' ideas or even an attempt to know what student have learnt as the result of engaging in the activity. Also more freedom was given to children in activities related to space and shape for constructing different figures and creating artistic objects rather than to traditional topics of teaching algorithms of division.

The responses above suggest that trying out activities by teachers who may not believe in their usefulness may lead to new experiences and reflection by teachers and perhaps make them more open to exploring new activities. So, teachers might appreciate good activities and problems in the curriculum after trying them out. However, this does not mean change in beliefs or method of teaching since they still considered practice as the main method for learning and thus constructed similar problems for practice even after doing the activity. Although some teachers were able to appreciate creative ideas of students, they was not able to use them centrally in learning mathematics. This indicates that there is potential for teachers to develop knowledge about student thinking through use of activities but may need further support and experience of working with students' responses. Thus, as discussed earlier, learning mathematics through the use of activities can be termed as a peripheral practice.

Inferences about beliefs

The descriptions of activities indicated that teachers used activities to introduce concepts using visual representations or contexts rather than using it to elicit and work on student ideas. Thus, activities were used in accordance with the transmission view of teaching. Use of activities, thus, seem to be a more frequent practice for primary teachers since they have been using it before as compared to middle school teachers. Therefore, it cannot be considered as a core practice since teachers beliefs and practice are not consistent on one hand and the practice is not held at priority by teachers as compared to other practices like showing procedures and repeated practice. With the inclusion in the textbook and being made compulsory by the administration, teachers are compelled to use it. However, interview data suggests that they still rely on drilling practice of procedures to make students learn mathematics and there was no scope for students' ideas to be expressed or discussed. This indicates that although teachers have tried making activities central o their practice, their approach towards it is still based on a transmission view of teaching and focus is still on procedures. Only primary teachers gave examples of activities designed by them. Middle school teachers relied on textbooks for activities and had limited knowledge of using or designing activities. Thus, use of activities is termed as peripheral practice.

4.3.2.2 Focus on explanation and justification in mathematics

There were several questions in the questionnaire which probed teachers' focus on explanation and justification while teaching mathematics and teachers had responded positively to them. Although most teachers focused on the teaching of procedures, some teachers did mention the focus on the explanation and justification in the interview.

Table 4.11: Questionnaire response for focus on explanation and justification in teaching

No.	Item	% Almost never	%Some of the time	%Half of the time	%Most of the time	% Almost Always
2.11	I ask my students to write explanations and justifications through words or pictures in their notebook.	12	35	0	19	35
2.22	I ask students the reason or justification for using a particular method to solve a problem.	8	19	12	42	19

Teachers mostly focused on talking about teaching procedures in the interview, but there were some instances when teachers recognized the need to focus on explanations and justifications. This was seen mostly in middle school teachers as most primary school teachers considered engaging in explanations and justifications as beyond the capability of primary students.

M5 discussed how he would discuss the reason for the solution being wrong for 'rectification' of students' error, calling telling the procedure in response to error as encouraging rote memorization.

Excerpt 4.24

> ...means reasons behind it (wrong answer) should also be there otherwise it will be rote memory. (M5, personal interview, May 29, 2009)

M6 also discussed how everything in mathematics has a reason behind it and thus there are always explanations and justifications for all the rules in mathematics. However, she felt that young students of Grade 5 or 6 will not be able to appreciate these explanations and for them, it would be fascinating as patterns and discoveries. This indicated that even when teachers know the explanation and justification they may not engage students in such discussion because of their beliefs about students' capability for understanding.

Excerpt 4.25

> Anything you do, have a reason behind it.. We talk of divisibility rules.. you have an explanation for it. But is the explanation necessary for the child. (This) is the question we have to think of answering. The child may not appreciate the explanation. We try to go to his level, reason it out with him but to a fifth class child 100 tens [means] 10 hundred he is not going to appreciate. So for him, it is through a series of patterns and discoveries. And we find that there is room for it. (M6, personal interview, May 29, 2009)

Among the primary teachers, only P2 mentioned the importance of explanation and reasoning in mathematics. But it was not reflected in the description of a typical lesson or other examples she gave from her teaching. She was talking about the "Analysis-Synthesis" method for teaching mathematics that she was taught in her B. Ed course. However, this was not reflected in her report of teaching primary level students.

Excerpt 4.26

> When we give theorems, they have data in it. So first we have to discuss "What is asked and what is given". You have to take help of other hypothesis. So we have to give a reason for every step that why you have written this step. Without reasoning, you can't write any step any rule. Analysis-synthesis method involves that through these small steps you come to a conclusion. (P2, personal interview, May 25, 2009)

The teachers' response to an interview question about explaining and justifying the division algorithm or the Pythagoras theorem is discussed in a later section on mathematical explanations (4.8). The responses indicate that teachers themselves had limited knowledge of how to justify steps in mathematical procedures and connecting them to underlying concepts.

Although teachers exhibited a positive attitude for focusing on the explanation and justification for the teaching of mathematics in the questionnaire, this cannot be corroborated with what teachers described in their interview in terms of practices preferred or reported. Teachers' beliefs about students' capability could be one of the reasons for 'focus on explanation and justification' being only a peripheral practice.

4.3.2.3 Connections to daily life

In the questionnaire, items 2.5, 2.6, 2.15 (See Table 4.12) probed teachers' views about using daily life contexts. The large majority of respondents indicated that they use knowledge of daily life and common objects from daily life to teach mathematics most of the time or almost always. However, in response to Q.10 in the interview (Appendix 2), about giving details about the way they establish connections with daily life fewer teachers reported the use of daily life contexts apart from textbook examples .Their examples were worthy of the application of mathematics rather than of using contexts to develop mathematical ideas.

Table 4.12: Questionnaire responses to practices related to connections to daily life

No.	Item	% Almost never	%Some of the time	%Half of the time	%Most of the time	% Almost Always
2.5	I use knowledge about students' daily life and culture for teaching mathematics.	0	8	15	31	46
2.6	Students in my class give examples of the application of mathematical concept being taught from their daily life.	8	19	12	46	15
2.15	I use commonly available objects from daily life while teaching.	0	8	4	50	38

In order to draw connections with daily life, teachers' interviews indicate the use of component practices like applying maths ideas to daily life contexts, asking students to give examples and using daily life contexts to elicit students' knowledge. While teachers gave positive responses to these practices in the questionnaire, in the interviews, teachers reported that they draw connections to daily life by giving examples of applications and did not mention other ways to establish connections.

Table 4.13: Comparison of questionnaire and interview responses of primary and middle school teachers for the practice of establishing connections with daily life

No.	Practice description	Questionnaire item number	Primary teachers' interview response* (N=5)	Primary teachers' questionnaire response* (N=5)	Middle school teachers' Interview response* (N= 6)	Middle school teachers' questionnaire response* (N= 6)
1	Showing the application of a mathematical idea in daily life	2.5	4	5	6	4
2	Asking students to give daily life examples	2.6	0	3	0	3
3	Using daily life contexts to elicit students knowledge	2.15	0	4	0	6

* See explanatory note for Table 4.3 on page 120.

In the interview, teachers were asked if they believed in using knowledge of students' daily life and culture for teaching mathematics. Most teachers agreed with this and gave examples of the way they would use *their* knowledge of daily life for teaching. However, most examples were cursory mentions of the phenomena in daily life where a mathematical concept could be applied. For some teachers, use of daily life examples was not central to doing mathematics but was useful to "create interest" (P3). **M3** felt that connecting mathematics to daily life meant that one needs to emphasize the importance of mathematical values like punctuality and accuracy in daily life, for e.g., coming to school on time and finding the accurate amount to be paid to shopkeeper (**M3**). Other examples were measuring the heights of all members of the family using a length scale (P2) and finding the height of the tree using trigonometry (**M2**). Teacher **P1** related knowing mathematics to proficiency in solving daily life problems like shopping, cooking, estimating the cloth needed for making a cushion, estimating the distance from the time taken to reach a place, etc.

No teachers talked about how daily life phenomena can be explored to elicit students' knowledge of mathematics gained from their daily experience or using the contexts in daily life for teaching mathematics.

Some teachers interpreted that connections to daily lives can be established by giving real life examples as representations/ models or as data for mathematical concepts. M6 described how, while teaching graphs to students he asked them to represent events from real life using graphs as in the excerpt below.

Excerpt 4.27

> You went to birthday party some got cake some pastry and some got ice cream so have to show the percentage on pie chart… real life situation like temperature shown after news…. So, the temperature is given for whole month… So, they represent in histogram and frequency polygon. If temperature very high, may be it was Jun, If low may be January…. This children correlate with life experience…. [I] liked doing all these things… children also enjoyed. (M6, personal interview, May 25, 2009)

P2 explained that she is not able to use daily life examples always, because of lack of time. She said, "If I am teaching one topic, 45 ideas will come and we get only 6 periods for maths in a week".

Inferences about beliefs

Teachers all agreed that mathematics should be connected to daily life but they also had doubts that it might get confusing for students or time consuming. The examples or contexts shared by teachers in the interview indicated that very few teachers used contexts to elicit students' knowledge of daily life or to discuss key mathematical concepts and meanings. Only a few were able to provide examples that illustrated meaningful connections between mathematics and daily life indicating that it is a peripheral practice which teachers find difficult to align with their focus on teaching procedures.

4.3.2.4 Equity in classroom participation

NCF 2005 promotes a vision of teachers engaging "every child" in doing mathematics, thus underlining equity as an important theme for mathematics teaching. The document identifies the "fear and failure" (p.14) for mathematics as one of the central problems that needs to be addressed. This is especially true for the way some students are perceived to be failures because of the social discrimination based on caste, class or gender. The issues of equity are related not only to the bias teachers hold against some students which influence their behavior towards them, but also to the opportunities teachers make available for all the students to participate in the classroom discussions and the extent of autonomy granted to students while solving problems. Teachers' responses to questionnaire items indicate that teachers, at least sometimes, do give opportunities to students to share their ideas and allow the use of mother tongue to express their ideas.

Table 4.14: Questionnaire responses to practices related to equity in classroom

No.	Item	% Almost never	%Some of the time	%Half of the time	%Most of the time	% Almost Always
2.4	Students come up with interesting ideas of their own to solve a problem.	4	62	8	23	4
2.10	Students express their ideas or reply to questions using their mother tongue.	27	31	19	15	4
2.11	I ask my students to write explanations and justifications through words or pictures in their notebook.	12	35	0	19	35
2.16	I tell students not to talk among themselves but ask their questions to me.	12	23	4	31	31

2.20	I ask students to discuss problems in groups.	4	12	12	38	35

The teachers' response to the interview for Q.16 and 17 (Appendix 2) threw light on equity issues in the classroom. The analysis of interviews revealed component practices that either supported or constrained establishing equity in classroom participation. For e.g., giving an opportunity to students to share ideas and giving attention and support to students from a socioeconomically weak background, supported establishing of equity. On the other hand, practices like labelling students, giving lower level questions to weak students to practice so that they pass the exams constrained the establishment of equity.

Teachers' response to questionnaire items indicated that they have a positive attitude towards adopting practices to support students' autonomy. However, interview responses indicated that they did not prioritize these practices. Only a few teachers were able to give examples of students' ideas that came up in the classroom. Those who did give examples believed that the students must have got these ideas from somebody older at home or elsewhere, indicating that they don't really believe that students can come up with their own ideas. However, 46% of the teachers shared that students are able to give examples of the mathematical concept being taught from daily life (2.6).

72% of the teachers also said that they ask students to work in groups most of the time (2.20) although few teachers reported this in the interview. Working in groups may or may not support equity depending on whether all the students get the opportunity to voice their thoughts but in comparison to a scenario where students hardly get any opportunity to speak up in whole class discussion, use of groups might provide more opportunities for sharing their ideas. 68% of the teachers reported asking students to show their solution to the class on the blackboard most of the time (2.24). However, if only bright students are called to the blackboard it may go against establishing equity although this is a better situation than students getting no opportunity to show their solution as the teacher shows the solution. Teachers' practices of asking students to give examples, showing their solutions, working in groups and using the mother tongue with the intention of eliciting students' ideas and giving equal opportunity to all students to share their ideas may be influencing the extent to which students came up with ideas of their own.

Teachers' practices for how they pose the problem determine whether students get the chance to come up with their own ideas. For e.g., 42% of the teachers responded that they give challenging problems to students but only some of the time (2.21). 34% of the teachers shared that they ask students to explain and justify using pictures some of the time, whereas 34% of the teachers did it most of the time (2.11). In the interview, only primary teachers reported using pictures.

Given so much focus on showing procedure, teachers were not able to provide an opportunity to students to share their own ideas or solutions. Teachers **M3**, **M4** and P2, said in the interview that students could come up with the procedures of their own. But deeper probing revealed that they felt that students can come to know different procedures from elsewhere (magazines, parents). They believed that only a few intelligent students can discover procedures and thus most students have to be taught.

Excerpt 4.28

> Children bring their original ideas, their world is very big. He learns from a lot of things other than the teacher in the school, e.g. internet, father in a specific profession. Very few children able to give justification and explanation on their own. (**P1**, personal interview, May 25, 2009)
>
> Justification and explanation they can arrive [at] if the child is bright. (P2, personal interview, May 25, 2009)

These excerpts indicate that teachers' beliefs about students capability hinder the way in which teachers conceptualize opportunities for students to share their ideas or solutions.

Teachers shared how they try to help students from poor homes by giving them special attention during free periods, making them sit on first benches. Some primary teachers said that they give emphasis on these students having correct records of work done in class in their notebooks by emphasizing that they copy correctly from the blackboard or from some bright student's notebook. Their talk also showed how they believed that these students are not interested in studies and disturb others.

Excerpt 4.29

> [poor students] I make them sit on the front bench only, and I make them write things, otherwise they don't listen. If they are sitting at the back, then they disturb others. They are least bothered about maths and any other subject and they don't want to copy anything. So always we used to make them sit in front bench....They won't listen but still I try to [make them] complete the notes at least, so that their parents or tuition teacher is able to help them. About 5-6 children are there in the class of this type. I used to tell good students to make them copy their notes at least. They won't write [by] themselves. So I have to find some students to make them write. (P2, personal interview, May 25, 2009)
>
> In class work done need to be revised at home. At home too, many distractions for

> the child. When we say the student is not learning, they say we have sent them to tuition. Because of time constraints and too many children, it's not possible to give personal attention to each and every child. We expect whatever done on the blackboard, step by step, the child should follow those instructions, so that, at least there is no mistake in the notebook record. The child should also be aware that, whatever child writes in the notebook, that should be as per teacher's instruction. That children don't do. Teachers are blamed that they have taught wrong. The teacher wrote wrong on the blackboard. It's not that. Student does it wrong while copying from the blackboard. The child should copy from blackboard with attention. (P3, personal interview, May 26, 2009)

Inferences about beliefs

Interviews indicated that many teachers were aware of issues faced by students coming from poor homes like lack of help at home but they limited their role to help in completing their notebooks. Some teachers also felt that girls find mathematics difficult since they did not talk much in the class. Teachers talked about how they try to help students whom they perceive to be at risk for failure by giving special attention and making them practice selected portions of the curriculum. The practices, however, indicated that teachers had biases against these students by having low expectations and believed that they are not capable of doing "difficult" mathematics. Thus bringing about equity in classroom participation was a peripheral practice as teachers' beliefs about students' capabilities from certain sections of society influenced teachers' behavior.

4.3.3 Conclusion

The change in curriculum did not bring about a significant change in teachers' practice since teachers had integrated new practices of doing activities and increasing student participation without moving beyond the focus on procedures.The core practices for teaching mathematics remained the same, with the focus on procedures, repeated practice, focus on speedy solutions, and following the textbook. Although teachers followed the textbook, they did so in the terms of using the content and problems and not necessarily the using the ideas of making sense of mathematics using contexts and representations which were implied by curriculum designers. There was hardly any talk by teachers of eliciting students' reasoning or giving them an opportunity to think independently.

These practices indicated that the core beliefs held by teachers about mathematics, teaching and students were more consistent with the transmission view of teaching and procedural view of mathematics. Teachers' focus on telling procedures and expectations for students to listen carefully indicated that teachers' believed that knowledge is transmitted. Teachers' insistence on practice for learning mathematics also pointed towards the belief of learning mathematics through memorization. Telling the steps of procedure as an explanation and when students make errors, again points to the importance teachers gave to teaching procedures.

Teachers' talk about the use of activities and connections to daily life to increase student interest and participation indicated the belief that mathematics can be made attractive and simpler through the use of activities and contexts from daily lives. I have termed these practices as peripheral since teachers prioritized covering of textbook material and students' performance in the examination as more important goals. They used the core practices of showing procedures and repeated practice to achieve these goals. These peripheral practices were used to generate interest rather than for developing understanding. Some teachers used activities to demonstrate procedures or reverted to showing procedures and practice after using activities. Here activities and contexts served to facilitate memorization rather than engaging students in doing mathematics.

Teachers did not believe that challenging problems can be given to students right at the beginning. They believed that one should move from simple to complex problems. In trying to adhere to this principle they avoided giving challenging problems to students. Another reason could be that they avoided student errors and giving challenging problems was considered as a risk which might create confusion in students' minds. They expected students from poor socioeconomic sections to perform poorly in mathematics. Although teachers expressed that good students can come up with solutions, shortcuts or explanations on their own, they believed that students come to know about them because this has been taught or told to them by other older people.

Although efforts have been made through educational reform to influence teachers' practice to move towards student-centered view of teaching and processes in mathematics, the core practices and beliefs of teachers remain aligned to transmissionist view of teaching and focus on procedures. In the following sections discussions on teachers' beliefs about teaching, and learning mathematics, nature of mathematics, students' capability and self are done.

4.4 Beliefs about teaching and learning mathematics

In the previous sections, it was discussed that the teachers' preferred practice were aligned with beliefs consistent with a transmissionist view of teaching and learning mathematics. The interaction between practices and beliefs has also been discussed in the earlier section. The questionnaire and the interview also included the probes that addressed teachers' beliefs directly. In this section, and in sections 4.5 and 4.6 to follow, teachers' responses to these direct probes concerning their beliefs have been analyzed. The discussion of the similarities and differences between responses obtained in the questionnaire and the interview is also included in this section. To an extent, repetition of the themes already discussed in the sections devoted to practice above is unavoidable. However, comparing these slightly different parts of the data is necessary to round out the picture about teachers' beliefs.

This section discusses specifically responses related to beliefs concerning the teaching and learning of mathematics. These beliefs will be discussed with regard to the following themes:

1. Teaching by transmitting procedures
2. Learning through memorization
3. Learning through discussion of ideas
4. Beliefs about good students
5. Beliefs about tasks

For each theme, comparison of teachers' responses related to practices is related to responses to questions specifically about teachers' beliefs regarding the teaching and learning of mathematics. Where there is strong agreement and consistency across beliefs and practices, the beliefs are inferred to have greater strength and thus as being core beliefs. However, if teachers indicate strong agreement for a belief but little or no evidence is found in either reported practices in the questionnaire or during the interview, then it is inferred to be a core belief but supporting a peripheral practice due to constraints in the system or lack of resources for the teacher.

4.4.1 Teaching by transmitting procedures

Data from the interview and questionnaire show that teaching of mathematics was focused on showing or explaining procedures and examples to students thus indicating that it is a core practice. Response to questionnaire items probing beliefs related to this practice (See Table 4.15) about teaching and learning mathematics indicate a varied pattern of responses.

Table 4.15: Beliefs about how to teach mathematics

No.	Item	SA%[2]	A%	U%	D%	SD%
4.1	The best way to teach mathematics is to clearly show the procedures (methods) to solve the mathematics problems.	12	44	8	32	4
4.3	Listening carefully to the teacher explain the mathematics lesson is the most effective way to learn mathematics.	8	36	16	36	4
4.9	The best way to teach mathematics is to show students how to solve some example problems.	16	28	12	36	8
4.14	The teacher need not explain how to solve all the different problem types or problem variations.	8	28	4	56	4
4.13	A teacher should explain things carefully in the beginning so that students can avoid mistakes.	24	68	0	8	0

For the first three items in Table 4.15, there is a spectrum of responses of the teachers from agreement to disagreement. The last two items show a more skewed pattern of response. A very large percentage (92%) of the teachers believed that the teacher should explain everything carefully in the beginning (4.13) so as to avoid students making mistakes. 60% of them disagreed that the teacher need not explain all problem variations. This points to a strong belief that the teacher must show students the solutions, rather than expect them to solve on their own, despite the varied response to items 4.1 and 4.9 above. This is consistent with their responses concerning practice, which indicated a procedural focus. For example, Table 4.14 shows that 96% considered the telling procedures as explanation (2.23) and showing procedures as an effective way to avoid students' mistakes (2.2). In the interview too, most teachers had reported showing procedures either before solving the problem or after giving students chance to think and respond to the question.

2 % SA= Percentage of teachers who strongly agreed to the item
% A= Percentage of teachers who agreed to the item
% U = Percentage of teachers who were unsure
%D = Percentage of teachers who disagreed to the item
% S= Percentage of teachers who strongly disagreed to the item

4.4.2 Learning through memorization

If one recalls the analysis about teachers' preferred practices, it revealed that most teachers engaged students in repeated practice of solving problems. However, the responses to related beliefs showed that some teachers agreed, while others did not.

Table 4.16: Beliefs about practice and memorization

No.	Item	SA%	A%	U%	D%	SD%
4.4	If student practices solving all the problems in the textbook two or three times, that is the best way to learn mathematics.	8	24	12	44	12
4.18	The key to learning mathematics well is to repeat the textbook exercises two or three times (or more).	4	28	12	44	12
4.20	The best way to teach mathematics is to explain one procedure at a time on the blackboard and then to make students practice it.	8	40	4	36	4
4.22	When students make errors, the best remedy is to make them repeatedly practice these types of problems.	20	48	16	8	8

Table 4.16 shows that about 66% of the teachers disagree that practice and repetition are important. However, in the context of students making errors, 68% of the teachers thought that repeated practice was important. The teachers' response to the practice part of the questionnaire (2.1, 2.8 in Table 4.4) showed that many teachers (50-66%) agree that they engage students in the practice of similar problems most of the time. However, the interview responses clearly showed that teachers thought that practice was very important in learning mathematics. Again, only one teacher (**P1**) expressed critical views concerning the memorization of procedures.

For the teachers who emphasized practice, they believed that it helps in "remembering" procedures, doing "fast calculation" and being "thorough" and developing students' confidence. For some teachers, practice of similar problems was a way to ensure marks for students in examinations. They focused on giving the practice of types of problems and the kinds of answers that are expected in examinations. Also, it was evident that some teachers show the solutions on the blackboard and then give similar problems to check if the students have "understood".

Excerpt 4.30

> Practice is required to keep what is learnt in memory so that they don't forget by the next chapter. (**M2**, personal interview, May 27, 2009)
>
> After practice, they understand better e.g. Multiplication table. I tell them to go

> through the table every morning and before sleeping. (P2, personal interview, May 25, 2009)
>
> After doing 3-4 sums you will get confidence and clarity.... After that student will be bored …and through practice he knows that I can do it successfully.... So practice is important to build up student confidence. (P5, personal interview, May 30, 2009)

One of the teachers, P5, felt that by practicing several problems of subtraction student can develop an idea that the result of the subtraction is always less than the minuend as a generalization.

Excerpt 4.31

> There are 14 toffees and you ate 6 toffees....How many of these toffees are left? Its a subtraction. Out of 14 toffees 6 are eaten so number of toffees is certainly less of that... but you cannot just hit upon [the number]...Here the child has to learn counting from 6 to 14 or back counting from 14...count six number back...so this they will get by practice only. (P5, personal interview, May 30, 2009)

However, there were teachers like **P1**, P4 and M5 who were critical of this practice of rote memorization through repeated practice. There were, however, subtle differences in teachers' views about 'what' is rote memorized as P4 and M5 talked about doing the same or same type of problem as rote memorization while **P1** talked about doing procedures without understanding conceptual basis as amounting to rote memorization. P4 and M5 shared how students even rote memorize the numerical aspects of the problem thus failing to solve a similar problem in exam with different numbers.

Excerpt 4.32

> Practice is needed but the extra practice makes the mind of the student blunt[...] Many students memorize the question in the textbook. If we change it in exam, they say that answer of this question in the book is this and this answer is not coming so question is wrong.... This means they have not understood the question... because it is by rote memory.... They have memorized through practice. That approach is wrong (M5, personal interview, May 29, 2009)
>
> Some teachers what they do, they give the same sum given in the class in the test. There is no application. That is rote learning. Your children will get 100% mark and [you will get] 100% result but what about their future? (P4, personal interview, May 26, 2009)

On the other hand, **P1** was critical of memorizing the procedure without understanding the conceptual aspects. She termed the practice of teaching mathematics by memorizing the procedure as "rote memorization" while giving the example of "putting a zero" for multiplying by tens in column method of multiplication without connecting it to the concept of place value.

Excerpt 4.33

> I don't repeat problems done in class. This practice is there in coaching classes which emphasize rote learning. (**P1**, personal interview, May 25, 2009)
>
> Tens ones were not clear. In our time "put 2 zeros" when multiplying by 100. This was not explained. If it is the second number [in column multiplication] then start after one zero. If you have to start for the third number then put 2 zeros or put a cross. We were told this. We rote memorized. We used to blindly follow. (**P1**, personal interview, May 25, 2009)Learning through discussion of students' ideas

Since showing procedures and memorization of the procedures are the core beliefs of the teachers guiding their teaching, it is likely that these beliefs will be inconsistent with the idea that students can share important mathematical ideas which would be worthy of discussion in the classroom. Teachers' response to beliefs and practice part of the questionnaire related to giving space and an opportunity to students to share their ideas is discussed below.

Table 4.17: Beliefs about students finding solutions on their own

No.	item	SA%	A%	U%	D%	SD%
4.6	It is essential that students express their ideas in classrooms to help them learn mathematics better.	56	44	0	0	0
4.7	A teacher should teach each topic from the beginning assuming that the students know nothing.	16	44	4	24	12
4.11	Students learn best if they figure things out for themselves instead of getting explanations from the teacher.	32	36	12	16	4
4.14	Only one method should be taught to students for solving otherwise they get confused.	8	28	4	56	4
4.16	Students learn better by discussing their ideas in the classroom.	32	64	0	4	0
4.21	Students should be encouraged to find different methods to solve a problem.	40	56	4	0	0
3.5	Given a chance most students can discover correct procedures (methods) for calculation although these may be different from standard procedures.	16	64	20	0	0
3.16	Students cannot discover procedures (methods) for calculation on their own. They need to be taught these procedures. (There may be rare exceptions.)	4	48	12	24	12

Teachers' responses in practices as well as beliefs about teaching and learning part of the questionnaire indicate that teachers believed and gave opportunities to students to share ideas. The majority of the teachers agreed that opportunities to express ideas (4.6, 4.16), figuring things out

(4.11), finding out different methods to solve problems (4.21) help in learning mathematics. With regard to practice, a majority of teachers did share that they ask students to give justification and explanation of the methods used by them (2.11), and discuss in groups (2.20) but did not allow students talking among themselves (2.16) (See Table 4.14).

However, teachers believed that students rarely come up with interesting ideas on their own and most teachers were not able to give examples of students' ideas. Only 36% of them disagreed with this statement. Interview responses, as well as responses to other items, suggested that teachers did not really believe that students can discover their own procedures. Their response to item 3.5 and 3.16 indicate that although 80% believed that given a chance students can discover procedures, 52% also agreed that they need to be taught these procedures since they cannot discover procedures on their own. This inconsistency between responses to these two items indicates that teachers do not believe strongly about students' capability. This is further confirmed by teachers' responses that 60% of the teachers taught assuming that students know nothing (4.7). This indicates that they do not take into consideration students' ideas for planning or discussion. Most teachers did not allow use of mother tongue by students to answer the questions, which indicates the limited opportunities for students to share their ideas in their classrooms. The discussion of interview responses in the section on peripheral practices (Section 4.3.2.4) on establishing equity in classroom participation further confirms that tensions existed between giving students' opportunities to share ideas, and considering students capable enough to have ideas of their own. Teachers' mostly expected students to share the ideas or procedures that they have come across in class or from elsewhere. This indicated that while majority of teachers believed in giving opportunity to students to share their ideas, they do not expect them to share any original ideas of their own.

4.4.3 Beliefs about good student of mathematics

Some teachers determined a student as being good in mathematics based on their performance in exams, consistency in performance and giving correct answers quickly (**M3**, **M4**, P2, P4, P5). (See section 4.2.1.3: focus on speed and shortcuts.) Although some teachers like P5 admitted that "all children are good, each may have weaknesses", they still thought that a few children have an innate "ability" to do mathematics due to which they are able to respond with answers quickly.

Excerpt 4.34

> Some children answer a little bit senselessly... very few only will tell, but they will tell the correct answer.... [The] method they will take this way that way. So especially this mixed fraction question quickly they will do. I will go and see how they got... and then application problem when you are teaching then how quickly they are grasping. (**M3**, personal interview, May 28, 2009)

There were very few teachers who considered reasoning and giving justifications, independent thinking, correlating with daily life as indicators of being good in mathematics.

In contrast, a few teachers who said that they value reasoning as an indicator of being a good student. **M2** shared that as a student she was motivated by being first in class but now thought that reasoning and thinking were indications of being a good student. M5 felt that being able to solve the problem does not indicate that a student is good in maths but the student should be able to correlate with daily life and give justification. **M1** thought "thinking, use of mind" as the aspects related to doing maths and a good student was one who, "asked thought provoking questions" and "will not simply accept what we tell them". P3 considered that besides accuracy, doing solution by different methods and independent thinking were characteristics of a student who was good in mathematics.

In the questionnaire, in response to Item 3.7, "Being good in mathematics means responding to a problem quickly and accurately", 48% of the teachers either agreed or strongly agreed with the statement. In Section 4.2.1.3, focusing on speedy solutions was discussed as a core practice. On the other hand, focus on reasoning was found to be a peripheral practice because teachers did not believe in students' capability. So although a few teachers might have a different conception of good student of mathematics, the predominant conception seem to be closely aligned to the capability of solving problems quickly. This indicates that most teachers believed strongly that solving problem quickly is an indication of being good in mathematics. However, it is difficult to categorize the belief about good students being quick as either core or peripheral belief for the group as a whole due to variations.

4.4.4 Beliefs about tasks

Some teachers believed that one should move from doing simple to complex tasks in the teaching of mathematics. 80% of the teachers agreed that one should not give difficult and challenging exercises to students in the beginning (4.12). There were a variety of views regarding asking probing questions to students that might create uncertainty and confusion among students. In general teachers avoided tasks or practices that might confuse students since they tried to avoid errors. Their efforts were to make things simpler or easier for students by helping them identify the operation or what is to be done in a problem, and use manipulative/drawings or contexts to make maths interesting. This is related to their sense of being a successful teacher as the one who is able to create interest, explain clearly to students and help students get good marks in the examination.

Teachers' belief about moving from simple to complex tasks and avoiding complex tasks in the beginning was evident in the interview when they discussed activities and questions from the textbook. For e.g., P4 was puzzled as to why 4 digit numbers were used in textbooks for finding fractions while students are not competent to deal with calculations with four digits numbers at that level (for e.g., students needed to find 1/3 of 6000 in the fifth grade textbook). This indicates that the teacher was not aware that the purpose of such problems in textbook is to develop students' number sense using certain numbers and support development of strategies rather than teaching calculation. It indicated a lack of understanding of the purpose and philosophy on which new textbooks are written where students are encouraged to explore different numbers using their reasoning from experience with smaller numbers. This is because in the earlier textbooks the sequence for teaching was prescribed as moving from operations with two digit number to three digit numbers and then to four digit numbers. Since most teachers felt that only good students can arrive at a solution on their own without being taught, they were not comfortable with giving challenging problems without telling students beforehand how they could be solved.

The above discussion indicates that teachers strongly believed that mathematics should be made simple and interesting for students to engage with so that confusion and errors can be avoided. This can be termed as one of the core beliefs of the teachers.

4.4.5 Conclusion

The analysis of beliefs through responses to questionnaire items on practices and beliefs, as well as to the interview indicated that beliefs like showing procedures and examples, learning by memorization and practice, and solving quickly are the core beliefs of the participating group of teachers. Sometimes teachers' responses to explicit belief statements in the questionnaire may lead one to think that they espouse student-centered beliefs. However, the broad pattern of responses, the emphasis that they placed on certain practices in their interview responses indicate otherwise. When teachers referred to "memorization", they interpreted it in different ways: ranging from memorizing the solution of a certain problem along with the calculation, or memorizing the procedure to solve certain type of problem, or memorizing the steps of an algorithm or a formula, all of which were considered important. This affirms that teachers aligned towards the transmission view of teaching. Further, a view of mathematics as procedures, discussed in the next section, also further reinforces these core beliefs and practice.

These core beliefs about teaching through transmission and learning through memorization had strong connection with teachers' belief about student capability which includes their perceptions about students from low socio-economic status, gender and perceptions of who they considered as a good student of mathematics. Teachers' appreciated correct and quick solutions by students. It appears that teachers' belief about avoiding errors among students is the main motivator for them engaging in the practice of showing procedures. This could be due to teachers' idea about being a good teacher might be dependent on their students getting good marks in examination. It could also be the result of teachers' feeling of 'care' towards students by avoiding putting students through the discomfort of perceiving maths to be difficult. This belief of mathematics being a difficult subject, exists in popular culture. Teachers too expressed that they perceive mathematics to be a difficult subject for students and feel responsible for making it simple and interesting for students. Thus beliefs about teaching of mathematics are closely related to the beliefs about mathematics, which is discussed below.

4.5 Beliefs about nature of mathematics

Teachers' belief about the nature of mathematics can have powerful implications for practice. Drawing from the research literature discussed in section 2.4.1 in Chapter 2 about beliefs and integrating the themes from the interviews, three dimensions to describe teachers' belief about nature of mathematics have been recognized. The first dimension refers to considering mathematics as a body of procedures involving calculations as compared to being a body of interconnected conceptual knowledge based on reasoning and justification. The second dimension refers to considering mathematics as an abstract esoteric subject without any connections to daily life. Items in part 3 of the questionnaire to probe teachers' beliefs about mathematics, were designed keeping these two dimensions in mind. The third dimension which emerged from the analysis of the responses is considering mathematics as easy or a difficult subject.

Teachers' ideas about mathematics were indicated through responses to interview questions that directly probed the teachers' view of "what is mathematics?" and if there has been any change in their views since their school education or since they have been working as a teacher. Other sources of teachers' views about mathematics were their ideas about good students of mathematics, their ideas about important tasks and best ways of teaching mathematics, which emerged as they described their typical mathematics lesson or the lesson that they liked the most.

4.5.1 Maths as body of procedures vs. interconnected knowledge

The questionnaire responses indicated that most teachers see school mathematics as based on or capable of justification, with a focus on reasoning. Their responses to items 3.1 (knowing why procedure works), 3.3 (maths having proper justifications), 3.9 (student justifying and checking procedure in case of doubt), 3.14 (understanding why tools work) indicated agreement by majority of the teachers. However, a majority of teachers also indicated in the questionnaire and interview that they strongly hold the view that mathematics is restricted to doing "4 operations" in calculations. Such views were more common among the primary school teachers rather than the middle school teachers. The responses to questionnaire items indicate that 76% of the teachers agreed or strongly agreed that mathematics is basically the four number operations (3.10). However, all teachers also agreed or strongly agreed with statements that "In mathematics, we can give proper reason or justification for all statements and procedures." (3.3). All teachers agreed with the statement that students need to understand why procedures work (3.1, 3.14). However only 56% believed that if they have a doubt, students can check and find justifications on their own (3.9). In the interview, however, teachers did not consider "why procedures work" as part of their explanation or that such justification of the procedure was needed. Neither did they indicate in their articulation that students can think of justifications and explanations on their own. This could be because either teachers had limited knowledge of why the procedure works, or considered it to be conceptually challenging for students. Teachers' responses in the interview on mathematical explanations, discussed in the next section, indicated that they could not give adequate conceptual explanations.

Table 4.18: Beliefs about mathematics as body of procedures vs. interconnected knowledge

No.	Item	SA%	A%	U%	D%	SD%
3.1	It is important for students to not only know procedures (methods) for calculation but also	68	32	0	0	0

	why the procedures work.					
3.3	In mathematics, we can give the proper reason or justification for all statements and procedures.	52	48	0	0	0
3.9	In mathematics, if the student has a doubt about a mathematical statement or procedure she or he can check and justify it on her/his own.	4	52	20	20	4
3.10	Mathematics is basically the four number operations (addition, subtraction, multiplication and division) and application of these.	16	60	12	4	8
3.13	Mathematical concepts have to be taught one at a time, there is not much interconnection between mathematical concepts.	12	16	4	64	4
3.14	In school mathematics, students must not only learn mathematical tools but also understand and be able to justify why the tools work.	52	44	4	0	0

As discussed earlier, teachers' focus on calculations and procedures was evident in their reported practice, thereby, implying an underlying belief that they are the important aspects of mathematics. There were some teachers who considered mathematics as limited to calculation like **M3**, M5 and **M4**, while there were other teachers who had experienced a change in their views due to higher education or due to change in the curriculum.

Excerpt 4.35

(Mathematics) It is fun....It is a plain subject which deals with numbers. (M5)

M2 talked about how she used to think of mathematics as "calculations" in her school days and in fact, had enjoyed being the first in class to find the answer. She talked about how school mathematics has undergone a change from calculation to focus on mathematical concepts.

Excerpt 4.36

> Maths is not just calculation. It's different concepts. like when you go higher, calculation doesn't come into the picture at all... You start with calculation. But then that's not the end. That is just the beginning. (**M2**, personal interview, May 27, 2009)

She described how the change in the curriculum has created tensions for her in the teaching of mathematics.

Excerpt 4.37

> When we were studying, we used to think if you are good at calculation you are good in mathematics. Now I think the concepts are to be clear – like why we are doing this, why is it so; how the child is thinking like even.… That's why we feel

> too much depth confuses children but maybe it leads to conception. (**M2**, personal interview, May 25, 2009)

M3 also affirmed her change in views from calculation to focus on hows and whys in mathematics and understanding the problems that children are facing. She said that the change in views of mathematics happened only after the graduate level education. **M4**, however, still held on to the view of mathematics as calculation with numbers.

Analysis of interview responses revealed that the beliefs held by teachers about mathematics were more complex and interconnected than what was suggested by the questionnaire responses. Calculations and procedures were the main aspects of mathematics that teachers had focused on throughout their career and during their education. However, the recent curriculum change had introduced tensions in teachers' views about mathematics, as the textbooks now contained reasoning based tasks, focus on concepts and use of multiple and informal methods before introducing algorithms. As a result, some teachers exhibited tensions between focusing on procedure versus concepts, some appreciated the change in mathematics, whereas others resisted this change by emphasizing the usefulness of knowing procedures to get correct answers. Many teachers expressed the change in curriculum as a change in "mathematics", as it radically differed from the mathematics that they had experienced in their school education. Teachers' belief about students' capability and their limited knowledge of conceptual explanations and justifications played a role in the tensions experienced by teachers. They either constrained change in mathematics teaching practice by sticking to focus on procedures or provided avenues to teachers for learning and using new ideas in teaching.

4.5.2 Maths as abstract vs. connected to daily life

Teachers' references to mathematics as abstract or being connected to daily life could be gleaned under themes where teachers talked about mathematics as being easy or difficult, use of concrete manipulative and using contexts from daily lives to talk about mathematics. As discussed earlier, teachers showed a positive attitude towards using contexts from daily life but used them as descriptions of the problem and rarely focused on the mathematical meanings within contexts.

Discussions about mathematics being easy or difficult illustrated how teachers make efforts to use concrete ways or connections to daily lives to teach the abstractions of mathematics. Primary teachers gave more examples of contexts from daily lives as compared to middle school teachers. **P1** was critical of her school experience that she was not taught "how to correlate mathematics with life" and thus she failed in solving real life problems like estimating the cloth needed for stitching a cushion. She said that she uses contexts, concrete materials and pictures as a regular feature in her teaching. She talked about how her views about mathematics have changed due to teaching and studying new textbooks.

Excerpt 4.38

> [About views of mathematics] there is a lot of difference. There [earlier] we used to think that we have to multiply only numbers and then find the area but how area is to be used [for estimating cloth] that we did not know.... Whether the area is needed or perimeter is needed all that I learnt while I am teaching the students. Every day I am learning, [My views have changed] and are changing further.... Earlier I just learnt the formula, learnt how to play with the calculation but I did not know how to use it in daily life. (**P1**, personal interview, May 25, 2009)

However, these views were not very common among primary teachers as some teachers insisted on focusing on calculations, giving them greater value than the understanding of contexts where mathematics is used. P2 shared that her views that the nature of school mathematics had changed from focusing on just calculations to "it can be applied to daily life". However, it was clear that she valued calculations as she liked the chapter in which there were a lot of practice sums based on the theme of "fish" ranging from 4 operations to topics like money, shapes, banking, speed and even fractions. She made many similar questions based on the questions given in the textbook.

In the questionnaire, teachers' views were probed for the connection of mathematics with the real world vs. mathematics being abstract. Most teachers' responses to contradictory items 3.4, 3.6 and 3.8 are in disagreement, indicating that teachers do recognize that real world examples should be discussed in mathematics class but not to a great extent. Perhaps, teachers held a more middle ground position regarding this belief. Strong disagreement to all three contradictory items indicates inconsistency. Thus establishing connections between mathematics and the real world emerges as a peripheral belief as compared to emphasizing calculations.

Table 4.19: Beliefs about connection of mathematics with real world

No.	Item	SA%	A%	U%	D%	SD%
3.4	Mathematics has strong connections with real world applications. These connections must be emphasized whenever we teach mathematics.	60	40	0	0	0
3.6	We should not emphasize real life examples too much	0	12	12	44	32

	in mathematics because they distract the students.					
3.8	Mathematics is abstract; there is not much connection between mathematics and the real world.	0	4	12	44	40

4.5.3 Maths as easy vs. difficult

Teachers frequently referred to mathematics as being easy or difficult either in reference to their students or in reference to their own experience of doing mathematics. Most teachers perceived mathematics to be a difficult subject and this belief guided their efforts to make the subject simpler for students. **M1** said that in school she used to teach students who used to find maths difficult and that she tries to make maths "simpler" for students by explaining from "A to Z...even in 8^{th} class, I will start from primary level" and thus "laying the foundation" to proceed to the topic.

A few teachers considered mathematics up to primary level as easy as it can more easily be represented using a concrete material as compared to higher mathematics. P3 considered representing calculations with concrete material as easy but considered algebra as difficult while **M3** considered Geometry as difficult.

Excerpt 4.39

> Mathematics means we used to think about calculation but geometry questions are little difficult like how to prove [theorems] yourself…. What is children's problem – teachers should know that.... If a teacher knows that and explains children will be very much interested in maths. (**M3**, personal interview, May 28, 2009)

She acknowledged that some students considered mathematics as a "fun" subject "dealing with numbers" but some were afraid of mathematics. Teachers like **M3**, **M4** and M5 expected that students good in mathematics can be good at anything, indicating a belief that being good in maths denotes some innate ability.

4.5.4 Conclusion

Along the dimension of procedural vs. interconnectedness, teachers' beliefs about mathematics were aligned more towards the procedural view indicating it as a core belief. However, teachers did experience tensions between these views of mathematics because of the change in curriculum and textbook, which expected students to engage in reasoning and understand interconnections between procedures and concepts. The belief that discussion on conceptual aspects and interconnections will confuse students acted as a deterrent.

Regarding the second dimension, primary teachers considered mathematics to be more connected to daily life as compared to middle school teachers and gave more examples of it. However, middle school teachers considered mathematics to be difficult for students and thus tried to make it simpler.

4.6 Beliefs about students

Decisions about what to teach and how to teach a certain topic are hugely influenced by what teachers assumed their students to be capable of. Teachers' beliefs about students from socio-economically disadvantaged background, girls and students who were weak in mathematics influenced their behavior and the practices that they adopted. Teachers had lower expectations and focused on repeated practice and memorization of expected problems in exams to make weak students pass. They justified this practice saying that these students' understanding is limited as well as that they do not get much help at their home.

4.6.1 Beliefs about students from socio-economically disadvantaged background

The questionnaire items in part 6 probed teachers' views about students, among which some items were related to their attitude and beliefs about the capability of students from a economically weak background (See Table 4.20, Items 6.10, 6.11, 6.13). 58% of the teachers agreed that students who do not have well-educated parents face the same level of difficulty in learning mathematics as compared to those with well-educated parents. 25% of the teachers felt that (6.13) students from poor homes can perform well because of being familiar with shopping in daily life, while 33% of the teachers were unsure and 42% disagreed. The items 6.8 and 6.9 probed how much the teacher perceives her role in helping students face difficulties to do well in mathematics. 53% of the teachers disagreed or strongly disagreed that some students face so many difficulties that teachers are unable to help them. 86% agreed that with the teachers' help students can overcome the negative influence of the home environment and do well in mathematics. These responses indicate positive beliefs towards students from poor homes. Teachers acknowledged that the students will be needing mathematics in their future careers (66%) and 53% agreed that most of their students are likely to study up to the degree level. However, the interview responses to Q. 16 and 17 (Appendix 2) showed how teachers had low expectations from these students.

Table 4.20: Beliefs about students from weak socioeconomic background

No.	item	SA%	A%	U%	D%	SD%
6.8	Some students face so many difficulties in mathematics that teachers are unable to help them.	4	29	8	50	8
6.9	With the teacher's help students can overcome the negative influence of their home environment and do well in mathematics.	0	0	4	67	29
6.10	Students whose parents are well educated and students whose parents are not well educated face the same level of difficulty in learning mathematics.	4	54	21	21	0
6.11	Students from poor homes tend to struggle in mathematics.	13	21	17	42	8
6.13	Students from poor homes can perform well in mathematics because they are used to buying things and doing other such work in their daily life.	4	21	33	38	4

In the interview, teachers were asked whether they have students from a poor financial background in their classes. Most of the teachers replied that they have a considerable number of students from poor financial background and parents of many students are not educated or not well educated. They considered it as a disadvantage since these parents are not able to help students in their work. Many teachers equated poor students with weak students in mathematics.

Excerpt 4.40

> Their basics are very poor. Retaining power is very less. Even if I want to teach them right from beginning, they will not retain that. They forget it very fast. Homework also they don't do. (**M1**, personal interview, May 27, 2009)
>
> Other children will do, but 10-15 children are from such background that they just throw the bag when they get home. We write in their notebook, note to their parents, who do not do anything, but we have to do it so that we can tell the principal when she comes for observation. If these children have not done their homework then I tell other children to help because I cannot waste time because of these 5 children… (P2, personal interview, May 25, 2009)

Some teachers thought that one or two students from among such students do well but most of them don't. However, there were a few teachers who believed in these students based on their observation or their identification with such students. For e.g., **P1** felt that students from weak financial backgrounds are better at mental maths.

Excerpt 4.41

> I don't face many problem in teaching [poor students]. It is not that they are coming from such homes, that is why they are not able to learn maths. At times they are better at mental maths… (**P1**, personal interview, May 25, 2009)
>
> Some are very intelligent. They do an activity and come to the blackboard. (P4, personal interview, May 25, 2009)
>
> I have a lot of children in my class from a poor financial background. I show sympathetic behavior towards them because I myself have come through the same stage. Their parents don't help them and if I also don't help them then they will not get help from anywhere. I call them personally, separately and teach or I bring him in contact with a good student. (**M4**, personal interview, May 25, 2009)

Teachers believed that parental help is an important factor contributing towards a child's successful performance in school, which students from poor homes do not have access to. Teachers compensated by giving special attention. But the teachers' focus was on avoiding errors and the practice of selective content to pass the exam, which indicates teachers' belief about the low capability of students from poor homes. Such beliefs colored their expectations and hence their efforts to engage these students in doing mathematics.

4.6.2 Beliefs about gender

In the questionnaire in part 6, four items probed teachers views about gender: 6.3, 6.6, 6.12 and 6.14. The questionnaire data indicate that more teachers are gender sensitive than not as 66% of the teachers disagreed or strongly disagreed with the statement that "For some reason, boys are better than girls in mathematics". More than 60% of the teachers disagreed that boys are more interested in maths than girls or that girls rote learn maths. However, 66% disagreed or strongly disagreed with the statement that "Boys and girls are equally fast in grasping mathematical concepts." This indicated that while teachers did not perceive any difference in interest or performance, still they felt that boys grasp maths more quickly than girls. In interviews, however, teachers acknowledged their biases while agreeing that boys and girls are to be treated equally. Thus there were tensions evident among what was expected from them as a teacher and what they really felt.

Table 4.21: Beliefs about gender differences in Students' capability

No.	item	SA%	A%	U%	D%	SD%
6.3	For some reason, boys are better at doing mathematics than girls.	0	25	8	42	25

6.6	Boys are more interested in mathematics than girls.	0	13	13	63	13
6.12	Boys and girls are equally fast in grasping mathematical concepts.	0	21	13	54	13
6.14	Girls find it difficult to understand mathematics and so they rote learn it.	0	17	17	50	17

Some teachers' views about the performance of students in mathematics and beliefs about their capability were gendered. Almost all primary teachers (except P4) acknowledged that girls were doing well at the primary level. However many thought that ultimately boys will perform better at the secondary level. The reason for this difference was given as girls may be hard working but boys are more intelligent. These views were expressed by middle school teachers also.

Excerpt 4.42

> Generally, we used to say that boys are good in maths but in my class, it is not like that... Girls are listening and doing. But in higher class boys will come up...Generally, nowadays I feel girls are doing better in maths than boys at primary level. (P2, personal interview, May 25, 2009)
>
> It is same... girls are industrious... but mostly what happens girls do the same type of problems...monotonous...that they are able to do..but I think boys have a better understanding than girls...but not always but sometimes. (P5, personal interview, May 30, 2009)
>
> Girls are more sincere. I would say boys are intelligent but they will not be very regular in completing their work. [Performance wise] Same only equal (**M1**, personal interview, May 27, 2009)
>
> I don't like the same sort of boys repeating the answer[i.e. speaking up in class]. I give equal chance to everyone in the class. (M5, personal interview, May 29, 2009)
>
> Girls are very hardworking, that is there. But whatever said and done they lag behind. (**M2**, personal interview, May 27, 2009)

Teachers **P1** and P3 however instead of using "intelligence" as the only explanatory principle for the difference in performance considered social aspects. They spoke of differences in the way girls behave as they grow up because of the influence of expectations communicated to them directly or indirectly for being a girl or the types of career choices that are portrayed as typical for girls. P3 had shared in her interview that she liked maths in school and was successful but did not take it up in college as all her friends were taking biology.

Teacher M6 felt that girls are better in mathematics based on the performance and pass percentage of girls in his class and considering results for board exams which generally show that girls have performed better than boys.

Excerpt 4.43

> Girls in my section/ for whom I am the class teacher/ girls are far better than boys... 4 or 5 girls are lagging behind somewhat and other girls are very good. (M6, personal interview, May 29, 2009)

There were only two teachers **M4** and M6 (both male middle school teachers) who indicated that gender had nothing to do with mathematics and indicated "studying" mathematics was important for doing well in it. **M4** talked about how effort can help in learning mathematics when he shared in his interview how he taught his own daughter.

Excerpt 4.44

> [Difference in boys and girls] Nothing like that. Those who study, whether they are boys or girls all are good. (**M4**, personal interview, May 28, 2009)
>
> My daughter was doing engineering. So the first year there was higher maths.... She told me, papa, I am not able to understand anything. After that 5 years continuously we both studied mathematics together. She got 90% marks. (**M4**, personal interview, May 25, 2009)

Most teachers considered girls as sincere and hard working but attributed innate intelligence in boys as a reason for their being good in mathematics. This is related to their perception of a good student of mathematics which was defined by speed and accuracy of calculations. Very few teachers thought that gender does not play a role in learning mathematics or emphasized the role of effort in learning mathematics.

4.6.3 Conclusion

Teachers indicated that they considered being good in mathematics or "thinking differently" as an "inborn ability" that only some students have. As a result, they believed that boys and students with better socio-economic status are more likely to be good in maths, indicating bias against students from poor homes and girls. Teachers' beliefs about students capability was closely connected with their belief about mathematics as procedures and learning as memorization since they decided students' 'level' of mathematical attainment based on their ability to remember procedures.

4.7 Beliefs about self as a maths teacher

Teachers' beliefs about the self were assessed along several dimensions including their perceived relation with mathematics, relation with the textbook, the role of the assessment system, confidence in one's knowledge, relation with administrators and what teachers consider as sources for their learning. Part 5 of the questionnaire had 25 items related to the teachers' beliefs about self along these dimensions. Most teachers responded positively to these items showing confidence in being mathematics teachers. The more interesting aspects about self were revealed in the interview. These aspects are discussed in detail below.

4.7.1 Role of Teachers' own experience of school mathematics

Some teachers had negative experiences of mathematics in their own schooling by expectations placed on them to learn everything by heart and had experienced mathematics anxiety like "dreadful nights just before mathematics examinations". They appreciated the "activity method" of teaching as they felt that it had helped reduced the fear of mathematics among the students (P4, P5).

Excerpt 4.45

> Mathematics was taught in such a way that it was scary because you have to by heart things and the teacher used to beat if you forget…. In our school days maths was taught 'this is like this' only. (P4, personal interview, May 26, 2009)

Some teachers (**P1**, M6, **M3**) were critical about their school experiences of rote learning procedures without understanding conceptual bases or its connections with daily life. They appreciated the new textbooks for enlightening them about these aspects. **M3** felt that the new textbooks are "more explicit" while the earlier textbooks had many things "hidden".

Excerpt 4.46

> Tens, ones was not clear. In our time "put 2 zeros" when multiplying by 100. This was not explained.... We were told this. We rote memorized. We used to blindly follow... (**P1**, personal interview, May 26, 2009)

However, there were a few teachers who had mostly had positive experiences of mathematics in their school education and held on to the view of mathematics as calculation. They recounted how they experienced success through "learning the rules by heart" (P3), "doing lot of practice" (P3, **M2**) and being rewarded for quick and accurate calculations (**M1**, P6). **M4** emphasized that maths should be taught through examples (solving example problems) thus indicating a preference to not use activities.

P2 who had had positive experiences of mathematics and calculations in her school days shared how she feels pressured to do activities as they are mentioned in the textbook.

Excerpt 4.47

> Some activity we have to do but we are not forced to do the activity [with the earlier textbook]. [Now] Questions and textbooks are designed like that. In earlier textbooks, only theory is given so that you do. (**P2**, personal interview, May 25, 2009)

Among the teachers who considered activities as a welcome change was P4, who had talked about her fear of mathematics due to the earlier method of teaching. She shared how it had helped in getting the students to think that mathematics was an easy subject.

Excerpt 4.48

> No, no, before that we use to just take chalk and teach it and slow learners and average learners they didn't used to like it. They thought maths is a difficult subject. So they would not do so much…. Now we teach mathematics in such a simple way, through activity method, that all children like it.... We still have calculation but the method of teaching has improved. Now children don't have to by heart things. (P4, personal interview, May 26, 2009)

The interview excerpts indicated that some primary teachers had a negative experience of learning mathematics and embraced the ideas in the new textbooks while some teachers with positive experience of mathematics resisted these ideas. The middle school teachers were not open to use of activities citing lack of time and pressure of completing the syllabus, although a few of them (**M3**, M5, M6) shared that they do activities from the textbook some of the time.

4.7.2 Confidence as a mathematics teacher

Teachers' openness to new ideas depends on the confidence a teacher feels in teaching mathematics using a particular textbook. Many of the teachers who were resistant to new ideas might have felt more confident teaching using the old textbooks rather than the new ones, which required them to learn new ways of doing mathematics and teaching.

In the questionnaire, Part 5 was about teachers' beliefs about self and the confidence they had in mathematics and in teaching mathematics. Items 5.3, 5.18 and 5.24 probed teachers' confidence in doing mathematics. Around 87 % of the teachers disagreed or strongly disagreed with a statement that they would give away teaching the subject of mathematics permanently. Around 90% of the teachers agreed or strongly agreed to finding many mathematics problems interesting and challenging. 25% of the teachers agreed that they avoided mathematics in their education wherever there was an option.

In Part 5 of the questionnaire, items 5.1, 5.11, 5.12, 5.16, 5.20, 5.23 and 5.25 (see Table 4.22) probed teachers' dependency on the textbook for teaching. Almost half of the teachers followed the textbook closely and felt that textbooks should give all the steps in detail. Half of the teachers said that they do not depend totally on the textbook for explanation. More than 80% of the teachers agreed or strongly agreed that they were confident to work out solutions to problems on their own and that while they go through the textbook, they prefer to teach in their own way.

Table 22: Teachers' response to Part 5 of the questionnaire about their use of the textbook

No.	Item	Strongly Agree	Agree	Unsure	Disagree	Strongly Disagree
5.1	When I teach mathematics, I generally follow whatever is given in the textbook.	8	46	0	42	4
5.11	If the maths textbook does not explain something clearly, it creates a serious problem for the teachers.	4	42	8	33	12
5.12	If something is not clear in the mathematics textbook, I am confident that I can work it out on my own.	38	50	8	4	0
5.16	I go through the textbook but prefer to teach in my own way.	21	58	8	4	8
5.20	In my view, maths textbook authors should give all the steps in a problem without skipping any step.	12	42	8	17	21
5.23	I don't depend much on the textbook.	8	50	12	21	8
5.25	There is no point in trying to doubt what is given in the mathematics textbook.	17	17	4	54	8

While describing themselves as mathematics teachers, many teachers described how their students, parents, peers, principals and inspectors have evaluated them as good teachers of mathematics. For most teachers the affection showed by students was a dominant part of their descriptions of their identity as a mathematics teacher.

Excerpt 4.49

> Teaching maths is my passion.... They (students) are not afraid of me .They are very friendly with me. They would say I am approachable. They say they like maths. This answer shows that they understand. (**M1**, personal interview, May 28, 2009)
>
> Students like me because sixth class students come and tell that they find maths very simple and they got 20/20 (since she was their primary teacher who had taught

> them in previous years). (P2, personal interview, May 25, 2009)
>
> May be because I am thorough in maths I can satisfy the children so when asked which is interesting subject – they say mathematics..... My students like me as a teacher but side by side they will say that she is very tough and strict. (**M3**, personal interview, May 28, 2009)
>
> I have always got very good during inspection. (**M4**, personal interview, May 27, 2009)
>
> As a mathematics teacher, students may not like me because I emphasize on practice, accuracy and neatness. (P3, personal interview, May 26, 2009)

The above excerpts indicate that teachers' confidence about being a good teacher depended on external sources of validation like students or authority figures and the academic performance of their students. It partly explains why teachers focusing on procedures were resistant to new ideas of teaching mathematics since the assessment system favored teaching of procedures rather than reasoning skills.

There were several occasions during the interview when teachers indicated that their focus is on students getting good marks in exams despite some teachers feeling that getting good marks in exams is not indicative of knowing mathematics. Some teachers like M6 even described themselves as a good teacher on the basis of the pass percentage of students they taught.

Excerpt 4.50

> In fact, my pass percentage has increased…. It was 90% last year. This year it is 97.5%. It is increasing slowly and slowly. It was never below 90%. It is always above 90% But in last few years, it has even been 100%. (M6, personal interview, May 29, 2009)

M2 described the lesson that she really liked was teaching heights and distances using figures and identifying the angle of elevation and the angle of depression by calling children to the board. She elaborated why she liked this lesson.

Excerpt 4.51

> Once student start making the figure they start getting the answer. This question carries 6 marks in Board exam. So when they are able to do I am happy that they are prepared for 6 marks. (**M2**, personal interview, May 26, 2009)

M1 talked about the pressure to get 100% result and how she perceives it as a constraint to focus on concepts while teaching.

Excerpt 4.52

> If you want to implement then exams-tests should not be there. After every one month, unit test is there. It will be common question paper set by others. Then the

> teacher has to get 100% result. It's a rat race. (**M1**, personal interview, May 26, 2009)

Although textbooks guide what gets done in classrooms, assessment plays a huge role in defining teachers' priorities and goals for teaching and thus affecting the learning opportunities that students get in the class. The assessment results contribute towards teachers' identity by being recognized as a good teacher by the society on its basis. Thus, teachers with mathematics background were more confident and maintained their focus on procedures since it leads to good performance in tests.

4.7.3 Role of administrators

In the interview, some of the teachers shared their experiences of interacting with inspectors, head teachers and principals who had observed their classrooms. These interactions had created a significant impression on the teachers' minds since they could recollect in detail their preparations for teaching, the appreciation and criticism that they received and the details of the lesson. Most teachers shared the lesson that they had prepared for inspection as the lesson that they had liked the most. However, it was unclear if it had any significant impact on daily practice. The lessons prepared for inspection were special since teachers put in a lot of extra effort to prepare them.

Excerpt 4.53

> [To create interest] I had to take activities and it so happened that HM came to my class and I made 4-5 children do sums on the blackboard and they could not do it... So finally she was also angry that why they are not able to do it...you send them back to their old class. So again I had to give remedial and so I started writing whatever I was doing with the children. So I wrote that almost for a month and even now I am writing… I don't know with this class what is going to happen. One and half month [I have been trying]... but the principal came to my class and that time I was introducing division. In division, we were going a little slow and he asked me you have to finish the syllabus and you are way behind the syllabus. I told him I will be able to complete it but this particular concept is taking time so I have to give them time. (P5, personal interview, May 30, 2009)

> I came to school after my mothers' death, and I came to know that there is inspection tomorrow. My diary was not complete. My mind was not working. I left it on God. When it is just tomorrow then I can't do anything. When I reached class, I tried to do equivalent fraction and unit fraction. It was the first day. Then, I used the chapati and explained the fractional number on board. Like you have eaten 2 chapatis and you are a little bit hungry. Then, you say give half more chapati. So I drew that on board and explained it to students. Then, I told them to do by tearing the paper. Then, that Inspector wrote a very nice note. The notebook that he picked

> was the weakest child in the class. I had not done his correction properly. He wrote, "it was related to life problem". At that time I had thought on my own to teach this way. That time the books were not like that. He did not highlight the correction work that was not proper...that gives encouragement. (**P1**, personal interview, May 25, 2009)

Teachers shared how the feedback is mostly about the pace of syllabus, student participation, delivery of the lesson and correction work. It was rarely about the content that the teacher was teaching or about ways to develop students' understanding.

4.7.4 Sources for learning

Most teachers did not talk about learning about teaching during the interviews but a few teachers hinted at the sources that they thought contributed towards learning for teaching. Among these sources, surprisingly in-service teacher education efforts were not mentioned. Teachers considered interaction with peers, students and parents as well as their own teaching as the source of their learning.

P1 viewed their growth as teachers as a result of learning from teaching. **P1** also talked about how she had developed preference over the years for teaching mathematics using pictures and representations and indicated how her teaching method had changed over the years. **M4** felt that teaching had helped him clarify the concepts in mathematics. M5 talked about how she had learnt to tolerate student mistakes over the years and had learnt from interacting with teachers and parents.

Excerpt 4.54

> That's what I said. In every way, we have changed. Dealing with students, I have changed. I was very strict earlier I never used to think that student can make mistakes. For me making silly errors and all these were all intolerable. I can't tolerate such mistakes. I used to expect quite a bit, but then over the years, I used to think that even for small problems, you can have a lot of discussions, their ways, you know interacting with many teachers. Many teachers we interacted, both within the school and outside the school, sometimes in train...different ways parents view maths from us. So these all things changed my mind. (M5, personal interview, May 29, 2009)

> I understood after I started teaching... that maths is nothing but very easy... but the only thing is understanding the concepts [and] making others understand what is children's problem... teachers should know that. They will be having this problem in this chapter on this topic. If teacher knows that and explains, children will be very much interested in maths. (**M3**, personal interview, May 27, 2009)

> My concepts have become clearer in maths. Earlier 9th, 10th, or B. Sc. Maths

> when we were doing, things are not that clear. Now, when I am teaching, now it is clear. (**M4**, personal interview, May 27, 2009)

4.7.5 Conclusion

Teachers' identity as mathematics teachers was influenced by the relation that they had with mathematics during their education, how much they depended on textbooks for making classroom decisions about content and teaching, and confidence in their knowledge of mathematics and teaching. Some primary teachers had a negative experience of mathematics and they welcomed the new ideas proposed in the new textbooks. Teachers viewed teaching and interactions with peers and students as a source for their learning about teaching.

4.8 Mathematical explanations

In the interview, primary teachers were asked to explain the division algorithm while middle school teachers were given a choice of giving an explanation of either the division algorithm or of the Pythagoras theorem. While all the five primary teachers gave explanations to the division algorithm, only three of the six middle school teachers gave explanations for the Pythagoras theorem. The remaining three middle school teachers chose to explain the division algorithm. Teachers' responses to the division algorithm can be summarized in the form of three observations. The first observation is that most teachers considered describing the steps of the standard algorithm as an explanation which has been discussed in the section on practices (4.3). Secondly, the conceptual connections with the procedure were not adequately addressed in the explanation and teachers were not able to come up with adequate justification for the use of procedure or why it works. Lastly, teachers rarely considered the contexts or used the real world examples in their explanations or considered using students' knowledge.

The interviews revealed that the procedures that were focused by the teachers were mostly standard algorithms. Most of the primary teachers gave the steps of division algorithm when asked to explain division. Only two primary teachers suggested the use of contexts for developing the meaning of division, while others explained the procedure focusing on digits rather than the quantity that the number signified. Teachers were aware of the alternative explanation given in the new textbooks by decomposing the numbers based on the place value of the digits and adding the partial quotients, which several teachers felt could be potentially confusing for students. Teachers also exhibited limited knowledge of alternative or informal methods and lacked knowledge of why algorithms worked. Teachers who had come to know about alternative methods because of the new textbooks, still preferred to teach algorithms since they felt that knowing many methods can confuse students. Some other teachers who knew of other explanations or methods from the textbook preferred not to use them for fear of potentially confusing students. For e.g., P5 knew the alternative methods to the standard short division where a student has the freedom to take away the divisor as many times as possible but she used it only as an introduction to division. Since it could be very lengthy, she asked students to practice problems using short division. P4 and P2 too felt that the emphasis on place value in the division was potentially confusing to students.

Excerpt 4.55

> One is the useful conventional method. Other method is there but I will teach by only one method because children will get confused especially small children. If expanded form we will use. Hundred-thousand method (based on place value), I won't teach them. It is very difficult for them to understand.... (P2, personal interview, May 25, 2009)

While explaining the short division procedure, primary teachers used digits of the number rather than focusing on the number as a whole or on the basis of digits and sometimes gave mathematically problematic explanations of the procedure. For e.g., P5 explained that one takes 36 (referring to the first two digits on the left) instead of 3 while dividing 36036 by 9 because one cannot take 9 toffees from 3 toffees.

Excerpt 4.56

> First I will say, this is 3, which is less than 9, so we cannot divide. Suppose 3 chocolates are there, we cannot take 9 chocolates from that. That student will understand quickly. So take 36. For 36, how many 9's are there? For that, they will say the table of 9 and up to this they have to tell the table. That is how explanation is given. (P2, personal interview, May 25, 2009)
>
> Just like you have names, these digits have names as ones, tens, etc.... In addition, we start from ones but for the division, we start from left to right...3 is less than 9 so you can't divide because 3 doesn't come in 9's table. So then you should take 2 numbers... small classes if you say about tens place, hundreds place, children will get confused... but you can say for addition and number names... We have to see that child is not confused and the child gets confident. (P4, personal interview, May 26, 2009)

In the explanation given by P2, it is mathematically problematic to say that "9 chocolates cannot be taken from 3" since the 3 there represents thirty thousand and not 3. In their efforts to make the algorithm easier to remember and use, teachers gave mathematically problematic explanations while focusing on digits of the numbers rather than their place values. This indicated that either they did not know connections between division procedure and place value or believed that such an explanation would be inaccessible or unimportant.

The three middle school teachers who described the division algorithm described the procedure in a manner similar to P2. The other three middle school teachers proposed the use of activity for Pythagoras theorem for verifying the theorem by e.g. measuring the sides, using different paper cut outs of squares having sides equal to the sides of the right angled triangle. The teachers did not go beyond the verification aspect of the theorem. **M3** felt that one will have to tell students the relationship between sides of the right angled triangle after doing the activity of measuring sides and then squaring them. **M4** proposed to use the activity method but was not very confident in explaining how he would do the activity with students and what he would do after students had measured the side and verified the Pythagoras theorem.

Excerpt 4.57

> ...whether the square of smaller two sides [of paper cutouts] is coming equal to the square of the hypotenuse or not. That means Pythagoras theorem is verified... (**M4**, personal interview, May 26, 2009)

Teachers' explanations of the Pythagoras theorem indicate that the main focus is on verification using particular examples and conceptual aspects like similarity, the area of triangles and their relation to their sides, the generality of the theorem, justification and proof are not discussed in detail.

Use of context/ real world examples in explanations

The questionnaire responses indicated that while all the teachers agreed that mathematics has real world connections that need to be emphasized, some teachers also believed that real world examples can distract students. This indicated the challenge that teachers faced in connecting and developing mathematical ideas using real world artifacts. Similarly, teachers rarely used or referred to a context to explain the problem or the procedure and focused only on the calculation. As described above, when primary teachers were asked to elaborate how they would give explanation and justification for a division question (36036 divided by 9), most primary teachers resorted to telling the steps of the procedure for this problem. **P1** used the context of sharing to help make sense of the division. She said that she would use this context if the student did not understand after doing the standard algorithm of the short division as she believed that both concept and procedure should be discussed.

Excerpt 4.58

> ... if you divided this amongst people then how much you will get. See again has everybody got equally? [...]Then even if he cross checks by his own method even if it is the addition, he should see what has happened to the number, whether he got the number. (**P1**, personal interview, May 25, 2009)

Here **P1** gave the example of how she would use sharing context to explain to a student who has not understood the division algorithm and would allow students to use their own method if they can justify it. Such thinking was rare amongst primary teachers, although the new textbooks laid emphasis on the use of contexts, reasoning and use of informal methods. A case study of teaching by **P1**(Pseudonym- Nupur) is described in the Sub-study 3 in Chapter 6.

The procedural focus was so dominant in teaching that teachers conflated showing the steps of the procedure as the conceptual explanation itself. During the interviews, teachers themselves did not know the conceptual explanation or considered conceptual explanation as potentially confusing for students. Some primary teachers did use contexts or examples to explain a problem, while middle school teachers' explanation remained in the realm of mathematical explanation. Explanations given by both primary and middle school teachers were mostly mathematically inadequate since they did not consider what the digits in the number represented and what is the effect of the operation on it. Although most teachers disagreed with the statement that there are not many interconnections between mathematical topics, many teachers stated that only one procedure or concept should be discussed at a time, otherwise students may get confused. They also did not themselves discuss the interconnections between the concepts and procedures. Thus teachers' practice of telling steps of the procedure as an explanation is supported by the belief that conceptual or contextual explanation might be confusing or distracting for students. This aligns with the transmission view of teaching and procedural view of mathematics. Although teachers indicated sensitivity towards reasoning and developing an understanding of mathematical concepts, their inadequate knowledge of key underlying concepts in procedures and their interconnections relegated the practice of reasoning as a peripheral one while telling the steps of the procedure seems to be the core practice which aligns well with the view of mathematics as procedures.

4.9 Discussion

4.9.1 Relation between core and peripheral practices

Teachers' responses to the questionnaire and interviews indicated that in spite of educational reform and the introduction of new textbooks to bring about the focus on making sense of mathematics, the focus is still largely on the teaching of procedures through memorization. This is indicated more by preferred practices reported by the teachers in the interviews as compared to responses to the questionnaire about beliefs or frequency of practices.

The practices reported by the teachers in the interviews and the questionnaire included the emphasis on memorization and practice of procedures, showing examples of solution before asking students to solve a similar problem, focusing on speedy solutions and following the textbooks closely. Thus, one can conclude that these practices lie at the core of the teachers' teaching of mathematics since most teachers strongly agreed to the items related to these practices in the questionnaire as well as indicated by their agreement in their interview responses. There was an integration of some new practices like group work and activities to increase student participation, which can be inferred as lying at the periphery of the teachers' practices since the purpose of these practices was still learning of the procedures and teachers avoided discussion of conceptual aspects while implementing them. Similarly focus on reasoning, connection to daily lives and equity in student participation were also practices that were peripheral since the teachers do not prioritize them over practices for teaching procedures.

The findings indicate that practices and beliefs lie on a continuum with transmission and student-centered beliefs and practices at the two ends of the continuum. The findings discussed in this chapter suggests that the core practices that the teachers reported may align closely to a transmission view of learning mathematics that emphasizes obtaining correct solutions, explicit teaching of solutions by showing, repeated practice, and avoiding or dealing with errors by emphasizing correct solutions.

Within this group of teachers, there might be a few teachers like **P1** who indicated beliefs towards student-centered end of the continuum, since they allowed the use of alternative procedures in class, valued reasoning of procedures, and used activities for conceptual discussion. Other teachers too indicated some of these beliefs, but it was not consistent across questionnaire and interview and between beliefs articulated and reported practice in the interview. However, even **P1** reported the use of transmission based practices like showing procedures, the practice of problems and following the textbook closely.

4.9.2 Why some practices are core and other peripheral?

The core practices identified are likely to be shaped by the routines that teachers have developed over years of teaching. In-service teachers with many years of experience are bound to develop these routines along with the repertoire of component activities which they use depending on the context. In the interview, many teachers also justified why they preferred the core practices by elaborating on how they helped in learning mathematics for e.g., memorization and building confidence through practice and speed through shortcuts. Core practices may be supported by more than one core belief. For e.g., showing procedures as a core practice can be supported by core beliefs of mathematics as procedures as well as the belief that students are not capable of coming up with solutions on their own and need to be told the procedure. These aspects make core practices stable and difficult to change in the light of educational reform.

The findings from this chapter indicate that core beliefs together form a coherent stable structure, as these beliefs are in alignment with each other and support the adoption of practices that align with these beliefs. For e.g., the practice of showing procedures and examples aligns with the that of having students practice similar problems to learn the procedure or to copy the solutions from the blackboard and also to give the steps of procedure as explanation again and again if a student has not understood. These core practices are connected to each other by the core belief that mathematics consists of as procedures and learning as memorization. Further, these practices and beliefs are at the core of teachers' identity as they construct their sense of self from their students' performance on the tests and exams which evaluate their capacity to remember the procedure to solve a particular problem. Teachers' years of experience of learning and teaching mathematics focused on procedures and knowledge of teaching in the manner, which supports the transmissionist view further, adds the stability to this core structure. This makes this beliefs structure resistant to educational reform efforts where change is sought through the change in textbooks. The beliefs held by most teachers constrain their change in practice, thus having the impact of reform efforts on the way mathematics is actually taught in the classroom.

Peripheral practices, on the other hand, could be the result of peripheral beliefs which are weakly held by the teachers, as a result of conflict between teachers' closely held beliefs and beliefs embedded in adopting a practice. Conversely, a teacher may hold a strong belief but may not be able to take up that practice frequently because of not having resources or because of constraints in the system for implementation. Beliefs and practices may not be aligned to each other thereby creating conflicts for teachers and thus resulting in certain practices remaining peripheral. The peripheral practices discussed in the chapter like the use of activities, concrete materials, connections with daily life do not align with the deeply held beliefs about mathematics and teaching-learning related to teaching procedures and thus remain peripheral in the teachers' use for largely superficial compliance with the reform agenda. Alternatively, teachers may use these practices, but as some interview data suggests, they may use it for the purpose of aiding memorization or making mathematics interesting.

4.9.3 Relation between beliefs and practice

There are various factors that play a role in the type of practices engaged by the teachers in teaching mathematics. The context of curriculum reform plays a role in the incorporation of the peripheral practices discussed in this chapter in the teachers' repertoire of practices. The various efforts for curricular reform did introduce tensions in teachers' beliefs about nature of mathematics and its teaching. However, strong belief about the importance of learning procedures, belief about teaching by telling, belief about the limited capacity for students to think and reason, limited knowledge of specialized content knowledge and pedagogical content knowledge, and the textbook culture in schools contributed towards constraining the change in beliefs and practices of teachers. On the other hand, teachers' belief about mathematics being an abstract subject which needs to be made easier and interesting to students, helped in integrating the introduction of activities, teaching aids and connections to daily life in their teaching. However, the intention was not towards eliciting students' prior knowledge or building their conceptual understanding as intended by the curricular reform. Instead, teachers hoped that adopting these practices will help in increasing student participation and interest in learning mathematics and will make memorization of procedures easier. These efforts to increase students' participation in the classroom did not necessarily transform into opportunities for students to express their ideas or invent informal methods.

Teachers' belief about mathematics as an abstract subject made them look for ways to make it simpler. This was also supported by low expectations from students. These beliefs made it easier to integrate the practice of using activities and teaching aids since teachers believed that they would make it simpler. Teachers' low expectations from students may have led to them believing that students cannot arrive at solutions on their own, making mathematics appear as a fixed body of knowledge which needs to be learnt and not discovered. Further, their belief about poor students not being good at mathematics, made them select lower level content for students from poor homes. The expectations were also gendered and teachers exhibited a variety of views about performances of girls, some considering them to be inferior, others considered no difference while some cited how girls are performing well in mathematics in board exams every year. These low expectations could result in blame being attributed to these students rather than the teacher when they are not able to perform successfully. Thus, these beliefs about students, mathematics and teaching together may have contributed towards teaching by transmission even when doing activities or using contexts from daily life.

4.9.4 Relation among beliefs

The difference between the beliefs that teachers indicated in the questionnaire and the interview and the beliefs inferred from the reported practices during the interview is similar to the difference between "espoused beliefs" and "enacted beliefs" (Even & Ball, 2009), building on the idea of espoused theories and theories in action proposed by Argyrus and Schon (1978). The data shows that teachers agreed to both transmission and student-centered view on practices, although with different strengths, in the questionnaire and the interview. However, the beliefs inferred from reported practice indicated a much stronger influence of transmission beliefs. To explain teachers' practice and beliefs and their relation with respect to the strength of relation, the distinction between practices has been introduced as core and peripheral practice as well as between core and peripheral beliefs.

The reported practice are cognitive images of how teachers view their practice rather than objective descriptions of their practice. Therefore they are indicative of beliefs held by teachers since it involves some generalization and reflection by the teacher to report their teaching. Core beliefs are reflected in the core practices, while articulated beliefs which are not reflected in practice or were not given due importance might be more peripheral in nature. If teachers reverted to a particular practice or considered a particular practice to be more useful than others, then it too can be termed as a core practice guided by core belief.

Core beliefs get reflected in practice and thus guide the action of teachers while peripheral beliefs are enacted based on the priority of the goals held by the teachers. The idea of articulated and enacted beliefs help in describing the relation between beliefs and practice, but the idea of core beliefs and practice helps in further explaining why some articulated beliefs get enacted (as they belong to the core) while others are just articulated or enacted with much lesser frequency since they are peripheral in nature. The idea of core and peripheral nature of beliefs has been proposed earlier by Pajares (1992). However, the teachers' articulations indicate that beliefs, knowledge and practice are intricately connected and thus need categories like the core and peripheral practices corresponding to core and peripheral beliefs.

Teacher beliefs, past experience of mathematics and knowledge acted as filters to determine the purpose for which practices and resources were selected by teachers for use in the classroom, sometimes different from the purpose intended by curriculum designers . Teachers' own identity as "curriculum implementers", defined the way they responded to questions as they felt obliged to align with the ideas embedded in new curriculum framework although they did not believe or had insufficient knowledge to support implementation of these activities (Kumar & Subramaniam, 2013). Teachers' exposure to ideas of curriculum and textbooks could be the reason why some teachers agreed with student-centered view in the questionnaire but were unable to enact them in practice as indicated in the interviews. Because of the social position of teachers as "curriculum implementers", it is difficult for them to acknowledge views different from the ones prescribed by the national curriculum. This further increases the complexity for ascertaining beliefs as articulated beliefs might not be "true" beliefs held by teachers while teachers' practice might be a better reflection of the beliefs held by teachers in form of "enacted beliefs".

Several studies have indicated that different contradictory beliefs can co-exist. This is indicated by the tensions experienced by several teachers between the focus on calculations and reasoning skills. Teachers who were against rote memorization appreciated the ideas proposed in the new textbooks and were more open to change their practices as compared to those who believed that repeated practice of similar problems was necessary to learn mathematics. The latter set of teachers critiqued the lack of enough practice sums for mathematics in the new textbooks which they consider as the key strategy to learn mathematics. Use of activities had provided teachers with new examples of how students can be creative in mathematics, paving the way to develop beliefs conducive to allow opportunities for exploration and imagination. Tensions faced by teachers to develop reasoning also have a potential to develop beliefs conducive to reasoning if support is provided to teachers to develop their own reasoning, and to develop questions and tasks that promote reasoning and assessing student answers for their depth of understanding of mathematics.

4.9.5 Role of knowledge in the relation between beliefs and practice

Another aspect that plays a role in defining whether a practice gets adopted as a core or remains at the periphery is the teachers' knowledge of mathematics required for use of that practice as well as knowledge of how to use it with students. This corresponds to pedagogical content knowledge and knowledge of students' thinking. Practices like engaging in reasoning, justification, communication, connecting within and across mathematical topics and with daily lives require knowledge of mathematical processes while teachers' themselves perhaps have been using predominantly the knowledge of procedures and how to teach them. For e.g., although alternative procedures to the standard division algorithm have been included in the maths textbook, some teachers believed them to be confusing for students and thus did not give them much emphasis while teaching. Teachers' lack of knowledge of conceptual connections with procedures, limited knowledge of alternative procedures, why procedures work, representations, connections between representations and designing activities for eliciting and building on students' thinking could also be the reasons why these practices remain peripheral. These knowledge gaps constrain teachers from adopting and even trying out practices like focusing on reasoning and justification, use of contexts or establishing equitable classroom participation.

One of the major reasons why tensions experienced in beliefs did not impact practice was the knowledge of representations and why procedures work which was found lacking in all teachers as discussed in section (4.8) on mathematical explanations. They also had limited knowledge of student thinking as indicated by how teachers' responded to student errors – instead of addressing conceptual gaps in students' errors, teachers repeated the steps of the procedure to help students arrive at the correct solution. Specialized content knowledge and pedagogical content knowledge have been discussed in the teacher education literature as an important part of mathematical knowledge for teaching and thus may have an impact on teachers' beliefs and practice. The new textbooks have taken a step towards developing this knowledge by including informal methods and multiple methods before discussing algorithms in such a way that understanding of why procedures work can be developed. However, teachers still preferred standard algorithms for their efficiency since informal methods can be lengthy. Teachers thus failed to appreciate how this knowledge is useful in learning mathematics and needed exemplars that demonstrated the value of knowing different methods and being able to connect these different methods. Thus the relationship between beliefs and practice might be rendered more complex by this additional factor – knowledge about mathematics, and about its teaching and learning, which may play the role of an important intervening variable in bringing about belief change. This calls for studies where teachers' development of knowledge is seen in relationship with the change in "enacted beliefs" that are held by the teacher and exploring the process through which such knowledge can be developed. Sub-study 4 of this thesis is denoted to explaining this issue.

4.9.6 Role of Systemic factors

The discussion in the sections above also indicates that part of the reason why teachers, though aware of alternative practices suggested in new textbooks, still prefer to focus on the teaching of procedures and practice is to follow the centralized timetable in completing the syllabus and the pressure to teach to the test. The beliefs of mathematics as being limited to calculation and knowing procedures and learning by memorization of procedures are the core beliefs that support each other and have developed as a result of teachers' own experience of mathematics in their own education as well as the beliefs about mathematics prevalent in the culture. Mathematics textbooks for many years have focused on these aspects of mathematics, thus building an understanding amongst the general public and teachers that knowing procedures are important. This has been further exacerbated by the assessment system emphasizing these aspects. These remarks indicate that just by exposure to a different curriculum or textbooks it may not be possible to change teachers' "enacted beliefs".

4.9.7 Methodological insights

Finally, Sub-study 1 also raises questions on the use of questionnaires to assess beliefs of teachers. The questionnaires afford assessment of a large number of teachers but fail to capture the complexity of beliefs as to how different beliefs are connected with each other and the factors that might influence the activation of some beliefs thereby influencing teaching. Agreeing to an item in a questionnaire does not rule out the existence of other conditional factors or other connected beliefs that exist. For e.g., teachers agreed that students can come up with their own procedures but probing revealed that they were thinking only of a classroom situation where students might share procedure learnt from magazines, parents or other sources. For other items where teachers agreed with a more constructivist view in the questionnaire, the data from the interview was inconsistent with beliefs indicated in the questionnaire. The analysis indicates that much of the inconsistency, conflict and tension between beliefs can be inferred even from reports of practice, not only from observations of actual practice. In later chapters, it will be discussed how the beliefs and practices reported here connect to discussions in teacher development workshops and to actual classroom practices observed.

The findings described here as well as findings from studies conducted elsewhere suggest that teachers' enacted beliefs are more resistant to change than their explicit assent to reform-oriented views. Further, the findings indicate how practices, beliefs and knowledge interact with one another. This may have implications for teacher development programs, where there is a need not only to create spaces for reflection on and revisiting of beliefs but also for strengthening teachers' mathematical knowledge. This issue will be focus of Sub-study 2 of the thesis.

5

Professional Development Workshop – Design and Enactment

5.1 Introduction

In the previous chapter, teachers' practices and beliefs related to mathematics and its teaching and learning of mathematics, students as well as self-efficacy were discussed. The discussion highlighted how, in the context of educational reform, teachers' core practices and beliefs are still rooted in a procedural view of mathematics. Teachers assume that mathematical knowledge can be transmitted to students by showing the steps of procedure or solution of an example followed by repeated practice of problems. Adopting the situative perspective on professional development, the professional development (PD) workshop provided opportunities to teachers to reflect on their teaching practices and beliefs as well as provide examples and vision of alternative approaches for teaching mathematics. So, instead of explicitly asking teachers to change their beliefs and practices, the workshop implicitly provided teachers food for thought to motivate them to reflect, explore and change their practices. The analysis of design and enactment of the workshop constitute Sub-study 2 in the larger study. In the larger study, this PD workshop performed an important function of preparing the ground for the Sub-study 3 and Sub-study 4, through an intervention that made teachers more aware of their own beliefs and relation between their beliefs and practices as well as awareness of alternative approaches for teaching mathematics.

In this chapter, the background of teacher professional development workshops conducted in the Indian context is discussed with the need for rethinking the goals of the workshops in the light of new policy initiatives. I describe the goals, principles and the framework adopted for design and enactment of the PD workshop along with descriptions and examples of the different types of tasks and sessions planned for the workshop. The design aspects were reconstructed through an analysis of the workshop data supplemented with discussions among the researchers who designed the intervention. The design aspects were not explicitly articulated prior to the workshop, but emerged in the course of the analysis of data. The chapter describes the three principles which were prominent in the design and enactment of the workshop – (i) situatedness in the work of teaching, (ii) offering challenges to teachers to revisit their knowledge and beliefs, and (iii) developing a sense of belonging to a professional community. Subsequently analysis of workshop sessions is presented with examples of episodes to illustrate – (a) how the three principles and goals of the workshop design shaped the tasks and enactments of those tasks, (b) how authenticity of the tasks and enactment lead to exercise of agency by teachers and teacher educators and (c) how exercising agency contributed to teachers' reflection on their beliefs and practices supported by their sensitivity towards students' thinking.

The analysis unpacks the precise nature of the teacher professional development (TPD) intervention, and provides the backdrop to interpret the findings of the subsequent sub-studies. Further, the analysis demonstrates a model of implementation of TPD workshop that has underpinning it a philosophy of teacher development that stands in contrast to the dominant approach prevalent in the Indian context. The aim of this chapter is not to evaluate the workshop for its effectiveness but to analyze and identify the episodes in the workshop which can illuminate our understanding of (I) teachers' beliefs and knowledge and (ii) how they engage in a social setting of professional development workshop to change them.

5.2 In-service TPD programs in India

As emphasized by the National Curriculum Framework for Teacher Education (NCFTE 2009) discussed in Chapter 1, there is a need to develop a clear vision of the goals that programs must achieve and the means by which they can be achieved. Most in-service TPD programs in India are designed in response to the need of curriculum reform and view teachers as agents of the state, who implement the reforms rather than as participants in the process of reconstruction of the curriculum. Underlying this is the assumption that teaching can be changed by directing changes in the content or structure of interactions in classrooms while not directly addressing the teacher's own conceptions of teaching, learning and mathematics. In-service TPD is seen as training for content or pedagogy, mostly revolving around the changed curriculum, but not necessarily as important for continuous teacher development. Content-focused interventions often consist of lectures delivered by "experts" and the mathematical content is typically divorced from the context of teaching and learning. Another common focus of TPD programs is "how to teach a particular topic". This may appear to be close to the work of teaching and hence directly relevant to teachers. However, there is a large variation in the contexts and life experiences that students bring to the classroom and teachers need to be flexible and adaptive in addressing the needs of a student (NCTE, 2009). Instruction to teachers during professional development is largely is guided by a *transmission model*, where recommendations on how to teach a topic tend to be recipe-like. As the literature reviewed in Chapter 2 indicates, the effectiveness of such an approach is limited. Further, the approach is not consistent with the vision articulated in the NCFTE 2009. Teachers need to develop their own vision of the changed goals of instruction and adapt their teaching in self-determined ways to meet these changed goals.

As discussed in Chapter 1, workshops are an important component of TPD programs on which time, effort and government resources are spent. The vision underlying most of these programs restrict teachers' agency to implementing a new textbook, a pre-designed pedagogy or a prescribed assessment technique. In my view, however, TPD programs need to have a broader vision of the needs of a teacher as a developing professional, view the teacher as an 'active learner', and must address issues of knowledge, beliefs, attitudes and practices in a comprehensive manner, rather than in the narrow context of a particular reform.

The review of literature in Chapter 2 indicates that research studies in other countries have pointed to pedagogical content knowledge and subject matter knowledge as useful constructs to describe essential knowledge for teaching (Shulman, 1986; Ma, 1999). However, it is rarely the central focus of any phase of teacher education in India (Naik, 2008; Kumar, Dewan & Subramaniam, 2012). Thus, providing opportunities for deepening teachers' knowledge of mathematics and of pedagogy revolving around mathematical practices can be considered to be one of the central goals for TPD programs.

Bringing about change in teachers' knowledge of mathematics relevant to teaching is clearly a challenging task, but only partly addresses the TPD need. As discussed in Chapter 2, teachers' beliefs also strongly influence teaching practice and determine what teachers notice in the classroom (Thompson, 1992; Phillip, 2007). In the Indian context, commonly held views include the belief that mathematics is a body of knowledge consisting of known solutions to a well-defined set of problems and that not all children are capable of learning mathematics (Kumar & Subramaniam, 2013). A study by Dewan (2009) indicates that such beliefs, which stand in contrast to the ones envisioned in the National Curriculum Framework, are held by not only teachers but even administrators, faculty members and directors of teacher education institutions, thereby indicating the extent of the challenge to implement the new framework. This points to the need to create spaces where teachers articulate and reflect on the beliefs that they hold while their professional identities are respected. Within such spaces, teachers need to not only experience alternative ways of doing mathematics, but also to build an awareness of and sensitivity to students' mathematical thinking (Kumar, Subramaniam & Naik, 2013).

Further, as seen in Chapter 2, research studies have illustrated how the development of professional learning communities contribute to teachers' professional growth, by providing a site for articulation and reflection on the beliefs, for sharing the knowledge held and practice adopted by the teachers.

As discussed in Chapter 1, in the Indian context as elsewhere, the goals of TPD programs need to include

- Enabling teachers to develop a vision for the changed goals of instruction and become "active learners",
- Providing opportunities to make teachers' knowledge and beliefs explicit,
- Strengthening teachers' knowledge integrating content and pedagogy,
- Building on beliefs through reflection and engagement, and
- Fostering professional communities as spaces for developing shared understanding about teaching and learning of mathematics.

In this chapter, how the components and interaction in a teacher professional development workshop can be shaped to address these goals has been discussed. The design, as well as the enactment of the workshop, contribute towards meeting these goals. Hence a framework is developed that illuminates both these aspects. The framework, where development was guided by the research group's experience of supporting TPD, the literature on teacher development and guiding policy documents in the Indian context such as the NCFTE, emerged from reflection on and analysis of the workshop data. The analysis of interaction episodes from the workshop is presented to illustrate how the framework illuminates the task design and the agency of the participating teachers and teacher educators in addressing the workshop goals.

5.3 Study design

The PD workshop sessions were led by four different teacher educators and were structured based on workshop goals and the agenda set for the particular session. The research team comprised teacher educators, researchers as well as project staff. Two teacher educators also had the role of researchers. The teacher educators observed each others' sessions as well as made comments when relevant to the discussion in the session. The same team worked with middle school teachers in Year 2 of the study on the topic of integers, the results of which are described in Chapter 7. In the project, I played the role of researcher and engaged in participant observation of different sessions in the workshop.

The data from the workshop was mainly available in the form of videos of sessions and notes of the sessions made by the researcher. Transcripts of all the sessions of the workshop were prepared from the video records of the sessions. Two researchers at first worked on two sessions from the workshop and developed the initial coding scheme based on open coding. These two sessions were led by two different teacher educators, one fairly experienced and another with less experience. The broad objective behind the selection of these sessions was to understand both teachers' and teacher educators' agency from the perspective of the framework adopted, which will be described below. The tasks in the two sessions, although structured differently required teachers to analyze and identify conceptual gaps in student thinking and think about the underlying causes of student errors. The coding process was adapted from Miles and Huberman (1994). The purpose of coding was to identify events that illustrated the beliefs, preferred practice and knowledge held by teachers, articulations that indicated reflection on them or conflicts among diverse views. Subsequently, was matched, discussed and consensus was established for the final codes, similar events were identified across other sessions. Those events have been selected for analysis with this chapter, where teachers' expressed their beliefs, and knowledge, reflected on their beliefs, or where they represented instances of teacher learning through task design features and teacher educator moves. The starting of an event is marked by an articulation that indicates, for e.g., a held belief by the teacher or a question posed by the teacher educator to the teachers to engage in a task or give their views and the end is identified by the resolution of the task or shift in the theme of the interaction. Further analysis was done to identify the task design features and teacher educator moves that led to such articulations by teachers as the latter was considered to be a step towards teacher learning. The codes were broadly from three categories: the task design features, the facilitation features and teachers' explorations and reflections. Some of the codes for task design features included "Analyzing student error", "Analyzing remedial strategy", "Anticipating student response" "Eliciting beliefs about mathematics and its teaching-learning" and "Identifying key concepts". The facilitation features codes included "probing", "asking for elaboration", "supporting", "revoicing", "connecting with other ideas", "raising questions", "raising larger issues of education". The last category of teachers' explorations and reflections included codes like "affirming", "challenging", "making conjectures" "reflection on teaching", "explanation of student errors", etc. These categories were identified and consensus about the coding was established by discussion among the coders based on the coding of two sessions by two different teacher educators. To see the full list of the codes used see Appendix 7. After the initial coding, the themes explicated in this chapter were arrived at keeping in view the framework presented in Table 5.1 in section 5.4 below.

After the initial coding, the researcher coded other sessions alone, using this coding scheme and selecting events, that were significant for the themes identified. A few additions to the coding scheme and modifications were made based on comparison across sessions and identifying new themes and revising the earlier selected themes. For e.g., sharing of dilemmas was identified as a new theme. The coding of the events helped in identifying the themes that were common and raised important issues about teacher thinking and learning across different sessions. Reliability of the inferences was ensured by selecting the relevant events which gave credible evidence based on how well they illustrate the theme as an example and multiple events were selected to illustrate the diversity of examples within that theme. Sometimes the same event is discussed under various themes as these events were coded for different themes.

5.4 Framework for analysis and research questions

In designing TPD workshops to address the goals described above, three guiding principles were considered as essential. These principles emerged on reflection on the workshop data but are related to the theoretical perspectives of situated learning theory (Lave & Wenger, 1991) and communities of practice (Wenger, 1998). The three principles, which, in my view, must inform the design and conduct of TPD workshops through all its activities in a comprehensive manner are

- Situatedness in the work of teaching,
- Offering challenges to teachers' to revisit their knowledge and beliefs, and
- Developing a sense of belonging to a professional community.

The aspect of situatedness is addressed through the choice of tasks as well as the mode of presentation of the task. The use of artifacts like students' errors, examples from textbooks, or examples emerging from live teaching or video records of classroom teaching with suitable questions, and prompts and examples used in the interaction can recall the context of teaching and learning. It is this aspect that allows teachers to make strong connections with their own practice thereby providing a stimulus for participation and reflection. Moreover, the use of artifacts from the daily activity of teaching has been emphasized in practice-based professional development (Ball and Cohen, 1999). The research related to use of artifacts and practice-based professional development has been discussed in Chapter 2.

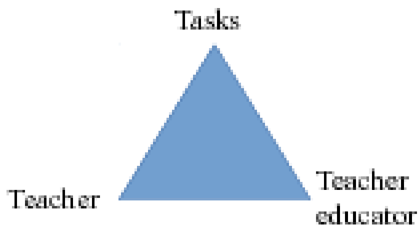
The second principle of challenging teachers' beliefs and knowledge needs to be built into the tasks chosen for the sessions and reflected in the actions by teacher educators such as re-voicing individual teachers' views for consideration by the participants, and providing counterarguments, explanations and questions to help teachers think about the tacit aspects of teaching and mathematical content. In the TPD workshops, such responses were made not only by teacher educators, but teachers on their own also reacted to their colleagues' articulations by making conjectures, arguments, assertions, counterarguments, explanations and reflective remarks.

The third principle of building a sense of a professional learning community acknowledges that teaching is a cultural activity, and the development of a teacher is not to be viewed in individual terms, but in the setting of a community. In this thesis, a broad view of community is adopted as encompassing teacher educators, researchers and teachers, all of whom are engaged in the enterprise of studying and improving teaching and learning. Opportunities were provided in workshop sessions for discussion, sharing and inter-animation of ideas to enable the emergence of a community. In the workshops organized by the research team, this aspect was addressed by posing tasks and questions for the whole group rather than to individual teachers. The teacher educators attempted to situate themselves as members of the larger teaching community by using "we" in their language as well as drawing on their own teaching experiences with students in the course of their research work. They adopted several words and categories commonly used by teachers and also elicited and acknowledged the teachers' knowledge about students and teaching gained through years of experience.

The three principles described above of challenge, situatedness and community building are interrelated. Focusing on the work of teaching helped in fostering the solidarity among teachers, who were regarded as knowledgeable members of the community as they are engaged in the work of teaching and were thus entitled to have and voice their views. Belonging to a community entails the work of making claims and conjectures, making arguments or counterarguments to support one's claims drawing on the knowledge gained from experience, and supporting the growth of knowledge in a community. Thus challenging beliefs and knowledge was an integral aspect of community building as much as situatedness in the work of teaching.

Understanding the role that interventions such as workshops play in the professional development of teachers requires consideration of not only design aspects, but also of enactment aspects. The affordances of the task that participants work on, and the interaction among the participants determine whether the workshop addresses the goals adequately. The framework outlined thus far includes goals and principles for the design and conduct of the workshop. To facilitate the analysis of the enactment of the workshop, further elements to this framework have been added which are relevant to key features of the interaction during a workshop. These elements are drawn from the notion of the teacher education triangle adapted from the didactic triangle (Goodchild & Sriraman, 2012) as shown in Figure 1. The interaction during a TPD session can be conceptualized as an interaction between the three elements of the task, the teachers and the teacher educators. The discussion focuses on the affordances of the tasks and on the agency of both teachers and teacher educators. Rather than viewing agency as "associated with the individual subject as a self-standing entity," the analysis indicates how this "arises out of engagement" (Wenger, 1998, p. 15). This engagement is with colleagues and teacher educators who are partners in the common enterprise of improving mathematics education in schools.

Figure 1: The teacher education triangle



These considerations led to the framework presented in Table 5.1, consisting of three broad categories: the workshop goals, the principles for designing workshop components and the interaction aspects. In the next section (section 5.5), the components of the workshop and how they relate to the goals and design principles in the framework are discussed. In Section 5.6, using the framework in Table 5.1, analysis of the interaction during the TPD workshop is presented under sections dealing with *the nature of the tasks, the agency of the teachers and the agency of the teacher educators*. The choice of the task, the communicative devices used by the teacher educators and the efforts to shape the interaction, all reveal the importance and inseparability of the aspects of situatedness and challenge. The efforts made to situate the discussion of teaching and learning both in the context of the work of classroom teaching and within the community of teachers have been discussed. In Section 5.7, teachers' explicit views on what they perceived as gains from the workshop have been discussed. In the light of the discussion in Chapter 4, an important question is whether the teachers show a greater alignment of their beliefs and practices with student centered teaching, which is also discussed in this section.

The questions that are specifically addressed in this chapter are the following:

1. What aspects of the workshop design and enactment are important from a TPD perspective?
2. How did the workshop tasks encode the design principles?
3. How was teachers' agency enabled in the course of the enactment of the workshop?
4. What aspects of the teacher educators' enactment of the task and interaction facilitated engagement by the teachers?
5. What were the learning gains from the PD workshop as perceived by the teachers?

Table 5.1: Framework for analyzing design and enactment of a TPD workshop

Workshop goals	• Strengthen teachers' knowledge relevant to mathematics teaching • Provide opportunities to articulate and reflect on beliefs relevant to teaching mathematics • Foster the development of professional communities of learning
Principles for designing components and tasks	• Situatedness • Challenge • Community building
Interaction aspects	• Task affordances • Teachers' agency • Teacher educators' agency

5.5 Components of the TPD workshop

The design of theTPD workshop in Year 1 was guided by the goals of providing opportunities to teachers to strengthen their knowledge for teaching mathematics, to reflect on their beliefs, and to foster the building of professional learning communities. The principles of situatedness, challenge and community building were implicit in the design of the workshop components, which included sessions involving the study of classroom teaching to learn about content rooted in pedagogy, learning about students' thinking from students' responses, working on mathematical problems and understanding relevant research on teacher learning and the work of teaching.

The following is a description of the components in the workshop, with an elaboration of their role in the development of mathematical knowledge required for teaching and in reflecting on beliefs and attitudes related to the teaching and learning of mathematics. The detailed time table of the workshop is given in the Appendix 3. The types of sessions were learning through problems, observing and analyzing teaching, analyzing curriculum materials, working on students' thinking and reading and presenting research articles. The purpose and the details of these kinds of sessions are described below.

Observing and analyzing teaching: In the workshop, the teachers viewed video excerpts of teaching as well as live teaching. They observed lessons live and discussed them over three consecutive days. The lessons were taught by one of the teacher educators to students participating in a vacation course. Teachers participated in the activity by contributing to the plan for the class, observing the lesson and then reflecting on it. The teaching observed was atypical, in its focus on eliciting students' ideas and building on them and thus was intended to be a source of reflection for teachers. These sessions provided teachers with a context for making their situated knowledge about pedagogical approaches and students' capability explicit. Prompts to elicit reflection on the lesson included inviting teachers to make conjectures about the intentions of the teacher in making specific moves, what children were thinking and what alternative pathways could have been taken at critical points in the lesson.

Learning through problems: Teachers worked on mathematical problems during these sessions, which were posed in contexts close to either their teaching practice or daily life. The problems were content specific and therefore separate sessions were conducted for different content topics such as number sense, fractions and ratios, and algebra. The main objective was to create distractions in a mathematics problem based on familiar alternative conceptions that teachers or students have, leading to cognitive conflict and eventually to reflective learning. The problem presented in Table 5.2 is an example.

Table 5.2: Example of a workshop problem task

A student in the class had added fractions like this: 3/7 + 2/3 = 5/10. Why do you think students add in this way?

When the teacher asked the student why she had done it in this way, the student said that her father had taught her. The teacher explained that this method was wrong. On the following day, there was a complaint from the father. He pointed out that the teacher had added exactly like his method. This was his example, Marks in history: 35/50. Marks in geography: 24/50. Total marks in social studies: 35/50 + 24/50 = 59/100.

How would you respond to the parent's criticism?

These sessions occasionally led to a deep exploration of mathematical concepts and making connections between various mathematical constructs, and providing a space for teachers to reflect and build upon their mathematical knowledge for teaching.

Working on students' thinking: These sessions included working on students' errors, uncovering students' thinking by analyzing strategies, and inferring potential misunderstanding underlying these errors. These led to discussions on issues such as – which questions are efficient in evaluating understanding of a specific concept, what do these errors tell us about students' and teachers' own conceptions including their beliefs about the nature of mathematics, and what do students' errors imply in terms of shaping the instruction. In a subsequent section, the interaction in the sessions of the workshop that involved working on students' errors has been discussed in detail.

Reading and presenting literature based on research: In these sessions, teachers in groups of three to four studied a research article from the field of mathematics education and made presentations to colleagues. The sessions were found to be valuable in fostering the sense that a teacher is a part of a community that systematically studies content and pedagogy with the goal of improving teaching and learning. The readings stretched the boundaries of the participant teacher community from the immediate peer group to the professional community of mathematics educators including researchers. An indicative list of the readings used in TPD workshops at HBCSE is given in the Appendix 4.

Analyzing curriculum material: These sessions were included to add connection, coherence and depth to teachers' comprehension of textbooks so as to use them more effectively. Teachers in groups of two to three analyzed textbooks from grade 3 to 6 for a specific mathematical topic for the following prompts – what is the hierarchical development of the topic, how are context and real life connections brought about, what is the role of the various examples provided, and how are representations used in the textbook. Although teachers use these textbooks on an everyday basis, these sessions provided an opportunity to distance themselves from the sole purpose of teaching and look at the textbook critically.

Expressing beliefs about teaching, students and mathematics: In this session held at the beginning of the workshop teachers completed a 6-part questionnaire based on the Likert-type scale, which provided them with an opportunity to reflect on their own beliefs about teaching, students, self and mathematics. Teachers worked individually on these questionnaires. The teachers' responses to the questionnaire and interviews have been discussed in the previous chapter. At times, the questionnaire items framed the discussions in subsequent sessions. Teachers also reported that the statements mentioned in the questionnaire made them think about issues that they had not thought of. Interview prompts that probed teachers' beliefs about teaching and learning mathematics, as well as their knowledge of algorithms, might have contributed to teachers' enhanced awareness of issues and a reflective stance during the workshop. At the end of the workshop, teachers were given parts of the belief questionnaire on mathematics, mathematics teaching and preferred practices and were asked to record the changes in their views, which are discussed on the section 5.7.2 below.

5.6 Workshop enactment

The sessions of the workshop were generally characterized by high levels of interaction and participation by teachers. In this section, how the enactment of the sessions reflect the principles and goals demarcated in the earlier sections[1] has been discussed. At first, I discuss how tasks were used as resources to help teachers share and reflect on their beliefs and practices. The tasks were designed to support teachers' reflections about students' thinking, about mathematics and about their own teaching practices. Then I discuss how teachers exhibited their agency during the sessions as a result of finding common issues or being challenged through tasks and discussions in the sessions. The subsequent section discusses how the agency of teacher educators drew on their belief-goals and knowledge in contributing to the teachers' articulations of their

1 Two sessions from the workshop have been chosen for detailed analysis from three main lenses offered by the framework: the affordances of the task, agency of the teacher and agency of the teacher educator. Additional examples from the sessions are also presented briefly to indicate the presence of these features across multiple sessions of the workshop.

beliefs, knowledge and practice while helping them to reflect and build on these through their engagement in the sessions.

5.6.1 The task as a resource for teacher education

The tasks in different sessions of the workshop were designed to engage teachers with instances situated in teaching to elicit their beliefs and knowledge, while attempting to provoke conflict or dissonance in teachers' beliefs or knowledge. These conflicts were induced by exhibiting examples of alternatives to traditional teaching practice, eliciting and discussing conceptual bases or raising issues of the larger goals of mathematics education. This helped in discussing the vision of teaching within the curriculum reform and provoked teachers to revisit their beliefs and reflect on their practice. A few tasks from some of the sessions in the workshop are discussed along with the efforts to make them authentic and challenging in order to provoke teachers' reflection. These tasks are discussed under the categories of different purposes they served, namely, tasks to reflect on students' thinking, tasks for doing mathematics and tasks for reflecting on the teaching of mathematics.

5.6.1.1 Tasks to reflect on students' thinking

Two tasks from two different sessions which engaged teachers in reflecting on students' thinking are discussed. In session 2.4, the task was to describe and explain student errors from looking at their responses to seven test items on the topics of number, place value and fractions (see Table 5.3 for sample items). At the beginning of the session, the teacher educator provided a set of prompts to guide the discussion for each question: identify competencies being tested, find all possible correct answers and understand what caused the student errors. The student errors were drawn from a pre-test of students participating in the vacation course, the same group of students whom the teachers knew they would be observing later. The task was thus authentic in engaging teachers in analyzing student errors, which is part of their day to day practice as well as being real data of students they were observing in the workshop.

Table 5.3: Example of test items and student errors shown to teachers in Session 2.4

Questions	Student error 1	Student error 2

Write the next three numbers: 3097, 3098, ------, ------, -------	3097, 3098, 3099, 30910, 30911	3097, 3098, 3099, 30100, 30101
14 tens + 23 ones	1423	14023
Draw 7/4	Diagram representing 4/7	Diagram representing 7/11

In Session 1.1, teachers were given a handout that described a student Mohsin's difficulty in writing numerals despite his familiarity with numbers (see Table 5.4). Here too, the task engaged teachers in identifying conceptual gaps in the student's thinking with the goal of identifying suitable teaching interventions to address it. The teachers discussed in groups and presented their suggestions about how Mohsin could be helped. The teacher educator informed teachers that Mohsin was actually a student whom he was teaching, and furnished details about Mohsin's responses to other tasks in the course of the discussion.

Table 5.4: Task given to teachers in Session 1.1

Mohsin is in class 5. He helps his father, who is a vegetable seller, with home deliveries. He can find the total amount a customer has to pay and often does the addition mentally. He also knows a lot about how much things cost: televisions, cycles, two wheelers, washing machines, etc. But when his teacher asked him to write "rupees two thousand twenty five" in numerals, he wrote "Rs 200025".
Think about what Mohsin's problem is and how his teacher can help him. How can the teacher make use of what he already knows so that he can learn something he doesn't know.

Teachers identified the students' responses presented in both the sessions as "common" responses. However, it required further probing on the part of teacher educators to make teachers think about students' thinking underlying the errors and the sources of the errors, beyond identifying them just as "common" errors.

The tasks chosen for the two sessions are situated in the context of teaching, while a challenge is introduced by requiring the teachers to think beyond the normal requirements of everyday teaching. In Session 2.4, this is achieved through the three prompts inviting teachers to uncover a deeper layer of students' thinking that can explain their responses. Teachers at first thought that the errors surfaced because of the non-typical questions asked in the test. For example, to explain why the student incorrectly showed 4/7 instead of 7/4, a teacher said, "because the numerator is greater than the denominator in the given fraction." Several teachers thought that it was not possible to represent improper fractions. The teachers interpreted fractions in terms of the "part-whole" meaning as a number of parts out of total parts, which made the representation of improper fractions awkward. The task thus challenged them to re-consider meaning attributed to fractions. This example will be discussed again later.

In Session 1.1, the teachers were asked to consider both what the child does and does not know so as to induce a sense of conflict by juxtaposing these together. For e.g., teachers identified that Mohsin knows numbers in the thousands, can add mentally and can read the price of a bicycle but cannot write the amount 2025 correctly. What accounts for the child's capability in the context of everyday calculations, and his profound lack of understanding of a related part of school mathematics? This tension sets a dialectic in motion allowing the teachers to revisit the relatively hidden and unspoken aspects of their everyday teaching. For example, teachers shared the different resources that they deemed fit for addressing Mohsin's problem like bundles of sticks, different denominations of money, Dienes' blocks for hundreds, tens and ones and the spike abacus along with the explanation they used in teaching place value to students using these resources.

The above two tasks were related to the challenges faced by teachers in teaching, which involves figuring out conceptions in the students' minds in order to promote students' understanding of mathematics. Embedding the tasks in the context of planning for teaching served the purpose of making teachers reflect on students' thinking.

5.6.1.2 Tasks for conceptual exploration

In several sessions, the tasks involved teachers in exploring mathematical concepts. Although giving mathematical tasks is a normal part of everyday teaching, teachers rarely get the chance to reflect on the mathematics involved in the task apart from considering its pedagogic function. The assumption was that solving problems along with peers might help in thinking about concepts and also experience the thinking involved in doing the problem while adopting a learners' point of view.

In Session 2.2 on "Measurement and Fractions," teachers solved problems based on measuring and sharing context to identify the fractions. Teachers' response to an earlier task revealed that they used the procedure of counting shaded parts to determine numerator and number of equal parts to determine denominator to identify a fraction rather than identifying the unit to measure the part of the whole. The research literature (Lamon, 2007) has emphasized that one needs to understand the role of the unit in determining a fraction rather than treating numerator and denominator as whole numbers. As a result of the conception that fractions are identified by counting the number of parts shaded as numerator and total parts as denominator, a teacher in an earlier session had protested that one cannot find the fraction for representing a part if the given whole is divided into unequal parts. The measuring context used in Session 2.5 required teachers to identify a suitable unit to measure a part of *Chikki* (rectangular slab of sweet toffee) that is eaten. Teachers used different units to denote the piece of chikki eaten thus coming up with different fractions as the answer. This created a situation in which teachers realized that different answers for the single problem were possible and that it depends on the unit which was selected to measure the whole. It helped in emphasizing the role of the unit in identifying the fraction rather than identification by counting the parts.

In another task in Session 2.2 teachers tried to find a solution for a situation where a student who was on a vacation in a village needed to make a chart with specified dimensions, but was not able to find any length scale in a village. Teachers came up the with the idea of using a one rupee coin to make a scale through iterations of the size of the coin and dividing it further into subunits. This helped in deconstructing how a scale is built by iterations of the unit and subdivisions to increase the precision. In doing these tasks teachers engaged as learners and reflected on the concepts involved in developing the understanding of fraction and measurement. A teacher (**P1**) felt that through this activity students can understand the importance of a standard unit while P5 felt that they will be able to understand how the scale is constructed through iterating a unit.

In another session (4.2) on learning from problems on the topic of ratio and proportion, teachers debated solutions of problems and discussed the application of the concept of proportion in solving problems. For e.g., in one problem one had to identify the time taken for towels hung out to dry if the number of towels increases. Teachers shared and debated the unspecified conditions of the environment where towels were drying as the determinant of the correct answer like whether the towels were drying in a room or in the open, the number of towels in the room, humidity, etc. The discussion led to the conclusion that although in classrooms one accepts a mathematically correct answer, in reality, mathematics may provide only estimates as answers to real problems. They discussed the connection between mathematics and the real world and how students mechanically work with numbers in such problems instead of using their knowledge of real world situations. Teachers felt that some challenging problems might be given to students to make them think rather than operating with numbers in problems mechanically. Earlier they had felt that students should be given clear explanations of how to solve a problem and should not be confused. Thus, solving problems and reflecting on connections of mathematics with reality made teachers reflect on the lack of mathematical thinking by students in their classroom and how it could be fostered by giving challenging problems.

Engaging in problem-solving from the learners' point of view had its advantages for teachers' reflection and learning. Doing these tasks from the viewpoint of the learner provided an opportunity for teachers to work on their knowledge of mathematics and its processes while also reflecting on their teaching and anticipating ways that students would engage with the problem. Teachers thought about how students would think and react to these problems while sharing their own solutions, thereby thinking at the level of the learner as well as of the teacher. Thus, discussion following tasks on exploring mathematical concepts led to teachers reflecting on the teaching of mathematics. However, some tasks, which were specifically designed to make teachers reflect on their teaching, are discussed below.

5.6.1.3 Tasks for reflecting on teaching

There were several sessions in which teachers watched and discussed teaching in different modes including live teaching, mock teaching and watching videos. In these sessions, teachers came across examples of teaching which were close to their own experience, and they acknowledged using similar practices and having similar experiences with students. They evaluated the episodes of teaching that they witnessed, made conjectures about students' learning, gave reasons or justification for evaluating a practice and made suggestions on how to modify it. These articulations indicated their underlying beliefs about mathematics, teaching, students and self and the priorities they have while teaching. For e.g., in a session (1.3) where they watched videos of the teaching of fractions, some teachers shared that fifth-grade students should be engaged in activity with concrete materials like making parts of the paper for understanding fractions. This indicated their belief about the need to use concrete materials while teaching mathematics.

In the sessions on observing live teaching, teachers experienced teaching which exhibited practices that were radically different from their normal practice such as giving problems without showing solutions first, asking students for their ideas, accepting pictorial/visual solutions, discussing students' reasoning and deviating from the lesson plan to address conceptual gaps. These sessions were helpful in provoking teachers to share their beliefs, notice examples of alternative teaching-learning and further to reflect on their own teaching. In Session 3.3, teachers had made the lesson plan together with the teacher educator and watched him teaching. Teachers noticed and appreciated that students were able to come up with pictorial solutions using their own reasoning instead of calculations and wondered if students will be able to use such reasoning to solve questions with bigger numbers. They also appreciated how students shared their solutions and constructed terms like "half of quarter" and "quarter's quarter" and how the teacher discussed them. The activity used for representing the size of different strips using fractions provoked a discussion on whether challenging questions should be posed to students in class. These discussions helped in understanding what teachers noticed in alternative teaching practice, finding parallels or differences from their own practice. Teachers also shared whether they found the alternative practices helpful in student learning which promoted reflection on opportunities for students' learning in different classroom cultures.

Teachers analyzed the textbook in some sessions where they compared one chapter from the new reform based textbook with the earlier version of the textbook. It brought out the issues that teachers face when using the textbook and highlighted aspects of the textbook that influenced teaching. Primary teachers noticed the lack of activities, visual representations in the old textbooks and appreciated the colorful illustrations and layout, activities and questions related to daily lives of students in the new textbook. Teachers felt the need of more questions for revision and practice in the new textbooks. The middle school teachers pointed to a chapter in the new textbooks that they felt was conceptually dense, and where the difficulty level of questions too high for middle school students. In both cases, teachers' beliefs influenced their perception and use of textbooks. Since primary teachers felt that practice is necessary for teaching maths, they compensated for the paucity of exercises in the new textbook by forming their own questions. Middle school teachers felt that students of sixth grade cannot engage in reasoning and thus perceived the difficulty level of questions as high.

5.6.1.4 Summarizing the role of tasks in the workshops

The tasks used in the workshop worked as a vehicle for reflection and engagement on the part of teachers by articulating their beliefs and knowledge. The tasks in the workshops engaged teachers with examples of alternative practice and alternate interpretations from traditional teaching as well as illustrations of or connections between conceptual aspects. In several sessions, teachers analyzed student errors, engaged in lesson planning, analyzing teaching and problem solving which are all part of authentic teaching practice which they are engaged with in everyday teaching. However, engaging with practice within the professional development context provided opportunities to hear alternative viewpoints and reflect on beliefs underlying the practices considered as the norm. Examples of alternative practice through live teaching, videos and descriptions in research papers provided teachers with concrete images of how practices for engaging students thinking can be used as well as evidence of how students respond to practices like asking students to solve problems using a context without first telling them how to solve the problem. Elaboration and discussion of conceptual aspects underlying the tasks helped in teachers sharing their knowledge as well as building on their knowledge. Even when the task was focused on a conceptual aspect, teachers invariably made connections with the pedagogy of teaching that concept and the way students might respond to certain explanations. The discussion moved beyond the immediate demands of the tasks to broader concerns like connecting teaching of mathematics with out-of-school experiences or considering students' thinking underlying their mistakes as a resource for teaching mathematics. Thus, the design and implementation of tasks for teachers needs to accommodate the possibility that the immediate task at hand may recede from focus, opening up a space for deeper engagement, where teachers can share and critically reflect on what they know, understand, believe and practice. The role of teachers' and teacher educators' agency in engaging with the task and the emergent issues will be discussed in the sections below.

5.6.2 Teachers' agency in engaging with knowledge and beliefs

An important part of understanding the work of teaching as a profession is a shared agreement about the specialized knowledge and expertise that informs the work of teaching. Professional development programs need to elicit and build on such knowledge, much as teachers elicit and build on students' knowledge in the classroom. This process also allows the community of teachers and teacher educators to develop a shared view of the contours of such knowledge. Providing opportunities for the process of eliciting teachers' knowledge and beliefs lead to understanding teachers' agency. The teachers' agency is defined as initiatives taken and autonomy expressed by teachers during the course of interaction to assert and justify their beliefs and articulating and applying their knowledge to make pedagogical judgements. In this section, how teachers' agency was expressed in the course of the interaction during selected episodes has been discussed.

5.6.2.1 Anticipating students' responses

Requests to teachers to anticipate and predict student responses were either built into the task itself or were made by the teacher educator in the course of the discussion. This aspect is embedded in teachers' everyday work of teaching. Over the years teachers develop an implicit knowledge about typical and atypical student responses. In the PD context, making this knowledge explicit works as a resource in building the shared knowledge between teachers and teacher educators and providing ways to discuss students' thinking.

In section 5.5.1.1, teachers' responses to two tasks have been discussed. For the task on student errors presented in Table 5.3, teachers were able to anticipate some student errors for the questions, which paved the way for discussing student thinking and indicated the knowledge that teachers had about students. Similarly, in the context of discussing the error made by Mohsin (Table 5.4), the teacher educator asked teachers to predict Mohsin's strategy to find the cost of 10 kg potato given the cost of 1 kg. The teachers anticipated that Mohsin would repeatedly add the unit cost to arrive at the cost of 10 kg, which the teacher educator confirmed was what Mohsin actually did. These questions were significant as they directed teachers' attention towards what the student did know at a point when they were focusing only on his incapability. Sharing the anticipations paved the way for discussion about the differences between the mathematics that students learn outside the school and in the school and the need to bridge the gap between the two. These episodes indicated the knowledge of students' thinking and the sensitivity towards it that the in-service teachers had developed over the years.

In another session (2.2) teachers were asked to predict how students from different grades would respond to a problem on measurement which required students to measure the length with a broken scale. Data from an actual survey was then shared which showed poorer performance than was anticipated by teachers. Teachers were then asked to think about the reasons for students' poor performance on the task. Engaging in anticipating student responses and discussing authentic data from a student survey created cognitive conflict for teachers to try to make sense of why students respond in a certain manner.

5.6.2.2 Elaborating and building on conceptual bases

The discussions in several sessions led to the elaboration of the conceptual basis of mathematical procedures and teaching aids. These elaborations were not restricted to teacher educators as teachers themselves shared the conceptual explanations and built on the conceptual bases that were elaborated in the discussions, thus exhibiting their agency. At times, teachers contributed centrally to the goal of building mathematical knowledge for teaching. In Session 1.1, while trying to elaborate why Mohsin made the error of writing two thousand twenty five as 200025, a teacher (P5) explained that understanding the "meaning of zero" involves understanding how it changes the value of a number with change in position – it produces no change in value when written in the leftmost position of the numeral and if written in other positions, it alters the place value of the some of the digits in the numeral. Her explanation of the concept of the position of zero can be characterized as a "key knowledge piece" (Ma, 1999) that is important in understanding place value of a number. Her intervention led other teachers to also identify the conceptual gap in the student's thinking and a teacher [**P1**] asserted, "he knows 2000 and he knows 25 but how to write [2025] he doesn't know".

In Session 3.2, during observation of teaching, teachers built on the discussion that they had had on units and iterations and breaking of the units into subunits to indicate the measure using fractions while discussing students' responses. They discussed a task that students had worked on which was "The road from Mankhurd to Cheetah Camp is 2 km long. If a road roller tars ½ km road every day, in how many days will it tar the complete road?". M6 and P5 discussed how instead of using unitary method or calculations with numbers, students used ½ km road as a unit to find out the time required to build 2 km road by iteration of a unit taken as ½ km. **P1** commented that students have developed an understanding of ½ through their environment linking mathematics to daily life, an issue discussed during Session 1.1.

5.6.2.3 Conjecturing underlying causes

Returning to the student errors discussed in section 5.5.1.1, for the fraction 7/4, some students had drawn part-whole representations of 4/7 or 7/11. Teachers tried to explain the thinking that might underlie these responses. As discussed earlier, initially teachers identified the cause of the error as unfamiliarity with the question. In the course of the discussion, a teacher put forth an alternative explanation of why students drew 7/11 to show the fraction 7/4. The student, he argued, may have interpreted 7/4 to mean "7 shaded and 4 unshaded parts" thus making a total of

11 parts of which 7 were shaded. Thus teachers began to engage with the reasoning that the student must have applied to create such representations. The discussion moved to how counting of shaded parts (using whole numbers) was generally over-emphasized while teaching fractions. Thus, teachers also reflected on their own teaching as a possible cause of student difficulties in learning.

To cite another example, when a question was raised in Session 1.1 about why students are not able to learn mathematics even after five years of schooling while they learn quickly outside the school, a teacher observed that "we do not correlate mathematics taught in school with everyday life". In one of the presentations on research readings from a chapter on multiplication from the book *Knowing and Teaching Elementary Mathematics* by Liping Ma (Ma, 1999), a primary teacher P3 reflected on why students make mistakes in column multiplication. She attributed these mistakes to the way multiplication is taught to students by asking them to put a zero when multiplying with the digit at tens place. She explained using the reading how the digit 4 at tens place is 40 and thus the answer has a zero at units place. The reading helped the teacher to build on her conceptual understanding of the multiplication procedure and led to conjectures about the cause of student mistakes.

In Session 2.2, teachers were asked to think about reasons for students' poor performance on the broken scale measurement tasks across several cities in the country. While citing several reasons, teachers attributed not knowing how to use the scale as the main reason. The teachers' explanation focused on the procedure to use the scale and reading from it and some even suggested that students fail to understand that subtraction needs to be used in the broken scale to find the length. However, very few responses indicated the need to understand the unit of the scale and being able to find the length by iterations of the unit. In line with their explanations, teachers suggested that giving lots of practice with use of the scale might help in improving students' performance. These articulations indicated that even though teachers know about certain student errors, their understanding of difficulties faced by students is constrained by the limitations of their own knowledge of concepts and procedures and connections between them.

5.6.2.4 Articulating and contesting beliefs

The occasions when teachers' beliefs were explicitly articulated have been considered as important moments in the workshop. Frequently, these were provoked by observations of alternative teaching or practice. Such articulations, where teachers defended their own practice while observing a different kind of practice, were more common in the initial sessions than in later sessions. One such example is from Session 1.2 when teachers resisted engaging with alternative procedures for arithmetic operations by ignoring requests to solve using such procedures. They explicitly indicated their preference for the standard procedure over alternative procedures, suggesting that knowing and understanding alternative procedures was not valued by them. The teachers also indicated that they considered standard procedures as easier than alternative ones and their explanations while teaching emphasized "telling students rules clearly". The teachers viewed mathematics as a set of rules which have to be memorized and thus teaching as a process of telling rules clearly. At other times, they expressed the belief that following a procedure from memory is easier than understanding a procedure and its conceptual aspects. They judged most conceptual or alternative explanations as not being accessible to primary or middle school children and mentioned the possibility of children getting confused or making mistakes. This is connected with the beliefs that these teachers had about the capability of students with regard to whether they can arrive at their own procedures/ explanations or solutions without being actually taught. This theme had emerged in the interviews (discussed in the previous chapter), where on being probed, almost all teachers felt that students come to know about solutions from sources other than the teacher or textbook and thus a child may not have arrived at it on his/her own.

In another instance, in Session 1.3 in responding to a video of teaching where students were engaged in solving challenging questions without being told how to solve, the teachers shared that they believed that mathematics teaching should progress from the simple to the complex. They felt that students in the video are either intelligent or belonged to good socio-economic background or were already taught how to solve prior to the episode shown. This showed how teachers believed that most students are not capable of reasoning or figuring out solutions on their own and thus one should only give simple tasks in the beginning before introducing complex tasks.

Teachers expressions as described above provided access to teachers' conceptions and opinions based on which dialogue could be initiated. Teachers' articulations of beliefs sometimes took the form of reflection suggesting a revisiting of beliefs. For e.g., in Session 2.3, after discussing student errors and students' thinking underlying these, a teacher reflected "in fact we know their mistakes but we don't really see into their thinking". Another mode of articulating beliefs was contesting and challenging views articulated not only by their colleagues but also by teacher educators. For e.g., in Session 2.3, the question, "Add 337 + 33700" was discussed, where students had made an error in vertically aligning the digits. A teacher reacted to the example and said "Addition questions should not be given in horizontal manner as it will lead to error" thus indicating her belief that errors should be avoided during instruction. Similarly, during a discussion of teaching aids for teaching place value, a teacher voiced his opinion that using teaching aids will cause a lot of confusion and it would be better if students are told the rules and asked to practice. Another teacher contested this by saying that they themselves (teachers) had learnt mathematics by rote when they studied in school but it is important now to emphasize understanding concepts. At a point in the discussion, a teacher asserted that the abacus was easier for students to learn place value. Another teacher responded by explaining how stick bundles representing different units (tens and hundreds) can build the understanding of place value better. Thus voicing of assertions led to sharing of alternative viewpoints, which created a need for justification, thus deepening the engagement in the workshop. In this instance, the difference between the stick bundles and the abacus led to exploring the difference between the grouping principle and the positional value principle discussed later. These articulations were important moments in the sessions, which provided a window into teachers' thinking as well as created a space for revisiting and reflecting on beliefs relevant to teaching and learning. Teachers also assessed their own learning in the workshop as reflected in the appreciative comments that they made at the end of the workshop. The active interventions by the teachers described above were indicative of teachers' agency as they were not merely involved in affirming or contesting what the teacher educator or other teachers were saying, but were engaging in their own sense-making about the aspects discussed related to the task.

5.6.2.5 Sharing and challenging pedagogical approaches/ explanations

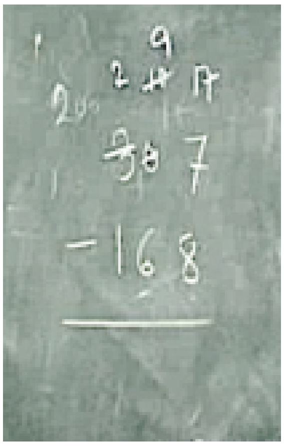
In several sessions teachers shared the pedagogical approaches that they followed in their classrooms for teaching as a response to the task posed to them. For e.g., in Session 1.2 teachers shared how they used the borrowing construct to explain the algorithm of subtraction in response to the request to subtract using an alternative method. They referred to single digits as numbers and tended to treat the "1" borrowed from digits of different place values as equivalent. Teachers showed the procedure of subtraction while describing how the rules are to be followed like "...we tell children that borrow, only when upper number [i.e., digit of the subtrahend] is smaller than the lower number [i.e., digit of the minuend]" or "You have to always borrow one only".

Teachers used the student response as the proxy to share as well as challenge the alternative pedagogical approaches that were shared by teacher educators. For e.g., during discussion on the subtraction problem 307 –168 teachers discussed what students do while subtracting. Although, they were talking about how students solve the problem, they were also sharing the explanation for the subtraction procedure which eventually led to discussion on what is a better explanation for subtraction.

Excerpt 5.1

> Let me show you the method that our student do. I like the method that children use. They cut this [7] and write 17 here (see Figure 5.1) and then cut here [3] and write 2 and then cut here [0] and write 10. Children are able to do well by this method.... Student think that we will borrow one from here, but there is zero there, so cannot borrow one, so they go to next step [digit] and borrow one from there, they cut [3] and write 2 there and then cut [0] and write 10 here...This is the way subtraction is taught... students find this method easier (**M4**).

Figure 5.2: Pedagogical explanation of the vertical subtraction algorithm using the borrowing construct



Most teachers acknowledged that this is the way (Fig. 5.2) subtraction is taught in the classroom. The common feature underlying this presentation is the lack of explanation of why for example one should "cut and borrow" or "borrow 1 from neighboring digit". Thus the explanation was rule-based with emphasis on subtraction of digits rather than considering the place value of digits. Also, there is a conceptual problem in treating 1 borrowed from digits at different place value as the same and then combining them to form numbers without explaining their place value.

Pedagogical approaches were shared also to illustrate teachers' beliefs and conviction about their usefulness. In one of the mock teaching sessions, teachers planned and taught fractions to other peer teachers as teams. In this session, both teams of teachers used concrete materials and in many instances, the audience response based on perceptual similarity for half, quarter, three fourth was taken as indicative of understanding without being probed further. This illustrated their belief that concrete materials are useful in learning. However, their acceptance of answers based on perceptual similarity indicated that these stereotypes of common fractions like half and quarter might be focused in teaching of fractions.

5.6.2.6 Articulating dilemmas of teaching

Discussion on several issues brought forward the conflicts faced by teachers while teaching leading to the articulation of dilemmas of teaching. A dilemma that was discussed was the way a teacher should deal with incorrect answers. After watching a video in which a teacher ignored the student giving a wrong answer and appreciated the one giving a correct answer, a teacher M6 pointed out that the teacher did not try to know why that particular student had given the wrong answer (session 1.5). Another teacher P2 commented that teacher should have corrected the child's answer. Later in the discussion, this teacher shared that teachers should "mould students' answer towards correct answer" as the student will be motivated to doing mathematics only when they get a correct answer. However, there were other teachers like **P1** who appreciated another video in which a teacher had accepted different answers and then encouraged students to find out which one is correct. She also asserted that teachers tend to use stereotypical questions and standard ways to find their answer which leads to rote memorization, therefore not encouraging students to think. These articulations suggested a tension between teaching towards getting a correct answer by avoiding students making errors and encouraging them to give different answers and finding out which one is correct through discussion.

5.6.3 Agency of the teacher educator: Inter-animation, knowledge and beliefs

In a TPD context to what extent teachers' knowledge and understanding are elicited, what aspects of knowledge are negotiated and in what direction the discussion moves during a session are critically dependent on the interventions made by the teacher educator. Not only is the participants' engagement crucial, but also the degree of inter-animation of ideas. Mortimer and

Scott (2003) have used Bakhtin's idea of inter-animation to illustrate how an interaction in the classroom is "functionally dialogic" when more than one point of view about an issue is represented as well as explored. Scott and Mortimer define low inter-animation as just listing of the varied responses shared in the group while high inter-animation means that there is an engagement with the different views expressed by the participants as a group. In the context of TPD too, it was found that the aspect of inter-animation of teachers' responses was crucial in how teachers' perceived their roles in the session. Two sessions were identified that were fairly different in the way teachers' participated in the session and also in the way the teacher educator responded to teachers' articulation and responses in the session. Session 1.1 was the first session of the workshop and an experienced teacher educator took the session with the purpose of breaking the ice and setting the tone of the workshop. This was the session in which teachers worked on the task about a student Mohsin described in section 5.5.1.1. The second session was 2.3 where in students errors were discussed. It was taken by a comparatively less experienced teacher educator. This was the session in which the teachers worked on the task presented in Table 5.3 in section 5.5.1.1 above. The selection and comparison of these two sessions allowed the identification of the moves of the teacher educator and elements of the session that promote teachers' learning through encouraging articulations, connections and reflection on the practice of teaching mathematics. In Session 2.3, it was observed that when teachers discussed the thinking underlying student errors (discussed above), inter-animation was low. The teacher educator in session 2.3 used a presentation to pose questions to teachers and after teachers gave their responses, explained the correct response. An excerpt from the session 2.3 is given below to illustrate the nature of interaction.

Excerpt 5.2

1. TE 2 showed the question "Fill in the blanks by writing the next three numbers: 3097, 3098, ___,___,___"

2. TE2: What is getting checked in this question?

3. T1: Place value

4. T2: Ascending order

5. TE 2: So in this question [writes, 113, 11 followed by three blanks], what will happen is that transition from teens to twenties will not take place. So the place value for the last digit is not changing, where as in the above question students will have to think about the change in place value while writing the sequence.

In the excerpt above, the nature of interaction resembled the I-R-E pattern of interaction (Mehan 1985) found in many educational settings. The initiation is through the task posed by the teacher educator, followed by response and then explanation by the TE. Although, teachers articulated their responses in more detail during the session, the teacher educator almost always ended the discussion of an error by explaining what she felt was the correct description of the conceptual gap. Using common student errors had great potential for connecting with teachers' practice, which did happen to some extent when teachers acknowledged the errors as common mistakes. However TE 2 did not use different responses of teachers to generate a discussion or use their responses to probe and elicit the nature of teaching related to the concepts discussed around the student errors.

In Session 1.1, the teacher educator re-voiced the various conjectures and opinions put forth by the teachers and posed them for consideration by the whole group. Not only did teachers contest the views of other participants but they also contested views expressed by teacher educator leading to discussion. The moves by teacher educator that led to high inter-animation in Session 1.1 included inviting teachers to respond to each other, problematizing teachers' responses to be discussed by the group and the use of open-ended questions. The excerpt below is from session 1.1 after the groups of teachers had discussed and had written their responses on the blackboard for the task posed by the teacher educator on Mohsin's difficulty with numbers (see section 5.5.1.1). The responses included descriptions of teaching approaches that might help Mohsin. TE1 had asked the teachers to identify similarities and differences in the proposal listed by different groups. He then attempted to categorize all the resources listed by the groups with the help of teachers' responses in terms of exercises, teaching aids, concrete materials, etc. He elicited teachers' ideas about the place value and the use of abacus to teach place value. Then he tried to focus teachers' attention to what Mohsin knows based on the description provided in the task.

Excerpt 5.3

1. TE1: Lets talk about what Mohsin knows and how he might have come to know about it.
2. T1: He learnt from environment.
3. TE1: He mostly know about money…
4. T2:...by selling vegetables.

5. TE1: He does accounting and he does the home delivery and he has to collect the money. His father may be asking him to calculate and make a bill, if it is 2-3 items.

6. T3: Up to 100 it is easy to add and subtract

7. TE1: Yes, he must be knowing calculation of small numbers.

8. T4: He does not know how to use columns.

9. TE1: He can mentally calculate and 1 kg, ½ kg he can multiply.

10. T5: He knows rates also.

11. TE1: He knows little bit about the real world like TV and how much it costs be cause he may be interacting with older children and plays with them or may be through news papers ads. She [T1] had mentioned that he knows thousand.

12. T1: Yes.

13. TE1: He knows because then how can he know about TV prices. But he has problem as she said in writing the number only.

In the above excerpt, one can see that TE1 and the teachers are brainstorming over what Mohsin might know based on the information given. The teacher educator revoices teachers' articulation in lines 3, 7, 11 and 13 when he either builds on a teacher's response or makes a claim and connects it with an earlier claim made by a teacher. The question in this section too was about the student's error as the basis of the discussion but the teacher educator was able to elicit teachers' ideas about students' previous knowledge, their accepted explanation of the place value concept, the teaching aids considered useful by teachers for addressing the concept of place value as well as predicting Mohsin's strategies for problems of multiplication based on an interpretation of what he knows. The inter-animation is high as the teacher educator used the teachers' responses to make important points in the discussion and made arguments using what the teachers had articulated. Instead of posing as an authority for validating correct or incorrect responses, the teacher educator was able to invite different responses from the teachers and connected his own argument with the teachers' points shared earlier.

In the sub-sections below, I briefly discuss the moves made by the teacher educators that reflect the goals and the principles outlined in the workshop design framework. Further, I note that the teacher educators' interventions during the course of the interaction, are guided by their own beliefs and the knowledge that they bring to bear on the discussions. Thus, the interaction between the teacher educators' beliefs and knowledge has been briefly discussed as reflected in the teacher educators' interventions.

5.6.3.1 Teacher educators' moves in alignment with the goals of TPD

In session 1.1, the teacher educator frequently invited teachers to respond to the views expressed by their colleagues communicating that the teacher educator is not solely responsible for evaluating teachers' responses but that it has to be decided by the deliberation of the whole group. This stance was also adapted by other teacher educators in the workshop. This led to situations where the teacher educators had to handle disagreement and conflict of views. Teacher educators usually welcomed disagreement and considered it a healthy sign of engagement, which allowed teachers to articulate their beliefs, which could then be taken up for discussion. The teacher educator sometimes restated a view in more general terms by placing it in the broader educational context. In Session 1.1, when there was disagreement over whether it is better to teach students rules for algorithms, the teacher educator contrasted learning inside and outside the school. (Translated and summarized: "if students hold strongly what they learn from outside the school, but they are not able to hold on to what is learned at school, why it is not held we must think and talk about it"). This can be interpreted as an attempt to build a sense of community by inviting teachers to reflect on their beliefs in the context of larger educational debates.

The use of open-ended or probing questions by the teacher educator elicited responses that were more in number as well as variety from the teachers leading to richer discussion strands. There were several examples of this in Session 1.1. The teacher educator asked teachers to suggest a variety of learning aids that could be useful for Mohsin. He then invited them to think about which teaching aids are better and why. Teachers' responses to such questions were often elaborate with some teachers recounting their own teaching experiences. Another significant move by the teacher educator was asking for clarification of the meaning of terms used by teachers and moving towards shared meaning and vocabulary. For e.g., in Session 1.1 the teacher educator asked for a deeper probing of the meaning of "place value", a term that occurred frequently in the discussion.

5.6.3.2 Interaction between teacher educators' beliefs and knowledge

The teacher educators held and acted on beliefs that were at times different from those held by teachers. Since the teacher educators' actions were guided by an expectation that teachers accept these beliefs, they could be considered to be belief goals for the TPD program. Some of the beliefs had to do with the emphasis or value ascribed to elements in the teaching-learning context. For example, the teacher educators believed that what a student knows is more important or at least as important as what he or she does not know. The emphasis placed on this was prominent in Session 1.1, when the facilitator repeatedly brought teachers' focus back to what the student (Mohsin) knew when the discussion turned to what he did not know. In Session 2.4, the following student error was discussed. In response to the question "show the number made of 14 tens and 23 ones," a student wrote "14023". The teachers' comment about this student was that he did not have a concept of place value. A teacher educator present in the audience pointed out that in fact, it did show a partial understanding of place value since the student knew that 14 tens can be written as 140 and 23 ones as 23.

Another visible belief of the teacher educators was the value ascribed to students' thinking as a resource for teaching, which was consistent with the emphasis on what the student knows in contrast to what she does not know. This belief interacts with knowledge about students' ways of thinking in enhancing teachers' awareness and sensitivity. The teacher educators attempted at times to lead the discussion into understanding students' responses more deeply. In the example discussed earlier of why some students incorrectly represented the fraction 7/4 by drawing a picture for 7/11, a teacher educator conjectured that it could be due to excessive emphasis on the part-whole representation of fractions by making as many parts as indicated by the denominator and shading as many parts as indicated by the numerator. Thus instruction treats numerator and denominator as separate whole numbers. Taking the discussion further, a teacher educator in the audience proposed an analysis of the fraction concept, by distinguishing between counting and measuring contexts. He suggested that measuring rather than counting is a better context to understand a fraction as a single number indicating a particular quantity. Counting contexts tend to reinforce the idea that a fraction is made up of two numbers. These interventions not only led to a deeper probing of students' thinking, but also to understanding how the choice of a teaching approach may play an important role in students developing certain conceptions.

Another belief held by the teacher educators, which is related to valuing students' thinking, is the belief in the efficacy of using students' previous knowledge (especially knowledge acquired from the everyday life/culture) as a resource for teaching. This was foregrounded in Session 1.1 by asking teachers to anticipate Mohsin's responses while using information about the daily life activities in which he engages and raising the question about the need to bridge the gap between the mathematics learned outside and inside the school.

The teacher educators attempted to communicate that umbrella concepts like "place value" need to be understood in detail, in terms of how they play a role in specific contexts of learning related concepts, of problem-solving, or of understanding an algorithm. For the place value concept, to become more useful, it needs to be decomposed into the sub-concepts of "grouping principle" and "positional value principle". The grouping principle determines that in the decimal number system, 10 units form the next higher unit in the sequence of units, tens, hundreds and so on. Number words encode the grouping principle by naming the different powers of ten: "four thousand", "six hundred", etc. Thus the grouping principle already underlies the system of number words. The positional principle, in contrast, comes into play only for written numerals – it determines that in the written numeral the value of a "digit" depends on its position. This principle is essential to understand written numerals and includes the understanding that when a zero appears at a certain position, it indicates that there are no units corresponding to that position. Both the grouping and the positional value principles help to reconstruct the number from the written numeral. This distinction was important in Session 1.1 in understanding what Mohsin knew (prices of articles and composing money in terms of currency units) and what he did not know (writing a number). The distinction between the grouping and positional value principles was new to many teachers, at least in an explicit sense.

The teacher educators believed that the usefulness of a "teaching aid" depends on the context and specific needs/difficulties faced by students. In Session 1.1, to help address Mohsin's difficulty, teachers had selected aids embodying both the grouping principle and the positional principle. This did not take account of the fact that while Mohsin had a weak understanding of the positional principle, he had a strong grasp of the grouping of multi-units because of his familiarity with money. The teacher educator was able to use teachers' responses to explicate how understanding of place value embodies both principles and how the abacus specifically supports the understanding of the positional principle.

5.6.3.3 Providing alternative viewpoints

In sessions when some artifact of teaching was shared in the form of a video, a resource for teaching or a sample of students' work, then teachers interpreted it in their own way and shared their observations. Teacher educators, on these occasions, used the opportunity to voice alternative viewpoints which teachers may have failed to notice or think about in their observations as.

In session 2.2, in response to a question, teachers used the method of cross multiplication to compare fractions. However, the teacher educator showed how the notion of the unit fraction can be used to compare fractions as unit fractions denoted the size of the unit and numerator indicated the number of iterations of the unit. So to compare 3/5 with 3/7, one knows that the size of the unit 1/5 is bigger than 1/7 and hence 3 measures of unit size 1/5 will be bigger than 3 measures of 1/7.

While discussing the reading of the chapter on multiplication from the book *Knowing and Teaching Elementary Mathematics* (Ma, 1999), teachers insisted that students need to know the rules of multiplication since it leads to "fast and accurate answer". The teacher educators then argued for the understanding of the multiplication process like distributive property and suggested that knowledge of alternative methods can have a greater contribution to students' understanding of multiplication. Later, some teachers shared how students make mistakes when they blindly follow the rules instead of understanding them, leading to a discussion on how ideas like the distributive property can be made accessible to students.

5.6.3.4 Establishing Connections

One of the most important roles of the teacher educator was establishing connections between the broad themes that were discussed in the workshop. These include connections among different concepts, between mathematics and teaching, between research and practice and between sessions in workshops and goals of mathematics education.

The connections that the teacher educators attempted to establish among different concepts like measurement and fractions in several sessions have been discussed above. Connections were also established between the concept of place value and various procedures for arithmetic operations. An example is a discussion on the multiplication algorithm and the concept of place value described above. A connection between arithmetic and algebra was made in one of the problem-solving sessions when teachers compared arithmetic and algebraic strategies to solve the same problems and realized that while some problems can be solved arithmetically, there are problems where the algebraic solution makes more sense and is more useful.

Connections between mathematics and teaching emphasize the role of pedagogy in making mathematics accessible to students. Teachers were aware of the efficiency of algorithms but were not aware of how they can be unpacked and made accessible to students so that they understand the method and are able to adapt it rather than relying on it as a rule (Kumar & Subramaniam 2012b). Alternative methods and their conceptual underpinnings were discussed in several sessions along with the pedagogy of teaching arithmetic operations. Similarly, for the topic of fractions, teachers engaged with interpreting the meaning of fractions in various contexts and reflecting on the pedagogy based on measuring and sharing contexts.

Connections between classroom practice and research were sought to be established by discussing research-based readings with teachers and teachers sharing their experiences. Teachers read and presented readings, which contained discussions of student work, tasks or pedagogical approaches on the topic of operations, fractions, geometry, ratio and proportion. Teacher educators also shared their research with teachers on the topic of fractions along with their teaching experiences through videos.

5.7 Teachers' learning from the workshop

In this section, I address the question whether the teachers' beliefs showed increased alignment with a student-centered view at the end of the PD workshop. I discuss the teachers' own reflections about their learning from the workshop from four sources: (i) oral feedback that teachers gave in the last session of the workshop, (ii) written feedback about the workshop given on the last day (iii) direct feedback about the workshop in their interview responses and (iv) revisions teachers made to their responses to parts 2, 3 and 4 of the questionnaire, which were about preferred practices, beliefs concerning nature of mathematics and beliefs concerning the teaching and learning of mathematics. For part 2 of the questionnaire, which dealt with preferred practices, they were asked to indicate the frequency of the practices that they would like to implement in future in their classrooms.

5.7.1 Feedback on the workshop

In this subsection, I discuss the teachers' feedback about the workshop expressed orally, in written form, and in their interview responses. Most teachers appreciated the sessions of live teaching, research readings and maths activity sessions. Although the teachers explicitly mentioned these sessions, the foregoing discussion reveals the role of other workshop sessions in meeting the workshop goals. They used words like "eye-opening", "thought provoking" and "child-centric learning" to describe the sessions in the workshop. Teachers stated that the sessions had thrown light about aspects of student thinking, the role of research in the field of mathematics education and its usefulness in classroom teaching.

In their written feedback at the end of the TPD workshop, many teachers wrote about how the various sessions had made them reflect about how students think and how important this is in planning for teaching or in addressing errors and misconceptions while teaching. In the written feedback P5 had shared that

Excerpt 5.4

> We as a teacher know content knowledge and methods to teach a particular topic in maths, but the teacher has to go beyond that and understand how the child learns and looks at a problem in his own perspective. We do not take this perspective in our mind while teaching. And the success of this course lies in throwing light on the child's way of understanding the particular problem. (P5, Written feedback, 3 June, 2009)

In the oral feedback session, P5 said,

Excerpt 5.5

> We use activity method but this particular insight that the child might do the sum in a different way, this thinking might be different, this idea has never struck our minds.... Which is – being like this is a – we are really enlightened us in that particular way that the child might have a different idea and the child has written wrong is not always wrong. (P5, oral feedback session, 3 June, 2009)

Middle school teachers too appreciated how the workshop had highlighted the child's point of view and its role in teaching. **M2** wrote in her feedback that

Excerpt 5.6

> This program is different from the other trainings where we mostly used to discuss the content whereas here we were to think of the child's angle. The model classes we observed here surely will influence us to conduct child-centric learning in different perspective. Each and every child is unique, each one has the role in the classroom. I learnt how to make learning challenging, how to relate the learning process to the real life situations and so on. (**M2**, written feedback, 3 June, 2009)

Another teacher M6 wrote about how it had helped her to understand different aspects of teaching while also providing an example of child-centered teaching. She said in the oral feedback session,

Excerpt 5.7

> ...I think this workshop has opened a new window for teachers to understand the

> different aspects of the teaching of maths and also an opportunity to beautify the subject in a child-friendly way. Classroom session with children gave altogether different angle where children are doing and thinking, more than simply listening to the teacher and copying things mechanically. (M6, oral feedback session, 33 June, 2009)

P1 talked about how the teacher paid attention to each student's thinking while teaching in the live teaching session. P5 elaborated her insight about how she had never thought that students might solve a problem in a way different from what a teacher expects.

Excerpt 5.7

> He had an eye on each student that why that student gave that answer. We are trying to find that 'why'.... It is good and also that we are trying to give personal attention to each student. (**P1**, Interview excerpt, 27 May, 2009)
>
> Children make mistakes. I will view their mistakes in a different way. Maybe their mistakes are – instead of straight away telling them the answer, I might ask them, "why have you done that; what they think about this?" I have to take that child in confidence and then that child [will] slowly tell me. This way, I have thought about it. (P5, Interview excerpt, 2 June, 2009)

Another middle school teacher M5 suggested that the session of live teaching was helpful in thinking deeply about student responses, although they as teachers have been anticipating and listening to student responses for years. He said that discussing student response in the professional development set up was useful since it unearthed many issues about students' thinking and teaching.

The session on research readings was appreciated since they provided insight into the field of mathematics education on the one hand and on the other hand made teachers realize the common problems faced by teachers across the world. Teachers were able to appreciate the need of research through discussing these papers. A primary teacher P4 appreciated the different contexts that were discussed for the teaching of fractions.

Excerpt 5.8

> Fraction connecting with so many things – cake, distance, which we are not doing. We should be giving more examples of this type and connect it so that the concept is clear. (P4, oral feedback session, 3 June, 2009)

Teachers also appreciated the kind of interactions with teacher educators and with other teachers during the workshop. A middle school teacher **M1** said

Excerpt 5.9

> First of all, atmosphere is very free. It's not like you know other places they are more concerned about attendance, where lectures would be there, where you have to listen. Here it is interactive and you can say anything you want and everybody is putting their own opinion. So we come to know so many things not just the speaker's views. (**M1**, Interview excerpt, 27 May, 2009)

A primary teacher **P1** shared how these interactions and the presentations that teachers did of readings, textbook analysis and classroom experiences helped in developing her confidence.

Excerpt 5.10

> ...To manage oneself and one's own answer helped me learn a lot here. Earlier I had thought of looking at the textbook deeply but never got the time. [It] helped me rebuild my confidence. Problem sessions were good. At first, I felt burdened when so many pages were given for reading but now I think it is necessary for us to read and understand what efforts are being done in teaching across the world. I liked that he [teacher educator who taught students during the lesson observation] asked about suggestions for improvement and if something was amiss in teaching… (**P1**, Interview excerpt, 3 June, 2009)

5.7.2 Shifts in teachers' responses to the questionnaire

In addition to the explicit feedback from the teachers about the impact of the workshop, teachers were also asked to mark their responses once again to the beliefs and practices questionnaire on the last day of the workshop to indicate any change in their beliefs. The responses to the preferred practices part of the questionnaire were to be interpreted as practices that they would prefer in the future. In this sub-section, the analysis of the shifts in teachers' responses to the questionnaire is presented.

Analyzing the responses of teachers to the questionnaire on the last day of the workshop and comparing it with the responses on the first day through a paired t-test, I found that there were statistically significant differences in the responses for some of the items. Tables 5.5, 5.6 and 5.7 show responses to items in parts 2, 3 and 4 of the Questionnaire, which were respectively about practices, belief about mathematics and beliefs about its teaching and learning. Only items for which there was a significant difference between pre-workshop and the post-workshop responses of the 13 teachers who participated in the teacher professional development workshop are included in the tables.

Table 5.5: Shifts in the teachers' responses to Part 2 of the questionnaire on preferred practices (1 – Almost never; 5 – Almost always)

Item no.	Statement	Pre-workshop mean	Post-workshop mean	Difference between Means (Significance)
2.1	In the beginning of the class, I show students how to solve a particular problem and then give similar problems to practice from the textbook.	3.31	2.08	– 1.23**
2.2	If any student does not understand what was taught, I explain to the student once or twice by repeating the steps in detail slowly.	4.46	3.00	– 1.46**
2.8	I ask students to practice the problems very similar to the one done in class as home work.	3.77	2.69	– 1.08**
2.23	To explain mathematical concept, I show the steps of the procedure to solve the problem.	4.62	3.69	– 0.93**

** Significant at 0.05 level.

Table 5.5 indicates that the means of the items presented showed an increase when teachers responded to the questionnaire again at the end of the workshop. Since the score of the negatively worded items were reversed, the decrease (indicated by a negative difference) in means post workshop indicate that teachers' beliefs pertaining to these items were aligned towards the student-centered end of the continuum rather than towards the teacher or procedure-centered practice. The negatively worded items are shown with a grey highlight in the table. The table shows that difference between the means of the teachers' response before workshop and post-workshop is significant at 0.05 level for items 2.1, 2.2, 2.8 and 2.23. This indicates that there is a high probability that the differences in their responses indicate true difference in their views and the probability is less than 5% that this difference occurred by chance. These items correspond to the themes related to showing procedures, doing similar problems, telling the steps of the procedure in case of error, all of which have been described as core practices among the teachers in Chapter 4. Teachers anticipating less frequent use of these practices in future as indicated by the post workshop responses suggests that they were reflecting about the use of these practices in the classroom and their role in learning mathematics. These shifts suggest that the workshop was able to create some dissonance for teachers related to the core practices followed by them and made them aware of other alternatives. The major difference in these items could have been due to the discussion in the workshops as well as the teachers' observations during the demonstration lessons of students' ideas and the solutions that students may arrive at on their own given the chance. Also, the emphasis on analyzing student errors and understanding their thinking may have made teachers more sensitive to analyzing students' errors and giving them more time to solve the problems. Although there were differences in the responses to other items in the questionnaire, these were not significant.

In Table 5.6, t test results of the items on beliefs about maths which showed a significant difference between the teachers' responses in pre-workshop and post- workshop questionnaire are presented.

Table 5.6: Shifts in the average of teachers' responses to the part 3 of the questionnaire on beliefs held about mathematics (1 – Disagree strongly; 5 – Agree strongly)

Item no.	Statement	Pre-workshop mean	Post-workshop mean	Difference between mean (significance)
3.3	In mathematics, we can give proper reason or justification for all statements and procedures.	4.54	4.00	– 0.54**
3.9	In mathematics, if the student has a doubt about a mathematical statement or procedure she or he can check and justify it on her/his own.	3.15	3.85	0.70**
3.15	It is important to teach students "shortcuts" or "thumb rules" for solving mathematical problems.	3.85	3.46	– 0.39**

** Significant at 0.05 level. Items are listed in the order of significance.

In Table 5.7 the most significant difference in teachers' response is visible for the Item nos. 3.3, 3.9 and 3.15. Items 3.3 and 3.9 are about the importance of the justification of the procedures used to solve maths problems. The difference between means for Item 3.3, unexpectedly, is negative and significant. It is difficult to reliably explain this unexpected difference. One may note that the pre-workshop mean was quite high (4.54), indicating that teachers already strongly believed the statement. The shift indicates that teachers initially expressed strong belief about giving reasons and justifications and then later showed less strong alignment. A possible reason might be a shift in teachers' understanding of what it means to give reasons and justifications of procedures. In the workshop, many teachers justified procedures like the subtraction and multiplication algorithm on the basis of rules, which were not considered as justifications by teacher educators and justifications on the basis of place value were discussed. It is possible that the workshop discussions made some teachers realize that they were unaware of justifications of other procedures or hold more strongly to belief that it is not possible to give justifications of all procedures.

The response to Item 3.15, on the other hand, shows a significant shift among teachers that shortcuts and thumb rules are not important. Teachers were also asked to justify and explain the shortcuts during the workshop and connect it with the other longer procedures for solving as well as with the underlying concepts. This may have lead to realization that connections between concepts and procedures is more important rather than memorization of the shortcuts. In consonance with this view, there is a significant increase in belief that students can check/ justify their solutions on their own indicating an acknowledgement of the importance of justifications in learning mathematics. Another item, Item 3.10, about the view of mathematics as limited to 4 operations, also showed a difference of +0.39 in the means, but is not significantly different.

Table 5.7: Shifts in the teachers' responses to the part 4 of the questionnaire on beliefs about teaching and learning (1 – Disagree strongly; 5 – Agree strongly)

Item no.	Statement	Pre-workshop mean	Post-workshop mean	Difference between means (Significance)
4.20	The best way to teach mathematics is to explain one procedure at a time on the blackboard and then to make students practice it.	3.15	2.08	– 1.07***
4.4	If a student practices all the problems in the textbook two or three times, that is the best way to learn mathematics	2.54	2.15	– 0.39***
4.14	Only one method should be taught to students otherwise they get confused	3.15	2.91	– 0.24***
4.7	A teacher should teach each topic from the beginning assuming that the students know nothing	3.69	3.34	– 0.35**
4.5	By asking probing questions that create uncertainty and confusion in the students' minds, we can help them learn better.	3.00	3.85	0.85**

*** Significant at 0.01 level. ** Significant at 0.05 level.. Items are listed in the order of significance.

Table 5.7 shows that the most significant difference is for item 4.20 which is about repeated practice and a core practice identified in Chapter 4 among teachers. Other significant items like 4.4 also indicate that teachers now agreed less with using repeated practice. The significant difference between the means for item 4.7 and 4.14 indicate that teachers may have started to think about their role – from telling and teaching standard methods to solving a problem considering a diversity of methods. However, the differences are not significant for practice of textbook problems (4.18) and practice in case of errors (4.22). Although there was change in the teachers' response for items describing the role of teacher for showing procedures and explaining (4.1, 4.9 and 4.13) the change was not significant. Although the change is not significant for asking challenging questions to students in the beginning of the topic (4.12), the change is significant for asking probing questions to students (4.5).

The overall pattern in the shifts in responses to questionnaire items indicate that some dissonance occurred as a result of workshop. Many of the changes are not significant and one significant change is opposite to the expectation that the workshop might have led teachers closer to the student-centered end of the belief continuum. However, it is difficult to arrive at reliable conclusions on the basis of questionnaire responses alone, as was seen in Chapter 4. There was no possibility of interviewing teachers after the PD workshop, and hence additional data that could be triangulated with the responses to arrive at more robust interpretations of the shift in teachers' beliefs is not available. The analysis indicates that the core practices supported by core beliefs were resistant to change. Where there are changes, I interpret them as indicating a developing sensitivity towards students' thinking as they had witnessed examples of how students can come up with solutions on their own and had discussed how students' errors reveal their thinking. This also made them open to respond more positively to the use of multiple methods for finding solutions and accepting mistakes as a natural part of learning. I do not claim that these shifts in teachers' responses after the workshop indicate a change in belief. I interpret these shifts as indicating a greater awareness of alternatives and adopting a reflective stance towards the practices used and beliefs held.

5.8 Discussion and conclusion

The effectiveness of TPD workshops depends both on design and enactment aspects (Kumar, Subramaniam & Naik, 2015a). I have attempted here to present a framework that can aid in the understanding of both aspects and to illustrate how the framework may be applied to an analysis of the components of a workshop and interaction episodes. The framework assumes that the central goals are to address teachers' knowledge and beliefs relevant to mathematics teaching. The framework does not describe what constitutes knowledge for teaching mathematics, nor does it elaborate on the nature of beliefs conducive to teaching for understanding. A framework that elaborates on the specifics of knowledge and beliefs relevant to teaching mathematics will need to be contextualized with regard to topics and to teacher communities. The framework presented here, in contrast, identifies certain principles that are important for the design of tasks and their enactment in workshop sessions. The study reported here does not aim to provide evidence for the effectiveness of a TPD intervention. The framework proposed here, I believe, is useful in identifying and providing rich descriptions of elements that are important in a TPD intervention.

The principles considered important are situatedness, challenge and community building. The components of the workshop and the tasks worked on were chosen to implement the goals of strengthening teachers' knowledge and reflection on beliefs, and implicitly embody these principles. As the analysis of interactions in the two episodes supported by other examples across the sessions shows, not only is the design of tasks important but also how interactions between teachers and teacher educators are shaped to support teacher learning. I have used the teacher education triangle having the three corners as task, teachers' agency and teacher educators' agency, as a framework to analyze the interaction aspects. The task incorporated contexts and artifacts that are situated in the work of teaching thus facilitating the involvement of teachers by identifying elements common o their own teaching practice and engaging in a deeper exploration of the contexts and artifacts. In the episodes discussed above, a prominent artifact was students' errors or responses. Discussion centered on these led teachers to analyze the conceptual gaps, the alternative explanation of errors and develop a perspective of explaining student errors by thinking about students' sense-making efforts.

The evidence of the types of teacher engagement that occurred during the episodes throw light on the kind of opportunities that arise for teacher learning. Teachers' engagement took the form of anticipating and predicting students' responses, identifying key knowledge pieces, conjecturing underlying causes, articulating and contesting beliefs and assessing a teaching resource or a teaching approach. Such engagement was crucial in building shared understanding not only among teachers, who rarely get opportunities to reflect collectively about teaching in their schools and professional development contexts but also for teacher educators by providing windows into teacher thinking. Teachers' assertions, counterarguments, alternative explanations and assessments were also a resource, which deepened fellow teachers' and teacher educators' understanding about mathematics teaching as it takes place in classrooms. The key knowledge piece of the meaning of zero, identified by an expert teacher, was important in deepening the participants' understanding of the conceptual gap that needs to be addressed to help Mohsin in writing numbers correctly.

I have elaborated on the principles of the situatedness of tasks, challenges and development of the community as guiding the design and enactment of the sessions. These aspects inform the decisions of the teacher educators about how interventions are to be made in sessions to facilitate active learning of the teacher. The belief goals of the teacher educators helped in guiding what interventions are to be made in terms of prompts presented to the teachers and in identifying aspects of teachers' responses that could be problematized. The agency exercised by the teacher educator is important to not only actualize the opportunities afforded by tasks but also in guiding discussions beyond the resolution of the tasks in order to relate to the broader goals of teaching mathematics. The actions of the teacher educator like inviting teachers to respond to each other, handling disagreement and conflicts by posing issues as more general questions, use of open-ended questions and building shared vocabulary paved the way for building a sense of community while challenging teachers to explain and justify their thinking.

I claim that while designing workshops for teachers it is essential to not only focus on the aspects that need to be discussed with teachers but also how the session needs to be enacted to allow teachers to exercise their agency and take ownership of their own learning rather than looking for answers from outside. This is important because I need to provide ways through which teachers can build on the knowledge of students and mathematics teaching that they already have rather than merely providing knowledge, which teachers may or may not find useful in their own classroom contexts. This point is important for designing workshops for in-service teachers as they have already developed an identity as well as the situated knowledge of students and teaching which must be respected and built upon.

Teachers' self-reports of their learning during the workshop throw light on how they interpreted gains from the workshop. The shifts in their responses to the part 2, 3 and 4 of the questionnaire, especially with regard to the items related to focusing on procedures and doing similar problems and telling the procedure in case of error were significant. However, no conclusive interpretations can be made of these shifts. I interpret these as indicating broadly a developing sensitivity to students' thinking. The chapter that follows will illustrate the take up from this workshop using a case study of a teacher who was open to changing her practices towards teaching maths for understanding.

6

Role of Beliefs and Knowledge in Teaching Fractions: Case Study of Nupur

6.1 Introduction

The practice of teaching mathematics tends to acquire a stable pattern for teachers in the course of their careers influenced by their own experience of teaching, the cultural beliefs and norms about teaching in society, the school's priorities and the teachers' own personal beliefs. However, it is important for teachers to engage in avenues for lifelong professional growth so that the teaching does not become mechanical and the teacher is aware of decisions and the impact of decisions taken by her on the learning of students. Periods of educational reform and professional development efforts like workshops and collaboration with researcher provide opportunities for teachers to reflect and re-evaluate their present practices, become aware of alternative ideas and practices that exist, which, in turn may provide motivation for teachers to engage in their own professional growth. However, change in practice does not necessarily result from educational reform efforts imposed from above or from directives. Teachers' practice is impacted by the way teachers interpret these advisories, the knowledge they have and the opportunities and support they get to extend their knowledge and explore new practices. It is the teachers' agency to reflect and take initiative for change that lead to changes in practice. Workshops, resources and collaboration may act as catalysts but the decision is made by the teacher. Teachers' identity plays an important role in teachers recognizing the scope for exercising their agency while responding to pressures from different aspects of school life like completion of syllabus, performance in exams, etc. Teachers' practice in school is thus influenced by the beliefs held by teachers, their perceived identity, the knowledge they hold for teaching mathematics and the repertoire of the practices acquired over the years as important factors.

Post the professional development workshop described in the previous chapter, visits were made to the classrooms of a few teachers with the dual purpose of understanding the impact of the workshop on teachers in terms of their beliefs and practices and understanding the constraints and affordances that teachers experience in bringing about the intended change. The researcher also collaborated with the teachers in the classroom to help in bringing about intended changes. As discussed in Chapter 3 (section 3.1), I followed a participant observation methodology, which included collaboration with the teacher in the classroom. The goal of collaborating with the teachers in the classroom was to analyze how different factors come into play in supporting or constraining teachers as they try to change their practice in the classroom and to understand the role of beliefs and knowledge in bringing about this change.

6.2 Case study of Nupur

In this chapter, I present the case study of a primary teacher *Nupur* (pseudonym), belonging to the same nationwide system of schools from which teachers had participated in the orientation workshop described in Chapter 5. Nupur was one of the participants and has been named as the teacher P1 in Chapters 4 and 5 respectively. Nupur came across as a consistently reflective teacher who held beliefs close to a student centered view of teaching. She was open to changing her practice to focus more on developing understanding of mathematics amongst students.

6.2.1 Research questions

The research questions that guided this Sub-study are

1. What changes did Nupur try to bring in her classroom practice specifically with regard to the way in which tasks for students were framed and implemented? How did her beliefs and knowledge support and constrain the changes that she tried to implement?
2. What textbook resources were available to her to support her teaching and how did she make use of these resources?
3. What was the role of the researcher as a collaborator in Nupur's teaching?

6.2.2 Why case study?

Case study was chosen as the appropriate research method in this Sub-study, since the larger study aims at exploring in detail the relation between teachers' beliefs, knowledge and practice. According to Eisenhardt (1989) case studies are suited for studying phenomena for developing theoretical insights. Case studies have been found to be useful to answer how and why questions about phenomena when either the boundary between the phenomena and the context is not clear or the phenomena itself is complex and the parameters are beyond the control of the researcher (Yin, 2009). Teachers' struggle to change practice or explore new practice is a complex phenomenon, which involves interaction among several aspects like teachers' beliefs, knowledge, practice as well as the beliefs and knowledge of students. Case study was also considered appropriate since it would help in triangulating the data from the questionnaire, interviews, teachers' participation in the workshop as well as classroom teaching. The case study of Nupur was considered as potentially useful since it could provide insights into the extent to which holding positive beliefs for student centered teaching can motivate a teacher to explore new practices and the kinds of challenges such a teacher might face.

6.2.3 About Nupur

Nupur was 46 years old at the time of study and had an MSc Degree in Chemistry and a BEd degree. She had been teaching maths and science at the primary school level for 21 years. She appreciated the need for developing the ability to reason about mathematics, talked about the need to connect mathematics with daily life and to focus on concepts rather than on rote memorization of procedures. Among the participants in the study she was judged to be most likely to appreciate a focus on mathematical concepts and connection in teaching, based on her interview and participation in the workshop and was hence chosen as a case study for follow up in the classroom. During the period of observation she was the class teacher of Grade 5 consisting of 45 students and had a good rapport with her colleagues. The majority of the students in her class belonged to the lower middle class and according to her, many of their parents were not even educated till the 8th grade. She knew that many students of her class attended coaching classes but felt that there they were taught by poorly qualified teachers who focus on rote memorization.

6.2.3.1 Nupur's beliefs and practices

As Nupur indicated in her response to interview questions and the questionnaire, she believed that mathematics concepts should be focused in teaching to limit the rote memorization. of procedures and that mathematics done in class should be connected to the student's everyday experience. She had indicated that her practice was mostly activity oriented and she most often used contexts, concrete materials, representations to engage students in doing mathematics and encouraged students to share their ideas. She believed that weak students in mathematics need more time to understand and need more focused attention of the teacher. Her response when asked for an explanation of the long division procedure and her participation in the workshop indicated that she did have knowledge of concepts and procedures and could predict students' spontaneous strategies and common errors.

Teaching Procedures

Nupur considered mathematics as one of her favorite subjects. A core belief of hers seemed to be about the need to make connections between mathematical procedure and concepts while teaching. In the interview, it was evident that she did have some knowledge of alternative procedures and conceptual explanations, appropriate contexts and manipulative that could be used for teaching certain mathematical concepts. She pointed out the need to discuss the role of place value in understanding the multiplication algorithm and multiplying with powers of ten. She used the context of sharing while explaining the long division algorithm. In the interview, Nupur indicated that she valued student autonomy as she shared that she tries to get students to answer on their own but in the end she has to explain or show how to solve so as to achieve the target of the syllabus.

Excerpt 6.1

> If they are not getting, even after explaining, then we solve it. Mostly, we first explain to them, five times, several times, then oral answer comes from students. If not we show by solving…. We are not able to keep a lot of patience because we have to go along with the syllabus…. If we give hint and the student does it, then it is all right, but in primary class we have to explain to the general class, at least once. Otherwise, we ask the student to tell how did he arrive at the answer. But in general we have to tell, so that we complete the target of that day. (Interview excerpt, May 25, 2009)

The excerpt indicates that she experienced tension between her belief to allow students to come up with their own ideas and need to complete the target of the syllabus within the specified time.

Role of practice in learning maths

Nupur believed that learning mathematics by practicing similar problems a number of times amounts to rote memorization. However, she felt that practice is necessary as it helps to "fix the concept" and helps to make calculation fast. She felt that shortcuts similarly help in fast calculation but are not useful for everybody as not everybody is able to understand them. So, from the interview data her practices and beliefs for teaching mathematics by focusing on concepts seem to be at the core. However, these might be in tension with other beliefs and values like completing the syllabus.

Alternative methods

Nupur felt that allowing different methods of solution helps in understanding as one might prefer one method over another. She gave the example of a Grade 3 student who solved the multiplication problem with addition. She felt that it is all right for him to do that since he is comfortable with it and sooner or later would have to shift to using the standard multiplication algorithm. She also said that she feels happy when students are able to solve problems after her explanation. So while she considered teacher's explanation as important for students learning, she also believed in providing space to students for giving alternative solutions.

Nupur laid emphasis on assessing students' understanding through adopting various practices like calling students to the board, asking probing questions and constructing problems different from those in the textbook, etc. If the students did not understand or gave a wrong answer she said she would point out which steps were wrong; give similar situations, explain the problem by focusing on what is asked and then let the students try it on their own without telling them the steps.

Belief about connecting mathematics to daily life

She had a critical view of the school mathematics that had been mainstream during her own school education. She felt it was unduly focused on memorizing procedures without understanding and that teachers did not use "teaching aids" or establish connections with daily life in teaching.

> There we used to feel that, we only have to multiply the numbers and find the area but didn't know where area is going to be used.... We were never taught how to correlate with life. (Interview excerpt, May 25, 2009)

As a result, she found it difficult to apply mathematics to her own everyday experience, for e.g., to estimate the size of bed or the cushion cover. She felt that "when you will learn all these things practically then your concepts would be stronger". To buttress her point, she cited examples given in the new textbook that connected mathematics to daily life tasks related to shopping, cooking time, etc. Therefore connecting mathematics with daily life seemed to be a belief that can be considered as one of her core beliefs, although almost all the examples that she gave for using daily life contexts came from the new reform oriented textbook. She acknowledged that she herself had learnt a lot of mathematics while relating maths to daily life as a teacher, for e.g., while buying bedsheets she has to keep in mind "whether area is needed or the perimeter". She elaborated this by saying how it is important to know the dimensions of length and breadth of the bed so that it covers the bed. If she used area, the dimensions could be different and the bed sheet may not cover the bed properly.

Beliefs towards equity

Nupur seemed to be sensitive to students from weak socio-economic backgrounds although she sometimes expressed ambivalence towards girls' performance. She felt that girls do well in mathematics in primary school but become quieter as they grow up as a result of social pressures and expectations that they start to internalize. She said that she takes time to "know" the students in a new class as some students may appear to have understood but their written work might indicate otherwise. She felt that some students have inborn ability to be good in mathematics although it does not depend on the socio-economic background of the student. She found that students from poor homes were good in mental maths since they are used to doing shopping related chores. She believed that some students might just need more time than others to understand the concept contending that all are capable of doing mathematics. She appreciated the efforts done at her school for helping weak students in mathematics by providing help from older grade students in a "Maths Clinic" and an outdoor mathematics lab for students discussed below in section 6.2.4.

Conclusion

Nupur's sense of identity as a teacher was fostered by helping students understand mathematics through the use of manipulative and connecting mathematics to daily life. She felt satisfied when there was more student participation in the classroom and when her explanation was able to help a student understand a concept that the student had difficulty in understanding. Her critical stance towards rote memorization. made her look for ways to help students understand mathematics through representations and illustrations and made her appreciate the new textbook. Although she acknowledged that some students from poor homes were not able to cope, she emphasized the role of teacher in giving more time and attention to help them. All these practices indicated that her belief and practices for teaching and learning mathematics were close to the student centered end of the continuum.

6.2.3.2 Nupur's beliefs and knowledge indicated in the workshop sessions

Nupur participated actively in most of the workshop sessions. In her responses, Nupur strongly expressed her belief that students learn not only from the teacher and within the school but also from their own environment. In response to a task, she was able to anticipate that a student, who was also a vegetable vendor would do repeated addition rather than append a zero when multiplying with 10 and would use money as a representation to find the solution. This task has been discussed in Chapter 5.

She often expressed her belief that conceptual aspects of mathematics were important. She explained how place value plays a role in the subtraction procedure, regrouping, aligning digits in vertical addition, etc. She was also able to identify the conceptual gaps underlying student errors. For e.g., writing 6 tens + 3 Thousand + 8 ones as "368" indicates that the student did not have the concept that place values must be represented by zero when there are no corresponding units. While describing the student's thinking underlying an error, she not only identified conceptual gaps but also what the student did know. She was able to think of alternative strategies for comparing fractions or carrying out division by referring to the context of equal sharing. This indicated that she had knowledge of the connections between procedures and concepts for the topics discussed in the workshop since she gave conceptual explanations, identified conceptual gaps, identified the concept that was assessed by a question and the contexts that could be used to discuss the meaning of mathematical concepts.

Her sensitivity towards students was reflected in her responses as she was generally appreciative of students' ideas. As described in Chapter 5, teachers were engaged in reading and presenting a selected research article in the workshop. Nupur made a presentation on the reading assigned to her – "A day in the life of one cognitively guided instruction classroom" (Hiebert, Carpenter, Fenemma, Fuson, Wearne, Murray, Oliver & Human, 1997). She appreciated how teachers in the project attempted to build on students' knowledge by giving hints and asking questions. She asserted that problem solving for daily life as described in the article should be a part of teaching, for e.g., finding out how many students are absent or sharing the food during lunch. She also appreciated the description of the classroom in the reading – how there was flexibility in the use of methods and numbers and that the focus was on explaining how one got the answer rather than on the correct answer.

6.2.4 Setting of the school

The school in which Nupur was a primary teacher was situated in an armed forces facility and catered to families of personnel from the armed forces, families from adjacent areas and also some families of employees in other government establishments. According to the teachers of the school most students belonged to the lower middle class, and their parents were not highly educated. (Several parents had not completed 12 years of schooling.) The primary section of the school (Grades 1 to 5) was supervised by the headmistress who reported to the principal who had the responsibility of the supervision of the whole school. The principal of the school, at the time of the study, had a background in mathematics and had taken several steps to encourage students' learning of mathematics. The school had partially converted an open play area into a maths lab by building concrete structures of various 3d geometric shapes as pots for plants and had mensuration formulae written on them. On one wall different formulae associated with various geometric shapes like, circle, square and rectangle were written. The children used to play in this area during the recess and the principal and teachers felt that looking at these shapes and formulae again and again while playing would help them remember the formulae.

Another initiative of the school was a "Maths clinic" in the primary section where once a week students of Grades 11 and 12 taught mathematics to selected weak students in the primary classes. The school felt that the students were weak because they had not been able to get enough individual attention in class, since the class size was above 40 students, and the Grade 11-12 students could give individual attention to them in their free period. However, the researcher did not see this happening during the times she visited the school. The school had an activity room for primary graders. The room contained some materials for learning mathematics which was kept locked in the cupboards. Students were mostly shown films in the activity room. The students sat on a mat on the floor and there was no blackboard in the activity room.

Meetings of all the teaching staff of the school were held on the last working day of the month where the headmistress and principal discussed the circulars issued by the authorities and how they were to be implemented in the classroom by the teachers. The teachers were also questioned during these meetings about the responsibilities that they had been allotted like sending students for quiz competitions, preparation of extra curricular activities, etc. The circulars were seen to be part of the efforts at implementing the reform initiatives – National Curriculum Framework (NCF 2005) or the Right to Education Act (RTE 2009). One of the circulars that was discussed while I was present in the meeting was making Saturday a "no bag" day. This meant that students would not bring textbooks or notebooks on that day, which led to the teachers being asked to make and submit worksheets for all Saturdays in the year. In another meeting, the principal had asked primary teachers if they were showing films distributed to them by the "Films Division" of the government, which they were expected to do as part of NCF 2005. In another meeting, the Principal discussed how it was now (post educational reform) easy for teachers to teach as they can make a "powerpoint presentation" and show it to students and thus also integrate Information and Communication Technology (ICT) into teaching.

Nupur mostly sat with other primary teachers in the staff room. So while working with her I had the opportunity to listen to the staffroom conversations and debates and develop an acquaintance with other primary teachers. In time, they would freely approach me to discuss or seek guidance for some mathematical topic and some even invited me into their classrooms to "help" them with the challenges they were facing in developing students' understanding.

The classroom observations were done in the year 2009 as well as 2010. In 2010, the school system authorities had issued circulars to improve the education system at the primary level under the heading called "common minimum program". The main issues that the circular addressed was to have a resource room for primary classes with audio/video facility along with the teaching aids. A contingency fund was allotted for teachers to spend on constructing relevant teaching aids. The Headmaster/Headmistress was expected to monitor and evaluate teaching as well as "lesson notes, activity plan book and worksheets" for primary teachers. Further he/she had to coordinate "demo lessons, interacting sessions and short duration workshops". It was recognized that "formation of strategy, identification of classroom activities and advance preparation of TLM (Teaching Learning Material)" are important and it was suggested to hold subject committee meetings regularly for "sharing of ideas among teachers of same subject" and to create a pool of necessary TLM. Suggestions for holding at least two one-day workshops for teachers to discuss "every aspect of teaching learning" was given in the circular. However, conversation with teachers revealed that the subject committee meetings were used to largely discuss the "split-up syllabus" (i.e., time wise distribution of lessons across the school year) and what portions are to be included in the upcoming test, rather than discussing the issues related to teaching.

The records of the teaching aids as well as worksheets used by the teacher were kept by the headmistress of the school and teachers were asked to submit worksheets in advance. The evaluation of teaching was done once a month by the headmistress by observing lessons. The lesson observation format consisted of categories like teaching-learning objectives, methodology, classroom interaction, blackboard work, use of teaching aids, students' response, teacher's performance, notebook checking and suggestion for the overall lesson. The interaction was analyzed based on whether the students were engaged in individual/ group work, problem solving and whether the introduction was done using students' previous knowledge and if recap was done at the end. Students' responses were analyzed for whether they were given in oral or written form and whether the student used one word answer or full sentences. The teacher's performance was analyzed based on whether the students were active and enthusiastic and whether the teacher asked questions to weak students and made them repeat the answer. There was no category in the lesson observation format for the content or assessment of understanding of content among the students. Teachers were required to fill up the daily plan sheet in which they had to write in one line or a phrase the topic taught or work done and the teaching aids used for each period in each class. These efforts were likely aimed at "developing accountability" among the teachers for teaching, but they did not lead to teachers actually thinking about and planning for teaching.

Most of the students in Nupur's classroom belonged to low socioeconomic background. Three sections of the Grade 5 class were surveyed by asking questions orally. I briefly interviewed each student and asked questions about the criteria listed in Table 6.1. Nupur taught mathematics in two of the sections and corroborated the students' responses. Table 6.1 depicts the criteria based on which students socio-economic background was estimated like mother and fathers' profession, the kind of amenities at home and whether they have a house-help or not. Information about whether the student goes for tuitions or not was also sought along with data about whether somebody from their family makes an effort regularly to help them in studying. Another question was asked about whether students liked maths or not. While some students might have felt pressurized to give a positive answer to this last question, there were a few students who explicitly stated their dislike.

Table 6.1: Socio-economic status indications of the students in the Grade 5 of the school

Section (No. of students)		A (41)	B (38)	C (32)
Father's profession		23 in defence, 9 in other govt. job, 9 self employed / private	17 in defence, 12 in other govt. job, 9 others	18 in defence, 7 in govt. job, 7 others
Mother's profession		38 housewife, 3 self employed	33 housewives,2 self-employed, 3 govt. job	30 housewives, 2 govt. job

Amenities at home	TV	41	38	31
	2 wheeler	12	19	13
	Car	1	3	3
	Computer	11	15	15
	House help	2	5	5
	Tuition	19	17	13
Who helps with studies		11 mother, 30 father	4 self, 8 mother, 17 father, 9 others	7 mother, 16 father, 9 others
Like maths		36	37	30

6.2.5 Data collection and analysis

A total of 15 lessons taught by Nupur in the first year and 24 in the second year were observed. All the lessons were audio-recorded with the recorder placed on the teacher's table in the front of the class. The researcher wrote logs of the interactions during the lesson, personal reflections about the lesson and made notes of discussions with the teacher Nupur after the lesson. Notes were also made by the researcher in the role of participant observer of the informal interactions with the teachers and authorities in the school. Other documents related to school like circulars were also collected.

For analysis, at first, day-wise descriptions of the lessons and discussions with the teacher were constructed using the field notes of the researcher, logs of the classroom and notes about the discussion with the teacher after some of the lessons. Audio records were reviewed to annotate and add relevant transcripts. Interactions of the teacher with the students were analyzed for the practices adopted by the teacher. Since most of the interactions reported here were bilingual (Hindi and English) the Hindi utterances have been translated into English. The teacher researcher discussions were analyzed for the themes that emerged. This helped in identifying the challenges faced by the teacher.

In this chapter, I have focused on the lessons taught by Nupur on the topic of equivalent fractions. These lessons have been selected for study since the topic of fractions was dealt with in the TPD workshop which Nupur had attended and lessons on the topic were observed in both years. It was hypothesized that the analysis of these lessons will give insight into what aspects of the workshop were taken up by the teacher and the challenges that she faced in using those ideas. The topic of fractions has been considered as a challenging one which students find difficult. Therefore understanding the teachers' efforts to develop understanding and the challenges experienced can contribute towards the knowledge for teaching of fractions along with aspects that need to be focused in professional development. Further details on the methods adopted to analyze the selected lessons are discussed after the framework of analysis is presented in section 6.3.

6.2.5.1 Visits to Nupur's classroom

I visited Nupur's school in both Year 1 and Year 2 of the study, in Aug 2009 and in July-Aug 2010. A total of 39 lessons of 35 minutes each were observed (15 and 24 respectively in each year) as indicated in Table 6.2. Informed consent of the teacher was taken for collecting and using the data records for research and educational purposes. As described earlier, an attempt was made to understand the school context by interacting with other teachers, the headmistress, looking at the circulars issued to teachers as well as understanding the duties that the teachers had to perform.

Table 6.2: Nupur's lessons observed in Year 1 and 2

Year	Period	Number of lessons observed	Grade	Topics
1	Aug-sept. 2009	15	5	fractions
2	July-August 2010	20, 4	5	Area, fractions

In this chapter, I discuss Nupur's teaching of fractions. All the 15 lessons observed in the first year and 4 of the 24 lessons observed in the second year were on the topic of fractions. Detailed analysis is presented for the first three lessons (2 double periods and a single period) in the first year and three lessons (1 double period and 1 single lesson) in the second year on the topic of fractions. The lessons in the first year are in the very initial days following the TPD workshop, when Nupur was exploring new practices and was faced with several challenges. These will be discussed in the following sections. Nupur taught the chapter on fractions titled "parts and wholes". Since the topic of fractions was one of the concepts that was focused in the orientation workshop that Nupur had attended, the classroom observations gave insight about what aspects/features/issues discussed in the workshop were taken up by the teacher and what kind of challenges the teacher faced when trying to use new practices in her teaching. The case study describes the changes in pedagogy as well as the way mathematics was focused in her teaching.

The significant events for discussion in the case study were selected from the detailed descriptions and notes of all the lessons on fractions, which depicted moments of struggle for the teacher and the efforts taken by her to resolve the tensions. However, the first three lessons of teaching fractions were fully transcribed and detailed analysis was done because they represent an interesting sequence of lessons reflecting the kind of challenges faced in bringing about intended changes. After Nupur had taught the topic of fractions over the period of a month in year 1, the researcher collaborated with her to develop a test on the topic for the students. The teacher and researcher's joint reflections on the students' performance on the test were presented to other primary teachers and the Headmistress of the school through a joint presentation by teacher and researcher in a 2 hour workshop in the school.

In the second year, observations were done for the chapter on "Area" followed by a few lessons on the topic of fractions. Two lessons on the topic of fractions have been fully transcribed and analyzed in detail as they represent the stage at which Nupur had reached in her journey of making her teaching more student centered, providing them more autonomy and developing an understanding of mathematics. As she attempted to implement changes, she faced new challenges that surfaced in the course of her teaching. I describe how she encountered and dealt with some of these challenges as she collaborated with the researcher and attempted to align her teaching to the goal of sense-making.

In the next section (6.3), I describe the framework used to analyze Nupur's teaching. Section 6.4 presents an analysis of the textbook chapter on fractions. The analysis of the classroom teaching will be presented in the section 6.5. In subsection 6.5.1 brief descriptions of the three lessons in the first year and 2 lessons from the second year are presented to establish their significance in illuminating Nupur's teaching and justify their selection. Sections 6.5.2 and 6.5.3 respectively deal with the task framing and implementation in these lessons. In the next section, 6.6, the role of the researcher as a collaborator is discussed. In section 6.7 the challenges that Nupur faced in engaging students in sense making and how she recognized some of those challenges through reflection are discussed.

6.3 Framework for analyzing teaching

As discussed in Chapter 2 (Section 2.4.1.2), the relation between teachers' beliefs, knowledge and practice is complex in nature as beliefs and knowledge influence practice on the one hand, but on the other hand, practice can lead to development of knowledge and beliefs. Ball, Thames and Phelps (2008) have argued that mathematical knowledge for teaching is an important component of the knowledge required by teachers to teach mathematics, which includes not only knowledge of the subject matter but also pedagogical content knowledge, knowledge of students and of the curriculum. As discussed in Chapter 2 (section 2.4.2.1), there is need for framework that one can use to identify the knowledge which is used and reflected in teaching through the process of task selection and implementation. The lessons that are analyzed in this chapter are on fractions. An important theory that illuminates pedagogical aspects of the teaching and learning of fractions is the sub-construct theory of fractions proposed by Kieren (1988). The sub-construct theory highlights the different interpretations of fractions that come to the fore in various contexts and tasks which involve fractions. Given Nupur's keen desire to connect the mathematics of the classroom with students' everyday experience, the sub-construct theory is appropriate since it focuses on the meaning of fractions in contexts. Thus, we have used the sub-construct theory has been used as a framework to analyze the tasks that Nupur set for the students and the way in which she implemented them in the lessons. The framework has also been used to analyze the concepts dealt with and the tasks presented in the Grade 5 textbook chapter on fractions that Nupur taught from.

Since Nupur had a goal of connecting school mathematics with everyday experience, the sub-construct theory would have been a useful framework for her to learn and apply. Regretfully, there was no opportunity to systematically and explicitly learn or apply this theory in or after the TPD workshop. The ideas contained in the sub-construct theory however sporadically came up in the workshop and in the discussions between Nupur and the researcher. Some of the tasks based on share and measure sub-constructs were discussed in the TPD workshop, but the theory of fraction sub-constructs was not explicitly discussed with the teachers. Nevertheless, analyzing teaching based on the underlying sub-construct was hypothesized to be helpful in illuminating the challenges that Nupur faced as she attempted to introduce changes in her teaching that were more responsive to student thinking, and began to increasingly emphasize reasoning and justification.

The sub-construct theory is based on the assumption that interpretations of fractions within different contexts vary significantly and that different contexts may involve different sub-constructs. Kieren claimed that students faced difficulty in learning fractions because the curriculum exposed them only to limited types of contexts and interpretations, leading to an impoverished concept of fractions. For fractions, Kieren identified five sub-constructs namely, part-whole, measure, quotient, operator and ratio. The part-whole meaning which is the most commonly used interpretation, is often depicted with an area representation by dividing a whole into equal parts and shading some of them. Here, the fractions are named based on the number of equal parts of the whole. The measure interpretation is most often using in the contexts of measuring quantities and where the units are divided into subunits to denote the quantity more precisely, for e.g., 2 ½ km, 5 ¼ litres of petrol, etc. When a fraction a/b is interpreted as the share that one gets by dividing a quantity 'a' into 'b' number of equal shares, this involves the quotient sub-construct. The operator meaning corresponds to multiplication by a fraction and denotes taking a multiple or fraction times a certain quantity, for e.g. water needed to cook rice is 2 ½ times the volume of rice. The ratio interpretation involves the relation between the numerator and denominator of the fraction which denote two different quantities which may or may not have the same measure (For more discussion, see Subramaniam, 2013). Kieren (1988) and Behr, Harel, Post and Lesh (1992) have argued that the teaching of fractions needs to include greater variety of contexts that relate to different sub-constructs discussed above so as to develop a more robust understanding of fractions. An excessive emphasis on the part-whole sub-construct not only impoverishes the concept of fraction that is learned, but also often restricts children's thinking within a "whole number bias". This includes treating the parts that a whole is divided into as discrete entities by themselves, rather than seeing them as sub-units of the whole. When a teacher emphasizes double counting – for e.g., counting the total number of parts and the number of parts which are shaded – without attending to the relation between the part and the whole, this is a degenerate form of the part-whole sub-construct, that remains within whole number thinking. The phrase "*m parts out of n parts*" has been used to indicate this form of the part-whole sub-construct.

Two aspects of the teacher's practice have been analyzed in this study – task framing and task implementation. Task framing refers to the type of problem posed and seeks to infer the goals and beliefs of the teacher embedded in the selection of certain types of problem. However, task implementation analysis reveals how the interactions between the teacher and the students contributed towards learning, and the goals and beliefs implicit in the actions and decisions taken by the teacher while teaching.

The claim is that fraction sub-constructs play an important role in both task framing and task implementation, and shape the way students and the teacher come to understand and reason about fractions. Hence it is important for teachers to be aware of and use fraction sub-constructs flexibly and appropriately in framing and implementing tasks. This suggests that the fraction sub-construct theory may be a component of the specialized content knowledge in the mathematical knowledge for teaching framework proposed by Ball and colleagues (2005) for the specific topic of fractions. This chapter throws light on how teacher's knowledge of interpretations of fraction can guide framing of task and its implementation, and the role it plays in supporting students' learning.

The framework given in Table 6.3 provides information about the aspects of task framing and task implementation during teaching that were analyzed to identify the common practices of the teacher with respect to questioning, responding to wrong answer of students and explanation. These teachers actions indicate a conceptual orientation on the part of the teacher as discussed in Chapter 2. The transcripts of the five lessons selected were reviewed to identify the tasks that were discussed in the lessons. The task formed the unit of analysis and the task framing aspects were coded as per the characteristics presented in Table 6. 3. For the analysis of task implementation, we focused on interaction episodes in the lessons were analyzed to identify practices that shaped the way tasks were discussed in the classroom. A detailed discussion of selected episodes was thought to be appropriate to analyze task implementation aspects. Accordingly, one episode each from four of the five lessons has been selected for detailed discussion. For Lesson 4, I do not discuss an episode in detail, but briefly discuss how tasks were implemented in this lesson.

Table 6.3: Framework for analyzing task framing and task implementation in the lessons

Types of interaction	**Characteristics**
Teacher's framing of the task (Task as unit of analysis)	1. Source of the task (textbook based/designed by the teacher) and Rationale for the task 2. Nature of task: (open/closed; calculation/reasoning based; whether context based) 3. Fraction sub-construct foregrounded
Task implementation	1. Questioning and evaluation practices: Nature of questions posed, equity in classroom participation, positive or negative evaluation, suspending the evaluation, asking students to evaluate

	2. Explanatory practices: Explanation based on procedures or representation, focus on correct answer versus reasoning, using variety of constructs and contexts for developing explanation, connecting symbolic procedures with visual representations

Task Framing: The way a task is framed to be posed in the class, may point to underlying beliefs and knowledge held by the teacher as well as the teacher's goals. It also partly determines the nature of classroom interaction and constrains the kind of mathematics that is focused in the classroom. Three aspects of task framing have been identified for analysis, namely, source and rationale of the task, nature of the task and the sub-construct used for the task. The source of the tasks can be from the textbook or designed by the teacher. The rationale for selecting the task from the textbook or constructing the task while teaching can be based on students' response or other factors considered important by the teacher. A second dimension is the nature of the task. A task can be framed so that students just have to do calculation with the numbers, or may call for reasoning on the part of the students. A predominance of calculation based tasks indicates that the teacher's goal is to develop proficiency in calculations. Other aspects analyzed in the nature of the task is whether the task is open ended or closed and whether or not the task is context based. The third dimension is fraction sub-construct foregrounded in the task. The teachers' implicit or explicit awareness of the difference between fraction sub-constructs is reflected in the task.

Task Implementation: How students are engaged in completing the tasks determines the nature of classroom interaction as well as the learning of mathematics that occurs as a result of engagement. Therefore, the portion of transcript dealing with the implementation of a single task was analyzed for the following aspects. .

Firstly, the transcript was analyzed to identify the questioning practices of the teacher. Questions were analyzed for being open or closed, the way contexts are used or the focus on calculation, whether the question demanded performing a learnt procedure or reasoning and whether opportunity arises for all students to be engaged in the questioning. (Although these aspects overlap with that of task framing, the attributes of questions are discussed separately from the attributes discussed under task framing, because questions are largely impromptu and arise in the course of task implementation.) The teacher may pose a question to a single student and then open up the discussion to others. It is possible that even when a question or task is posed to the whole class, only a few students may engage. However, if the teacher focuses only on a single student, and does not open up the discussion to the whole class, then it is a loss of opportunity to equitably engage all students. The teacher's explicit efforts to include the weak or less vocal students in class by calling them to solve the problem can be considered as examples of the teacher trying to have equitable participation in the class discussion and thus contributing to learning of *all* students. If the teacher expects the students to solve tasks on their own without telling the procedure, it indicates that the teacher is using the task to elicit students' ideas and strategies. However, if the teacher asks students to work quickly, it could be an indication of belief that students should be able to solve mathematical problems quickly and works against establishing equity in classroom participation. Task implementation also involves evaluation of students' response which can further shape the interaction. Positive or negative evaluation may move the discussion to next tasks while suspending the evaluation or asking probing question may move the discussion towards developing an explanation.

The discussion of the solution also involves developing explanations. When the task implementation involves a demonstration of a procedure, or using a procedure used or discussed earlier in a similar problem or arriving at the solution through funnel type questions (Wood, 1998), then the explanation developed is mostly procedural. The task can be framed using a certain representation involving a particular sub-construct, but the representation discussed or sub-construct foregrounded in the classroom may be different from the one in which the task was framed. The number of different representations used in discussing a task points towards the connections that can get established, helping students to develop meaning of the concept. Also, one can analyze if the connections between the different representations that were discussed were established and to what extent the key concepts and ideas embedded in the representations were made explicit.

Analysis of the practices related to questioning, explanation and evaluation contributes towards understanding the challenges that constrain the change in practice that the teacher is trying as well as identifying the efforts made by the teacher in bringing about change in her practice.

6.4 The textbook chapter on fractions

Two of the three characteristics mentioned under task framing (nature of task and nature of sub-construct foregrounded) have been used to analyze the tasks included in the textbook. The textbook chapter on the topic of fractions for Grade 5 contained 23 problems, many of which were based on contexts and included activities, games and puzzles. This chapter in the new textbook was a departure from the old one which had predominantly representations of fractions with shaded parts of rectangles or circles and exercises in which students were expected to operate on symbolic fractions using procedures. The new textbook did not have exercises containing a series of computation based tasks, which was very typical of earlier textbooks. The textbook was also different from the earlier one in having a variety of tasks as well as having many colorful illustrations.

Many of the tasks given in the textbook were open ended. For e.g., students were asked to design the flag of their choice and find fractions that represent different parts. In other questions students were asked to divide a geometric figure into a number of equal parts in different ways (p. 52, 54, 60). At another point students were asked to color a grid in different ways and infer that the same fraction (8/16) can be represented in different ways by coloring different parts which may or may not be contiguous (p. 57). In one of the games for coloring a circle divided into 12 parts, students could see how different fractions can combine to make one whole (p. 60). A note to the teacher about this activity suggested discussing students' responses to develop conceptual understanding. Thus, there were many open ended questions to encourage students to see how a variety of answers could be correct responses to a question.

The chapter adopted a problem solving approach as many of the tasks in the textbook were in the form of problems situated in some context or the other and a few problems were given as part of "practice time". Thus the focus was on eliciting and developing meaning of fractions in different contexts rather than asking students to use learnt procedures to solve problems. Several questions encouraged students to reason about quantities and fractions. For e.g., students were asked if 1/6 of the bigger rectangle will be bigger than 1/6 of the smaller rectangle and to justify their answer (p. 54). Suggestions to the teacher for the flag activity (p. 50) encouraged the teacher to help students see that some parts of the figure could be identified as less than or more than a fraction, thereby developing precision in the language for describing parts using fractions. In one of the tasks (p. 61) where the student had to name the fraction of the colored part, the part was colored but the whole was not divided into equal parts. In a game given in the textbook (p. 60), students took turns to pick up a token from a lot that had a fraction written on it and color the part corresponding to the fraction on their individual fraction disc. The student who was the first to color the whole disc won the game. This game helped in consolidation of students' conception about comparisons of fraction, identifying equivalent fractions and formation of the whole as students were expected to color the parts denoted by the fraction on the token they picked from a lot. In the process they could compare it with the fraction colored by a playmate and identify when the fraction on the token would complete a whole. All these activities indicated that the objective of the chapter was to make students think about the relation between the part and whole while naming the fraction and careful consideration was given to address the misconceptions in understanding fractions.

A range of contexts were used in the textbook which were related to shopping, making parts of objects or collections for categorization or sharing, partitioning the field for growing different vegetables, time spent on daily activities, relation between distance travelled and time taken, etc. The chapter started with the context of studying the Indian flag and the fraction denoted by the three colored bands in the flag. This was followed by examples of other flags and tasks to name the parts of the flag with different fractions. The activity of making a magic top involved the use of composite fractions by asking students to color 2/8 of the circle. The context also allowed students to discuss a science concept – why does the magic top turn white on spinning. There were also tasks to make students think about how different fractions could combine to form one whole – for e.g., when in the context of a humorous story the students had to figure out how many slaps would ½, 1/5 and 2/5 of a prize of 100 slaps be (See description of lesson 2 below). The chapter had suggestions for several activities, games and concrete manipulative which could be used as an aid for developing understanding. The chapter had suggestions for setting up a math club in the school in which interesting activities could be taken up by the students like "puzzles, tangrams, maps of buildings and calculating area and perimeter of the school playground". The textbook recommended that activities like the coloring game on the fraction disc and use of concrete materials like matchsticks or fraction strips along with follow up activities would help in developing conceptual understanding. These suggestions indicate the attempts within the textbook to make mathematics relevant and interesting for students.

The tasks in the textbook foregrounded a variety of sub-constructs. The area representation and the part-whole sub-construct was predominant, i.e., shaded parts of various shapes were used in the chapter along with the fraction notation. However, there were also many problems which were framed in a situation of equal sharing or measurement indicating the use of share and measure sub-construct. The ratio sub-construct was also used to compare different quantities, for e.g., number of blue hats and red hats in a collection of hats (p. 53) and when comparing the number of hours a student engages in different activities during a day (p. 67). Both these contexts involve part-part comparison using the ratio sub-construct. The operator sub-construct emerged in tasks which engaged students in finding the cost of different quantities of vegetables purchased and finding 1/5 of 100 slaps in the story context. Connections were established between mathematical topics like fraction and area, and fraction and measurement. A connection between fraction and time was established by using a horizontal number bar for representing time.

The analysis of the textbook thus showed that the tasks given in the textbook had the underlying rationale to develop students understanding of fractions by relating fractions to different contexts, understanding the relation between part and the whole, engaging students in reasoning about fractions using different types of representations and used a variety of sub-constructs to frame the questions in the chapter.

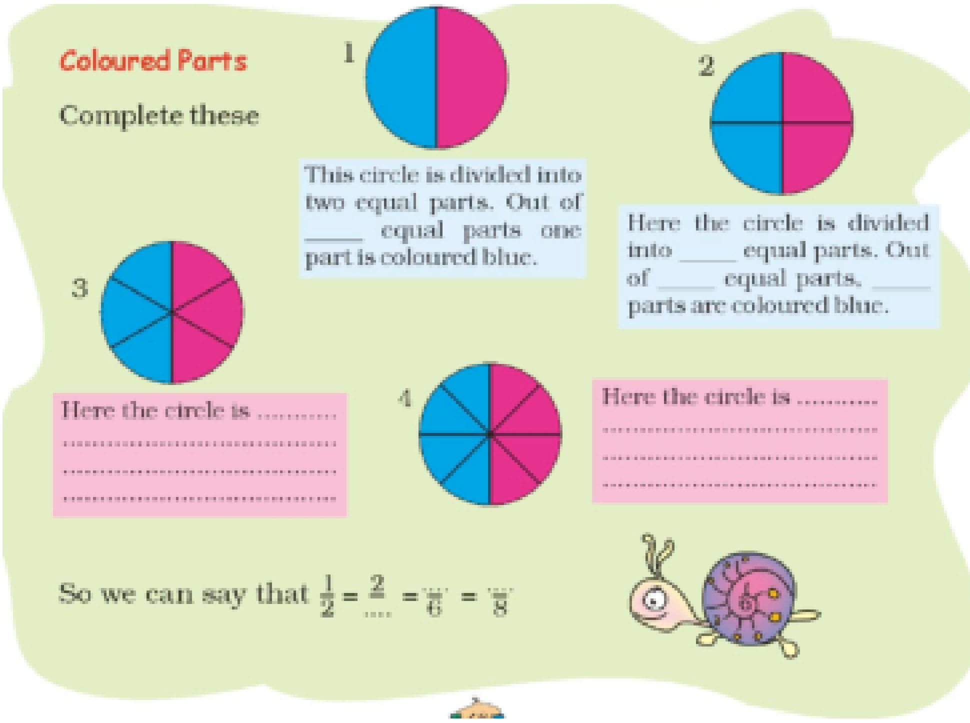
6.5 Analysis of Nupur's lessons

6.5.1 Summary of the five lessons

Summaries of the five lessons, which have been analyzed in detail for this chapter, are given below. The lessons 1 to 3 are from Year 1 and lessons 4 to 5 are from Year 2.

Lesson 1: Nupur had started teaching fractions before the lesson observation started. This was the first lesson that was observed by the researcher. In this double period lesson, Nupur started with the task of naming the fraction for the shaded part of a circle. She first used a multicolored disc to ask students to name the fraction for different colored parts using unit fractions. She then used the textbook task of naming equivalent fractions of half using circles with 4, 8 and 12 equal parts (see Figure 6.1). Some of the students in her class already knew the procedure of multiplying or dividing the numerator and denominator by the same number to obtain equivalent fractions, learned perhaps at home or in a 'tuition' class. She drew attention to fractions equivalent to half using terms like 'half moon'. On the teacher's invitation, the researcher interacted with the students and asked "Are 2/4 and ½ equal?". The students argued that they are not equal since 2/4 has an extra partitioning line and a different number of parts. The researcher then asked students to think about whether two ¼ pieces of a cake and a ½ piece of a cake kept on two sides of weighing balance would balance each other. In the beginning of the next lesson on the following day, all the students agreed that that two pieces would balance each other and that ½ and 2/4 would be equal.

Figure 6.1: Task from the textbook used by Nupur in lesson 1

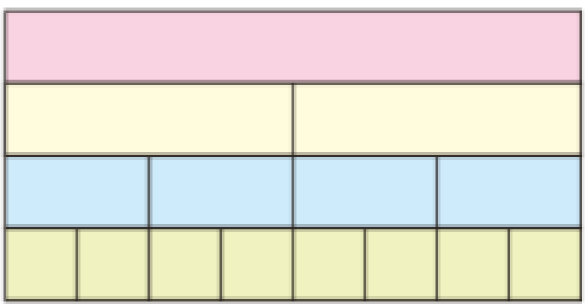


Lesson 2: This lesson occurred on the day after lesson 1. The teacher narrated a humorous story from the textbook about 'Birbal', a poet who wanted to meet the King. The gatekeepers at three different gates each asked for bribes – a share from whatever prize the King would give Birbal. The shares of the gatekeepers were respectively 1/10, 2/5 and ½ of the reward. To punish the gatekeepers, Birbal asked the king for 100 slaps as a reward. The teacher discussed how many slaps each gatekeeper would get. The students were able to orally arrive at the answer for 1/10 and ½, but tried to multiply 2/5 with 100 to get the answer and made errors. The teacher asked students to explain their calculations, which students were unable to do. The teacher then engaged students in identifying equivalent fractions of 2/5 asking them to guess the equivalent fraction with denominator 100. Some students were able to guess but other students said that they had not understood. The teacher then asked for advice from the researcher. The researcher

suggested making a representation of the question posed so that students are able to come up with their own strategies. However, Nupur proceeded to draw a representation of 2/5 of 100 by making first an array of 10 × 10 coins, and then circling 4 coins in each line to show that it would be 40/100. By the time she finished her explanation, the time was up.

Lesson 3: This lesson was also a double period, and occurred on the day after lesson 2. Based on the figure given in the textbook, the teacher brought paper strips of the same size and drew the representation of equal sized rectangles divided into different number of equal parts on the board (See Figure 6.2). In the lesson, she asked students to fold the strips into different numbers of parts (2, 4, 8, 3, 6 and 5). After folding, for each strip she asked students to name one part, and different partial lengths of the strip. She gave them two fractions and asked if they were equal, to which students responded by identifying equivalent fractions. The teacher asked students to identify the equivalent fractions of ½ using the representation. She then gave the task "3/4= */8 = */16" asking students to find the numerator. The students were able to give the correct answer for 6/8 but for the third equivalent fraction two fractions were proposed: 12/16 and 8/16. The teacher discussed both fractions. A student showed the correct calculation procedure of multiplying both numerator and denominator in 6/8 by 2 to get 12/16. Unsure if the students had understood, the teacher used the representation on the board to show how 12/16 was equivalent to ¾ pointing out that they were of same size. She then asked students to identify different unit fractions, composite fractions and equivalent fractions using the representation on the blackboard.

Figure 6.2: Representation used to discuss size of unit fractions and to identify equivalent fractions



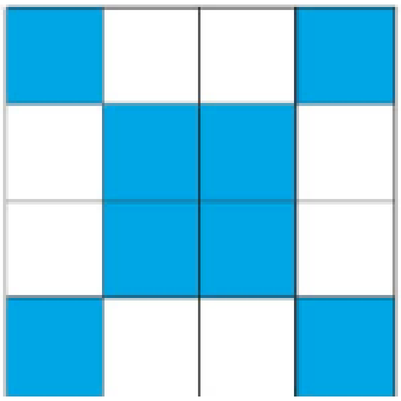
Year 2:

Lesson 4: This was Nupur's first lesson on fractions in Year 2. Her goal in the lesson was to consolidate students' learning of area and prepare the ground for teaching of fractions. She had asked students to make flags of their choice as homework. After looking at some flags, she asked students to draw a rectangle of length 12 cm and width 6 cm and divide it into 6 equal parts. Some students came to the board and showed 4 different ways to make 6 equal parts. One student incorrectly drew both diagonals and lines joining the midpoint of the opposite sides of the rectangle obtaining 8 equal parts instead of 6. Nupur discussed this solution with students about whether such partitions lead to equal parts or not. Nupur probed students for other ways of partitioning and also asked students about how they had drawn the partitioning lines. She helped a student articulate that when she took points at distances of 2 cm, she was able to get 6 equal parts. She also helped the student express that the total length of the rectangle could be obtained from combining the length of each part by repeated addition or multiplication. She then asked students to find the area of one part using the area formula and then estimate the area of two equal parts. The students were able to estimate the area for two and also for four equal parts by doubling. In her discussion with the researcher, Nupur said that she would use these ideas for teaching fractions.

Lesson 5: In this double period lesson, on the day after lesson 4, Nupur used an area representation from the textbook where non-contiguous parts were colored and asked students to find the fraction to represent the colored as well as non-colored portion, which students identified as 8/16 (see Figure 6.3). The teacher asked students if it would be considered as equal to 2/4. The students drew a figure depicting 2/4 and gave arguments like "among every 4 parts, 2 parts are colored" Some students objected that the figure should not be changed in this manner while others felt that it was valid to do that. To resolve the conflict, the teacher asked students to talk about the meaning of 2/4. One student conjectured that since 8 is half of 16, therefore one can write it as 2/4. Teacher asked students to compare 2/4 and ½ and researcher asked them to identify what is same in the fractions ½, 2/4 and 8/16. The students reported that numerator was half of denominator. In the follow up, the teacher stated that the shaded part is denoted by the numerator and the total number of equal parts of the *whole* are denoted by the denominator. When a student made a mistake of naming one part of the whole as "one", the researcher asked students to compare whether 8 or 8/16 is bigger. While discussing this, Nupur asked students to draw representations of 8, 1 and 1/8. With the help of the teacher, the students realized that these fractions represent 8 sheets of paper, 1 sheet of paper and one part of the paper respectively.

Nupur focused their attention on the area of one part denoted by the fraction 1/8.

Figure 6.3: Task posed to students to name and justify the fraction in lesson 5



In most of the lessons, Nupur took a task from the textbook followed by similar tasks constructed by her. The general pattern followed by Nupur in her lessons was:

- Ask some quick questions to recall the work done previously.
- Give a problem/ task/activity to the whole class and solicit answers from students.
- Students call out their answers.
- Evaluation of the answer and posing of the next task or question. Alternatively, she asked the student to explain his/her answer by showing the solution method..

6.5.2 Task framing in the lessons

In this section, the analysis of task framing in the 5 lessons that were selected has been discussed. The lessons have been analyzed to identify the way the teacher framed the tasks which were posed in the class according to the framework discussed in section 6.2. The categories of task framing and implementation sometimes overlap since the task framing also determines the way the task is implemented.

The framing of the tasks is important in revealing the teacher's objectives, beliefs, conceptual knowledge including the knowledge of the fraction sub-construct, and the type of mathematics considered important by the teacher. The tasks in the textbooks are framed keeping a particular pedagogy of mathematics in mind as they encourage multiple correct answers, focus on reasoning and use contexts. Research studies have indicated how there are differences in the intended curriculum and the implemented curriculum as the way the curriculum is interpreted is dependent on complex interaction of factors like teachers' beliefs, knowledge and common practices engaged by the teacher(Cronin-Jones, 1991) . The teachers' selection of tasks from the textbook and framing of tasks similar to the textbook might indicate teachers' beliefs and knowledge by looking at what aspects of the similarity is maintained.

Table 6.4 illustrates the nature of tasks and representations used by the teacher in the three lessons in the first year and the two lessons in the second year. The table is followed by a discussion of what the entries show in the following subsections on the source of task, the nature of task and the sub-constructs foregrounded in the selecting/constructing the task.

Table 6.4: Dimensions of task framing in Lesson 1 to 5

Lesson	Number and Nature of tasks	Representation used in the task/ solution
Lesson1 3/08/09 (2 periods)	7 tasks: 2 open, 5 closed; 1 textbook, 6 constructed	5 naming the fraction using circular area model, 1 making circular area model equivalent to half, 1 symbolic fraction notation
Lesson2 4/08/09 (1 period)	10 + 1 tasks: 1 non-math task, 1 open, 9 closed; 4 textbook, 6 constructed	7 identifying fraction of quantity using symbols, 2 writing fraction notation, 1 visual representation used by the teacher
Lesson3 5/08/09 (2 periods)	15 tasks: 13 open, 2 closed; 1 textbook, 1 adapted, 14 constructed	9 identifying fraction/equivalent fraction using visual/concrete representation, 2 making parts of strip, 2 finding fraction of quantity, 1 copying representation, 1 finding equivalent fraction using fraction notation
Lesson 4 29/07/10 (1 period)	3 tasks: 1 open, 2 closed; 1 textbook, 2 constructed	3 Rectangular area model used: 1 making 6 parts of a given rectangle of specified length and breadth, 2 finding area of number of parts knowing area of one part
Lesson5 4/08/10 (1 period)	6 Tasks: 2 open, 4 closed; 1 textbook, 5 constructed	4 identifying fraction using rectangular area model, 1 comparing fraction with whole/s using fraction notation and paper, 1 making representation for a given fraction

6.5.2.1 Source and rationale of tasks

The textbook seemed to be the main source for selection of tasks and sometimes Nupur considered what problems students would find interesting to engage with. However, the objective of the task was not necessarily the same as what was given in the textbook. As Table 6.4 shows, in all 5 lessons, the teacher gave tasks from the textbook but also gave tasks that she constructed involving a calculation similar to the one in the textbook. The purpose and rationale for the tasks varied. The task in lesson 1 was very similar to the textbook task, in which she asked students to come up with as many equivalent fractions of ½ as possible and then to construct the representation of half and one of its equivalent fractions of choice. She may have felt that by giving the choice to students, she had made it more student-centered. In Lesson 2, the initial task was from the textbook, which was rooted in the context of the culturally familiar Birbal story and the objective was to engage the students in problem solving. However, the six tasks later posed by the teacher all involved calculations to find the fraction of a quantity. This indicated that the teacher considered that the main purpose of the task was to learn how to find a specified fraction of a given quantity. As discussed earlier, Nupur's beliefs reflected a tension – she thought that the use of similar tasks leads to rote memorization., at the same time she thought that practice was necessary. In lesson 3 too Nupur introduced tasks not given in the textbook. For e.g., she used the representation given in the textbook to discuss naming of both unit and composite fractions while the task was given in the textbook was aimed at identifying equivalent fractions.

In lessons 4 and 5 in the second year she constructed tasks based on the misconceptions among students that she felt were important to address. There were fewer tasks per lessons than in the three lessons from Year 1. In lesson 4, she focused closely on building on students' understanding of area to make equal parts in different ways as a preparation to learn fractions, thus using the measure meaning of fraction. In lesson 5, she addressed students' difficulty of identifying fractions using a representation which had non-contiguous shaded parts, by focus on identifying the size of each part. Another misconception that she addressed was that students considered a fraction to be bigger if the denominator was bigger. She had an extended discussion to compare 8, 1 and 1/8 to help students develop a sense of relative size of the three numbers.

Thus, in the initial lessons the tasks given by the teacher were largely variations of the textbook tasks, but focused more on calculation. In later lessons most of the tasks were designed specifically to address student misconceptions. This is consistent with Nupur's remarks made to the researcher, which reflected a growing sensitivity to students' difficulties. This isdiscussed in section 6.6 below.

6.5.2.2 Nature of tasks

In Lesson 1, the teacher quickly shifted from using shaded figures to discuss ½ to asking students to give equivalent fractions of ½ using the pattern that the numerator is half of the denominator. In other lessons, the teacher spent more extended time, discussing the figures and attempted to establish the relation between part and whole through comparison and linking the visual representation to the symbolic notation of 8, 1 and 1/8. In the initial lessons, the focus was mainly on naming the fractions and finding equivalent fractions while later lessons were about naming the fraction, comparing the area of parts to the whole and grasping the relation between the part and the whole.

The textbook had many questions that expected students to reason using fraction representations. For e.g., in the story context in lesson 2, the share of the three gatekeepers together accounted for the whole and thus Birbal would not get any slaps he had asked for. However, since the lesson was largely devoted to trying to find 2/5 of 100, the teacher was not able to discuss this aspect.

We note from Table 6.4 shows that the majority of tasks in four out of the five lessons analyzed were closed in terms of having a single answer. Only lesson 3 had a large number of reasoning based tasks, which may be attributed to the topic that Nupur was dealing with, namely fining equivalent fractions. In the second year, the open ended task in Lesson 4 involved the students in dividing the rectangle of given length and breadth into 6 equal parts in different ways, followed by a discussion of 4 different solutions.

While the textbook favored a contextual problem-based approach along with activities and games for teaching mathematics, the teaching was still focused on using the area model representations where counting of parts was the procedure to identify the fraction from the whole followed by calculations using fraction notation. Although Nupur had indicated connections to daily life as important, she did not use any context from daily life to frame the problem for students apart from the contextual problems given in the textbook. The tasks framed by her illustrated the teacher's goal of making students proficient in identifying and making representations of fractions (unit fraction, composite fraction and equivalent fraction) using the area model and using the fraction notation, and being able to give equivalent fractions by understanding the pattern between the numerator and denominator.

6.5.2.3 Use of sub-constructs to frame the task

Nupur used the interpretation of fraction as 'shaded parts upon total parts' as the main idea to frame the tasks using area model in Lesson 1 when she asked students to identify the equivalent fraction by 'counting' the shaded parts and the total parts. In some instances, the focus was on identifying the equivalent fraction based on matching it visually with the "half moon". The identification of fraction is thus based on the perceptual features of the representation rather than the properties or relations embedded within the representation. The mathematics focused is narrowed down to writing the fraction that can represent the shaded part by counting. This reliance on counting is mathematically problematic since the parts may be of different sizes and the student may be led to think that fractions like 7/5 are not possible.

The sub-construct used in the first lesson is predominantly "*m parts out of n parts*" a degenerate form of the part-whole sub-construct. In the second lesson, when the teacher discussed the problem of finding the share of each of the gatekeepers from the 'Birbal story', lends itself naturally to the operator interpretation the operator sub-construct came to the fore. The operator sub-construct was also foregrounded in the follow up tasks given by Nupur to the class. However, when the teacher represented 2/5 of 100 slaps by drawing groups of five coins and circling 2 out of every 5 coins, it was the ratio sub-construct that was emphasized. In the third lesson, there was a predominant use of the measure sub-construct when she asked students to name the fraction based on the size of the part folded on the strip. The measure sub-construct was also predominant in the fourth and fifth lesson when the teacher discussed the area of the part in comparison to the whole. The quotient sub-construct which is based on equal sharing did not occur in these five lessons, nor did it play a prominent role in any of the other lessons on fractions.

6.5.3 Task Implementation

The analysis of task framing as discussed above indicates broadly the teacher's goals and her focus in her lessons. More about how the teacher's beliefs and knowledge interact with practice can be inferred from the way a particular task was implemented in the classroom. It is in the use of a certain task, in the teacher's responses to sometimes expected and sometimes unexpected student responses to the task, and in the actions taken to address the errors or to give explanations, that the richness and potential of the task, as well as the role of the teacher's beliefs and knowledge is revealed. The manner in which tasks are implemented can throw light on the purposes for which tasks are included in the textbook, and the way these are interpreted by the teachers.

For the analysis of task implementation, an excerpt has been selected from 4 of the 5 lessons. These excerpts are illustrative and reveal in some detail some of the practices that Nupur adopted. Three kinds of practices are focused – questioning practices, evaluatory practices and explanatory practices. After each excerpt, I discuss the practices that are reflected in it. These excerpts provide evidence that Nupur was trying to bring in new practices. At the same time they also reveal Nupur's struggles– moving the students from a procedural focus to learning with understanding, interpreting fraction tasks meaningfully, eliciting reasoning and problem-solving strategies that are spontaneous, connecting reasoning and spontaneous strategies with procedures and providing coherent explanations and justifications. The discussion of each excerpt highlights the efforts to incorporate new practices and the challenges faced by Nupur in doing so.

The sequence in which the excerpts are presented and discussed may suggest that Nupur's practices evolved towards being more responsive to students' thinking. Indeed, the researcher's presence in the classroom across the two years did enable her to see the gradual change in the culture of the class. The researcher also noticed that Nupur frequently fell back upon practices that were more procedure-focused or "transmissionist". However, it is not the aim of this analysis to establish that such a change took place. Given Nupur's strong beliefs and orientation, it is justifiable to assume that she was making efforts to change her practice. My focus rather, is on the challenges that she faced as she tried to bring in changes. Anattempt has been made to identify the hurdles that came in the way of adopting more effective practices of questioning, evaluation or explanation. The discussion of the excerpts will attempt to bring these to the fore.

6.5.3.1 Excerpt from lesson 1

Teachers' questions are the most important tool to engage students in a task, probe their thinking and evaluate their responses. The nature and quality of the question, however, needs to be taken into consideration along with the context in which the question is asked. Nupur's questioning practices showed variation over the course of collaboration in the nature of the questions that were asked, the way the representation was used to pose the task or discuss the solution and the variety of contexts used to pose the questions.

In the initial few lessons in the first year, Nupur often asked funnel type questions or closed ended questions for which only one correct answer was possible. In the research literature (Wood, 1998) these type of questions have been described as having the objective of narrowing the focus of conversation with the students so that they arrive at the correct answer. Using such questions, the teacher breaks the task for the student into smaller tasks with less cognitive demand, ultimately guiding the student towards getting the right answer.

In Excerpt 6.1, from lesson 1, the questions asked from line 9 to 19 are funnel type questions, the purpose of which seems to be to help students arrive at the correct answer. The larger discussion is about the equivalent fractions of half using the circular area representation.

Excerpt 6.1:

1. T: Sg1, tell me, how to name this part?
2. Sg1: 2/4
3. T: 2/4,Why 2/4?

1. Sb1: Ma'am, Ma'am, because...
2. T: No Sg1 will tell, if she is not able to tell, then you speak. Tell me why? Explain why this is 2/4?
3. Sb2: Ma'am
4. T: Keep quiet Sb2. Let her try. You people don't let the girls speak. If she does not speak then only you speak. She was telling and she was telling right. Sg1, tell *beta* [girl].
5. Sg1: [Silent]
6. T: How do you identify the number? What do you write first? How many parts are there? How many parts?
7. Sg1: 2
8. T: Are there 2 parts? How many total parts are there?
9. Sg1: 4
10. T: How many of them are shaded or eaten?
11. Sg1: 2
12. T: 2. So then what is the number?
13. Sg1: 2/4
14. Sb1: Ma'am, May I tell. 4/8 (Answer to the next question on the board).
15. Sb2: 8/4

16. T: 8/4 or 4/8?
17. Ss: Ma'am 4/8
18. Sb?: Ma'am, just like when marks are given.
19. Sb?: Ma'am, because we have halved it.
20. Sb?: Ma'am, we have shaded 4 out of 8. (Classroom excerpt, Date- 3/08/09)

Note: T: Teacher; Sg: Girl Student; Sb: Boy Student; Ss: several students; Sb?: Unidentified boy student

Prior to the segment in the excerpt, again through a set of funnel type questions, Nupur had established that to write a fraction for a shaded portion one should write the number of shaded (colored) parts upon total (equal) parts. Student responses of the type "2 green parts and total 8 parts" were taken as correct reasons for saying why a colored part was denoted as 2/8. In the above extract too, the teacher asks a student to name the shaded portion as a fraction and after the correct answer, asks the student to explain why the answer should be 2/8. However, the framing of the question to identify the fraction of the shaded portion without reference to area or what the whole represents indicates to the student that one needs to only count the shaded and total parts as numerator and denominator to identify the fraction. This type of "double counting" to name the fraction constrains the development of concept of fraction as the student might view fraction as composed of two numbers rather than thinking of it as one number indicating quantity. In Excerpt 6.1, from line 9 to 15, the teacher tries to help the student explain why the answer is 2/4. Here one can see the tension and challenge experienced by the teacher in focusing on understanding in teaching. On the one hand, she is asking students to give explanations, but on the other hand, the explanation that she expects is based on the procedure for counting the parts. Thus, her questions, evaluations and explanations have a strong procedural focus as seen in this excerpt.

As seen in line 19 in the above episode, Nupur used closed-choice questions often when a student responded with an upturned fraction indicating that she considered this response to be a careless mistake rather than indicating a conceptual gap. Another common error that she considered as a careless mistake was saying only the numerator in form of whole number to denote the shaded parts for example, saying that 2 parts are shaded rather than 2/8 part of the whole is shaded. To such a response from the students in the initial lessons, she often asked "Is it 2, or 2/8?", following which the students would switch to saying the fraction. However, in later lessons Nupur noticed this pattern in some of the students' responses, that they repeatedly avoided the use of fraction words, and responded in terms of whole numbers. She became aware of this error and considered it as an important misconception. Her discussion of 8, 8/1 and 1/8 in lesson 5 was aimed at addressing this error.

A problem that has been recognized with the use of funneling or closed choice questions is that it creates in the teacher an *illusion of competence* (Schoenfeld, 2007) on the part of students, as the teacher may misunderstand that students have understood the concept, since they have arrived at the correct answer with the teacher's help. However, when students are asked to do the task independently or if the task is slightly changed, they are not able to solve it. This is because, they have not really understood the concept but have learnt how to answer a particular type of question. Thus the teacher may fail to recognize the conceptual gaps among the students and assess student understanding incorrectly if they use such questions. In Nupur's case, however, her remarks to the researcher as discussed in section 6.6 indicate that she did notice the errors that students made and gradually became more sensitive to students' mistaken ways of thinking as her lessons on fractions progressed.

6.5.3.2 Excerpt from lesson 2

This excerpt is from lesson 2 when the teacher was discussing the share of one of the gate-keepers as 2/5 of the prize that Birbal (the character in a story) would get from the emperor. Birbal, had asked for 100 slaps as the prize.

Excerpt 6.2:

60 T: So the first gatekeeper got 10. 10 out of 100. What about second gatekeeper (2/5 of 100)?

61 Sb: 40

62 Sb: 20

...

64 T: Who is saying 40? Come Yogesh, Who is saying 20? How will you solve? 10 is also answer, come. Any other option? One more 40. You have to tell how?

65 S: Ma'am. Let me tell.

66 T: Just wait. Let them do. You should listen to others. Always don't focus on telling your answer.

[Students solve the problem on the board. A student shows 2/5 × 2= 40.]

85 T: You have written 5 there, so how did you get 40?

....

93 Sb1: Ma'am 5 into (multiply by) 2 if we will do and 2 into 2.

94 T: Leave into (multiply). You tell me, there was 100 so, how did we get out of 40 from it? How is 2/5 of 100, 40? How will you calculate?

….

96 Sb1: Ma'am 5 20's are 100 and 5 4's are 20.

98 T: Sometimes you say 20 sometimes 40. You go now.

99 Sb1: Ma'am correct answer is 20.

[Teacher writes 2/5 × 100 on the board and asks another student to solve. Another student Sb2 uses the multiplication fact 5 × 20 =100, so 20 which when multiplied by 2 gives 40. Teacher then turns back to the student who gave 20 as answer.]

118 T: Why you are getting 20 as answer? I don't know if the answer is correct.

119 Sb1: Ma'am we have to distribute (share).

120 T: We have to share. 2 out of 10 or 2 out of 5? Out of 10 or 5? (Emphasizes the numbers 10 and 5.)

[Another Student shows 2/5 × 100 =40 by cancelling 5 and 100 and writing 20 over 100.]

124 T: Ok. Go back to your seats. What is the meaning of 2/5? You tell me. Out of 5, how many parts did you get?

125 Ss: 2

126 T: So out of 10 how many will you get?

127 Ss: (Silence)

128 T: What is the meaning of 2/5?

129 T: Out of 5 you are getting 2. Like, from 5 toffees you are getting 2. So out of 10, how many will you get?

130 Ss: 4

131 T: Out of 100...

132 Ss: 40

133 T: So you can do it orally like this itself. (Classroom excerpt, 4/08/09)

The excerpt above is an instance of Nupur's practice of asking students to explain their answers, where instead of evaluating the answer from students right away, she asked students to show how they got the answer.

The excerpt above reflects both Nupur's struggles with the challenges faced as well as efforts to include new practices while teaching. The practice of asking students to share different solutions and discussing them is a new one, which Nupur had admitted in a post lesson discussion after the lesson. Nupur shared that she introduced this practice as she appreciated the opportunity given to students to share their answers on the blackboard and to discuss each other's answers in the live teaching sessions in the TPD workshop she had participated. In trying to establish this as a new norm, Nupur requested students to "listen to others" in line 66 rather than asking for a chance to tell the correct answer to the teacher. However, the teacher is struggling to move from a procedural to a meaning based understanding as she tries to engage with students' solutions and make sense of it. The solutions shared by the students were different procedures for finding 2/5 of 100, including an incorrect procedure. The students were making mistakes in calculation because they were not able to make sense of what it means to multiply a quantity by a fraction. Instead of immediately evaluating students' answers, the teacher asked for other solutions or tried to make sense of students' responses as in line 94, where she pointed out to the student that he had not written 100 anywhere in the problem solution, evidently to make the student think more carefully. This practice of not evaluating the answer immediately and asking for explanation or other solutions is a new practice that Nupur tried to incorporate in the lesson. When a student (before line 124) used the correct procedure to arrive at the answer, Nupur could have ended the discussion there, to move on to the next task. But perhaps she realized that most of the class would not have understood the reason behind multiplying 100 with 2/5 and also why 100 has to be divided by 5 but multiplied by 2. Nupur's closed choice question in line 120 is an attempt to make the student Sb2 think about the meaning of a fraction. The seemingly funnel type questioning in lines 129 to 133 is actually tracing the steps of an argument which indicate the teacher's effort to move from a procedural focus towards reasoning.

The explanation that Nupur developed post students' correct response from line 120 to 133 is based on the construct of equivalent fractions together with a ratio interpretation of fractions (2/5 = 4/10 = 40/100). In this explanation, the share of 2/5 is interpreted as a relation (ratio) between part and whole, which remains invariant as the whole is increased from 5 to 10 to 100. She was scaling up the total parts by a factor and the students were expected to scale the numerator by the same factor, thus implicitly using the notion of equal ratio. Her selection of number 10 and 100 as the quantities to scale up indicate the thought given by her for selecting numbers which would be easier for students to extrapolate. An alternative and more appropriate approach would have been to use the operator interpretation of multiplying by a fraction – showing how 100 can be divided into 5 equal parts and that two of these parts corresponding to

'2/5 of 100' equals 40. The operator interpretation, in this case, also offers a connection with the procedure of multiplying 2/5 × 100. The invariant ratio interpretation, in contrast, involves multiplying the numerator and denominator by the same number so as to make the denominator 100.

Post the episode illustrated in the excerpt 6.2, Nupur still believed that some students may not have understood the relation between 2/5 and 40 out of 100. She then asked for researcher's advice who suggested asking students to draw a representation of 2/5. The teacher then herself drew a representation of '2/5 of 100' on the board, by circling 2 coins out of every 5 coins and asked students to find how much coins would be circled if they considered all 100 coins drawn on board. Though she was able to lead the class to the correct answer using the representation in this manner, there was no discussion on whether 2/5 and 40/100 are equivalent or on why the ratio between the numerator and denominator remains the same, since the focus was on arriving at the correct answer. Nupur used a similar reasoning when she moved to the next task (line 133). The task that she gave students was to find the share of the gatekeeper if he would have asked for 3/5 of the prize.

In this episode, the teacher tried various moves in response to a wrong answer which included asking other students to share their response and explaining the procedure in different ways including using a visual representation. A tension is seen in the episode between teacher's attempt to simplify the procedure for students to arrive at the correct answer and to develop meaning of fractions. Knowing that students tend to calculate without attention to meaning, she opted for an oral and simpler method to arrive at the answer, but in the process mathematical idea about the meaning of fractions and finding the fraction of a quantity remained implicit. The challenges faced were engaging students in understanding the meaning of the fraction 2/5 and the above 2/5 of 100 even as some students quickly showed the procedure for multiplying 2/5 with 100, which they probably had learnt in tuition classes. Another challenge that is evident in excerpt 6.2 is the establishment of connection between symbolic representation of the procedure of multiplying 2/5 × 100 and the visual representation. Nupur made the visual representation but did not discuss how it was connected to the symbolic procedure. The limited pedagogical content knowledge on the teacher's part to select appropriate representation and lead a conceptual discussion could be the reason. This suggests gaps in the teachers' specified knowledge for teaching and ways in which they may be addressed.

In a subsequent discussion with the researcher on what the operation involved was, Nupur identified the process of combining every 2 coins out of 5 as addition of fractions. The researcher pointed out that it was similar to combining ratios and it would be problematic to represent it as addition of fractions. Teachers' knowledge of how a visual representation translates or can be represented using the symbolic representation is important in building conceptual understanding and developing a facility with using visual representation and symbolic representation flexibly to solve problems.

6.5.3.3 Excerpt from lesson 3

In lesson 3, Nupur used the fraction strips to compare fractions and find equivalent fractions. The students engaged with concrete material, answered open ended questions, connected the fractions named to the process of making a part through folding the fraction strips, and connected the concrete fraction strip representation to a visual representation in the form of a diagram. In the beginning of the lesson, Nupur took paper strips of the same size and asked students to fold them into different numbers of equal parts. She then asked them to name the size of a single part as a unit fraction and then asked them to combine the unit fractions to name the composite fractions. In the process students were able to identify the composite fractions which were equivalent to ½, ¼, and ¾, etc. This possibility of identifying different fractions with the same part made the nature of the question open to students and also afforded the opportunity for students to reason with fractions by comparing their size. For e.g., they were able to compare the size of unit fractions and identify the number of unit fractions needed to complete a whole.

In the excerpt given below from this lesson, the teacher used the fraction strips to justify which among several proposed answers were correct. This paved the way for using representations for justification and reasoning in the later lessons too. In the excerpt, Nupur asked students to find the equivalent fraction of ¾. In lines 210 to 238, the discussion is aimed at identifying the fraction equivalent to ¾ as distinct from the fraction equivalent to half.

Excerpt 6.3

176 T: How much is three fourth equal to? [Asking for a response to ¾ = __/8].

177 Sb: 6/8

178 Ss: 6/8 (many)

….

185 T: Without any calculation you tell me if it is 16 here [denominator] then what will come here [numerator]? [Response to ¾ = __/8 = __/16]

186 Most students: Ma'am 8

189 S: 12 (few students)

190 T: 12/16, any other answer?

191 Sb: Ma'am 8

192 T: 8 by 16, any other?

194 T: Two are saying 8/16. Any other answer? How many of you vote for 12/16? How many for 8/16?

(8 students raised their hand for 12/16 as the correct answer.)

197 T: How do we confirm that who is right? Sg1, you tell how you got 12/16 and Sb1, No, somebody else tell me how you got 8/16. You both explain to the whole class. If you want to make drawing, show by paper folding, use whatever method you want, if you want to do calculation [use that]. Do whatever method you want.

199 Sg1: 8 × 2 =16, We have colored 6 parts out of 8 so if we multiply by 2 then 12 will come. So 6 × 2 =12.

(Teacher asks her to write on the board.)

206 Sb1: Ma'am, My answer is wrong.

207 T: Sb1 has agreed that his is wrong, Sb2 you come and tell which is right. (Teacher asks student to explain to the other student who gave a wrong answer.)

210 Sb2: Ma'am, when it will be 24 here (denominator) then 12 will come (in numerator). (Student writes 8/16 =12/24.)

213 T: 8 '2' s are 16.

214 Sb3: Ma'am 8 is added double times then it would be 16.

219 T: Listen [to] what Sb3 is saying. When it will be 8 here, then it will be 16. And when this is...

220 Sb3: 24

221 T: Then what will come here? This is half of 16, No? Here.

222 Sb3: If it is 24 then only it will be 12.

223 T: Here you have made half part, *beta* [identifies the incorrect move made by the student].

224 Sb3: No Ma'am, It is ¾.

225 T: Ok, let's do it by paper folding. Let's take the one with 8 folds or let's make a fresh one.

228 T: I am making on board while he is folding. So, what should I do to make 16 (parts)?

230 Sb4: Ma'am, You can half it (Board already has a figure with 8 parts)

231 T: I should halve it, then it will double. We complete this first. You come for shading Sg2, shade equivalent to three fourth. First you count till half. Yes, this is half, what will be half of half?

....

237 T: How much has been shaded till now? Half, What she has to do to make ¾? [Teacher then makes the line of ¾ prominent and extends it downwards to meet the other paper strips.]

238 T: You have to shade more for it to be three fourth. (Classroom excerpt, 5/08/09)

In the above excerpt, Nupur's approach gives more autonomy to be students in evaluating an assertion than an approach where the teacher is the authority for assessing which answer is correct. Although the teacher moved to the next question when many students responded correctly to ¾ = __/8, the teacher suspended the evaluation when she got two different responses for the equivalent fraction of ¾ with denominator 16. There were two different answers namely 12/16 and 8/16. The teacher made efforts to discuss both the right and wrong answers of the students. The suspension of evaluation allowed opportunity to students to evaluate and support their answers with arguments, while it allowed the teacher to get a sense of what understanding the students were developing. In line 206, Sb1 agreed that his answer was wrong after watching a student give the correct solution, while Sb3 in line 224 insisted that the fraction 8/16 was equivalent to 3/4 even after the teacher's attempt to make him explain why his answer was wrong. The students' openness to say what they think indicates the quality of relationship that the teacher had developed with students, where they did not fear disagreeing with the teacher. To gain a better sense of students' understanding, she also asked the students to vote in line 194 for the two different answers given in the class. This was a practice that was first used by her in lesson 3 after researcher had used it in lesson 1 to discuss the equivalence between ½ and 2/4. She continued to use this practice into the second year.

To convince the students that 8/16 is not equal to 3/4, Nupur used the paper strips that she had already used earlier in the lesson to show how the size of 12/16 would be equal to ¾ and 8/16 would be equal to ½. Here Nupur introduces a new practice of checking the correctness of the answer using a concrete or visual representation rather than a calculation.

Another practice that isseen in the excerpt is to ask for explanations for the answers given.

Although the explanation given initially by the students is based on procedural calculation, Nupur pushed the students to use fraction strips to validate their answer and understand the equivalence between ¾ and 12/16. Through working with equal sized strips, which were folded into a varying number of equal parts, the students were able to relate equivalence of fractions to size. While Nupur had focused on perceptual features in Lesson 1 and mostly on procedures in Lesson 2, in Lesson 3 she attempted to make a close connection between the size of the piece and the name of the fraction assigned to it. Before asking for the equivalent fraction for ¾, she had asked students to name the fraction based on folding paper strips into equal sized parts and also discussed for example how parts of the size ¼ when combined make ½ and ¾, and how four such parts complete a whole. This foregrounded the measure sub-construct for fractions.

In the discussion following the episode described in Excerpt 6.3, Nupur discussed the fraction as a relation between part and whole, as far as onecan identify, for the first time. In contrast, in Lesson 1 she had asked students to match parts visually based on how half looks in a circular whole. The use of linear representation in the form of fraction strips in this lesson made students focus on the size of the part rather than the appearance of the part. It allowed for a discussion of fractions using the measure and part-whole interpretations. Later in this lesson, Nupur asked students to name the fraction by shading non-contiguous parts. Her purpose was that students could name the fraction, not on the basis of what it looks like but on the basis of how much is shaded of the whole.

Even though Nupur was successful in using the fraction strip representation, the problem remained for Nupur to connect procedures with fraction representations in the explanation. As mentioned earlier, some students in her class were already familiar with the procedure to obtain equivalent fractions by multiplying or dividing both numerator and denominator by the same number. Nupur attempted to respond to other students who struggled to understand and make sense of this procedure. She realized the need to connect these procedures to some form of fraction representation that she was using and tried to explain this to the students. However, she was not able to establish the connection between the representations and the procedures used. For e.g., in lesson 3, the same area was divided into more parts (from 2 to 16 parts) to represent equivalent fraction of ½ as 8/16. However, the connection between this action of partitioning a portion of the figure and the procedure of multiplying both numerator and denominator of ½ by 8 is only briefly established in line 231 in excerpt 6.3 and needs a fuller explanation (increase in the number of subdivisions by a factor of 8). In the absence of such an explanation, students may consider representations as a way of finding the answer un-connected to calculations, and will not develop deeper understanding of procedure.

Later in the teaching of fractions in Year 1, the teacher and students made connections between fraction operations and other representations. When students were finding it difficult to understand the meaning of multiplying a fraction by a number, the researcher suggested to Nupur the use of number line, which was also discussed in the textbook. The textbook had several problems in which one had to find the cost of a quantity of thing purchased when either the rate of 1 kg or other quantities was given. The researcher showed how the double number line could be used to depict both fractions as well as operations with fractions like addition, subtraction, multiplication and division. The researcher also modeled the use of number line in the class with the students. In the next class (25/8/09), students shared several ways to arrive at the solution when Nupur asked them to find the cost of buying potato in different quantities when cost of one kg is given (as Rs 24). Some students arrived at the cost of 3 kg by addition (Rs. 72) while other students used multiplication. Most students found it difficult to find the cost of half kg potato, while some students showed that the cost for 1 kg is to be multiplied with ½. The teacher then drew the number line, depicting the quantities ¼, ½, ¾, 1 and 1½ kg. With the help of this number line, Nupur elicited the price of different quantities from students. The students were able to find the price of ½ kg as Rs 12 by halving the price of 1 kg. Nupur then asked them to find the price for ¼ kg which they were able to find as Rs 6 by further halving the price for half kg, since they had already discussed in earlier lessons that ¼ is half of half. A student was then able to find the price of ¾ kg by adding 6 to the price of ½ kg which was 12. Another student showed that one can even multiply 6 three times to get the answer. Using the number line, students were able to find the price of other quantities like 1½ kg using the price of ¼ kg. Through the double number line, the teacher and students were able to integrate measure and operator interpretation of fractions meaningfully for halving. The students were able to relate that they could find the price by either repeated addition or by multiplication. The number line was thus able to elicit students' spontaneous strategies other than the standard procedure. Students could use the operations on their own to depict the relation between quantities and how quantities are changing across the number line. However, the connection between operations on a visual representation like number line with operations on symbolic representations was not established satisfactorily.

It is important to discuss connections between symbolic procedures and representations so that students become aware of the consistency of mathematical concepts across use of representations which may be visual or symbolic in nature. In absence of these discussions, students may come to view mathematics as a set of tools for solving problems and representations may be considered as yet another tool just like procedures. The discussion on these connections can potentially build understanding of underlying of mathematical ideas behind the use of procedures as well as develop the facility of representing a process done on a visual representation through a mathematical statement. The realistic approach of Freudenthal institute favors the latter approach, but in a situation faced by the teacher where students are already aware of procedures it is important to develop the knowledge in both ways i.e. from procedure to a visual representation and from a visual representation to the procedure.

In Lesson 4 in Year 2, students used the area representation to show how a rectangular whole could be divided into six equal parts in different ways. The students showed five different ways to make 6 equal parts of the rectangle. A student had made partitions at every 2 cm for a rectangle of length 12 cm to make 6 equal parts. When Nupur asked her why she selected the 2 cm distance, the student explained that adding 2 cm 6 times comes to exactly 12 cm. Nupur later shared with the researcher that she planned to connect these ideas later when students have to understand the fraction of a quantity, for e.g., area of one-sixth of the rectangle.

Using the distance at which partitions were made, the teacher asked students to calculate the area of the one-sixth part of the rectangle. Using the area of one part, students reasoned that the area of 2 equal parts would be double this area and the area of 4 equal parts would be double of area of 2 equal parts. The students were also able to express the relationships between these parts mathematically through addition or multiplication. The area of 4 equal parts could also be expressed as repeatedly adding the area of one part four times ($a + a + a + a$). However, it could also be expressed as area of one part multiplied by 4 ($a \times 4$). Nupur planned to use these ideas in teaching of fractions later, for e.g., when discussing how ½ is double of ¼. Here a more focused use of the measure interpretation of fraction has been done to develop students' understanding and attempt to connect symbolic notation to intuitive strategies.

6.5.3.4 Excerpt from lesson 5

In Lesson 5 (see summary of the lesson for the task posed), the teacher asked the students if the fraction 8/16 depicted by the shaded squares (non contiguous in the figure) is equivalent to 2/4. A student re-drew the representation of 8/16 by making the shaded parts contiguous and argued that it is equal to 2/4, since in every 4 parts, 2 parts are shaded. The teacher's discussion of this response is given below.

Excerpt 6.4

T: How many say that this is correct? How many wrong? Sb1, you think it is wrong. What is wrong, tell?

Sb1: Ma'am, she has made it topsy turvy [mixed it].

T: She has shifted the squares from here to here, therefore you are feeling wrong? If it was on the same place it would be right?

Sb1: Yes Ma'am

T: Could we have shown 2/4 by some other way?

(Students shout together for a chance to answer.)

Sb2: Ma'am, It should be all blue here, and all white here.

T: What is the meaning of 2/4?

Sg1: These 4 here, when we shade half of 16...

T: Can you write 2/4 as half?

S: Yes ma'am.

R[1]: Sg1 asked a question, may be everybody was not able to listen?

T: Yes, Sg1 asked a question, speak loudly *beta* (child).

Sg1: This is 16, half of which is 8, therefore we can write it as 2/4.

T: Half of 16 is 8 therefore we called it as 2/4. Therefore can we call it as ½.

Ss: Yes ma'am.

R: But why did you call it 2/4. What is same in 2/4 and 8/16?

Sg1: Ma'am, half of 4 is 2.

R: So, is something same as ½?

Sg1: Yes ma'am, half of 2 is 1. (Classroom excerpt, 4/08/10)

In this excerpt, the teacher is asking questions to elicit students' reasoning and their spontaneous strategies. As in excerpt 6.3, the focus is not on getting the correct answer but in trying to understand the different students' perspectives and interpretation of the visual representation. Instead of telling the correct answer or how to get the correct answer, teacher is trying to reason with the students about whether the same representation can be expressed in different ways.

In the above excerpt, the teacher engages in the practice of asking students to evaluate the

1 R stands for the researcher

answers, as in excerpt 6.3 from year 1. This is consistent with the researcher's observation that she established this practice as a norm in the second year with a new group of 5th grade students. The way students responded readily without hesitation indicated that it was an established practice, while in the Lesson 2 and 3 when she had initiated this practice, it took some time for students to start responding to the teachers' questions. The teacher started the second lesson on fractions with a non-contiguous figure as she wanted to deal with the students' misconceptions about the fractions directly and early in the teaching sequence. The question is also open in the way it is posed since the teacher allowed students to express their own ideas and perspectives.

In terms of explanation, the student is able to use the visual representation for the explanation rather than relying solely on the procedure. Here Sg1 is reasoning about the fractions which are equivalent to half by identifying the visual representation of half in different representations of equivalent fractions of half. She is also able to recognize that in each of the equivalent fraction of half, half of the total number of squares would be shaded and thus is able to abstract what is common in 2/4 and 8/16. This is in contrast to the way half was discussed in lesson 1 in Year 1 when students disagreed that ½ and 2/4 cannot be considered equal because of the partitioning line. Here the child is recognizing that many different representations can be equal to half since the amount of area occupied by the shaded parts is the same. Thus,the teacher continued to use in Year 2, the practices which she had started to use in the year 1 indicating that they were becoming more stable.

6.5.3.5 Equity in classroom participation

Nupur had shared with the researcher in the beginning that there were many students in her class who already knew the answers of the textbook questions since they had done the chapter in the tuition classes. This had created an issue for equitable student participation in her class since the students who were doing the problems for the first time did not get time to think or answer as the tuition going students would give away the answer too quickly. Nupur had also expressed concern that girls in her class did not speak up or take interest. While observing the teaching, the researcher too found that there were solutions written in some students' textbook prior to the lesson, which they read out when the teacher asked a question from the textbook. To address this problem of increasing student participation, she had made row-wise teams in the classroom and she would pose questions to the teams. However, when a student became overenthusiastic she would give a more challenging problem to him (mostly a boy), while she engaged the whole class in discussion. In order to address this issue, Nupur tried to call on students specifically to give answers after identifying the students who have not been speaking and would try to give them more time to think. There were times when she specifically posed a question to a girl and encouraged her to give the answer in whatever form she could (for e.g., see excerpt 6.1, line 5 & 7, in section 6.5.3.1) and gave her time to think and answer the question while asking other students to hold back their answers. However, the students would still shout their answers and would try to get the teacher's attention and her acknowledgement for their correct answers. Thus, although Nupur tried making the classroom interaction more equitable and participative it was challenging to engage all students since students' focus was on giving the correct answers rather than engaging in reasoning and listening to each other.

6.5.3.6 Conclusion

The analysis of Nupur's lessons in detail along with description of other events during the course of collaboration reveals the practices that Nupur was exploring and the challenges she was struggling with. Although the analysis reveals that Nupur was attempting to encourage student participation and reasoning, the focus on procedures was still prevalent through the use of funnel type questions and explanations focused on procedures rather than conceptual aspects in excerpt 6.1 and 6.2. The description of the types of questions posed by Nupur along with the kind of explanations developed indicate an increasing focus on developing meaning of fractions and understanding using representations, moving away from merely calculating with fraction symbols. The teacher's practice which earlier consisted predominantly of funnel type and closed choice questions, began to include questions asking for explanations and reasons. The teacher gave opportunities to students to develop explanations using representations. Initially students shared explanations based on procedure, but the teacher's persistence in valuing and engaging students in explanations based on representations led to students using representations to make sense of fractions. Although, at times the teacher reverted to older practices like evaluating the correct answer and giving the explanation to students when she faced challenges, the emphasis on reasoning was sustained. Another change that was observed was connecting symbolic procedures to visual representations and the students' intuitive strategies.

The back and forth shifts in practice between procedures and explanations and between symbols and other representations, were indicative of teachers' learning during teaching. These shifts in questions, explanations and sub-constructs used collectively point to shift in the teacher's goals towards teaching for understanding and responding to students' thinking. These events highlight the tension experienced by the Nupur indicating reflection and reorganization of the belief structure, the challenges faced by the teacher and the kind of knowledge and practice that needs to be developed by the teacher to bring a stable change in practice. Many of the changes were likely induced or supported through the collaboration with the researcher. In the next section, the nature of collaboration and the role of researcher is discussed.

6.6 Role of the researcher as collaborator

My role as a researcher in the collaboration for teaching was twofold. One of my functions was to identify the take up from the orientation workshop and the challenges faced by the teacher in implementing intended changes. The other role was to analyze the practice to triangulate the data about teachers' beliefs and preferences for practices. The collaboration also threw light on the knowledge that was involved while teaching and what type of professional development support is needed to overcome the challenges faced by the teacher.

The teacher and researcher met frequently after or before teaching (2-3 times a week for 3 weeks) to discuss students' responses in the class and the learning that was taking place and to think of ways to support student learning. The researcher asked the teacher if she was satisfied with the students' learning, about the decisions made while teaching and about the plan for the subsequent lessons. The teacher and researcher together made a test after completion of the fractions chapter in the first year and discussed the students' performance on the test. In this section, I present some of the critical events, including some excerpts, during the teacher-researcher discussion, that likely shaped the teacher's decisions and understanding about teaching of fraction. I also present excerpts from the one day workshop at the end of the observation period in Year 1, during which the teacher and the researcher together presented to other teachers in the school, their insights from the teaching on the topic of fractions.

In the course of the collaboration, the researcher frequently shared her observations with Nupur about students' understanding. Nupur too adopted the practice of sending students to the researcher to show their notebooks providing the researcher an opportunity to interact with them, thus acknowledging the collaborative relationship and the joint responsibility towards developing students' understanding[3]. She shared the tasks that she planned to take up in the next

3. After a set of lessons, Nupur and the researcher jointly developed a diagnostic test to be given

class with the researcher to get feedback. The researcher too shared the tasks and ideas that had been used by the research group for teaching fractions. Nupur used some of the tasks, while adopting some of the ideas shared with her, but mostly used tasks from the textbook. This was because she felt she had to cover the syllabus.

The role of the researcher during the collaboration was that of an active participant observer, since she not only interacted with the teacher but also with the students during the lesson. She framed tasks for students, engaged students in these tasks for short periods during some of the lessons and sometimes highlighted an interesting solution or conjecture shared by a student, which the teacher had missed in the course of interaction. The way I would like to portray my role and interactions during the collaboration is a process of implicit dialogue which involved using students' responses and tasks from the teachers' own lesson to debate about the ideas that can be used for teaching fractions. I refrained from telling the teacher directly to use ideas or tasks, but showed how they could be used to understand deeply how students are thinking about fractions. The teacher too commented about the researcher's role during the interview that "... you identify what point has been missed/ignored by us... we try to understand what is there in the book and teach... but you look at how student has got the solution and why he used this method".

As discussed earlier, at the end of lesson 1, on an invitation from the teacher, I asked the students if ½ and 2/4 were the same, to which the students said that they were not the same. The following day they responded that ½ of a cake and 2/4 of a cake will weigh the same. Reflecting on these responses, Nupur said

> That means student has not understood equivalent fraction. We will give stress on it tomorrow.... It means student do not understand through numbers... they have to do it with some object... we can try by making pieces of a paper rectangle. (Meeting Excerpt, 3/08/09)

Recognizing the challenge that the teacher needs to spend more time on planning, I discussed the topic that would be taken up in the next class and suggested resources that could be used in the classroom or pointed to the ideas which she felt as important to address while learning fractions. For e.g., I discussed the sharing and measure interpretations of fraction with the teacher, and resources like a worksheet on naming and comparing unit fractions which was later used by the teacher. The teacher used some of these ideas and resources in the classroom and found them

to the students.

to be useful. In lesson 3, as discussed earlier, she used equal sized fraction strips and emphasized the measure interpretation of fraction.

Other instances where the researcher intervened in the classroom teaching included lesson 2, when the teacher felt stuck in as students were not able to understand 2/5 of 100. The researcher suggested asking students to make a visual representation of the problem. However, the teacher chose to make the representation herself. On another occasion, I showed how the game played by the students by using different fractions to complete a whole could be represented as fraction addition. I also modeled how quantities and price could be represented on the double number line and used to find the cost of a larger or smaller quantity, which is discussed earlier.

Nupur felt that the resources shared by the researcher were helpful as they pointed out things to her that she had not been focusing on in her teaching like the idea of measure and comparing unit fractions using their size. She felt that during the course of the collaboration, the students had started participating more and were giving good answers as well as understanding what is wrong and why.

After six lessons in the first year while recounting her observation of students playing with fraction disc[4], she commented,

> Student is still understanding the number. Whatever number is there that student is shading. Student is not understanding what relation is there between the numbers. In full circle there are 12 parts and student does not relate with it. He is saying that 4 parts are there or 1 part is left to color. He is not understanding that it is a piece of the whole. (Meeting excerpt, 7-08-09)

The above articulation shows the growing sensitivity of the teacher to students' use of whole number language to talk about fractions and the impediments this could cause. After a few days in the first year, she shared the major changes that she felt had occurred in her teaching and her goals. She acknowledged the change in her teaching practice as she felt that she was now focusing more on the type of questions she asked to assess students' understanding.

Excerpt 6.7

> Now, I am focusing more on framing the right questions where stress is required. If something is tricky then I try other ways too....Like if the student is able to give ¼ of rupees, will he be able to give ¼ of kilogram?...This is based on my own expe-

4 A game discussed earlier as a circle divided into 12 parts which students have to colour depending on the token they lift from a pile having a fraction written on it. Whoever completes colouring the whole circle wins.

> rience.... Sometimes mothers give money to buy things from shop, so they should be able to calculate.... I try both types of question. If 20 paise is 1/5 of the rupee [is one type of question] then [ask a question of the type] what is 1/5 of a rupee?... If the question is not of the level of the child, we can change it to a more simpler problem... like calculation of amount of carrot to be bought for 18 Rs would have involved dividing 500 and 3 and answer in decimals.... So I changed the question. (Meeting excerpt, 24-08-09)

In the above remark, the teacher was referring to a lesson in which she gave the task of finding the amount of carrots that can be bought for 15 rupees when the cost of the carrot was 18 rupees per kg. This task was not given in the textbook but Nupur was following the advice given in the textbook to ask students to find the prices of different vegetables and was constructing calculation tasks for students. After giving this task, she realized that the task would be difficult for students. One student got stuck after finding that 9 rupees will get her half a kg of carrots but did not know what to do about the 6 rupees which were left. After struggling to explain the solution to students, Nupur resorted to changing the numbers. She changed the amount of money to buy vegetables to 27 rupees from 15 and then students were able to easily find that 1 and half kg of carrots could be bought. This made her careful and aware of the numbers selected to construct the problem since it might throw up challenges. However, this incident is also indicative of how Nupur avoided the challenge and discussion on the number and changed the problem to lower the cognitive demand for the learners.

Nupur was experiencing a tension between focus on procedure and engaging students in making sense of the procedures. The students used to share the procedure that they had learnt from tuition classes. However, in the discussion given below she realized the lack of connection between the use of procedures and understanding their meaning.

Excerpt 6.8

1. T: Students are focusing on the calculation rather than [doing it] mentally. May be this was stressed at home. We have also learnt like this. If I also do it without the textbook, then I will teach by this calculation only.
2. R: Are they able to understand that why are they multiplying here? Or why divide? Because, I don't know on what basis did they decide to divide here – may be they had done it at home.
3. T: I give these kind of problems often, where I have taught them that ¼ means they have to divide by 4. I have also put numerator and denominator word in their ear. Numerator and Denominator will be multiplied and then he is doing by practical. He has to correlate [these two].
4. R: That is what I want to know, that, when a student is dividing 1000 by 4, is

he understanding that 1000 is being divided into 4 parts and one of the part is 250 grams?

5. T: I had asked this question from students, that, 'What is half?' and then, 'What is one fourth? So, they know that one fourth is half of half. Both sides I have done, but whether he relates this [i.e., half of half and one out of four equal parts]?

6. R: That is what I also have a doubt.

7. T: He is understanding that half of half is one fourth, but here, [when dividing 1000 by 4] whether he is thinking about it in this way [not sure]. One child divided 1000 by 4. He was an intelligent one. He had divided nicely [Long division].

8. R: If they solve any other problem, would they know that dividing by 4 is making ¼ of the quantity?

9. T: I think they would. Lets see. We can do that tomorrow.

10. R: How do you decide that the student has understood?

11. T: I asked him this - 'Why are you making half?' When you are taking 1 kg then the quantity is more so you would need more money or if the quantity is reduced then money required would be less. So, he is understanding that if the quantity is reducing the price will also reduce, like this they say. We have to ask question in this manner.

12. R: What kind of explanation do you prefer from students?

13. T: It should be logical, but he has to learn this calculation also. Both are important.

14. R: If one student is explaining that half of half is one fourth and other has divided by 4 then what will you do?

15. T: Both are right. According to method nowadays [after curricular reform] both is right. But if we do not refer to the textbook and try to find the solution then all students will give the answer. They know the solution but how to explain and write in statement, that he is not able to understand. He is not able to express. You will express mathematics in the way it has been taught to you. (Meeting excerpt: 24/8/09)

In this conversation, I was trying to find out how Nupur was making decisions in the classroom and what kind of responses she considered as indicators of understanding. Weboth were aware that students were sharing procedures that they had previously learnt from tuition classes and that Nupur wanted to engage students in developing understanding. The teacher believed that students will have to know how to calculate and get the answer and this was something that she was familiar with. However, she was dealing with meanings and procedures in a separate manner, avoiding linking the two, perhaps since she herself was not clear about the linkages and the meanings.

As discussed earlier, after giving the students a test and reflecting on results together with the researcher, the teacher was able to understand better the problems associated with using proto-typical figures and using counting as the procedure to identify the fraction. One of the figures given in the test showed only ¾ of a circle as the whole, which was divided into three parts of which one part as shaded. The students had to identify the fraction for the shaded part. Students who were used to considering the full circle as a whole, considered the part as ¼ while a few students were able to identify that it would be 1/3. Another figure showing 1 and ½ circles shaded was represented by most students as 12/16, which would be correct if the whole consisted of two circles. The researcher had deliberately used these tasks to point out the problems related to using *m* parts out of *n* parts interpretation.

The students' responses to both these questions had made Nupur reflect on the importance of focusing on the whole and using non-typical questions in her teaching. This was evident when she spoke to other teachers in the school during the session held for them about difficulties students face in learning fractions and what a teacher needs to focus so that students do not face problems in learning fractions in higher classes. An excerpt from the workshop is given below.

Excerpt 6.9

> Because we have always given figures in example which are one full circle or band (rectangle). We have not given any other example. They should focus on the shape of the whole. We always call it one chapati, but we have to look at the shape of the whole and then see parts accordingly. We ourselves have given this idea to students that the whole is always full. Depending on the shape of the whole the part would be 1/3 [referring to the problem discussed above], but we are not ready to think of that shape as the whole. Here there are three equal parts but still we are considering the full circle as whole [when we give ¼ as answer].
>
> We taught the whole lesson and I never took fractions more than 1 (mixed fractions). So when this question was given, students counted the total parts and shaded parts and wrote 12/16, but when I asked them today using the example of chapati, they were able to say 1½. So it is these places that kids get stuck and then they face problems in higher classes. (Workshop excerpt, 31-7-2009)

In both these excerpts, it is clear that Nupur is realizing the importance of considering the whole while identifying the fraction and not just based on counting the parts. These two tasks served as a tool for teacher's reflection and were much more effective than the researcher counting the teacher about counting the parts for identifying the fraction.

6.7 Conclusion

The findings from the classroom observation reveal that Nupur made efforts to change her practice of teaching mathematics after the workshop. These changes were mainly through inclusion of practices like asking students for explanations, use of representation for reasoning, giving students more autonomy by asking them to evaluate the answers and establishing equity in the classroom participation (Kumar & Subramaniam 2012c).

Nupur's positive beliefs about student-centered practices and teaching for conceptual understand helped in adopting these new practices in her repertoire. Her belief that reasoning about the correct and incorrect answers should be understood by the students and that repeated practice of similar/same problems leads to rote memorization led her to spend more time on discussing reasons and explanations in class than on practicing problems. Her belief that understanding concepts takes time also helped as she had patience for students to develop conviction in their answers. She slowed the pace of the lesson and even took the discussion of a topic again in class when, after observing and discussing students' response with the researcher, she felt that students had not clearly understood the concept.

Nupur's efforts to overcome the challenges in her teaching to teach conceptually led to development of sensitivity for mathematics as well as sensitivity towards student understanding. With her increased sensitivity towards mathematics she was more careful in her choice and use of representations as mathematically appropriate as well as in listening closely for the mathematics in students' responses. She also become careful about the use of mathematical vocabulary and emphasized the relation between the 'part' and the 'whole' and when student responded to naming the fraction problem with whole numbers. As her teaching progressed she was sensitive to students' use of 'whole-number talk' instead of fractions, recognizing it as a conceptual issue.

Her sensitivity for students' understanding helped her in assessing whether students had understood the concepts. She did not base her decision on the correctness of the answer but on the kind of explanations that the student provided. She tried establishing equity in the classroom participation by trying to give a chance to many students to answer rather than restricting to the few who knew answers and methods as a result of going to tuitions. Initially she asked many funnel type questions and later she asked students to explain and provide justifications more often. Her efforts to discuss even the wrong responses of the students and suspending the evaluation of responses while inviting students to evaluate the response of their peers is indicative of the sensitivity she was developing to inquire and understand student thinking. Her sensitivity is indicated in the way she framed questions in the class while teaching as she moved from using textbook based questions followed by calculation questions to deciding questions on the basis of what misconceptions were elicited in students' responses, for e.g., the comparison between 8, 1 and 1/8 in lesson 5. She realized the value of using representations as tools for sense making although she was not able to use them to unpack the procedures used for finding equivalent fractions. Through an analysis of students' responses and reflection on the lessons with the researcher, the teacher was able to identify the conceptual gaps related to the interference of the whole number learning, focus on typical figures for teaching fractions and students not being able to understand the meaning of equivalent fraction.

Developing these sensitivities was a result of the teacher's experimentation with use of representations for equivalent fractions and reflection on student responses along with the researcher. The researcher not only provided additional information about students through her own interactions but also articulated alternative ways to make sense of what happened in the classroom. Student responses were the data used for analyses of students' understanding as well as making decisions about pedagogical interventions that can be used in the classroom. This data as well as teacher's experimentation were sources for researchers' learning also in the realm of student thinking and teacher learning.

Thus, the practices that were observed in Nupur's lessons included asking why questions, discussing different and wrong answers of students, building explanation based on students' responses, using representations and concrete materials to develop reasoning and pursuing students' responses and conjectures further to develop understanding of mathematics. However, the efforts to include these practices related to questioning, giving and building explanations and responding to students are beset with challenges that may, at times, lead to unproductive or problematic discussion or teacher reverting to telling students how to solve the problems.

One of the challenges discussed in the chapter is to identify and use appropriate representation as well as the sub-construct to discuss the representation. For e.g., in Lesson 2 Nupur represented the problem of $2/5 \times 100$ through a visual representation based on interpreting fractions as ratio rather than as an operator. Although, students arrived at the correct answer with teacher's help that 2/5 is equal to 40/100, it was not clear if they had understood that 2/5 of 100 slaps is 40 slaps. Further, there was no attempt made in the class to connect the procedure of multiplying 2/5 × 100 with the visual representation used. This is an indication that Nupur was not yet familiar with the various sub-constructs of fractions and the need to use them flexibly in different contexts. This is also indicated by the fact that although the textbook had more context based tasks and problems invoking a greater variety of sub-constructs, these were not fully explored in Nupur's lessons. Indeed, in many instances across the lessons, Nupur was unable to connect the operations on symbols with the operations done with other representations such as the fraction strips, the arrays, or the number line. A hypothesis that needs to be explored is if a more robust understanding of the variety of sub-constructs allows a teacher to make better and more appropriate connections between procedures with symbolic notations and operations on other representations.

Nupur recognized and acknowledged the fact that even though students counted parts to designate a fraction correctly, they may not understand the quantity that a fraction denotes. She addressed this challenge by taking up fraction strips and focusing on the size of the part in relation to whole in lesson 3 and in Lesson 4 and 5 on the relation between the area of the part in relation to other parts and to the whole. In this case, she used the measure sub-construct appropriately to discuss the magnitude of the fraction. It can be inferred that she realized the limitations of using the 'degenerate' part-whole sub-construct where in the focus is on counting the number of shaded parts and total parts corresponding to the numerator and the denominator.

Some of Nupur's beliefs also constrained her efforts to develop students' understanding. Her belief that primary students are too young to engage in challenging mathematics questions introduced tensions in her teaching between letting students find their own ways of solving problems and 'telling' students the 'simple ways' in which to arrive at the answer. While discussing issues to do with the students' understanding she said "There is lot of confusion, they are too young for this". This could also be because she did not want her students to come up with wrong answers. Although she tried not to provide evaluative feedback to students about their answers and even discussed wrong answers to explain the reasons why it was wrong, she was perturbed when students got the wrong answer. To her, the students' wrong answers were indicative of her failure in teaching. There were a few instances when students gave wrong answers and she reverted to telling ways to get the right answer. There were few instances when she gave a problem to students and then realized that it would be too difficult for students because of the numbers involved and then she changed the numbers so that student were able to solve successfully. I infer from these instances that student success at tasks might be important in sustaining change in practice.

The analysis of Nupur's teaching indicates that when a teacher starts to incorporate student-centered practices because of some conducive beliefs, support is needed to sustain the change in practices. An important form of support is the development of teachers' knowledge relevant to the topics that she is teaching. In the case of the topic of fractions, the findings suggest that knowledge of the various sub-constructs of fractions is needed by a teacher. Such knowledge may provide a foundation to develop other kinds of knowledge, including knowing how to act in situations which may be unanticipated and knowing the content in flexible ways to link with students' understanding. The teacher's knowledge helps the teacher to pursue the classroom discussion beyond just establishing the answer as right or wrong, pursuing students' conjectures and responses to develop important mathematical concepts. It allows the teacher to build explanations based on students' responses and above all helps in developing a form of teaching based on students' thinking rather than what is given in the textbook. For teaching to be determined by the student's thinking, a teacher has to be empowered to take decisions like selecting appropriate examples for teaching, recognizing opportunities from student's responses to develop important concepts, constructing questions for assessment of understanding and also determining what response of students has to be considered as an indication of understanding and what not.

A further lesson that can bedrawn fromthe analysis is concerning how a teacher may come to acquire the complex knowledge that is needed in the classroom. We suggest that the development of knowledge about aspects like sub-constructs of fractions, knowing students thinking, working with student's responses, framing questions to assess students' understanding needs reflection on classroom teaching. This suggests that professional development interventions like workshops need to be complemented by PD interventions that are centred around classroom teaching and reflection on teaching. Further, having a collaborator or participant observer, initially helps in reflection and thus may lead to development of such knowledge by considering students' responses as evidences to reflect upon teaching. In order to break the established routines of practices over the years, a teacher needs opportunities for continuous reflection and support in form of alternative ideas and framework to guide the teaching. In the next chapter, a preliminary step in the direction of evolving forms of teacher professional development that are closer to classroom teaching has been discussed.

7

Topic Focused Professional Development on the Teaching of Integers

7.1 Introduction

The main objective of the intervention that is the focus of the sub–study reported in this chapter was to support the development of teachers' knowledge and practices for teaching the topic of integers, as well as the associated beliefs conducive to effective teaching. The findings of the earlier sub-studies formed the background against which the design and enactment decisions for this Sub-study were taken. Earlier sub-studies indicated that teachers held tensions among beliefs for teaching rules versus developing conceptual understanding and that their knowledge repertoire in using contexts and representations to teach important concepts was limited. Although the first workshop had initiated thinking about different aspects of practice, as discussed in Chapter 5, the case study of the teacher Nupur in Chapter 6 indicated the challenges faced by the teacher due to limited knowledge of representations, of mathematical meanings in contexts, of the ideas underlying representations and connections between representations and procedures. It has been argued in the previous chapter that teachers need support while teaching and need topic specific resources in designing tasks and representations to discuss conceptual aspects. This sub–study traces the growth of teachers' knowledge and its impact on teaching as a result of participation in topic focused professional development workshops. These topic focused workshops were based on a situative perspective for professional development since they provided opportunities for teachers to engage in the activities similar to teaching using artifacts like textbooks, problems and representations used for teaching integers. The results show how collaborative planning and development of resources for teaching helped the teachers in developing knowledge of key ideas and meanings related to integers and their addition and subtraction, and motivating them to use these ideas in the classroom along with the resources based on these ideas. The use of these ideas brought about, to varying extents for different teachers, shifts in teaching from exclusive reliance on rules to focus on improving reasoning and understanding. This sub–study contributes to the main study of the model for professional development of mathematics teachers. In the first year, the professional development (PD) workshop and classroom based support were not topic specific and classroom based support was offered after the workshop by the researcher. The PD workshop took place during summer holidays and the classroom intervention occurred during the school year. In contrast, in the second year, the workshops were specifically on the topic of integers and occurred at regular intervals during the school year. Classroom teaching occurred in parallel along with support by the researcher. This model of teacher development provided an opportunity for teachers to share their concerns about teaching a difficult topic like integers, developing resources for teaching and then sharing experiences of teaching using those resources. The model provided for greater continuity between the teachers' professional development experience and their classroom teaching.

7.2 The study

7.2.1 Research questions

The main questions addressed in this Sub-study are

1. What were the teachers' concerns about the teaching of integers and how are they related to issues of meaning of integers?
2. How did teachers construct Specialized Content knowledge (SCK) for teaching integers using the framework of integer meanings through the exploration of contexts?
3. How did the criteria used by teachers for judging adequacy of representations evolve in the course of the topic study workshops?
4. What was the impact of the learnings from the topic study workshops on teaching of integers as reported and as observed?

Section 7.3 presents the framework of integer meanings that informs the analysis in this chapter. The research questions listed above are addressed in subsequent sections. Section 7.4 presents an analysis of the chapter on integers in the sixth grade textbook which teachers drew upon as an important source of knowledge. The next section (7.5) analyses teacher talk in the initial phase of the workshop to identify their concerns, beliefs, goals and preferred practices as well as indicate the specialized content knowledge for teaching of integers that the teachers initially possessed. I also attempt a characterization of the teachers' criteria for representational adequacy. Section 7.6 discusses the teachers' engagement with the meaning of integers and operations by discussing student errors, explanations of representations and finally by exploring contexts using the framework of meanings of integers and its operations. In the course of this engagement, teachers developed additional taken–as–shared criteria for representational adequacy based on meaningfulness and consistency moving their discourse around integers and their representations to a deeper level. In Section 7.7, the teaching of three of the four teachers participating in the workshops is described, highlighting the efforts taken by them to integrate ideas from the workshop in teaching and exploring new practices. Section 7.8 discusses the impact of participation in the workshops on teachers' beliefs, goals, practices and preferences for representation, the role of specialized content knowledge and evolution of criteria for representational adequacy.

7.2.2 Study participants

In the second year of the larger study, topic focused professional development was organized through workshops aimed at collaborative planning of resources for teaching. Two separate subgroups of teachers from the original group of 13 teachers: four primary teachers and four middle school teachers participated in the workshops. In this chapter, only the work of the middle school group of teachers in these topic focused workshops is discussed. Table 7.1 provides the details about the participants in the study[1]. The primary teachers group was not able to implement their plans in classrooms due to constraints of the timetable and upcoming examinations. A team of five members of the resource team planned and facilitated the meetings of the topic study workshops. Usually, two to three resource team members were present in each subgroup during the workshops.

Table 7.1: Background information of the participant teachers

Teachers' Names (pseudo names)	Age and Gender	Qualification	Teaching experience (Primary + Middle/Secondary)	Average number of students in the class 2007-2009 (Self reported)
Swati	42, F	M.Sc. Maths, B.Ed.	10 + 7	45
Anita	47, F	B.Sc. Maths, B.Ed.	20+3	40
Rajni	53, F	M.Sc. Maths, B.Ed.	0 + 23	45
Ajay	54, M	B.Sc. Maths, B.Ed.	0 + 22	40

1 The participant teachers are mentioned in Chapter 1 using following codes: M1-Swati, M2-Anita, M3- Rajni, M4- Ajay.

7.2.3 Timeline of the study

The aim of the workshops was to focus on a topic that was challenging to teach, to develop a deeper understanding of the topic, and plan for teaching. The middle school teachers' group chose the topic of integers to be taught in Grade 6. During the six one-day workshops spread over the period of 18 weeks teachers engaged in various activities which are illustrated in Table 7.2. For each workshop day, there were broad goals, leaving room for discussing questions raised by teachers. The design of next workshop was based on previous workshop interactions and insights about teachers gained from those interactions. The tasks were designed to promote teacher's learning while giving them opportunities to share classroom experiences and imagine alternative scenarios/ activities that could be interpreted in their lessons.

Table 7.2 presents the different phases of the six one-day workshops. The phases are convenient divisions with overlaps and elements of each phase present in the other phases. The initial phase (Days 1 and 2) is termed 'articulation and analysis phase' since opportunity was given to teachers to articulate the difficulties faced in teaching integers. Teachers described the textbook's and their own approaches to teaching integers using various representations like number line, neutralization model, etc. The second phase is termed 'development of theory' (Days 2 and 3) as the teachers were introduced in this phase to theoretical ideas like the meaning of integers and integer operations. The third phase is termed 'design and analysis phase' (Days 2, 3 and 4) during which teachers analyzed the examples, explanations and representations using meanings of integers and operation as the basis and then used the ideas to make individual lesson plans. The fourth phase of 'sharing and reflection' (Days 5 and 6) involved sharing insights about teaching integers using the ideas and representations developed in the workshops and preparing a presentation for another larger group of teachers. This presentation took place on Day 7. The first four workshops (Day 1 to Day 4) took place within the space of about one month. The next two workshops (Days 5 and 6) took place after a gap of two months, during which period, the teachers mostly completed their lessons on the topic of integers.

Table 7.2: Description of activities engaged in topic focused workshops

Meeting	Date	Duration	Activity
Day 1	30 July 2010 Articulation and	6	1.1 Selection of topic of integers for the study by the teachers justifying its difficulty. Discussion on students' common mistakes
			1.2 Sharing and analysis of pedagogical approaches used by

Meeting	Date	Duration	Activity
	Analysis Phase		teachers for the topic of integers
			1.3 Discussion on student errors and pedagogical approaches in the combined group of primary and middle school teachers.
Day 2	9 Aug 2010 Articulation and Analysis phase Development of theory Design-Analysis phase	6 hours	2.1 Discussion on student responses and common errors.
			2.3 Sharing the instructional explanations, activities and contexts used or designed for teaching integers.
			2.4 Worksheet on State, Change and Relation interpretation of integers. Exploring a variety of contexts and the interpretations of integers in them.
			2.5 Analyzing the textbook for the learning objectives, sequence of concepts and activities.
			2.6 Design of activities that can be used in the classroom for teaching integers
Day 3	10 aug 2010 Development of theory Design-analysis phase	6 hours	3.1 Exploring and addressing teachers' questions on dealing with students' responses/ questions
			3.2 Discussion of contexts and representations for addition and subtraction of integers through contexts designed by teachers and from textbooks.
			3.3 Framing and categorizing questions for addition and subtraction of whole numbers, as well as integers into the categories of 'Combine', 'Change' and 'Compare' for both whole numbers as well as integers.
			3.4 Brainstorming on tasks involving addition and subtraction of integers using contexts.
Day 4	30 Aug 2010 Design-Analysis Phase	6 hours	4.1 Identifying learning outcomes for teaching integers
			4.2 Analysis of the textbook questions in terms of representing state, change and relation notion of integers.
			4.3 Discussing a worksheet based on integer mall context along with a suggested sequence of questions.
			4.4 Development and discussion of individual lesson plans for ten days and revisions of plans.
Day 5	1 Nov 2010	6 hours	5.1 Two middle school teachers share experience of teaching integers in a common meeting. Analysis of student responses

Meeting	Date	Duration	Activity
	Sharing-reflection phase		incidents reported by them.
			5.2 Sharing of students' misconceptions related to integers, decimals and subtraction from the same schools by one of the researchers.
			5.3 Teachers planning for presentation to other teachers in a workshop.
Day 6	20 Nov 2010 Sharing-reflection phase	6 ½ hours	6.1 Common meeting about objectives for presentation to teachers in workshop
			6.2 Swati shares her experience of teaching integers
			6.3 Sharing of insights gained from teaching and participating in collaborative lesson planning
			6.4 Selection of materials and approaches that seem promising for students' learning for sharing with other teachers
			6.5 Preparing mock presentations and discussions
Day 7	24 Nov 2010 Participation as resource person	4 hours	Teachers' presentations to their peers in a workshop on the following themes: 7.1 Developing the sense of integers as state, change and relation through categorizing examples of situations and defining how integers are represented in the three senses.
			7.2 Developing contexts and questions different from the ones given in the textbook as an important resource in supporting students' thinking and providing teachers with insights into students' thinking and learning
			7.3 Use of contexts helped students in developing meaning and language that helped students in later working in abstract contexts of the number line and algebra
			7.4 Use of concrete representations like making pairs of opposite colored buttons to represent addition and subtraction can help students in making sense of rules used in addition and subtraction operation of integers.

7.2.4 Data analysis

The findings reported in this chapter are based on an analysis of teachers' talk in the workshops, the reflections that they shared with teacher colleagues, the lesson plans made by teachers for teaching integers and analysis of teaching done by three of the four teachers. The data was collected in form of video and audio recordings of the workshop sessions and teachers' classroom teaching. These recordings were transcribed by the researcher. A few lessons from the classroom teaching of Rajni and Anita could not be transcribed and researcher's notes were used along with reviews of audio recordings to analyze the teaching.

The data from the topic study workshops consisting of audio recordings included interactions between teachers and teacher educators in the workshop as well as teachers' reports and reflections on using resources developed in the workshop for teaching. Additional data included teachers' individual lesson plans and presentations made by teachers to their peers in the last meeting of the workshop. The main focus is on the participants' exploration of representations involving the use of integers, including contexts and models, and their developing understanding of the meaning of integers and of addition and subtraction of integers in relation to a variety of representations. This focus derives from the framework of integer meanings elaborated in the next section.

The discussion in all the days of the workshop was fully transcribed. Each utterance by a participant (i.e., each turn) was coded to identify aspects related to the topic of integers using the following categories: speaker, mathematical purpose, pedagogical purpose, integer meaning, operation meaning, type of representation, and specific model/context discussed. The category "mathematical purpose" coded for mathematical content, and included the codes "integer meaning," "integer order," or "addition–subtraction." The category "pedagogical purpose" described the pedagogical concern reflected in the talk and included the codes "student thinking" and "evaluating accessibility." It also included codes that captured teachers' engagement with the mathematical content without explicit reference to the teaching context such as, "explaining a mathematical point" and "evaluating mathematical consistency." The codes for integer meaning and operation meaning were obtained from the framework described in the next section. Types of representation were "symbol," "context," and "formal model." To see full list of codes used see Appendix 7.

The researcher and a colleague independently coded the transcripts of the initial workshop sessions. After initial coding, codes were merged and simplified to remove ambiguities. Differences between the two coders in coding were resolved through discussion; when they could not be resolved, both codes were marked together for the particular turn. Subsequently, the researcher completed the coding of the remaining transcripts and used the codes to collect together utterances related to a common theme, which indicated the broad features of the discussion. The codes were also used as filters to focus on specific aspects of interest and to validate the claims made. In the discussion of the results below, arguments are supported using extracts from the transcript guided by the coding scheme.

For the analysis of the lessons observed, an elaborate coding system was not used. Transcripts and researcher's notes were reviewed with a focus on the meanings and representations that emerged in the lesson. Attention was paid to tasks used by teachers for discussing meaning of integers and contexts, comparison of two negative integers and addition and subtraction of integers. The review was guided by the detailed analysis of the workshop discussions within the framework of meanings and representations. Episodes where the teachers adopted resources developed in the workshop were focused and analyzed in detail. The analysis of the lessons taught provides excerpts and details taken from this analysis.

7.3 Framework of integer meanings

7.3.1 Three components of integer meaning

Most mathematics teachers recognize the topic of integers as being difficult for learners. Also, the teaching and learning of integers has been a topic of research for several decades. Researchers have developed frameworks to understand student difficulties and guide teaching approaches. The review and the summary of research presented here is taken from Kumar, Subramaniam and Naik (2015b). Some frameworks emphasize the symbolic aspect, namely the "meaning" of the negative (and the positive) sign, while others emphasize the "meaning" of signed numbers or integers. Vlassis (2004, 2008), adopting a Vygotskian perspective, emphasizes the symbolic aspect and focuses on the multiple meanings or uses of the minus sign. She lists three uses: *the unary, the binary* and *the symmetric* functions. In the unary function, the minus sign is attached to a number to form a negative number, as in "–6". The second use, relates to the use of the minus sign for the binary operation of subtraction in arithmetic or algebra as, for example in "5 – 3". The first two uses identified by Vlassis correspond to the distinction emphasized in other studies between the use of the minus sign to indicate a signed or directed number as opposed to the use of the minus sign to indicate the subtraction operation (Glaeser, 1981). The third use of the minus sign refers to the unary operation or function of taking the additive inverse of a number as, for example, in –(–6). This unary function is a symmetric function. This sense is less frequently emphasized in other studies. It is noted that "taking the inverse" is of particular importance in the context of letter numbers and algebra. When an expression such as "– x + 3" is supposed to be evaluated for x = –3, the symmetric function interpretation of the "–" symbol comes to the fore.

As distinct from the meaning of the minus sign, frameworks developed by other researchers focus on the meaning of signed numbers or integers. Researchers commonly distinguish between the interpretation of a signed number as a property or characteristic of an object and as a transformation or change (Thompson & Dreyfus, 1988). Vergnaud (1982) uses a three–way distinction in the meaning of a signed number, as *state, transformation* and *static relationship*. As state, an integer may, for example, specify the ambient temperature. The change or transformation in temperature from hour to hour may also be represented by an integer. One may also use integers to represent the temperature of one place relative to another to indicate how much hotter or colder it is – a static relation. The operations of integer addition and subtraction may represent contexts of *combine, change* or *compare*. One may combine positive and negative scores on a test to obtain a net score. One may use the subtraction operation to find the change in temperature, or to compare two temperatures. It may also be noted that a change or relation may be represented both by an operation and an integer. For example, if the temperature fell from 30^{0}C to 16^{0}C in a few hours, then the change may be represented by the operation "16–30" or the result of the operation, "–14^{0}C". Thus, while the state may be represented by integers, change and the relation may be represented by both integers and the addition/subtraction operations. The analysis of the three components of meaning – of the minus sign, of signed numbers and of the addition and subtraction operations – complement one another to constitute a more complete account of the meaning of integers.

7.3.2 Contexts and models used in teaching integers

Besides the meaning of integers, research studies have explored the use of *contexts* and *models* for the teaching of integers. Contexts refer to situations that may be real or realistic (sufficiently real to the students) which involve the use of signed numbers and operations. These may, for example, be about profit and loss, assets and debts, height above and below sea level, or people entering and leaving a bus (for a review, see Schwarz, Kohn & Resnick, 1994). In contrast to realistic contexts, models are more formal in nature, and are thought of as "a way to support students organizing their thinking that can be modeled/inscribed in the form of physical tools and symbols" (Stephan & Akyuz, 2012, p. 431). Based on their review of several studies, Stephan and Akyuz categorize the models used to represent integers as neutralization or as number line models. In the neutralization model, there are positive and negative quantities and cancellation is a salient operation. Contexts to which the neutralization model readily applies are positive and negative electric charges, or assets and debts. The number line model makes the order aspect more salient. Contexts such as height above and below sea level, floors in a building, are examples where the number line model readily applies. One of the characteristics of the number line model is that it does not readily make sense to add two states, that is, two points on the number line (such as two floor numbers), while subtraction can be readily interpreted (as, for example, the directed distance between two floors).

A deeper examination of a context may reveal the relevance of multiple models. For example, debts and assets seem to be best described by the neutralization model since a debt and an asset of equal value cancel one another. However, combining assets and debts makes sense only in relation to the notion of "net worth", which is the sum of the assets and debts taken with their proper sign. "Net worth" is a state variable and fits more closely with a number line model, where each distinct state represents a point on the number line. It does not make any sense to add two points on the number line (two net worths) unless one changes the context to one where two entities with different net worths are merged. Analogously, in the context of electric charges, while equal positive and negative charges cancel one another, combining charges only makes sense in relation to a notion of "net charge", which is closer to a number line model.

The various contexts that have been explored by researchers or have appeared in instructional materials have varying instructional possibilities and potential (Schwarz, Kohn & Resnick, 1994). It needs to be emphasized however that a context typically packs in more mathematical possibilities than may appear at first sight. In other words, it may allow multiple interpretations of signed quantities and the application of both, the number line or the neutralization model. The example of temperatures at different times of the day has been mentioned earlier, where temperature as well as a change in temperature can be represented using integers. Here, the salient quantity, namely, ambient temperature is always positive under tropical conditions. But the context allows one to speak of a "derived" quantity, namely, change in temperature, which may be positive or negative. Allowing for the possibility of defining such derived quantities makes the contexts richer in mathematical meaning, and integers have a role in expressing and computing with such derived quantities.

The framework outlined above consisting of the meaning of the minus sign, integers and integer operations, together with the formal models of the number line and neutralization is illustrated in Table 7.3. It is claimed that the framework points to critical constituents of the *specialized content knowledge* (SCK) needed to teach the topic of integers. See section 2.4.2.1 from chapter 2 for a discussion on mathematical knowledge for teaching and SCK. I hope to show through the analysis of the interaction among teachers in the study that the framework is useful in supporting teachers' construction and exploration of contexts. The framework allows teachers to identify derived quantities in situations and to deepen the mathematical meaning embodied in them. As teachers explore contexts and models, they build the repertoire of representations that is accessible to them while teaching the topic of integers. From the evidence presented in this chapter, it is argued further that, teachers see such knowledge as relevant, important and interesting. They explicitly utilize the framework that was outlined based on research and the teaching experiences of the research group, in constructing, exploring and extending the knowledge making and using representations.

Table 7.3: Specialized Content Knowledge framework for integers

Meaning of the negative sign	**Meaning of integers**	**Meaning of addition subtraction of integers**
Unary function, Binary function, Symmetric function	State, Change, Relation	Combine, Change, Compare
Models: Number line models / neutralization models		
Contexts: Eliciting salient quantities and derived quantities		

7.3.3 Pedagogical representations for teaching integers

Pape and Tchoshanov (2001, p.120) state that "representation is an inherently social activity" in the sense that the process of representing allows learners to construct, interpret and communicate both internal and external representations in individual and social activity so as to develop shared meanings with others. The understanding and interpretation of representation thus get transformed in the process of communicating one's representation to others and negotiating meaning. There is recognition that representations are not "transparent" and require significant amount of negotiation amongst members to be able to support shared meanings (Cobb, Yackel & Wood, 1992). For example, it might not be easy for a person to see the "mapping" between a concrete model and an arithmetic operation without elaborating on and showing how they are connected (Pape & Tchoshanov, 2001). Several studies have identified the challenge that teachers face in transforming mathematical ideas into representations (Ball, 1990, 1992) thus pointing towards a possible knowledge gap in making "translations" between multiple representations. Bruner (1966) categorized representations into enactive, iconic and symbolic, while proponents of dual coding theory have categorized them as verbal or visual (Clark & Pavio, 1991). Clements (1999) has categorized representations as verbal, pictorial and symbolic. Lesh, Post and Behr (1987) categorized the representations into five types – manipulatives, models, pictures, written symbols, oral language and real world situations, while advocating development of students' ability to translate between representations for strengthening the concepts. For the purpose of this study, representations have been categorized for teaching integers into three types: *symbols, models and contexts* since these categories were implicit in teachers' talk. Symbolic representations include numeric and algebraic forms. Models can be visual representations like the number line or manipulative like tiles of two colors for integer addition and subtraction. As discussed above, for integers, two types of models are frequently referred to in the research literature – the number line model and the neutralization model (Stephan & Akyuz, 2012). Number line models represent integers arranged sequentially and operations of integers can be represented by movement on the line. In contrast, neutralization models, exemplified in manipulative like tiles of two colors, represent positive and negative integers as two distinct quantities. Combining equal positive and negative quantities results in neutralization represented as zero.

Contexts refer to realistic situations which illustrate the use of the relevant mathematical concepts. Contexts, models and symbols represent increasing degrees of abstractness. Contexts are concrete and will need identification and interpretation of essential properties of the mathematical concept to be a useful representation, whereas models illustrate the essential properties of the concept although they need not be transparent to students. Symbolic representations are more abstract and formal in nature than models or contexts. The structures and actions within the context or a model can be interpreted and represented as symbolic expressions. There can be many contexts which can correspond to a model and thus model is more abstract than contexts. For instance, cancellation of positive and negative electric charge corresponds to the neutralization model used for integer addition while a car traveling on a highway can be represented on a number line. However, according to Clements (1999) different representations chosen judiciously, should be taught in parallel rather than sequentially so as to enable students to make connections between them.

Through the analysis of data from discussions with teachers during the topic focused workshops, the implicit criteria used by teachers to select, design and evaluate representations have been made explicit, which include contexts, models and symbols. I call these as the criteria for determining representational adequacy, that is, the appropriateness of a representation for its use in teaching. I have identified three dimensions of representational adequacy which are *translatability, meaningfulness and consistency–coherence* and provide evidences for the application of these criteria by teachers. One may have a surface level concern for representational adequacy in applying one or more of these three criteria or one may have a deeper level concern for representational adequacy applying all three criteria in a coherent manner. A model of how surface level and a deeper level of representational adequacy may be reflected in teachers' use of the three criteria is illustrated in Table 7.4 below.

Translatability criterion refers to the feasibility (as perceived by the teachers) of translating one form of representation into another, for example, from symbolic form to a model. Translation is dependent on the feasibility of representing different facets of a concept in a representation, be it a symbol, a model or a context. These facets can be sequential order of positive and negative integers (ordinality), comparing values of integers (cardinality), meaning of zero as balance between positive and negative integers and as a reference point for relative position of positive and negative integers, and addition and subtraction of integers. The translation from one form of representation to another is possible for both representations of integers as well as operations of integers. Surface level concern for representational adequacy is reflected when translatability from one representation to the other is determined based on getting the same answers to a given problem. For example, solving a symbolically presented problem of ordering integers or operations with integers leads to the same answer as carrying out some manipulation in a model. When translation criteria are applied at the surface level, the manipulations are rule based and justifications and explanations are not explored. Frequently, the translations are carried out from symbolic representation to a model or context. A deeper level concern is reflected when structures and processes on representations have meanings consistent with the mathematical concept which is represented and the procedures performed using representations can be justified.

Using the criterion of meaningfulness requires one to acknowledge that symbols used in mathematics have meanings attached to them, which can range from one to several and are rooted in different situations that are mathematized. Kieren (1976) identified several sub–constructs for fractions that correspond to different meanings like part–whole, measure, share, operator and ratio which can be represented by fractions (See Chapter 6, Section 6.3). As indicated earlier, integers may similarly have diverse senses such as state, change and static relation in different contexts (Vergnaud, 1982). In the context of learning integers, several meanings need to be revisited to evaluate whether they can sensibly be represented using integers or integer operations. Such meanings may be distinct from meanings used in whole number arithmetic. For example, the meaning of zero as nothing left may not be as useful as the meaning of zero as representing a balance between positive and negative quantities. The operation of subtraction is predominantly represented as take away in earlier grades, which may not be meaningful while subtracting negative integers. The latter needs development of additional meanings for the operation of subtraction like comparison and adding the inverse. The meaning of addition as counting everything together has to be modified in the case of integers, where combining equal positive and negative quantities amounts to neutralization and thus zero. Considerations for representational adequacy remain at the surface level when these underlying meanings are not explored in the discourse and the representation is used merely as a vehicle to communicate procedures that can be performed using the representation. The representation with respect to its meaning is perhaps transparent to the person using it but this is not explicated to others in social activity. On the other hand, a deeper concern for meaningfulness acknowledges that meanings are not transparent in a representation and thus due consideration is paid to opportunities for exploration and negotiation of meaning. At the deeper level of discourse, meaningful connections are made between different translations and analysis of meaning is done for different representations discussed, including representations proposed by students. For example, one may state that addition of a positive integer leads to movement towards right on the number line as a rule. This statement by itself is procedural and not explanatory. In contrast, one may explain this rule saying that quantities increase by adding a positive integer (quantity) and integers are positioned on the number line in such a manner that the numbers increase as one moves towards the right. Thus addition of a positive integer indicates an increase in quantity.

The criterion of consistency–coherence refers to two types of consistency – mathematical consistency and consistency of meaning. Mathematical consistency indicates the consistent nature of mathematics and the way it is exhibited in the discourse while explaining mathematical manipulation with objects (e.g. whole numbers, integers) and operations (e.g. subtraction, addition). An example where mathematical consistency–coherence is not preserved is when one does not differentiate between the minus sign as referent of subtraction and as a referent of negative integer. When this reference is not made explicit, the equivalence between expressions like a – b and a + (–b) or a – (+b) is assumed rather than established. It indicates surface level concern for representational adequacy.

Consistency of meaning refers to how one applies considerations of meaning in the discourse around mathematical representations. Having different meanings for mathematical terms and concepts will lead to disruptions in discourse. In the analysis of teachers' talk, it wasfound that when students encounter negative integers, there is interference from certain beliefs that they may have developed due to experience with arithmetic like a bigger number cannot be subtracted from a smaller number. In the case of integers, one needs to either revisit these beliefs or provide an explanation to maintain consistency of meaning. For example, for subtraction of integers, teachers either need to use the compare meaning of subtraction which can be used consistently for whole numbers and integers or they may have to think of situations to show how a bigger quantity can be taken away from a smaller quantity. Still, it might be hard to find a situation to show taking away of negative numbers. A discourse that treats integer operation only symbolically and does not address these beliefs and meanings held by students will lead to disruption in consistency of meaning and thus indicates a surface level concern. When a teacher recognizes the meanings held by students and thinks of ways to build on those meanings or providing experience to develop new meanings, then the teacher is indicating a deeper concern for consistency of meaning. Concerns of consistency–coherence subsumes the knowledge and identification of meanings of integers and operations and in this sense one can show a deeper concern for consistency–coherence only if one already shows a deeper concern for meaningfulness.

The three criteria for representational adequacy, i.e., translatability, meaningfulness and consistency–coherence may be connected to each other in some discourses while they may not be connected in other discourses. It is possible to translate one representation to another without being consistent or meaningful but considerations of consistency do involve reflection on meaning and translatability. These criteria are more connected in discourse indicating deeper level concerns for representational adequacy than discourse motivated by surface level concerns. The characteristics listed for surface and deeper concerns for representational adequacy can be considered as a continuum rather than opposite categories where it is possible for a person to exhibit surface level concern at one point or for one representation and deeper concern at another time.

Table 7.4: Three criteria for determining surface level or deeper level representational ad-

equacy

Criteria for representational adequacy	Surface level application	Deeper level application
Translatability criterion	– Translatability determined by getting same answer using two different representations. – Steps in symbolic procedure correspond to some manipulation in the other representation. – Translation is rule based. – No need felt for justification.	– Translations are concept based. – Structures and processes can be mapped between symbolic and other representations in a meaningful manner. – Justifications are made explicit for why representation works.
Meaningfulness criterion	– Representation serves as a tool for communication. – Meanings are under-explored, may not even be explicitly discussed.	– Awareness of non–transparency of representations. – Awareness and recognition of range of meanings that correspond to the concept. – Connections are established with real/realistic context. – Representation serves as a tool for exploration of meaning. – Meaningful connections among translations.
Consistency criterion	– Equivalence between representations is assumed – Usage of symbols not consistent with meaning. – Meanings held by students are not explored or revised.	– Mathematically consistent – Equivalence between representations is established. – Consistency with meanings held by students or connecting new meanings with the old ones.

7.4 Textbook resources for teaching – paucity of reasoning opportunities

The Grade 6 chapter on integers in the textbook (NCERT, 2006b) was analyzed by the researcher to identify what it offered as resources for teaching and specifically for the opportunities to make meaning and reason with mathematics. The sequence of topics in the chapter comprises need for integers, ordering and comparison of integers, addition and finally subtraction of integers. Representations, examples, explanations, activities and exercises mentioned in the textbook for each of these topics are reviewed below.

Need for integers

The textbook introduces integers through the context of "borrowing". Negative integers were needed, the textbook points out, to differentiate the quantity borrowed from the quantity possessed. This is followed by activity of assigning signs to quantities in contexts followed by a brief discussion of the number system and place of integers in it.

Integer ordering and comparison

The number line and a realistic context of steps of a well is used to discuss integer ordering. The textbook observes that 7 is to the right of 4 on the number line since 7 is greater than 4. Similarly, it deduces that –3 is less than zero since zero is to the right of –3. The observation is then generalized to say that numbers decrease as one moves left and increase as one moves right on the number line. This is followed by problems where movement from one number to another on the number line is given in terms of moving *n* steps towards right/ left direction. The first exercise in the chapter has a mix of problems asking students to represent integers in contexts, placing integers on the number line and comparison of integers based on whether they are towards right or left on the number line. Context based problems are used to deduce the sign that should be assigned to an integer. Comparison of integers is not discussed using contexts but only either with or without the number line.

Addition of integers

Addition of integers is introduced by asking students to represent the movement on the stairs in the form of addition of integers. This is repeated using the horizontal number line and a game using two dice – one die with + and – signs and the other with numbers. This is followed by an activity based on the neutralization model with black and white buttons. Students are asked to observe the results of addition of like integers and then an observation is given in bold that "You add when you have two positive numbers like (+3) + (+2) = +5 [= 3+2]. You also add when you have two negative numbers, but the answer will take minus (–) sign like (–2) + (–1) = – (2+1) = –3." (NCERT, 2005, p. 125). The addition of positive and negative integers is demonstrated by asking students to remove black and white pairs since "(+1)+(–1) =0" (NCERT, 2005, p. 125). Again the observation is expressed that "When you have one positive and one negative integer, you must subtract but the answer will take the sign of the bigger integer" (NCERT, 2005, p. 125). Thus rules are stated as observations in the textbook. Addition on the number line is demonstrated through movement towards the right when integers are positive or towards left when they are negative. The problems discussed all had positive and negative integers with explicit signs enclosed in brackets with addition sign between integers. e.g. (–5) + (+3). After declaring that addition of a positive integer leads to increase and that of negative integer leads to decrease, the idea of additive inverse is discussed as the number which when added gives zero. This is followed by problems on how to find a number x more than number y using movement on a number line.

Subtraction of integers

Subtraction of integers is first discussed using the number line where it is reasoned why the answer to a problem 6 – (–2) will be 8 as it cannot be same as the answer of problem 6 – 2, which is 4. The textbook observes that subtraction of a negative integer leads to a greater integer as the answer. After a few more examples it is declared that "To subtract an integer from another integer it is enough to add the additive inverse of the integer that is being subtracted" (NCERT, 2005, p. 130). The exercise that follows consists of only numerical problems for which no explicit method has been suggested to be used by students. At the end of the chapter, a summary of points discussed in the chapter along with rules for addition and subtraction is given.

Thus explanations for addition and subtraction were based on models (neutralization and number line models), with a limited justification as discussed above for procedures or actions performed on them. The procedure for solving integer addition and subtraction problems using models was explained in the textbook through examples. In the exercises, students were explicitly asked to solve problems using the number line. Students were expected to move from using the number line model to find answers to addition or subtraction problems to finding answers "without the use of number line" (NCERT, 2005, p. 131). This implied use of rules for addition or subtraction of integers. The rules are procedural, like for adding a negative and a positive integer, one has to subtract and put the sign of the bigger number. The horizontal number line and numerical expressions were the most dominant representations used in the chapter, except for two examples briefly described in the text where the vertical number line was used along with a context (water level in a well; going up and down the stairs). The tasks did not require students to study the connection between a context and a model. Tasks expecting students to use representations like the number line were also procedural in nature since students were expected to follow the worked out examples given in the textbook. All the exercise tasks of the chapter were closed ended having a single correct answer while only two tasks expected students to engage in reasoning by evaluating statements or generalizations about integers and giving examples to justify their reasoning.

As discussed below, the analysis indicated that teachers' goals and use of representations, tasks and assessment were determined by what was given in the textbook since parallels existed between goals and approaches discussed by teachers in the initial workshop meetings and the goals and approaches given in the textbook. The teachers justified not engaging with alternative approaches in professional development by citing that they are not included in the textbook, which further indicates that teachers identified themselves as followers of the textbook.

7.5 Teachers' goals and concerns for teaching integers

Teachers in the initial phase (Phase 1) identified issues and concerns that they faced in teaching integers. In the discussion, the teacher educator's prompts included asking teachers about student difficulties and errors, asking them for the students' thinking underlying these, and how teachers addressed them in their teaching. Further inputs and discussions evolved on the basis of what teachers said. The predominant concerns that the teachers shared were about addressing student errors of integer computation and providing clear explanations using representations so that students get the correct answer.

The discussion indicated that teachers wanted to avoid student errors through strengthening remembering and recall of rules and procedures. This points to the goals that they considered important for teaching of integers as well as beliefs held by them about teaching and learning of mathematics. The major goals for teaching integers seemed to be memorization of rules for operation of integers, computational fluency, error free performance of students and "covering" the textbook in teaching (Kumar & Subramaniam, 2012a).

7.5.1 Teachers' concerns and issues of meaning

In the initial discussions, teachers discussed several student errors and underlying causes that were important to address. Most of the students' difficulties that teachers identified had to do with the integer operations of addition and subtraction using symbolic expressions. (Multiplication and division operations were not discussed since they were not included in the Grade 6 curriculum).

The teachers did not always explicitly connect students' difficulties with difficulties about the meaning of integers or integer operations. Initially, when the teacher educators suggested that students' difficulty with operations could be because they did not understand the meaning of integers, the teachers responded that students did not have a problem with the "meaning of integers". However, as pointed out below, several issues concerning the learning of integer addition and subtraction led to an exploration of underlying issues about the meaning of integers. Excerpts from the Topic Focused Group workshops are discussed, where the teachers raised issues centered around the teaching of procedures for integer operations using symbolic representations or formal models. The teachers' concerns were about addressing student errors and developing meaningful explanations using representations. Through an interpretation of excerpts from the discussion, it isshown how issues of meaning underlie teachers' concerns. Further, it is shown that teachers' SCK about integers was limited in terms of awareness of meanings associated with integers and operations, awareness of the distinction between minus as sign of integer and that of the subtraction operation, challenges faced in giving meaningful explanations for procedures on representations, and lack of knowledge of conventions of representations. In the sub-sections below, the framework of meanings outlined earlier is used to specifically focus on the distinct meanings of the minus sign, the meaning associated with the subtraction operation, and the conflict between focusing on rules and focusing on meaning.

The following excerpt 7.1 reports an exchange about a student error that occurred at the very beginning of the discussion on Day 1, and is indicative of the kinds of concerns that teachers had about the teaching of integers. The numerical code in the bracket indicates day and session number followed by the serial number of the utterance (turn). Thus "1.1; 17" indicates that the utterance is from Day 1, Session 1 and is number 17 in the sequence of turns in the discussion.

Excerpt 7.1

> Swati: If we write 7–6 they will say 1. If we say –7+6 then they will make 13, they may put negative sign... (1.1; 17)
>
> Anita: For subtraction they have to first convert it into addition, which children forget to do... Using buttons – that [subtraction] is not there, only addition is possible not subtraction... when subtraction problem comes they make this error (1.1; 18)
>
> Swati: Concept of subtraction-addition becomes confusing because both places they have this negative sign. In addition also… negative sign is there for integers. (1.1; 19)
>
> Rajni: They do *ulta* (reverse). (1.1; 20)
>
> Anita: They should remember, no? Though they know it very well.… But when

they have to do it they are in a hurry and they forget and make error. (1.1; 21)

(Workshop Excerpt, 30–7– 2010)

In the five consecutive turns quoted above, teachers are citing examples one after another, rather than responding to one another. Each of the three teachers Swati, Anita and Rajni identifies an error or offers an explanation for errors made by students. Swati identifies the error known as the detachment of the minus sign (–7+6 = –13) (Linchevski & Livneh, 1999), and follows it up with her explanation of the underlying reason why students make this error in 1.1; 19. Anita identifies the subtraction operation as difficult because students forget to convert it into addition. She follows this up with an explanation – students respond well to the two-color button (neutralization) model, but she thinks that the model applies only to addition and not to subtraction. Rajni identifies the problem of students' incorrectly reversing the order in subtraction as a problem that teachers need to address. Each of the error patterns identified in this brief excerpt connects to discussion threads that were about issues of meaning. These discussion threads were specifically about the meaning of the minus sign and the meaning of the subtraction operation, which is discussed below.

7.5.2 Distinct meanings of the minus sign

The discussion thread related to the distinct meanings of the minus sign shows a growing realization among teachers of the importance of the distinction between the sign for a negative integer and the sign for the subtraction operation. This realization is part of a general movement towards sensitivity to issues of meaning. The exchange in Excerpt 7.1 suggests that teachers were aware of the common errors that students generally make while computing with integers. They described student errors making reference only to the procedures that students need to use and did not think that the errors were due to the kinds of meaning that students made of the expressions. In Turns 1.1; 18 and 1.1; 21, Anita attributes students' errors to their forgetting rules or procedures. This was a frequently invoked explanation of students' mistakes. For example, Rajni, discussing the error cited by Swati in Turn 1.1; 17 above, remarked, "How much ever example we give... they should keep in mind –7 and +6, that minus they will forget" (1.1; 64). Swati, however, offers a different explanation for students' difficulty with integer operations in Turn 1.1; 19. She is pointing to the fact that students may be confused at seeing the minus sign in addition problems, which is not the case for whole numbers. She appears to suggest that the minus sign has a different meaning in the middle grades that students need to internalize, while in earlier grades they are only accustomed to interpreting the minus sign as indicating the subtraction operation. This indicated awareness of the challenge faced by students in distinguishing sign of minus as operation and integer but was articulated through an example rather than as a general difficulty faced by students.

The teachers' initial explanations for procedures associated with specific representations that they used in the classroom did not exhibit a distinction between minus as sign of operation and as integer, leading to possible misinterpretations. In session 1.2 during a discussion about modeling subtraction of integers on the number line, a teacher suggested that while moving on the number line, one must reverse the direction of movement on encountering a minus sign. This would ensure that for 3 – (–4), one starts from 3 and moves correctly towards the right. This was challenged with the example of –3 – 4, where moving from zero to –3 and then reversing the direction would lead to an incorrect answer. The teacher educator then made a suggestion that "You have to make a distinction between minus as the operation sign and minus as negative sign. (1.2; 68)".

This distinction was readily appreciated by the teachers and Ajay applied it to explain the difference between the minus sign in "4 – 2" and "– 4 + 4", as seen in the following excerpt.

Excerpt 7.2

Ajay: Here the meaning of – 2 is different.... The first minus [i.e., in "4 – 2"] is operation and the second [i.e., in "– 4 + 4"] is number (1.2; 93)

Rajni: Second is quantity.

Ajay: –4 is one number.... The first is a sign of operation, but the second is not operation.

Swati: Sir, this is clear to us, but for a 6th Grade child, this operation sign – he will not understand [the distinction?].

Ajay: This is the point that we have to explain in a simple way.

Rajni: We have to differentiate the two...

Swati: They don't know...

Rajni: Yes, they don't know that.

TE2: For them, probably all the signs mean operation.

Swati: It is the same for them, all [the signs] are addition, subtraction. (1.2; 102)

(Workshop Excerpt, 30–7– 2010)

The teachers thus acknowledged that students may not understand the distinction between minus as sign of operation and sign of integer and that hence this is an important issue to be dealt with in teaching. The discussion shows teachers thinking about the meanings of the minus sign held by students vis-a-vis the meanings invoked by teachers while explaining the procedures for integer operations using representations. However, at this point, the discussion of meanings was centered around symbolic expressions and teachers attempted to identify the features of an expression that can indicate whether the sign denotes operation or integer. Teachers attempted to articulate criteria using which they themselves identify the sign as indicating integer or operation. One suggestion was that "+" sign always meant operation, since the "+" sign is generally implicit for the positive integers. Another criterion suggested was that whenever two signs appear in succession, the first is the sign of the operation. The issue was not satisfactorily resolved at this point.

In the discussion above, one sees the teachers moving from an implicit recognition by one of them (Swati) of the distinct meanings of the "–" sign to explicit acknowledgement of the distinction and its importance, and to attempts to articulate criteria for identifying the distinct meanings in a symbolic expression. There is however, a flip side of the distinction, which is the question, "why is the same sign used for two distinct meanings, rather than two distinct signs?" This question surfaced when the teachers were discussing tasks for children around a shopping mall with floors above and below the ground (See Figure 7.1). One of the tasks was to ask students to number the floors in the mall, and it was expected that students would number floors below the ground starting with "–1". However, Ajay raised a question – why would students accept using the minus sign here, which they understand as the sign for subtraction. This led to an interesting discussion on the advantages and disadvantages of numbering basement floors with the letter "B" as opposed to the minus sign. A satisfactory resolution of this question calls for an explanation of why the "–" sign is used to denote both a "negative" quantity and the subtraction operation. This explanation emerged in a subsequent discussion in a specific context, which are described later. At this point the teachers thought of the meaning of the sign together with the meaning of the integer.

In moving towards acknowledging the importance of the distinct meanings of the minus sign, the teachers' discourse moves towards greater mathematical consistency of representations. Similarly, the discussion on why the minus sign is used to indicate basement floors reflects a concern for coherence of the meaning of representations. Both of these mark significant moments in the shift towards a deeper concern for representational adequacy.

7.5.3 Meaning of the subtraction operation

The meaning most commonly applied to subtraction of whole numbers is "taking away" a smaller quantity from a larger quantity. However, the "take-away" meaning has to be re-contextualized for understanding subtraction of integers, which may involve adding zero-pairs (+1–1) to preserve the value of minuend, before "taking away" the subtrahend. Other meanings of subtraction like difference or comparison can be used consistently across whole numbers and integers. The discussion on student errors led to teachers realizing that the issue of the meaning of the subtraction operation is important for the teaching of integers.

Returning to Excerpt 7.1, one notes that in Turn 1.1;20 Rajni is pointing to the order error in subtraction made by students, which she elaborated later: "What should be subtracted from what... that is the main mistake they [students] are making. Subtract 7 from 3, they will do 7–3.... That order only we have to teach... so many examples we should give" (2.1; 1). In Rajni's view, students did 7–3 instead of 3–7 because they had difficulty interpreting the English sentence "Subtract 7 from 3". She thought that the error of forgetting to reverse the order needed to be addressed while teaching and she mentioned it several times in the course of the discussion. A discussion of this error took place in Session 1 of Day 2, which was a combined session with primary and middle school teachers. A teacher educator (TE1) suggested a different explanation for the error by asking if students would make the order error for "subtract 2 from 5". Most teachers agreed that they would not do this because they were familiar with 5–2, which is taking away 2 from 5. However, they have a problem with taking away 7 from 3 or 3–7. As a primary teacher said, "In the primary classes, we fill it in [their] mind that you cannot take away 5 from 2. How can you take away? You can never take away 5 from 2." (2.1; 28) Teachers thus confronted the need for students to understand what "taking away" a bigger number from a smaller number actually means.

In Excerpt 7.1, Turn 1.1;18, Anita refers to the use of the two-color button neutralization model for the addition of integers. This model is discussed in the textbook and also taught by the teachers, but only for the addition operation and not for the subtraction operation. Anita apparently believed that the two-color button model does not work for subtraction. She thought that this might account for why students found the subtraction of integers especially difficult. TE1 took the opportunity to model the subtraction operation using two-color buttons, using the meaning of subtraction as "take away". Problems such as 7–3 and –6–(–3) are relatively straightforward since one can take away three positive buttons in the first case and three negative buttons in the second. Problematic cases are handled by introducing "zero pairs", that is, a pair of buttons with opposite colors. Thus 6–(–3) could be modeled as follows: introduce 3 "zero pairs" (i.e., 3 positive and 3 negative buttons) to the 6 positive buttons already laid out on the board. This does not change the value of the set of buttons on the board. Now take away 3 negative buttons, which leaves 9 positive buttons. This explanation was new to the teachers and led them to believe that the two-color button model could be used for subtraction.

Although the teachers thought that the two-color button (or card) model was useful, they did raise issues about the meaningfulness of specific actions carried out using this model. Thus, while discussing how addition of integers works with the two-color button model, Swati raised the question of why a red card and a black card cancel each other: "We say it is zero … it is +1 and –1 we know. [But] they start counting all of them… Here they can see +1 and –1 [but] they make it 2, they don't consider it zero." (1.1; 132,134)

Rajni had a different way of using the two-color buttons to model subtraction, which she explained in Session 2.1. She started with the rule that subtraction problems can be changed into addition problems by replacing the subtrahend with its additive inverse (or "opposite"). Thus, 3–4 could be rewritten as 3+(–4), and could be solved by adding four black cards to 3 red cards giving the result –1. Similarly for the problem 3–(–4), one could rewrite it as 3+4 and add 4 red cards to three red cards to get 7. However, both Swati and Anita objected saying that the problem was being changed. In the quote below, Anita expresses her dissatisfaction with suggested ways of modeling subtraction, both that of changing the color of the buttons, and that of adding zero-pairs.

> Anita: But in this method I feel you are manipulating the question... change the color... for zero you add 2 buttons... so you have changed the question... (2.1; 79)

Rajni had used the meaning of integer as opposite to explain the subtraction using two color buttons but it was not convincing to the other teachers. The discomfort seems to stem from modeling the procedure using two colors to get a correct answer without delving into explanation of why subtraction of an integer is equivalent to addition of its inverse. Also, the procedure for subtraction in the case of integers is not consistent with subtraction of whole numbers needing an explanation for why the procedure needs to be modified. The group however, did not go deeper into discussion about meanings connected with this representation.

The teachers were initially satisfied with the teacher educator's presentation of how the neutralization model can be used to represent the subtraction operation. This acceptance can be interpreted as based on the criterion of translatability. However, Swati and Anita raised questions of meaningfulness and consistency of the mathematical processes associated with this model pushing the discourse towards addressing deeper concerns of representational adequacy. The alternative procedure for subtraction using the neutralization model suggested by Rajni is also similarly questioned because it remains at the surface level of mere translatability. The negotiation of the meanings associated with representations and an examination of their their consistency-coherence leads teachers towards a more critical stance concerning the adequacy of representations.

7.5.4 Teaching rules versus teaching with representations

Teachers shared the representations that they used for teaching integers in Phase 1. Teachers' talk about representations made explicit their beliefs about teaching integers, their preferences for use of specific kinds of representations, the meanings used implicitly to identify features that can be represented by integers and the perceived role of rules in the teaching of integers. Besides the use of the formal two-colored button model to teach addition, the teachers used the formal number line model to teach integer addition and subtraction. In this case, addition and subtraction were typically interpreted in terms of movement on the number line, at times involving seemingly arbitrary rules about reversing the direction of the movement when encountering a minus sign. Teachers and teacher educators expressed the view that the rules seemed arbitrary and did not seem meaningful. Despite these reservations, teachers had almost exclusively used only formal models, and had not used contexts, to teach the operations of integer addition and subtraction.

Teachers mentioned contexts involving the use of integers while teaching, but only while introducing integers. The teachers made a distinction, between a sub-topic that they referred to as "need for integers" and the sub-topic of "operations with integers". This distinction is made in the textbook and the teachers were being faithful to it. They believed that contexts were useful to introduce students to the "need for integers", but were not very useful in teaching operations with integers. While introducing integers (also referred to as "giving the concept of integers"), teachers explained the meaning of positive and negative numbers by referring to contexts involving opposite quantities such as increase-decrease, depth-height and above-below. However, they generally did not draw on meanings or contexts while discussing integer operations.

Although the teachers used the two formal models of colored buttons and the number line to teach integer operations, the models were thought to be useful only initially, when the numbers dealt with were "small numbers". The students needed to eventually learn to operate with big numbers, and teachers felt that for this they needed to know the rules. All the teachers acknowledged that they explicitly teach the rules for operations with integers, but they differed in the importance they gave to rules. For Ajay, the main aim of the chapter was to know the "laws of integers" which are needed to solve problems with bigger numbers efficiently. However, Swati thought that students find it difficult to remember all the rules. Anita felt that rather than teaching rules explicitly, one should let students construct rules as they worked with models such as the two-colored button neutralization model. The excerpt 7.3 below reflects these tensions.

Excerpt 7.3

> Ajay: I think that when we give the concept of button, same time we can give concept of rule. (1.1; 83)
>
> Anita: No, why don't we make rules through buttons – if white color is more then it is positive... this rule can be constructed. (1.1; 85)
>
> Rajni: We are calling them for buttons, ultimately we should tell them rules otherwise big numbers they will face problems. Here only they are making mistakes. (1.1; 86)
>
> Ajay: The main thing that you have to tell in integer is rules. If they understand rules then they can do everything. (1.1; 87)
>
> Swati: They are not able to remember [rules] that you know. Every time that is the problem. (1.1; 88) (Workshop Excerpt, 30–7– 2010)

In thehe excerpt 7.3 above, talk about remembering or forgetting rules stands in contrast to and in tension with talk about understanding the meaning of symbols and operations. Teachers' talk about use of representations indicated that they preferred rules and symbolic representations over using contexts and models for teaching integers. This is perhaps because of their belief that representations like contexts and models are only useful for small numbers and for introducing students to integers but are not useful in developing fluency in computation using integers.

Teachers' talk in Phase 1 reflected their concerns about teaching integers, but indicated gaps in their SCK in terms of a limited repertoire of representations, limited knowledge of meanings which can be attributed to integers, signs and operations, lack of distinction between these meanings in their discourse, and inadequate explanations of procedures using representations. These knowledge gaps constrained teachers' understanding of student errors and the conceptual shifts needed in moving from whole numbers to integers. However, even in Phase 1, the discussions around student errors and the models used for teaching integers extended teachers' SCK in important ways. Two elements of the teachers' construction of SCK were (i) awareness of the distinction between the use of the minus sign for integer and for operation and of the importance of this distinction and (ii) awareness of the meaning of the addition and subtraction operations as applied to the neutralization model. These constructions occurred in the context of identifying the challenges faced by students in working with symbolic expressions or with the neutralization model. The teachers discussion also indicates a shift in the criteria for evaluating the representations that the teachers had been using, from surface level concerns to deeper concerns related to meaningfulness and consistency.

Further, the teachers' reliance on rules and formal models as the preferred representations to teach integer operations closely followed the textbook. Discussions on the meaning of integers was restricted to the introductory topic of "need for integers", again closely following the textbook. Ajay and Rajni were even skeptical of the usefulness of models and preferred to emphasize rules applied to addition and subtraction problems presented purely symbolically. They possibly represent a position that is even more emphatically focused on symbolic computation than the textbook or the other teachers.

7.6 Teachers' engagement with meaning of integers and operations

In the second phase (Days 2 and 3) the teachers systematically engaged with the meaning of integers through tasks that called for proposing contexts or interpreting contexts using the framework of meanings of integers and integer operations. The teachers were exposed to the integer meanings of state, change and relation through worksheet tasks (see Appendix 5). A large number of contexts were discussed, integer meanings explored and judgment was made about their pedagogical usefulness. Table 7.5 presents contexts, which were discussed over 10 or more turns. Prompts by the teacher educators included questions about which integer meaning applied to a given context, whether the use of integers was appropriate, and which contexts were useful for the classroom. Of the large variety of contexts discussed, the teachers found the following contexts to be useful for the classroom: integer mall, change in day temperature, scores on tests, and change in a baby's weight. Discussion of the many contexts helped deepen teachers' SCK for teaching integers. This claim is supported with two kinds of evidence. One kind of evidence is derived from analysis of teachers' talk, which reveals the aspects of SCK that were extended and built upon during the course of the Topic Focused workshop discussions, using their initial talk as a frame of reference to chart their growth. For this, examples are described of teachers' selection, interpretation, design and evaluation of contexts and highlight discussions of the meaning of integers and of integer addition and subtraction. The second kind of evidence is the teachers' self report of what they have learnt, and the changes in their teaching as a result of participation in the study, which are discussed in the Section 7.7.

Table 7.5: Turns of teacher and teacher educators' talk while discussing contexts for teaching integers

Contexts discussed	Total number of turns	Turns of teachers' talk	Turns of teacher educators' talk
Integer mall – floors in building and movement of lift	211	89	122
Temperature	96	51	45
Marks/score	53	32	21
Baby's weight	51	26	25
Profit/ loss	44	24	20
Mixing water at different temperatures	30	16	14
Ticket reservation	29	16	13
Loan taking/giving	28	17	11
Family size	26	16	10
Queue	26	12	14
Length of shadow	24	16	8
Water level	22	9	13
Journey by train	20	12	8
Steps	19	11	9
Altitude – heights of different vehicles	15	7	8
Speed of car	11	7	4

The first sub-section below describes teachers' engagement with the use of integers to represent change and relation, which marks the important shift from the use of integers to represent only state. The second subsection describes their attempts to interpret the addition and subtraction operations in contexts, which marks the next important shift from the exclusive use of formal models to also using context based representations to teach integer addition and subtraction.

7.6.1 Integers as representing change and relation

Most of the initial examples given by the teachers of contexts for teaching integers were of state, which became apparent to the teachers when they became explicitly aware of the alternative senses of change and relation that integers may denote. The teachers used integers to represent states in contexts that might be familiar to students, such as temperature, profit and loss, height above and below ground level, or position of floors in a building above the ground or in the basement. In these examples, the positive and negative integers were associated with opposite states like above-below, profit-loss, increase-decrease, etc. However there were occasions where they used integers to inappropriately represent "opposites" like number of boys and girls, number of children sitting and standing, without offering an interpretation of what canceling positive and negative quantities might mean in these contexts. Over the course of the workshops, the variability in the meanings represented with integers increased along with the awareness of different meanings. Teachers gave examples of contexts for representing integers; they evaluated and challenged examples, and proposed and designed activities for teaching based on the contexts that they had discussed.

7.6.1.1 Using integers to represent change

It was challenging for the teachers initially to identify features of representations that correspond to the change meaning of integers. The meaning of change was introduced when, during the initial discussion, Swati asked how one might explain that +1 and –1 cancel to give zero (1.1;127).

Excerpt 7.4

Swati: This concept zero, Why it becomes zero? [How] to explain?

TE1:Why it becomes zero? This is a good point. What do we mean by +1 and –1 or –2 and +2? I think of these as increase or decrease.

Rajni: Or take and give – earn. 2 rupees you give to somebody how much it is remaining

TE1: Take and give, when I take it increases, when I give it decreases. When I have +3, it increases the value. When I have –2, it decreases the value. So when I take +1 it increases by 1. [so] When I take –1, it decrease by 1, and so increase by 1 and decrease by 1 can be cancelled. So that is why you can say...
Rajni: Starting point only you are reaching

Swati: No, that is all right. Here [with colored buttons] they can see +1 and –1. They make it 2. They do not consider it zero.

TE1: That is right. +1 and –1,… This is a convention.
Swati: We say it is zero. It is +1 and –1. We know, [but] they start counting all of them.

TE1: Which is why we have 2 different colors. Why do we have 2 different colors? Because 1 represents increase, [so] positive. Other represents decrease, [so] negative, that is very important. That is the first idea they should get.
(1.1, 127 to 135, Workshop Excerpt, 30–7–2010)

TE1 suggested that one may interpret +1 as an increase of 1 and –1 as a decrease of 1, so they together cancel each other resulting in no change. However, initially teachers preferred to represent change only with operation rather than also as an integer. An engagement with this issue is reflected in Rajni's attempts to incorporate the change meaning into her explanation of addition and subtraction on the number line as movement. She now interpreted movement to the right as increase, and to the left as decrease. However, her association of the "increase-decrease" was still with the operations of addition or subtraction rather than with the positive and negative integers.

Excerpt 7.5

Rajni: We are telling them that add means right and subtract means left. Then they will ask 3+ (–4).... So we will go where? 3+ (–4)... it is equal to left only actually but it is increasing. Plus means it is increasing but –4, so negative weight is increasing

(1.2; 21, 25, Workshop Excerpt, 30–7–2010))

In this excerpt, Rajni is still connecting increase to the operation of addition, while the negative integer represents a state ("negative weight"). Teachers' resistance to the use of unary integer to represent change signals a limitation in underlying beliefs about what features of representation were appropriate to represent with integers.

In contrast to Rajni's seeming resistance to using integers to represent change, Anita designed a context to show addition of integers, where the integers represented change. She proposed a context where stones are added or removed from a bowl containing an unknown number of stones. The number of stones added and removed was represented using integers.

Excerpt 7.6

Anita:...already there is a collection of stones and you add some thing and by taking away these stones there will be decrease in the number of stones…. Like that similarly increase 2 and decrease 4, what is the result – like that. We will ask and get the number and convert these activities into mathematical form. We will see how they are going to write. There you can introduce minus 3.

(2.1; 116, Workshop Excerpt, 30–7–2010)

Here Anita has explicitly used unary integers to represent a transformation. Moreover, the context and the framing of the task make the change meaning salient, and the representation of a decrease using a negative integer meaningful. Similar contexts, where change is salient and is represented using integers, have been used by other researchers to support integer learning, for e.g., persons entering or leaving a discotheque (Linchevski & Williams, 1999). An explanation that Anita offered indicated how she understood integers as representing transformation in contrast to integers as representing state. She described her new idea as a recognition that "–2 is not always from zero". That is "–2" does not always mean 2 less than zero, but could mean 2 less than some arbitrary or unknown number. This prompted Swati to describe the unknown number as a "reference point" (2.1; 131). The idea of the reference point emerged as a key construct, which was useful in later discussions in distinguishing the meaning attributed to integer in different contexts. This as an important moment in the teachers' construction of SCK through an exploration of meanings and contexts.

Teachers appreciated the need to explicitly distinguish between the state and change senses of integers as indicated in the Swati's and Anita's comments below. Swati discusses how the meaning of state and change can be distinguished in the context of the integer mall (See Figure 7.1) as negative integers representing lower floors indicate state while the change is represented by integers depicting the movement of the elevator. Anita points out how in the context of representing change in water level in a tank the reference point will keep on changing since the initial level will be the reference point for determining the change denoted by the final level of the water in tank. She is able to reiterate her earlier argument that reference for negative integers is not always from zero by using the meaning of change in a context.

Excerpt 7.7

> Swati: Child has to understand the difference between the movement and the state because when we are writing numbers here –3 so he will say it is here [as basement floor], here when –3 is coming it is telling you that we [have moved] 3 floors down (4.2; 235)
>
> Anita:...Initial level can be the reference point always...let that be the reference point...initially 10 litre and then increase [represented by] + sign and decrease 10 litre [so] how much? (3.2; 335)
>
> (Workshop Excerpt, 10–8–2010, 30–8–2010)

This notion of reference point was initially proposed and taken up by Swati and Anita, however later Rajni and Ajay too understood the importance of this construct and used it in their discourse to distinguish different meanings of integers. In a session with peer teachers (to be discussed later), Rajni explained the significance of reference point to clarify how one determines the height (state) of an object using the sea level as a reference point.

Excerpt 7.8

> Rajni :...here in this question we have referred sea level for knowing the position of the bird and position of the diver... [if] simply we are telling 25 m above a bird is sitting on tree... so you don't know from where... from the ground from the depth of the sea or whatever we don't know that... so for referring this bird, sea level is the reference point... That is called as state…
>
> (Workshop Excerpt, 24–11–2010)

Swati, who had proposed the notion of a reference point, went on to use it in insightful ways. Here she uses the notion to distinguish the contexts where integers represent change from those where they represent state.

Excerpt 7.9

> Swati: [State is] where the reference point is not changing.... When we talk about change, reference point is changing every time. [In the context of change in profit from day to day].... We are comparing today with tomorrow and that day with the next day. So reference point is always changing. (2.2; 407)
>
> (Workshop Excerpt, 9–08–2010)

Using the meaning of integers as change, the teachers were also able to identify additional features in contexts that could be represented by integers. For example, an initial context constructed by the teachers to represent change using integers was that of test scores. Ajay had suggested the inclusion of negative points for a wrong answer as an example of using integers – one has to combine (add) the positive and negative points to get a total score. In a later discussion, Anita suggested modifying the task to record only the change in the test score of a student over a series of tests. The change would be positive or negative taking the score on the previous test as the reference point and the change can be combined (combine-change) to show the student's progress over time. Similarly, teachers discussed the contexts of weekly change in a baby's weight and hourly change in ambient temperature, for which they designed tasks of representing change using integers. (Note that in tropical conditions, there is no possibility of day temperatures falling below 0 degree Celsius.)

Excerpt 7.10

> (Following a discussion about weekly change in baby's weight.)
>
> TE1: Similarly [Any] other situation... you can think of which will be interesting to students?
>
> Swati: Change in temperature from morning till night.
>
> TE1: That's very nice.

Anita: Different time interval... 9 am [temperature increases from then on] then again in evening it decreases.

TE1: So we can give... rather than temperature you can give the change in temperature.

(2.2; 167-171, Workshop Excerpt, 9–08–2010)

In the task for students developed by teachers involving representing weekly change in weight of a newborn baby, they asked students to give reasons for an unexpected event (such as a negative change in the baby's weight) as it may help in meaning making.

Excerpt 7.11

TE1 (reading the task written by teachers): Represent the above data (i.e., change in baby's weight) as integers… What could be the reason for 200 gm. decrease in first week? What could be the reason for 500 gm. increase in 6th week? How much does the baby weigh in the second week?.... In which week did the child gain maximum weight? What is the total weight gain or loss in the first month?

(3.3; 254, Workshop Excerpt, 9–08–2010)

The teachers were able to adopt the change meaning for identifying features of representation to be represented by integer. This was facilitated by the explicit distinction between the meanings of state and change and through considering the possibility of representing by a unary integer what was usually represented by an operation. Thus a distinction was made at two levels – between operation and integer and between two different meanings that can be attributed to an integer. However, as Swati pointed out in the remark quoted earlier, the two meanings of state and change are also related in that both signify being less than or more than a reference state. In the state meaning, the reference point is fixed and taken to be the zero point. In the change meaning, the reference is to a previous state that might be designated by an integer different from zero. Swati and Anita used the distinction between state and change meaning in exploring new contexts. In the process, they extended their SCK by extending the range of meanings accessed for thinking about integers, which in turn led to a greater variety of contexts and context features being represented by integers.

7.6.1.2 Using integers to represent relation

As in the case of representing change, teachers initially preferred to represent relations using the subtraction operation rather than using unary integers. The worksheet presented two situations to prompt thinking about the use of integers as relations: "Me and my sister are standing in a queue to buy ice-cream. How far is my sister from me?" The second situation referred to two persons standing on different floors of a building. Teachers represented these situations using the operation of subtraction (or incorrectly, using addition).

Excerpt 7.12

> Anita: Me and my sister standing in the queue. How far is the sister away from me, there we have to do subtraction.... In the second case we have to add... to find the distance between [floors].
>
> TE1: In the first case, suppose I say how far is the sister from me.
>
> Anita: Yes, then it is subtraction
>
> (2.2; 13-15, Workshop Excerpt, 9–08–2010)

Another situation described by TE1 during the discussion led to the teachers accepting the use of unary integers to represent a relation. He described an airplane in the air with its instruments displaying the relative altitude of other planes nearby – heights above the plane indicated with a positive sign, and below with a negative sign. This made the relation salient and meaningful. Other contexts were discussed to illustrate the use of integers to represent relations – depth in water, temperature, and counting years in different eras in different cultures, relative position of runners in a race, relative differences in test scores.

Swati, who had earlier connected the meaning of state and change through the construct of reference point, made a similar insightful comment about representing relations using integers:

Excerpt 7.13

> Meera is standing 3 position[s] ahead [of me] and Radha 7 position[s] behind me so Radha here, me here, and Meera here. I want to know where I am standing from the starting position. So if you don't know the starting point it is not possible to find the position from starting point. Here it is only relation you are taking.... Generally we take 0 as the reference point … but in this particular question 0 is not a reference point.... We have taken something [else] as reference with respect to it we are finding the position so it is a relation.
>
> (Workshop Excerpt, 30–08–2010)

Hence, in the teachers' interpretation, the meanings of state and relation are similar, with an important difference of reference point. The reference point for state is fixed by convention, while the reference point in the case of relation is arbitrary. Thus I see the construct of reference point being used to distinguish different meanings of integers.

Figure 7.1: The integer mall

One context, the "integer mall" with a lift (elevator) containing only buttons marked as "+" and "–", was worked on in some detail since it contained features corresponding to all the three meanings of integers, as well as the various senses of integer addition and subtraction. The integer mall (Bajaj & Kumar, 2012) is illustrated in Figure 7.1. Note that floor numbers correspond to a state meaning of integers, while instructions for movement of the elevator, in the form of number of presses of the "+" or "–" buttons correspond to the change meaning. The position of any floor in relation to a given reference floor provides the relation interpretation. Thus teachers were able to identify different meanings within the same context and differentiate them based on the generalization they made about critical features that contributed to the meaning.

Use of only state meaning to identify features to be represented by integers may lead to limited understanding of integers as exhibited by teachers in their initial discourse in workshops when they gave inadequate explanations for how and why procedures using representation work to give the correct answer. Knowledge and use of other meanings increase the variety and flexibility in use of contexts while aiding in building meaningful explanations for procedures associated with particular representations. Through exploration of contexts involving integers and thinking about the meaning of integers explicitly, teachers were able to identify the critical features of the context that can be represented by integers and thus differentiate between the two meanings of integer as state and change. This led to not only an increase in the number of contexts identified as useful for teaching integers but also increased potential of using contexts for meaning making.

7.6.2 Addition and subtraction of integers

After the exploration of contexts using the framework of meanings of integers and signs, teachers engaged in tasks of exploring contexts where it would be meaningful to represent addition and subtraction of integers. They constructed problems where integers could be combined (added), and which involved change and relation that could be represented using addition and subtraction. This provided opportunities for teachers to connect meanings of integers with meanings of operations, identify which contexts are meaningful to represent integer addition and subtraction, and establish connection between different representations through meanings.

The discussion of a range of situations led teachers to realize that not all situations contain features that correspond to integer addition and subtraction. An interesting context that was discussed was mixing water at two different temperatures – a situation suggested by one of the teachers. Through a discussion led by the teacher educator, teachers realized that the resultant temperature is not the sum, but a weighted average of the two initial temperatures. Temperature change, on the other hand, could be represented using positive and negative integers, and could meaningfully be added. This was the case for the hourly change in ambient temperature, another context suggested by teachers.

The integer mall context (Figure 2) contained a feature in the form of buttons on the elevator marked "+" and "–". Addition could be used to find where the elevator would stop after a certain combination of positive or negative button presses. It meant understanding that equal number of positive followed by negative button presses will take one to the same floor, i.e., no change. Teachers felt that this notion of no change should be explored both from the ground floor as well as any other positive or negative floor. This corresponds to the neutralization model, where two opposite changes neutralize resulting in no change.

Excerpt 7.14

> Swati: We can include problem of from any floor pressing plus twice and minus twice … same number "+" times and same number "–" times so it will come back to the same floor. So those +2 have cancelled –2. So no change.
>
> (4.2; 3, Workshop Excerpt, 30–08–2010)

Here Swati is drawing a connection between the integer mall context and the neutralization model by identifying that equal amount of upward and downward movement cancel one another. As noted earlier, she had asked how one can convincingly explain why +1 and –1 sum to zero. Using the change meaning of integers and the combine meaning of addition, she is able to devise a satisfactory explanation for why equal and opposite integers cancel to yield zero, illustrating an important mathematical idea. Thus change meaning helped the teacher in constructing this explanation by connecting it with features in the integer mall representation. The use of integers to represent opposite changes made sense, so did the cancellation of additive inverses to yield zero. Using the meaning of combining change, addition of integers was easy to grasp, and teachers felt that students will be able to complete the addition of integers without recourse to rules.

The teachers also pointed out that students will have to represent multiple presses of the "+" and "–" buttons using integers.

Excerpt 7.15

> Swati: Two times plus coming to +2, so we are forming integers (4.2; 7)
>
> Swati: Two plus [es] is +2 and one minus –1 (4.2; 29)
>
> Ajay: When you press ++– you actually press +2 –1... (4.2; 30)
>
> Ajay: This is the essence of this chapter. (4.2; 34)
>
> (Workshop Excerpt, 30–08–2010)

In the context of the integer mall, teachers discussed how finding the directed distance between two floors could be represented using subtraction. The directed distance could be interpreted as the movement required to go to the target floor from the starting floor. Students could verify this distance easily from a visual representation of the context, and could associate it with the subtraction operation. Thus "a – b" could be interpreted as the movement required to reach floor a from floor b. Here the subtraction operation corresponds to the "compare" sense and yields the distance between two floors, which could be interpreted as a required or actual change, or as a relation. The sign of the integer obtained as the result of subtraction could be further confirmed by the direction of movement. For e.g., if moving from –2 (basement floor) to 5th floor, the movement can be expressed as 5 – (–2); students could verify from the picture that the distance between the two floors is 7 and since the movement is upwards and would require pressing + button 7 times, the answer would be +7.

Excerpt 7.16

> TE3: So how will we phrase it so that the [corresponding] mathematical expression is 3 – (–2)? (3.3; 126)
>
> Anita: From –2 you are moving up to 3rd floor. That means movement is upward 5 floors...(3.3; 131)
>
> TE1: So 3 – (–2) is +5 (3.3; 132)
>
> Anita: Up... because it is +5 (3.3; 133)
>
> Ajay: Answer is 5. But if students ask why then we will have to give a reply. (3.3; 134)
>
> Anita: Why is 3 – (–2) [equal to] +5? (3.3; 137)
>
> Swati: From –2nd to 3rd floor... 5 upwards. (3.3; 140)
>
> TE1: See, we can think of it as 2 steps. 3–0. If we reach the zeroth floor first, then I have to go from zero to 3. So how to reach zeroth floor? It is zero minus minus 2... zero minus minus 2 is always +2 because if I have to go from –2 to zero then I have to press +2... (3.3; 141)
>
> (Workshop Excerpt, 10–08–2010)

TE 1 is offering here an explanation of why subtracting an integer is the same as adding its additive inverse, by interpreting movement in two steps with the zeroth floor as an intermediate station. Thus connection was made between a mathematical idea – the equivalence of subtraction with addition of the inverse – and a transformation on a representation in the form of the integer mall context, illustrating an important piece of SCK. This addresses the issue raised earlier by teachers of how one can meaningfully interpret the rule of changing subtraction to addition of the inverse. Although there was no evidence of the teachers taking up this idea, it points to the possibilities contained in a rich context like the integer mall.

The teachers addressed the question about the meaningfulness of cancelling pairs of equal positive and negative numbers by relating it to a context. Further, concerns for consistency may be noted in the care taken to formulate the symbolic representations of combinations of lift movements and the progressive movement towards expressions representing the addition of integers. We may also note the careful reading of subtraction expressions and relating them to the movement required to go to a target floor from another floor. All of this indicates the teachers' engagement with deeper levels of representational adequacy. The teacher educator's suggestion of how the operation of changing the subtraction expression to an addition expression using the additive inverse may be interpreted in terms of the lift movement carries this concern further, although the teachers did not engage in this exploration.

The teachers' engagement during professional development in exploration and design of contexts for use in teaching helped in constructing important aspects of SCK for teaching integers. These aspects include identifying contexts that can be meaningfully modeled by integers and integer operations. Within these contexts, the teachers were able to identify critical features that correspond to particular meanings of integer. In particular, using integers to represent change made it possible to include many contexts involving the addition operation, as well as to identify derived quantities that could be represented and operated with. Expanding the meaning of integers to change and relation, as well as incorporating the change and compare meanings of subtraction, made it possible to meaningfully model the addition and subtraction of integers on contextual representations like the integer mall. Thus, teachers were able to use the potential of a context to a fuller extent by identifying multiple meanings of integers within a context. Further, teachers were able to make connections between contexts and other representations like models using meanings of integer and their operation as a framework and making connections between meanings. They were able to interpret movement on the number line as increase or decrease, and to provide a more meaningful explanation of why additive inverses sum to zero. Thus meanings helped in bringing coherence among different representations that could be used for teaching integers as well as an increase in variety of contexts for teaching.

7.6.3 Teachers' reflections on engaging with integer meanings

Teachers' exploration of representations using the framework of meanings helped them realize the importance of considering meanings associated with representations used for teaching. This was indicated in teachers' reflections on their learning, in their use of meanings to make sense of other representations and concepts, and in their design of tasks for peer teachers. In the last phase of the workshop, teachers reflected on their own learning while designing a workshop session for peer teachers on integers. Swati reflected how students need to understand a very basic but difficult point that "Just like 3 is a whole number, –3 is also a number" and thus needed to develop a sense of minus as representing integer. Rajni, on the other hand, shared "I thought of integers always as numbers and never connected it with any situation. After coming here I thought like this". Anita added that they always considered the reference point for integers as zero. She realized that any number can serve as a reference point for representing an integer opening up various possibilities for contexts to be represented as integer. Their planning of workshop presentations for peer teachers reflected that they now wanted to discuss the meaning of integer and operation framework with other teachers, explore their connection with contexts, use it for analyzing students' mistakes and responses from classroom experiences, and for designing tasks beyond the textbook for better engagement of students. For e.g. Ajay designed a task for engaging teachers in exploring contexts and representing them with integers. Swati suggested that they can ask about situations where integers meaningfully be applied and then can ask teachers to categorize them as situations representing state, change and relation. Teachers reflected that it would take time for teachers and students to change their way of thinking just as their own thinking had changed because of participation and has led them to use the knowledge in other topics and grades.

Excerpt 7.17

> Swati: It will take time because students have their mindset, we have our mindset. Now, after coming for so many days here, our mindset has changed a little. Like, when I am teaching other classes, [then] also I am able to apply.
>
> (Workshop Excerpt, 20–11–2010)

Teachers were also able to use their knowledge of meanings to make sense of new representations not discussed in the workshop. Their discussions indicated that they had developed a shared value of meaningfulness of representations and sought explanations and justifications of new representations that they encountered. As discussed earlier, Swati came up with an explanation using the construct of reference point to differentiate meanings of integer as state, change and relation. Anita also showed initiative in bringing new representations into the discussion and constructing valid and meaningful explanations associated with the representation. She showed how twin number lines could be used for integer as well as algebraic operation, stating that she had found this idea on the internet. She wanted to know how and why the representation works. She shared how one scale was used as a reference scale and other scale was for representing the operation. She talked about how they represented addition on twin number line by aligning 3 of reference number line with zero on second number line and then moving by 2 in the positive direction, thus representing $3 + 2 = 5$, which is the point on the reference scale corresponding to 2. She recounted her conversation with a teacher in her school who believed that "Negative numbers will always be less than zero" and was not convinced that one can take an arbitrary point as a reference point to represent integers. Thus, she realized how understanding the role of the reference point and representation of change as integer are significant conceptual attainments in learning integers.

During the presentation to their peers, the teachers asked for examples of contexts which can be represented by integers and were able to give justifications for appropriateness of contexts based on meanings. An example of a blood group given from a teacher in audience was not considered appropriate by Swati teacher as she felt it was "just a label and does not have any magnitude" and shared that such examples are given by students also. This indicated a deeper degree of concern for translatability and meaningfulness.

7.7 Teaching of integers

In this section, the teaching done by three middle school teachers, Swati, Rajni and Anita is discussed. All the lessons on the topic of integers by Swati and Anita and four lessons of Rajni were observed. Rajni, had already taught some of the portion before the observations began, so some of the topics in the first observed lesson were done as revision. All the lessons by Rajni and Anita and the first five lessons by Swati took place in the two months between Day 4 and Day 5 of the topic focused workshops. Lessons 6 to 14 by Swati took place between Day 5 and Day 6 of the workshops. Table 7.6 gives brief outlines of the lessons taught by the three teachers.

Table 7.6: Lessons taken by Rajni, Swati and Anita for teaching integers

Lesson no.	Rajni	Swati	Anita
Lesson 1	Introduction to integers ; Integer mall context– state and movement; ordering integers on number line; Comparison of integers	Introduction of negative integers using temperature and integer mall contexts	Introduction to negative integer using integer mall and temperature context
Lesson 2	Addition of integers using integer mall context, neutralization model and rules	Ordering integers using number line; Using integer mall context to represent floor number and movement	Need of negative integers, milestones context and opposite meaning for ordering integers, comparison of integers
Lesson 3	Addition of integers using integer mall context, neutralization model and rules	Ordering integer using number line and doing textbook exercises	Number systems: whole numbers, natural numbers and integers; context to represent quantity and movement in opposite directions as integers;

Lesson no.	Rajni	Swati	Anita
			comparison of integers
Lesson 4	Subtraction of integers using integer mall context and rules	Addition using number line and rules	Addition of integers using integer mall context and game context; comparison of integers; contexts to represent integers
Lesson 5		Addition using number line and integer mall context	Addition of integers using integer mall context; additive inverse
Lesson 6		Addition using number line and comparison of integers	Rules for addition; addition using integer mall context; neutralization model
Lesson 7		Comparison of integers and addition using number line	Addition using neutralization model and student writing observations
Lesson 8		Addition using neutralization model	Addition using neutralization model; rewriting the expression using additive inverse
Lesson 9		Addition using neutralization model and rules, using just numbers	Addition using numbers; integer mall context, rewriting the expression using additive inverse *; comparison of integers
Lesson 10		Addition using rules	rewriting the expression using additive inverse; interpreting expression using integer mall context
Lesson 11		Addition using integer mall context, additive inverse	Commutativity and converting subtraction problem to addition problem; interpreting subtraction using integer mall context
Lesson 12		Subtraction using number line, convert subtraction problem to addition problem using additive	

Lesson no.	Rajni	Swati	Anita
		inverse	
Lesson 13		Subtraction using number line, integer mall context, rules and additive inverse	
Lesson 14		Converting subtraction to addition using additive inverse	

The classroom observation of the three teachers while teaching the topic of integers is discussed in the following subsections. The analysis of observation of teaching is based on the framework of meanings of integers, which was introduced in the workshop to teachers. Table 7.6 indicates the topics and representations used by the teachers for teaching integers in the lessons that were observed. There were differences among the teachers in the emphasis on the use of contexts for discussion, the way meanings were discussed, discussion on rules and the extent of engagement of students during discussion. All the teachers used the integer mall to discuss introduction, comparison and addition-subtraction of integers. They also used the meaning of state and change to discuss representations implicitly or explicitly while the relation meaning was used to a limited extent. What was noticeable in all the teachers' lessons was the attempt to explain procedures using different representations and contexts and the back and forth movement between presenting rules and developing explanations.

7.7.1.1 Introduction of integers

Introduction to integers is an important topic since the students encounter integers for the first time in the sixth grade. The teachers had shared concerns in the workshop that students are not able to make sense of negative integers, mistaking the minus sign to be that of the subtraction operation. Swati started with a short discussion on using integers in contexts followed by textbook exercises. Anita on the other hand spent three lessons to introduce integers and engage students in discussing meaning of integers using different contexts, which are discussed below.

Anita began by discussing the meaning of integers in the context of the integer mall. She discussed both the state and the change meanings in this context. In the integer mall context, the meaning of positive and negative integer was explained using zero as a reference point – as the integer denoting the number of floors above or below zero thus depicting state. The lift buttons +1 and –1 were introduced as controls to move in opposite directions and she discussed how change in position could be represented through integers. Anita pointed out that addition and subtraction are opposites of each other, although she did not discuss how the subtraction sign and the negative integer sign are different. She used the relation meaning of integer when she asked students to use integers to identify the position of different floors using a given floor as a reference. She also discussed the change meaning in other contexts like temperature, amount of money in the bank, etc. She asked students to describe situations which could be represented using negative integers, thus giving them an opportunity to share their ideas. Students came up with interesting examples like the buttons on a TV remote control and prescriptions for spectacles where negative sign/ integers can be used. She encouraged students to give reasons for using negative integers in the situation that they described, getting them to use both the change as well as state interpretation. She developed her own classroom tasks based on these meanings and gave textbook exercises as homework.

The other two teachers, Swati and Rajni also used contexts to introduce integers and their meaning. Swati used contexts like temperature and the integer mall to show how both positive and negative integers can be used to depict state or change in quantity. Rajni gave examples of contexts such as sea floor level and basement floors where negative integers depict state and downward movement in the integer mall context and in the school building to depict the change in position. Thus all the teachers used both the meaning of state and change while discussing integers in contexts. However, Rajni and Swati spent much less time on this than Anita.

Rajni and Anita discussed integers in relation to the family of number systems. Rajni started the lesson on introduction with a discussion on the relation of integers to the whole numbers and natural numbers while Anita discussed this only after discussing comparison of integers. Both discussed how whole numbers and natural numbers are contained in the set of Integers. Anita drew a Venn diagram to explain that the set of integers is the larger set which contains natural numbers and whole numbers as sets within it.

7.7.1.2 Ordering and comparison of integers

All the three teachers used both the horizontal and the vertical number line, as well as the integer mall context to discuss the ordering of integers. A picture of the integer mall was used for the first few lessons, after which the teachers represented the integer mall through the vertical number line. All of them associated the idea of direction with the sign of an integer – positive integers to the right or above and negative to the left or below. It may be noted note that comparison between a positive and a negative integer and between two positive integers is easier than comparison of negative integers and teachers spent a significant amount of time in the lessons on the latter.

Discussion on ordering and comparison was initiated by all teachers by asking about the placement of –1 on the number line, which is also discussed in the textbook. Rajni asked students to predict where negative integers or –1 should be placed on the number line. When the students answered that it should be placed to the left of zero, she asked them for the reason. Without waiting for students' responses, she explained that they are placed to the left of zero since they are less than zero. Rajni then stated a generalization that when comparing two integers, the number placed towards the right on number line is always bigger. Using the number line, students reasoned that negative number –2 is greater than –3 since it lies towards the right.

Swati started the discussion by saying that as one moves right on the number line the numbers are increasing and as one moves left they decrease. Like Rajni, Swati gave this generalization as a rule for students to follow. She then gave the example of temperature and represented negative temperatures below zero using a vertical number line. She also used the integer mall, emphasizing the state meaning by referring to the position of negative integers as below or to the left [of zero] on the vertical and horizontal number line respectively. Swati used this rule to discuss all the textbook questions on comparing integers, which was done only symbolically without discussion on contextual meanings. However, Swati revisited these topics again soon after Day 5 of the workshop and dealt with them in a different manner, which will be discussed below.

Anita began discussing the ordering of integers by asking students how negative numbers can be placed on the number line but her approach was slightly different from the other two teachers as she focused on using context to develop meanings rather than giving a generalization. When a student was not able to place –1 on the number line, she introduced the context of milestones on a road and asked the student to think of how a movement of 1 km in opposite directions could be represented. She then used the meaning of positive and negative integers as opposites of each other to persuade students that –1 should be placed to the left of zero as it is at the same distance as +1, but in the opposite direction. Here the integers are placed according to the relation they have with zero. Thus, she addressed why the sequence of negative integers should start with –1 to the left of 0, since negative numbers denoted an equal distance from 0 in the direction opposite to the distance of the corresponding positive number. When students made mistakes like placing –10 before –2, Anita took this up for discussion. She explained that as the temperature drops by 1 degree Centigrade successively from zero, the temperature becomes –1 deg C, then –2 deg C, and so on eventually reaching the state of –10 deg C. The state of –10 deg C would be achieved much later than a temperature of –2 deg C and would be less warm in comparison to –2 deg C. In this explanation the teacher used the change meaning (–1 deg C at each step) as well as the state meaning (reaching a temperature of –10 deg C), and also indicated the magnitude in relation to 0, thus using all three meanings of integer. Students too started using the relation with zero for comparison. Thus, Anita's use of the idea that integer denotes directed distance from zero helped the students in placing the numbers on the number line and in making judgements of ordering and comparison.

Although Anita had focused on the development of meanings for the most part of her teaching, she discussed rules just before the students attempted a test. However, her focus on developing meaning did result in students giving reasons to compare integers. For e.g., one of the students observed that values are opposite in negative integers, i.e., 4 is greater than 3 so –4 is less than –3. Another student reasoned that since –1 is the greatest negative integer, any negative integer nearer to –1 will have greater value. Here student has used –1 as a reference point to establish relation with other negative integers.

Rajni attempted to use the context of borrowing to make it easier for students to compare negative numbers. However, this was not very successful, probably because she did not use the notion of "net worth". Without this notion, it is unclear what the borrowed amount refers to, because it is money in the hand of the borrower, to which assigning a negative sign may not seem appropriate to students. However, for the most part she had students rely on whether numbers were to the right or left on the number line to make judgements of comparison.

After a long gap, teachers reconvened for the topic focused workshops on Day 5, when they discussed student responses and misconceptions while teaching integers in the classroom. After this, of her own accord, Swati revisited the comparison of integers in her class. In the meeting, it was discussed how the number line forms a basis for discussing rational numbers in higher grades and how the idea of measurement is inherent in the construction of number line as the positive and negative integers are at regular and opposite distances from zero. Swati discussed some examples of student errors from her class. While labelling the floors of the integer mall, they placed –1 at the lowest floor instead of the floor just below zero. She shared how students keep confusing the magnitude of negative integer viewing –3 as greater than –1. The group, based on Anita's classroom experience, discussed how it would be more meaningful for students to understand that negative integers are the same unit less than zero as its inverse positive integer is greater than zero. When Swati revisited comparison of integers in her classroom after this discussion, she did not accept the reason for integers being smaller or larger based on the direction of placement on the number line, as she had done earlier. Instead, she pushed students towards identifying that negative numbers are less than zero since they are certain "*units*" less than zero. For e.g., she asked students to explain how many units is –7 less than 0 and how many units is –2 less than 0.

This as a significant episode which involved a clear interaction between discussion in the topic study workshop and teaching in the classroom. The discussion during the one-day workshop had convinced Swati that students had not understood comparison properly. The shift to comparing integers based on value and use of units to talk about integers was an important shift, since the teacher took the initiative on her own to revisit a topic which she had taught earlier while using the idea discussed in the workshop.

I now cite another example of how a classroom episode generated an interesting discussion between the teacher and the researcher and in the workshop. In Anita's class, while comparing negative integers, a student stated that –3 is greater than –2. Anita asked other students if they agreed or disagreed with the answer. A student disagreeing with the answer, reasoned that –2 is greater than –3 because when 2 and 3 are subtracted from a number say 20, the amount left in the case of –2 (18) is more than for –3 (17). Anita responded that they were discussing negative numbers while the student was talking about subtraction of whole numbers which is different. After the class, there was extensive discussion between the teacher and the researcher about how one should have responded to this statement and if the reasoning was valid. Anita felt that it was possible that the student was thinking of numbers as negative integers since "20 – 2" can be interpreted in terms of "20 + – 2" and the student knew that instead of 20 any number could be taken. She argued that the student was interpreting integers as a certain quantity less than/ more than a reference quantity. However, she regretted that she did not probe deeper to get a clearer idea of the students' thinking. In subsequent lessons she paid closer attention to the student's arguments and eliciting their thinking. When this episode was shared and discussed on Day 5 of the topic study workshop, some in the group argued that it was not clear whether the student was interpreting the minus sign as an operation sign or as indicating a negative integer.

The example served to highlight once again that one of the difficulties faced by students was the distinction between minus sign as a sign of number vs. operation.

Later in her teaching Anita used the context of the quiz to discuss addition of integers as well as comparison. She first asked students to play the quiz and kept the score for different teams. A positive score was given for a right answer in the game while a negative score was given for a wrong answer. After playing the game, Anita asked students to find out the total scores to identify which of the teams had won. In using this context, the individual scores at each turn represented the state and the task of comparison of states was integrated with the task of addition. This integration was useful since students were able to use it for reasoning during the comparison. For e.g., students argued that the score of –1 is higher (greater) than that of –6 because the team which had got –6 had made more mistakes.

Swati had expressed that one of the difficulties with the number line was that numbers like –100 could not be shown on it. After a discussion on this in the topic study workshop, Swati took up a representation proposed by the researcher of using one's own body as a vertical number line, considering zero at the chest. Using the floor as another reference, that students could stretch or shrink the "number line" flexibly while comparing integers, for e.g., –1 is greater than –100 since –100 would be closer to the floor while –1 would be closer to the chest. Swati found the device useful since students could visualize and compare negative numbers of up to 3 to 4 digits. She used a similar reasoning on the horizontal number line.

From the discussion above, we find two ideas that have been used for comparison of integers by the teachers – one is directed distance from a reference point to indicate value and the other is how much the value of an integer changes in reference to another integer as result of some change as in the case of temperature. All the teachers used 0 as a reference point for comparison at some point or the other and also discussed its value as being more than negative integers. This indicated implicit recognition amongst all teachers that the meaning of zero needs to be revisited in the context of learning integers. The teachers' discussion of comparison using the number line and contexts implicitly used the state, change and relation meanings of integers. The teachers' emphasis on discussing placement on the number line and comparison using direction and a reference integer served to develop understanding of ordinality. At the same time, when teachers discussed the value of integers and change in value in relation to contexts like the change of temperature, then this served to develop understanding of cardinality of the integers. The teachers' attempts to relate these contexts with the number line can be seen as attempts to integrate the ordinality and cardinality of integers.

7.7.1.3 Addition of integers

Extensive discussion took place during the workshops on how to represent addition and subtraction of integers using the meanings in different contexts and models. During the initial days of the workshop, movement on the number line in a particular direction to denote the negative sign of the integer and the minus sign of subtraction was recognized as potentially confusing. However, since the method was given in the textbook and a question about adding or subtracting integers using the number line was almost always asked in the term end exam, the teachers decided to use it in their lessons. Both Rajni and Swati first used the horizontal number line to discuss addition and subtraction of integers. Anita, in contrast, started by discussing only addition using the integer mall context. The discussion of addition and subtraction based on movement on the number line emerged as problematic in classroom discussion as the excerpt 7.18 below from Swati's lesson shows.

Swati posed a problem to the class to represent the movement on the number line from –8 to –13 through a mathematical expression. Students gave different answers like –8 + 5, –8 – (–5), 8 + 5, – 13 – (–8). Swati then tried to convince students that movement towards left should be represented by minus thus representing movement as –8 – 5 = –13.

Excerpt 7.18

> T: See you are starting at –8, so you have written –8, and you have to reach –13. So your answer has to be –13. How many places you are going? You are going 5. Now what should come here?
>
> Sb1: Minus.
>
> T: Why minus, or why plus?
>
> Sg: Ma'am, because we cannot minus (falters and laughs).
>
> Sb2: Ya Ma'am, plus.
>
> T: Plus?
>
> Sb1: But Ma'am, it was decreasing as we move towards left.
>
> Sg: No, because the bigger sign is in 8. So, Ma'am, the minus sign would come if we plus, plus ya. Ma'am, if we plus and 8 + 5 is 13 and bigger sign is...
>
> Sb3: Ma'am, –8 plus. No. –8 – (–5).
>
> Sg1: Ma'am, when we add the numbers of the same sign, the same sign will come.
>
> T: You are telling the rule. See what is happening? When you go to this side, it is decreasing. What sign should we use?

Sb4: Minus.

T: Decrease, when we were… if we have to go this side it is increasing, if we go this side it is decreasing, So you are adding or subtracting?

Sb1 (hesitating): Subtracting.

T: What you are doing?

Sb1: Subtracting.

….

Sg2: Ma'am, 8–5 is 3.

T: When you do 8–5, you use the number line.

Sb5: Ma'am, we are confused (loudly).

T: If you are doing 8–5 (shows on BB) To do 8 minus 5 where we have to go? This side (right) or this side (left)?

S: This side (left)...

T: So 1,2,3,4,5. We get 3. So if we have to minus, you go this side. If plus,… you go this side. Right, so there which direction are you moving?

Sb1: Left.

T: So left means what?

S: Minus...

Sg1: Ma'am we were giving the correct answer.

T: No, but you were not saying why. Reasoning is needed.

Sb1: Ma'am we were confused because in positive numbers, number and value both increase, but in negative numbers, the number increases, but the value decreases.

(Classroom excerpt, 1 Nov. 2010)

In the above excerpt 7.18, the teacher was trying to get the students to associate the movement towards left with the minus sign since it was decreasing. The students' confusion is reflected in the last line of the excerpt when the student articulates the difference between the positive and negative integers. It indicated that the students did not necessarily view the change from –8 to –13 as a decrease in quantity, therefore even when the teacher indicated that movement towards left is to be represented by minus sign, students did not seem convinced. The use of minus sign for both integer as well as for subtraction further creates confusion. Students knowledge of rules for addition of negative integers also interfered with what Swati was saying. By the rule students perceived the problem as addition of two negative integers i.e. –8 + –5= – 13, while the teacher was representing it as subtracting 5 from –8 by moving left. The teacher knew the flexibility of representing –8 – (+5)= –13 as –8 + (–5)= –13, which may not have been accessible to students. Since the discussion was done without using any context as reference, it was difficult for students to understand what is decreasing and to make sense of subtracting 5 from a negative number. Later, students protested that they did not know what to do since the teacher was not telling them how to solve the problem and was not accepting rules as a reason. As a result of students' protest, Swati then discussed the rules for addition of integers. For Swati, it was a significant shift from earlier classes since she had regularly used rules to justify solutions and was making an effort to try to discuss reasoning behind procedures and develop an explanation.

In the post-lesson discussion with the researcher, she shared that she had discussed rules in the class because she felt pressured by the students' expectations and needed to complete the syllabus. She felt that during the classroom discussion students are not interested in discussing reasons and just want the solutions. However, in the next class, she shifted to using the integer mall context in the middle of discussing the explanation for addition using a horizontal number line. (This event is described below.) In the post-lesson discussion she explained that this was due to the feeling that students are not able to understand how to represent addition mathematically using the movement on the number line.

Swati thus exhibited back and forth movement between the use of rules, developing an explanation and use of context which indicate the tensions experienced by her. Her use of the same explanation which was considered as problematic in workshop indicates that she was not totally convinced about the problems related to explanation. However, since she was aware that it may be problematic, she was observant of student difficulties and thus shifted to other ways to discuss addition. This also points to the need of consolidation of knowledge developed in the workshop with the experiences and observations of students' conceptions during teaching to bring about any significant change in the teacher's practice.

Addition using the meanings in the integer mall context

As mentioned earlier, Swati and Rajni shifted from using the horizontal number line for addition of integers to the integer mall context while Anita started with this context. The teachers used the lift feature of having + and – button to discuss movement up and down. Rajni represented movement of the lift by pressing +1 and –1 buttons, recording the presses in symbolic form like "– – – + + ". The negative and positive signs were then cancelled to find the final movement as well as direction. She next asked students to write the movement in numeric form. Thus in this approach, integers were used to represent change and the operation of addition of integers represented the final resulting change. Students were able to generalize that if equal amount of movement is done in opposite directions then the person will reach the starting point resulting in no change. At the end of lesson, she again revised the rules that had been discussed earlier for adding integers of same sign and opposite sign, showing that it still holds true in the integer mall context.

Swati translated the numbers in an addition expression into the features of the integer mall context. For e.g., the problem +9 +(–6) was illustrated by visualizing oneself at 9th floor and then pressing –1 button 6 times to reach the 3rd floor. Using the integer mall representation, she was able to convince students that even when one is on one of the basement floors (represented by a negative integer), one would move further down by pressing the minus button (represented as addition of a negative integer). In subsequent lessons, Swati used the idea of no change to discuss how equal and opposite movement cancel each other. In post lesson discussion, Swati expressed her happiness over the use of the integer mall context citing that it is helpful in engaging students in discussion as students started behaving as if "they have been awakened" after she started using the integer mall context. The students' enthusiastic participation motivated her to use this context in later lessons. Also, the feature of visualizing oneself as moving from one floor to another could have helped students in understanding what each symbol in the mathematical statement represents.

Anita engaged students in representing movements across floors in terms of symbols representing button presses and then converting this into numbers, similar to what Rajni had done. Then she asked students to interpret an expression of adding integers to depict movement in the integer mall context. In doing these problems, Anita like the other two teachers, used both the state as well as the change interpretation. Floor numbers represented state and button presses in the lift represented change and thus addition had the meaning of change of state. She also used these meanings in another context of playing a quiz game and keeping scores using integers (discussed in the previous section) which helped students in comparing integers. In Anita's lesson too, students used rules, sometimes incorrectly, to justify their answers. For e.g., a student felt that –2 – 5 will be +7 because "minus and minus is plus". Here, Anita invoked the context of the integer mall suggesting that the student need to think of –2 as going 2 floors down and then –5 as again going 5 floors down, with the total change being 7 floors down which can be represented as –7. This was convincing to students. Even while making a generalization, Anita used this idea to say that "When we are adding two integers of the same sign, that means we are moving further in the same direction and therefore the result will have the same sign". Anita challenged students to think carefully before responding when they forgot or made mistakes in putting the correct sign of integer. She explained that writing –8 as 8 would mean that there is no difference between them and pointed that they would represent different floors in the integer mall context and also movement in different directions. After doing problems involving addition with the same sign, Anita used problems in which the net result of movement was zero, e.g., +6 + (–6). She introduced the idea of additive inverse using these examples in which addition of equal and opposite numbers lead to zero and was careful in representing the implicit + signs in problems. In further examples of addition of positive and negative integers, Anita engaged students in identifying how equal and opposite movement cancel each other. The class developed the explanation for adding two integers with different signs that the result will have the sign of the bigger number since it would involve moving further in that direction after the effect of moving in the opposite direction is nullified. These examples formed the basis for generalization of rules and also for establishing connections with neutralization models later. While discussing the addition of integers with different signs she related it to addition using the neutralization model by asking students to relate the numbers cancelled with the movement that is nullified.

When Anita asked students to interpret movement on vertical number line when the problem is given, two different ways of interpreting a problem also came up in class. e.g. 5 + (–2) was interpreted as starting from the 5th floor and coming down 2 floors to reach +3 which is the answer. Anita showed how if the numbers are reversed like –2 + 5, the answer will still remain +3 since it would mean starting from –2 floor and coming up 5 floors to reach +3. In this way, she discussed commutativity of integer addition using a context. We may note however, that a better context to discuss commutativity would have been to depict the integers as change and then showing that combining the change in a different order gives the same resultant change.

Addition using neutralization models

All the three teachers also showed addition using the neutralization model using buttons of two colors since they felt that this would help students. Swati used this model to help the student that she thought was the “weakest”. She was very happy when this student was able to find the correct answer using the buttons. Both Swati and Rajni attempted to generalize rules by using results of addition of integers after using number line or neutralization model. After engaging students in doing the activity with neutralization model, Swati gave problems to students which had the same numbers but different signs as shown below.

1. $-4 + 2 = -2$
2. $4 + (-2) = +2$
1. $-4 + (-2) = -6$
2. $4 + (+2) = +6$

Using these problems and their answers, Swati revisited the rules for adding integers discussed before. She asked them to remember that the answer will always have the sign of the bigger number. She said that knowing these rules will be helpful since it would not be possible for them to always use buttons with bigger numbers. Rajni derived rules in a similar manner asking students to observe the answers to several examples of addition.

Anita too discussed when the answer to an addition problem would be positive or negative using the neutralization model by asking students to write their observations on the blackboard. Students arrived at observations like "when both numbers are of same sign, we use the buttons of same color" and "if we add two positive integer, the answer will also be positive integer". Through observations like "when we arrange 4 red (for positive) and 3 yellow (for negative) in two rows, the answer is +1 because the reds are more", students were able to explain addition of positive and negative integers. Anita asked them to explain how they got the answer and discuss why the numbers cancel each other. She recalled the earlier discussion of additive inverse where they learn that addition of +1 and –1 is zero. She showed how $-3 + 4$ can be written as $-3 + 3 + 1$, to show how equal and opposite integers form a pair and cancel each other out. She constructed problems like $8 + -5 = 3 + \_\_ + -5$ to make students think in terms of making integer pairs.

Anita introduced rules after discussing addition of integers using the integer mall and game contexts and using the number line. Similar to Rajni and Swati, she wrote expressions which had the same result except the sign, through which she generalized that one will always get positive integer when we add two positive and negative integer as result of adding two negatives. However, after this discussion she engaged students in interpreting expressions and using the idea of positive and negative integers as opposite and the idea of no change to reason with expressions. e.g. Anita asked students to use the fact that "5 + 11= 16" to think about what integers to add to get –16. Students were able to think of adding –5 and –11 to get –16. In another instance, when she asked a student to solve a problem of $2 + -3$, he solved the problem by subtracting 2 from 3 and putting the minus sign. Anita then wrote his solution as $2 + -3 = -(3 - 2)$ and agreed that answer would be –1. She asked students to think of it as an alternative way to express the addition of integers with different signs. She also showed how adding integers with same sign can be written in a different way: –2 + –7 = – (2 + 7) or 5 + 7 = + (5 + 7). Though Anita did not discuss in detail how these expressions are equal, these expressions created an opening for developing deeper understanding of rules.

7.7.1.4 Subtraction of integers

Teachers used two meanings for discussing subtraction of integers. One was to represent the movement from one floor to another as a difference between two floors and the other was to use the idea of additive inverse. Teachers used the term additive inverse and 'opposite' interchangeably in the discussion. Anita went one step further by discussing two expressions as opposites of each other when their results (values) are opposite integers. All the teachers used the brackets around the second integer in the expression to differentiate the sign of subtraction operation from the sign of the integer. Interestingly, these brackets were not consistently used by all the teachers when discussing addition.

Rajni introduced subtraction of integers by representing the movement from one floor to another mathematically and explained how an expression can be interpreted as "destination floor – starting floor". However, after discussing some examples she told students that if they get minus of minus in the problem they have to convert into addition using the additive inverse. After discussing a few problems with additive inverse, she asked students to solve the problems in the textbook.

Swati discussed the meaning of the minus sign and its different interpretations before talking about subtraction of integers. She discussed that minus can be used to depict the basement floors (state) as well as to depict the downward movement (change). She then explained that +2 and –2 are additive inverses of each other since they depict opposite movement. She pointed out that "numbers" are the same but the signs are different. She then asked students to interpret the expression 3 + (–2). She wanted students to focus on identifying whether the problem represented addition of integers or subtraction of integers. However, students used the rule for addition of positive and negative integers by "subtracting the numbers and putting the sign of bigger number". (This is problematic since students were still treating the numbers as whole numbers, thinking of the negative integer as 'bigger' than the positive integer, when its value is less than the positive integer). Swati then explained that the '+' sign denotes the operation and gave examples of expressions which represented addition or subtraction of integers making students identify whether it is an addition or subtraction problem. Thereafter, she asked students to represent the expression 3 – (–2) on the number line. When the student represented it as movement from 0 to 3 and then back 2 steps to reach 1, another student objected that it being same as 3 + (–2). He argued that it is wrong since it indicates that there is no meaning of minus in the particular expression as an operation sign if one accepts that result of 3+ (–2) and 3 – (–2) would be same. This was an interesting moment when the student was looking at consistency of meaning in expressions. Swati asked the student to repeat his statement and explained his point to students. Swati then used the integer mall context to represent the movement from starting floor to destination floor as subtraction of the two floor numbers, but students were not able to remember whether the minuend should be starting floor or destination floor. After students gave several responses, Swati realized that the students were getting confused. She explained that "subtracting an integer is same as adding its additive inverse/ opposite". She suggested that the latter can be used to convert the subtraction problem into an addition problem and gave several problems for practicing this conversion. Students kept quoting rules, but Swati explained that they need to understand this as it would help them understand the mathematics in higher grades.

In a discussion before teaching, Anita shared that she wanted to introduce the idea of the negative sign as the opposite of the positive sign and wanted to build students' understanding of expression based on this idea. In class, she discussed how the expression +4 – 2 and 2 – 4 are opposite since their result +2 and –2 are opposite of each other. She argued that therefore minus of –2 i.e. – (–2) is equal to +2. On being invited by the teacher, the researcher asked students to explain the expression 4 – 2. This expression was interpreted by students in different ways using the integer mall context. One interpretation was coming from 4th floor, 2 floors down i.e. reaching 2nd floor. Another interpreted that it is going up from 2nd floor to 4th floor, thus representing the movement of +2. Yet another interpretation was the distance between the floors. In the next lesson, a student again interpreted 4 – 6 as starting from –6 floor going four floors up to reach –2. In this interpretation the student was parsing the expression as +4 – 6 where +4 represented the movement while –6 represented the floor from which to start. Thus, unknowingly the student had converted the subtraction question into an addition question. Anita identified this and used this as an opportunity to discuss if the subtraction expression could be converted into an addition expression. She took two equivalent expressions as cases to discuss this idea: 4 – 6 and 4 + –6. She pointed out that in both cases the answers are same and the difference is in the way the expression is interpreted. In the first expression the operation is subtraction and 6 is assumed to be a positive integer, while in the second expression the operation between the integers is addition and 6 is interpreted as a negative integer. She asked students to solve the expression 10 – (–3) and asked them to explain the meaning of subtracting –3 from 10. A student, who gave the correct answer, cited the rule of "minus of minus as plus" as the reason. Anita, however, insisted on an explanation using integer mall context. When the student interpreted it as starting from 10 and coming 3 floors down, Anita pointed that it would be same as expression 10 –3 or 10 +–3. She asked students how the expression 6 – (–2) can be represented and explained that it is distance between the –2 floor and the +6 floor. She then asked them to interpret – 6 – (–2) which student interpreted as starting from –6 floor to –2 floor resulting in upward movement. She then used the expression –6 – (–2) and –6 + 2 to point out that for both the expression answer is equal to –4. She then explained that – (–2) means taking the additive inverse of –2 which is +2.

As the discussion above shows, teachers tried out different ideas for explaining subtraction of integers and found it challenging. They relied much less on the integer mall context as compared to other topics for discussion. There was also back and forth between the use of difference meaning and the additive inverse meaning, with teachers shifting towards additive inverse eventually. The discussion in all classes shifted towards manipulations of the expressions when teachers discussed conversion of a subtraction problem to an addition problem.

7.7.2 Impact of the topic study workshops on teaching

From the observation of teaching, it is evident that the teachers were attempting to introduce new elements in their teaching by way of new representations that foregrounded the different meanings of integers and integer operations. Thus they used the contexts and key ideas discussed in the workshops, although in different ways and with different degrees of integration with the textbook exercises and teaching of rules. While teaching, the teachers had to deal with two kinds of tensions. Firstly, there was an expectation from students that the teacher help them. When this help was withheld, the students were uncomfortable and pressurized the teacher to tell them the answer and resisted engaging in discussion based on meanings. A second tension was applying specialized knowledge about integers (discussed in earlier sections) in the classroom to flexibly deal with student responses and to facilitate student understanding. The teachers had understood the various interpretations of integers and were able to use it for designing tasks collaboratively. However, to use these ideas while teaching was fraught with issues like the level of teachers' confidence in the task and explanation used and knowledge of how to deal with unexpected student responses. Developing knowledge *in situ* needs time and experience. Teachers explored the ideas discussed, observing the issues that arise in teaching and students' misconceptions and evaluating the usefulness of the ideas discussed in the workshop implicitly. This explains the back and forth movement between the use of contexts and meanings and the use of rules. Teachers appreciated the level of students' engagement as a result of using the contexts including "weak" students and made sincere attempts to develop students' understanding using meanings.

In this section I analyze the impact of the discussion in the topic study workshops on the classroom teaching drawing on the lesson observations as well as the teachers' own reflections. These reflections were expressed by the teachers in the workshop and in the presentations that they made to peer teachers. What teachers chose to report in the workshops about their classroom experience is an indicator of what they found significant in their learning and thus what aspects of the workshop discussions they found to be useful.

7.7.2.1 Shifts in goals and beliefs

The teachers' initial goal was to teach rules and the textbook too indicated that students were expected to become fluent in solving integer problems given in the textbook exercises using rules. Therefore, the teachers focused on calculations and believed that contexts were not useful for learning integer operations. In their lesson plans and report of teaching experiences, the three teachers (Swati, Anita and Rajni) acknowledged a change in their approach from telling rules in the beginning to exploring contexts first with students and then inducing rules. However, they were not entirely successful in using this approach in their classroom. The tasks used in their lesson plans had a balance of questions which involved representing features or actions in a context mathematically as integers as well as questions on calculation with integers presented symbolically. Anita included questions for eliciting student meanings for the minus sign like "Give examples of situations from your daily life where you have seen use of the minus sign" (See Appendix 6 for a lesson plan by Anita). This indicated a shift in the goal from teaching of rules to the development of meaning using contexts. All the three teachers (Rajni, Swati and Anita) used contexts to discuss where and how integers are used as well as to discuss addition and to a lesser extent for subtraction of integers (as discussed in section 7.7.1).

Another shift in the goals was from avoiding student mistakes to understanding the thinking behind student errors. Teachers' initial talk in workshop indicated that they tried to avoid students' mistakes by giving clear explanations. However, there were differences among teachers in the extent of sensitivity they developed towards student thinking. Rajni and Ajay still insisted on many occasions that procedures should be told to students clearly to avoid mistakes. On the other hand, Swati and Anita discussed both the mistakes that students made in their classrooms and the possible underlying misconceptions. This indicated that they may have started to accept mistakes as the natural outcome of learning experiences and wanted to share the misconceptions and mistakes for discussion. This was reflected in Swati's comments about how one needs to focus on what students do and why rather than what student or teacher should have done. She said, "That the student should know is different, whether the child does is different. That is the problem, we are discussing here for what [the] child does". Swati shared her realization about how students typically overgeneralize and try to make connections between different concepts that are discussed in the classroom.

Excerpt 7.19

> Swati: They say if we are doing this here then why cannot we do this there. Then again, I think, this is how children connect you know. Whatever we are teaching, they do try to connect. They try to apply one thing in another context. That is why understanding, I think explaining them and understanding is more important you know than just practicing. So if both do not happen together, learning won't be proper. (Workshop Excerpt, 24–11–2010)

Covering the textbook and the exercises given in it seemed to be an important goal for teachers initially as they resisted approaches not discussed in textbook.

Excerpt 7.20

> Anita: Inverse method we can use that but exam point of view we will not be able to use that. In exam number line question only will come.
>
> (Workshop Excerpt, 30–07–2010)

The shift in beliefs about teaching integers through rules versus engaging students in reasoning involved teachers experiencing the struggle of managing classroom interactions to develop understanding while developing their own knowledge in order to support students' engagement with reasons. The expectations of completing the syllabus on time also added to the tension. Swati shared how it was challenging to engage students in reasoning because of lack of time and students' resistance to engage in making sense rather than solving the problem using rules.

Excerpt 7.21

> Swati: They had to apply only rules. Only one thing was there, there was not so much thought process – like they had to just learn the rule and it was like that. Why

> it is happening – that they didn't use to think. I know it is going to take a very long time. You know, first, we have to change ourselves. We have to, we also want to do things quickly. You know we are worried about the portion [completing the syllabus] so we want to do fast, then slowly and slowly if we do.... If not in all, in few classes if we do. Once the children start thinking, I think our job will really become easy. They will start applying their mind to all other things. Only they have to be motivated that we should think and do. (Workshop Excerpt, 24–11–2010)

Swati pointed how it is difficult to change practice in teaching only one topic in one of the classes while students experience traditional teaching in all other classes by other teachers. This explains the confusion and resistance among students to engage in reasoning or sense making using contexts that the teachers were trying to use in the class. Taking a larger perspective, this points to how efforts to reform teaching by a single teacher in a school makes her feel isolated. The teachers felt that other teachers and principals in the schools need to listen and understand the efforts that they were making. Anita tried to organize a workshop for mathematics teachers at her school in which she shared the resources and ideas that she was using in her teaching for integers. Teachers' taking up the role of resource person in a workshop, in which they shared not only their accomplishments but also their struggles is another expression of their felt need for a community to exchange ideas about teaching mathematics. They also expressed the desire to have collaborations with other teachers for planning the teaching of topics other than integers.

7.7.2.2 Going beyond the textbook by designing tasks

When teachers designed tasks and contexts on their own in the workshops, it motivated them to go beyond the textbooks and use these tasks in the classroom (Kumar & Subramaniam, 2015). Teachers' reflections indicate that they used these tasks to engage students in thinking and reasoning rather than having them solve the tasks mechanically using a known procedure. Thus the goal of covering the textbook made way for the use of tasks to engage students in thinking.

Excerpt 7.22

> Swati: Some children had already done. They knew the answer but when I asked them to explain they were not able to explain so taking such an activity made them also think. When I asked the reasoning they told Ma'am we do not know the reason but we know this is the answer. Because they have learnt the rules, they directly learnt it. Learning rules is easier and those bright children you know they are able to learn the rules very fast. What is the disadvantage [is that] they do not want to know the reason (...) so that is advantage of having something different in the class which is not there in the textbook (Workshop excerpt, 24–11–10)

To encourage student thinking she suggested the use of open ended questions and gave the example of how she had collaborated with the researcher to construct a test having open-ended questions. While Swati's earlier stance on learning mathematics was that practice of a number of problems was necessary to become fluent, in her presentation to peer teachers, she emphasized that development of understanding is as important as practicing exercise problems and without understanding a student may just blindly follow or over-generalize procedures.

This focus on reasoning also indicated her change of focus in assessment from evaluating whether the answer was correct or incorrect to evaluating whether the student is able to explain her/his answer. As noted earlier, teachers' resistance to alternative approaches was also determined by the assessment practice of giving only textbook questions in exams. Teachers' engagement in designing tasks different from the ones given in textbooks based on the meanings of integers led teachers to look critically at their assessment practice. Many students used to cover the textbook ahead of lessons in tuition classes, which resulted in them knowing the procedures and answers to most questions discussed from the textbook. They quickly solved the textbook problems and were considered as bright students by the teachers. Swati realized that this identification of students as "bright" based on their quick response to textbook questions is an artefact of the practice of using only textbook questions rather than an indication of real learning by students. While addressing peer teachers, she spoke about this common problem faced by many teachers across the education system.

Excerpt 7.23

> Swati: Students whom we call bright are not really bright because it's just that they have already done the chapter and thus know the answers but if we twist the question they are not able to answer. They do not know the basics but they will solve it. (Workshop excerpt, 26–11–10)

In the same vein, Swati talked about how it made children *think* instead of solving problems mechanically, especially when tuition classes drilled solving problems mechanically.

Excerpt 7.24

> Swati: So when you are taking a new example which is not there, they have not gone through that example, it catches their attention also and we can involve the whole class with such an activity. And actually, that is what happened when I took the lift case [i.e., integer mall context] in the class. There were some very bright children also who used to – you know when I used to ask them 2 – 3, five or six of them were ready with the answer. But when I took the lift problem they also tried to do and actually find the answer. They were also taking time to find the answer and this also gave an opportunity to other children who participated and we also framed questions and exercises which were not there in the book. The benefit of having different exercises, different examples is that they are motivated to think and answer,

they are not ready with the answer. (Workshop excerpt, Swati, 26–11–2010)

Anita acknowledged that "usually we follow the textbook method, this was entirely different as I used this button activity and lift context wherever required". She explained how students were able to explain their answers and give reasons using these contexts. Thus both teachers acknowledged that the tasks designed by the teachers helped in engaging students in thinking and reasoning with mathematics. They both also shared students' errors and unexpected interpretations that they came across in their teaching. Their reflections indicated that they had begun to accept that, while exploring models and contexts, students may interpret mathematical expressions differently and may over–generalize certain observations. For e.g., in Anita's class, a student interpreted the expression 4 – 3 as moving 4 floors up starting from the 5th floor and then coming down 3 floors. Although this was different from the movement that she intended the expression to convey (starting from the fourth floor and moving three floors down), she responded to the student by representing her answer as 5 + 4 – 3 where she discussed the resultant expression 5 +1=6 and in the context also discussed the net movement in terms of expression 4 – 3 being one. While reporting this episode, she pointed out how she had changed her "track according to student's answer" and shared how "this makes us also feel good in being able to interpret their answers. That is why it is a learning experience. Instead of saying their answer is wrong it is one way of thinking".

The most significant way in which teachers went beyond the textbook was in the way they used students' thinking as a basis to make pedagogical decisions rather than mechanically following the textbook. Swati shared the change that she had observed in her own teaching.

Excerpt 7.25

> Swati: Actually we did it in so much detail here so I could – I was more aware. I realized that the students need a clearer understanding of integer, otherwise we would clearly say 'no, not like this– do like this'. This is how we used to deal. So, that is the change in us I could observe. (Workshop excerpt, 20–11–10)

Anita gave examples of 'eliciting questions' that can lead to students' thinking like "How to represent the loss or gain of weight through integers?" and in this way "go beyond the textbook as such questions are not there in the textbook". She justified that it helps the student in "relating to their daily life situation" and gives students opportunities to speak and "not just sit and listen, they speak up, they tell their mind".

Teachers' sensitivity to students' thinking was also indicated in their arguments for evaluating contexts from their experiences of using them in their teaching. Swati felt that the borrowing–lending context was not that useful for comparing integers as representing the borrowed money is done in negative integers but students tend to think in terms of the amount of money borrowed, i.e., borrowing Rs 3 is more than borrowing Rs. 2 and thus think that –3 is more than –2. Rajni stated that one should ask the question to students in terms of how much money is there in hand rather than who borrowed more. Ajay felt that "who is more rich?" is a better question. He felt that the temperature context also has a similar problem because when the student compares –2 and –5 deg C she knows that –5 deg C is colder than –2 deg C and thus finds it problematic to believe that –5 is less than –2. Anita shared that the idea of getting positive and negative points for right and wrong answer was useful for comparing integers as students were able to reason that person getting –3 has a lower score than a person getting –2 as the first person has made more mistakes. Here the teachers are evaluating a representation as useful based on the meaningfulness of the contexts for students indicating that they may have internalized this criterion.

Teachers discussed and used student errors and responses as the point of focus to discuss issues about teaching integers in the workshop while discussing their classroom experiences and while playing the role of speaker in the workshop for peer teachers. Anita discussed how student interpretations of symbolic representations through the use of contexts and vice versa might be different from what the teacher intends and thus there is a need to have a discussion on how contexts can be represented through symbolic expressions and how symbolic expressions can be translated in contexts. Anita shared how it was enriching to hear student examples.

Excerpt 7.26

> Anita: I had to relate their answer to my questions. So that is what we should know immediately how to interpret their answers suitable to us. This makes us also feel good in being able to interpret their answers. That also gives us some satisfaction. That is why it is a learning experience, instead of saying their answer is wrong it is one way of thinking, makes us feel good. They gave so many examples [which] we never expected like that eyesight (prescription) number doctors give, increasing and decreasing the (TV) volume by remote. All this comes from their own contexts. So next year I am enriched with all these examples. (Workshop excerpt, 24–11–10)

While making a presentation to peer teachers, Ajay discussed the difficulty students faced when understanding that 5–3 cannot be equal to 3–5 since they have not developed the meaning of negative sign as the sign of integer and thus will say that "larger number cannot be subtracted from [smaller] integer". The fact that he chose to talk about challenges faced by the student rather than how rules should be told to students (which was the focus of his talk in the initial workshops) indicates the sensitivity he had developed for seeing the mathematics from the students' perspective.

Thus, in the teachers' talk there was indication of considering the students' thinking while making pedagogical decisions like selecting tasks, questions and explanations which would address challenges faced by students and can build upon students' thinking. Swati and Anita's teaching became more responsive to students as they voluntarily spent more time than was slotted officially for teaching the topic of integers in spite of the perceived lack of time. Swati shared that "Generally it takes more time, because when we go to the class, we interact with the children, then, many times we have to change what we have planned because this is what happened with me in the class."

The teachers also made connections between the learning of integers and the learning of topics in algebra. While all the teachers used both the horizontal and vertical number line, Swati briefly mentioned in her class the connection of these to the topic of Cartesian co-ordinates where students would use both horizontal and vertical number line, thus justifying studying both number lines. Anita gave the example of how she used the change meaning of integers in her teaching of algebra by connecting use of the phrases "more than" and "less than" with mathematical expressions. She shared how she had used this meaning while representing 3 more than –2 in the integer mall context. She found that it was very easy for students to understand and relate with the expression x +3 as 3 more than any number. Teachers discussed how students face problems when substituting integer values in identities like $(a+b)^2$, $(a-b)^2$ and (x+a)(x+b), which leads to confusion between the sign of integer and the operation. They felt that developing meanings of integer and operation will help them to make sense of these identities. Swati was able to identify a conceptual gap in a student's work on polynomials, when he was not able to understand that subtraction and changing the sign of the terms in an expression is equivalent and thus changed the sign as well as subtracted the terms. She felt that "It is not that he has mixed the method/ it is the concept which is wrong."

7.8 Conclusions and implications

The findings described above indicate shifts in the teachers' goals, beliefs, knowledge and practice. The teachers' discourse and reports of their teaching indicated a shift from the goal of telling rules to that of constructing rules through the use of contexts and models and to developing an understanding of meanings of integers and their operations, and a shift from following the textbook to designing tasks to provide opportunities for meaning making and reasoning. The initial goals were closely connected to the textbook in terms of emphasizing rules and computational problems, consistent with the teachers' role as followers of the textbook. The education system reinforced their role in following the textbook through the norm of giving textbook based questions for assessment.

In the initial teacher talk, attributing students' errors to "forgetting" indicated that they believed that memorization is important in learning mathematics and focused on helping students remember the procedures. The teaching goals of developing computational fluency and avoidance of errors also aligned with these beliefs along with preference for symbolic representations and rules. They spoke about student errors and their teaching concerns in a manner largely disconnected from issues of meaning and focused on procedures as conventions dissociated from meaning. They also explicitly disavowed that students' difficulties were with the meaning of integers.

7.8.1 Development of SCK for teaching integers

Analysis of the initial talk by teachers in the topic study workshops about student errors and representations indicated gaps in the teachers' SCK in terms of their limited repertoire of representations, the explanations associated with their use of representations and making connections between representations.

Teacher beliefs and SCK indicated a shift as shared criteria for determining representational adequacy developed in the group through the exploration of meanings in relation to contexts using the integer meanings framework. It led to an increase in the variety of situations that can be represented by integers. This contributes to increase in the richness of the example space (Watson and Mason, 2005) that teachers can access for generating tasks, guiding classroom interactions and assessing learners' understanding. The use of meanings and variety of contexts may contribute towards developing a more robust conceptual structure for teaching integers. Appreciating the change meaning of integers allowed the teachers to use contexts where "changes" could be combined in the form of integer addition, leading one of the teachers to design a context of a bowl containing an unknown number of stones, to which stones could be added (an increase represented by a positive integer), or from which stones could be removed (a decrease represented by a negative integer). In analyzing a variety of contexts, the teachers used integers to represent derived quantities that were different from the salient quantities – for example, change in temperature as opposed to temperature, change in baby's weight as opposed to weight, and relative position in the mall as opposed to floor number. This led teachers to design and adopt such contexts where integers represent change and hence could be added meaningfully – hourly change in temperature, weekly change in a baby's weight and movement of a lift in an "integer mall". Interpreting an integer as representing a "static relation" allowed further exploration of contexts and the possibility of modeling the subtraction operation using contexts. In the integer mall, for example, subtraction was used to find the movement required to move to a target floor from a given floor.

The teachers' movement from using integers to represent only states to representing transformation and relation is an important move, whose significance and challenge has been identified by other researchers (Thompson & Dreyfus, 1988). This is related to the move from representing transformation using the subtraction operation to representing it using an integer. The teachers initially chose to represent change by means of the subtraction operation rather than using integers. Representing the *process* of change using an integer is an essential step – reifying a transformation into an object that can be represented as a number. In some accounts, this is at the heart of algebraic ways of thinking (Sfard, 1991), which calls for flexibly interpreting symbols as representing both process and object. The move from representing transformations as operations to representing them as integers similarly reflects a flexible understanding of process–object duality.

Merely becoming aware of the various meanings of the minus sign, of integers and of integer addition and subtraction does not constitute SCK for teaching mathematics. Using the framework of meanings, teachers need to construct further elements of SCK by relating it on the one hand to teaching concerns and on the other to representations. Evidence was found of three ways in which such construction of SCK was made by teachers in the workshops. Firstly, as mentioned before, teachers identified features and processes associated with representations, especially contexts, that corresponded to one or the other meaning of integers. Secondly, teachers connected various meanings of integers through their insight about the key idea of a reference point. They noted that in contexts where a sequence of changes is represented by integers, the reference point is constantly shifting. They noted that to represent state using integers, they need to fix a "zero" as a reference point by convention, while to represent relations, the reference point is arbitrary. The teachers also made connections across different layers of meaning, by relating the distinction between the two meanings of the minus sign (integer and subtraction operation) to the distinction between the state and change meanings of integers. Ajay teacher raised the question, also raised by Anita's students, as to why basement floors are marked with a minus sign. At the heart of this question is an important mathematical idea, namely that the sign used for the subtraction operation is also the sign for a negative integer. Using the insight about the connection between the state and change meanings, Anita was able to explain that the floor number was related to the amount of change needed to reach the floor from the reference point of the zeroth floor. Finally, teachers used the framework of meanings to interpret student errors (the difficulty in extending the take away meaning of subtraction to "taking away" a negative integer), to offer explanations using representations (moving right on the number line corresponds to an increase) and finding new ways of modeling procedures for addition and subtraction using representations (subtraction using the neutralization model, or using the "integer mall").

The response and take–up by the teachers in the study supports the claim that the framework of integer meanings forms an important part of the SCK for teaching the topic of integers. Following exposure to and work with the different meanings of integers, I noted several key movements and shifts in the participating teachers' discourse. These included a movement towards relating teaching concerns with issues of meaning, a shift towards using context–based representations rather than the exclusive use of formal models to teach integer operations, developing lenses to analyze contexts in terms of the meanings of integers embedded in them, using such analyses to make judgements about the appropriateness of representations for teaching and learning, striving for consistency of meaning and designing contexts for teaching. These shifts occurred in parallel with the evolution of shared criteria for evaluating, using and designing representations based on their translatability among representations, meaningfulness and maintaining consistency of meaning in explanations while having a concern for meanings held by students. More importantly, we witnessed teachers constructing, for themselves, SCK for teaching integers through interconnecting different meanings associated with integers. Further evidence of the importance of the framework of meanings comes from teachers' take–up of resources and ideas from the workshop into their classrooms, and their self–reports concerning the relevance of integer meanings for teaching.

The teachers' construction of knowledge by probing meanings associated with representations indicates the importance of understanding the distinctions and connections between the several meanings of integers. This suggests that a framework that distinguishes different meanings may function as a foundation on which further elements of knowledge relevant to teaching could be built. In this study, I have chosen to support this claim with a fine–grained description of teachers' construction of elements of SCK, rather than probe what individual teachers' gains in SCK using objective measures. Using measures such as a paper–pencil test would not have made it possible to capture teachers' construction in a detailed manner. However, I believe that on the basis of detailed descriptions of teachers' constructions, it may be possible to develop further measures of SCK that are detailed and specific.

We may note that all the teachers were highly experienced, knowledgeable and resourceful. They had many years of teaching experience. They were aware of student errors and were familiar with the textbook and the curriculum. Given this fact, the lack of detailed attention paid to issues of meaning in the initial phase of the workshop was remarkable. It suggests that the knowledge encoded by the framework of integer meanings is an important part of SCK that is not gained directly through the practice of teaching alone. One reason for this might be that developing such distinctions and frameworks needs deep engagement with issues connected with both content and with the learning of content. Hence SCK elements such as integer meanings may be important bridges between the knowledge acquired through mathematics education research and the knowledge that is essential for effective teaching.

Observation of teaching by three of the four teachers revealed that there were variations in the knowledge and understanding exhibited by the teachers in the workshop context and the way teachers used ideas, contexts and representations in the classroom. This can be explained as the interaction of complex factors that come into play while teaching in the classrooms. These factors range from the practical ones like pressure for completing the syllabus on time, following the textbook and expectation of students for teaching rules and procedures to cognitive and social factors too. The cognitive factors can be ascertained by analyzing what differs in the context of the workshop with that of the classroom. While teaching in the classroom, teachers had to manage the students' responses, many of them unexpected, while fulfilling the responsibility of making them learn mathematics so that they get good grades in the exam. It is known that going from acquiring a piece of knowledge to using it effectively, involves a process of use and reflection and may require several iterations Teachers were in a position where they had to use newly acquired knowledge to replace the explanations and approaches that they had been using for years. Given the slow nature of change in developing knowledge to use representations meaningfully and to use practices to support meaning making in classroom, it is understandable that teachers showed back and forth movement using ideas developed in workshop and the ideas that they have been using for teaching integers.

The remarks above also suggest that SCK elements such as connections between meaning and representation are important for both pre–service and in–service teacher education. There is an under–emphasis on content knowledge in teacher preparation (Chazan & Ball 1999). The analysis points to how such content could be designed, at least for certain topics in school mathematics. Exploring distinctions and connections among meanings of mathematical objects and processes, between mathematical objects and various representations, may be important to include as part of the mathematical knowledge required to prepare teachers. Such "framework of meanings" may be important not only for the topic of integers, but also for other topics such as fractions, and operations with fractions and whole numbers (Kieren 1988; Ma 1999; Fuson 1992). Thus, the SCK elements identified in this chapter could expanded to other topics, and could form the basis for work with pre– and in–service teachers, exploring ways in which teachers construct SCK using meaning frameworks as the foundation. Even within the topic of integers, the framework that was developed did not include the meanings associated with integer multiplication and division. This is work that remains to be done.

7.8.2 Criteria for representational adequacy

It is in the nature of mathematics that it has connections with reality and can be used to make sense of real phenomena. Doing mathematics involves mathematization of reality to the effect that it abstracts/idealizes the reality. As a result, the mathematical model may not exactly correspond to reality but has certain essential characteristic related to the mathematical concept in focus. Thus representing realistic contexts mathematically is a challenge as not all contexts or all aspects of a context can correspond to a mathematical idea. Rather it might be meaningful to represent some contexts or aspects of a context mathematically but not others. It is the process of translation from contexts and models to symbolic representation and vice versa that creates opportunities for establishing correspondence, consistency and meaningfulness. In the number line model, it is not meaningful to represent addition of states (positions) but it is possible to add a movement to a state. It is possible to represent addition of integers meaningfully in contexts where quantities increase and decrease and to represent subtraction of integers as comparing quantities or magnitudes. Different realistic contexts may together correspond to the idealized mathematical rules, but it may be difficult to find a context which can represent all the facets of a mathematical concept meaningfully. Therein lies the challenge for teachers to build mathematical understanding by giving opportunities of generalization after experience with carefully selected contexts and reflecting about mathematical aspects that can be meaningfully represented.

Teachers' initial talk and criteria used for evaluating representations indicated that teachers tried translations between models and symbolic representations through arbitrary rules which did not have any justification and indicated surface level concerns for representational adequacy. These criteria indicated the beliefs that teachers held about representations, about mathematics and about teaching and learning of mathematics. Teachers believed that symbolic representations are more efficient than other more concrete representations like models or contexts. This preference for symbolic representations was also indicated by Chinese teachers (Cai, 2006) as concrete or visual approaches were believed to be not useful for representing larger numbers. Other studies have also found that teachers considered symbolic and numerical representations as more central to learning and doing mathematics as compared to visual which are termed as "informal" (Stylianou, 2010; Bergquist, 2005). Teachers' exploration of models and contexts made teachers experience how larger numbers and operations could be represented in them which they had earlier thought not to be possible. In the process, their criteria for judging representational adequacy began to include criteria of meaningfulness and consistency-coherence as discussed in Section 7.6.

The shift that teachers exhibited to a deeper level of concern for representational adequacy from a surface level could be due to the type of interactions in a collaborative setting. Teachers' explanation of representations were critiqued, challenged and subjected to analysis as to how they represent different facets of integer concept as well as discussions about why the particular representation works. Teachers also were exposed to a variety of representations and their explanation and justification given by their colleagues and the teacher educators. This pushed the need to consider their choice of representation more deeply. Further, teachers identified and analyzed meanings of integers embedded in different contexts and designed contexts to focus on particular meaning of integers or to teach integer addition and subtraction. As teachers themselves explored meanings of contextual representations, they became aware of how contextual representations are a tool for exploration of meanings of mathematical objects.

The negotiation of meaning and collective critique of the adequacy of representations indicate the development of meta–representational competence among teachers, which is the capability to critique and design representations. Hence, the collaborative discussions helped build both knowledge of representations and knowledge about representations. Teachers' exploration of meaning of integers and operations in contexts helped them in rethinking their focus in teaching and raise questions like "why negative sign should be given to basement floors?" It thus helped teachers in realizing that representations are not transparent and how meanings need to be explicitly discussed. Some teachers also thought about the opportunities where students can reason mathematically using contexts rather than using contexts as site of application of procedures. Thus, it is not enough that teachers use multiple representations but more important is *how* teachers use and connect these representations.

The study points to the significance of the criteria that teachers hold for selecting, designing and using representations. I have focused on mathematical criteria like translatability, consistency and meaningfulness. There are indications in data of the existence of other criteria that I have not explored like curricular limitations and beliefs about student capability. More research is needed to know about other mathematical and non-mathematical criteria that are used by teachers and how they influence the selection, design and evaluation of representations. I have not analyzed teachers' use of representations in classrooms and the criteria that govern their use in this paper. It would be interesting to explore whether the criteria remain the same or differ from what were used in professional development setting. It would also be interesting to look at criteria used by researchers, textbook writers and teacher educators since it can illuminate the differences in concerns of various people involved and can initiate dialogue about the important criteria that need to be focused. It is important since in-spite of the wide range of representations available for teaching certain topics in curriculum and research, we need to know more about how teachers actually use these representations. More work is needed to know how teachers can make these criteria explicit, develop sound criteria in professional development contexts and use it in classrooms. Developing these criteria and having awareness of them will help teachers in the long term in augmenting their own professional growth by studying and making decisions about representations.

8

Conclusion

8.1 Introduction

In India, various efforts have been made to implement the National Curriculum Framework, NCF 2005, which prioritizes child-centered teaching and learning with understanding (NCERT, 2005). The NCF 2005 places before teachers the challenges of providing children quality education through student centered pedagogy, assessing students comprehensively and continuously, and relating school subjects with daily lives of children. NCF 2005 has been criticized for being silent on how teachers are supposed to bring about the change in their classroom and for not addressing the much needed teacher development to support curriculum renewal (Batra, 2005). Efforts undertaken like changing textbooks and issuing directives to schools and teachers sidestep the issue of developing adequate beliefs and knowledge amongst teachers, which is needed to realize the vision portrayed in the new curriculum framework. Although many workshops have been conducted to "orient" the teachers to the new curriculum and textbooks, its impact on the classroom teaching is not adequately researched. The challenges faced by teachers in implementing the reform efforts and the support they need in meeting these challenges provided the overarching focus of the present study.

As discussed in the opening chapter, the NCF 2005 advocates a shift away from a textbook centered rote learning approach, to one that emphasizes the link between school learning and life outside school. It gives precedence to the goal of mathematical thinking or mathematization, rather than "knowing mathematics" as a set of rules and facts. It expects from the teacher a deeper understanding of subject matter as well as the teaching learning process, rather than merely adopting new techniques. Teachers in elementary and middle grades are expected to not only make their students fluent in computational mathematics but also address process goals in the learning of mathematics, such as reasoning, using multiple ways to solve problems, justifying their solution, making generalizations and conjectures, analyzing the mathematical work of others, etc. (NCERT, 2006). However, there have been few Teacher Professional Development (TPD) programs in India, which have focused on the beliefs and knowledge required to facilitate this kind of teaching. Studies elsewhere in the world have indicated that focus on change in teaching strategies without taking *teacher thinking* into consideration leads to teachers making superficial changes that do not bring about significant change in student learning opportunities (Cohen, 1990). It is therefore important to first understand the beliefs and practices that are prevalent among teachers in order to support reform in teaching that is not superficial.

This study has four sub-studies presented in Chapters 4 to 7. Chapter 4 discussed the teachers' preferred practices as well as beliefs at the beginning of the study, which is characterized as Sub-study 1. The findings of this sub-study serve as a background to the findings of the other sub-studies, in which teachers engaged in different types of professional development activities. Sub-study 2 in Chapter 5 described teachers' engagement in a professional development workshop by analyzing the tasks as well as interactions that occurred in the workshops. Sub-study 3 described in Chapter 6 is a case study of a teacher who participated in the orientation workshop and showed inclination to change her practices towards teaching mathematics with understanding. The sub-study highlights the challenges that arise when a teacher may agree with the philosophy of curriculum reform but still needs effort and relevant knowledge to engage students in developing an understanding of mathematics. Sub-study 4 highlighted the importance of developing specialized content knowledge on the topic of integers amongst teachers by engaging in topic-focused professional development. Teachers developed their knowledge of the meaning of integers and integer operation to construct and use tasks for teaching integers. All the sub-studies are mainly qualitative in nature and serve to enhance our understanding of how beliefs, practices and knowledge interact in the teaching of mathematics. In this chapter, I provide a summary of findings from the previous chapters and draw conclusions across the chapters about teachers' practices, beliefs and knowledge as well as the impact of professional development initiatives on participant teachers' beliefs and practices. I also discuss the implications for professional development initiatives and the limitations of the study and make suggestions for further studies.

Researchers in math education have studied TPD in a variety of settings and contexts. There exist several models of TPD emphasizing teachers' agency by providing opportunities for active engagement in different settings. However, how teachers' agency and interaction in different professional development settings impact the beliefs and knowledge held by teachers and influence the types of practices used in the classroom is addressed by this study. In this study, analysis shows how beliefs support or constrain the change process as well as analyzing the dialectical and interactional process between the beliefs, knowledge and the practices used by teachers in the classroom. Frameworks that emphasize mathematical knowledge required for teaching as well as the agency of teachers, guided the analysis as well as the design of tasks and enactment in the professional development space, while analysis of teachers' interaction in the workshops and in the classroom with students helped in identifying challenges faced by teachers as well as their learning in the classroom space when they tried exploring new practices.

8.2 Research study overview

The study reported here was situated in the context of a TPD intervention, whose overall aim was to promote change in teachers' practice towards teaching that is more responsive to the development of students' understanding. The question investigated in the study was: In the Indian context, characterized by reform efforts, what factors support teachers in adopting learner centered practices in the classroom and what factors inhibit or constrain them in doing so. This question was interpreted in terms of a framework that took teachers' beliefs, knowledge and goals as the core components of teacher learning, giving rise to specific research questions addressed in four sub-studies. The methodological approach followed was participant observation and the methods of analysis were qualitative including case studies, supplemented with quantitative analysis for the belief questionnaire.

The participants of the study were mathematics teachers, who participated in professional development opportunities in three different settings:

(i) Professional development workshop for ten days (including a non-working Sunday) using tasks situated in the work of teaching, posing challenges to teachers' thinking and building a sense of community,

(ii) Collaboration between the researcher and the teacher in the classroom with a view to support adoption of practices conducive for developing understanding of mathematics, and

(iii) Topic study group of teachers as an adapted form of lesson study where teachers explored meanings and representations of integers, made lesson plans, taught the topic in their respective classrooms and shared their learning in a workshop with each other and another group of teachers.

The first two sub-studies were located in the workshop setting and focused on (1) teachers' beliefs and (2) design and interaction in the workshop during year 1. Sub-study 3 focused on one teacher's attempts to change her practice and was located in the classroom collaboration setting across the two years. Sub-study 4 in year 2 studied teachers' engagement with beliefs and knowledge in a topic study group through use of a framework for meanings and representations of integers and their use of this knowledge in their instructional planning and teaching of integers. The research questions addressed in each of the sub-studies are recalled before summarizing the findings in the following sections.

8.3 Sub-study 1: Teachers' beliefs and practice

The sub-study described in Chapter 4 of the thesis analyses the questionnaire responses of 26 teachers and interview responses of 11 teachers from the same school system, who participated in the orientation workshop. The questionnaire and interview were designed to assess beliefs of teachers regarding mathematics, teaching of mathematics, students and self along with the practices preferred for teaching mathematics. The research questions addressed were:

1. What are the core and peripheral practices of the teachers in the sample with respect to the teaching of mathematics?
2. What beliefs are core or peripheral as indicated by the teachers' articulation and the practices preferred by the teachers?
3. What is the relation between beliefs expressed and the practices preferred by the teachers?
4. What is indicated about teachers' knowledge from their explanations? What is the relation between preferred practices and the knowledge held by teachers?

The answers to the research questions obtained from the analysis of teachers' responses are summarized below. Sub-sections 8.3.1 and 8.3.2 address research question 1. Research question 2 is answered in Sub-section 8.3.3, research question 3 in Sub-section 8.3.4 and research question 4 in Sub-section 8.3.5.

8.3.1 Framework for analyzing beliefs and practice

The framework that has been used to analyze teachers' practice posits a continuum from transmission based teaching to student-centered teaching, where the student-centered end views the teacher in an active role basing his/her pedagogical decisions on students' thinking. I introduce a distinction between core and peripheral practices based on the frequency with which the practices were reported and overall consistency with each other. The distinction is related to the distinction between core and peripheral beliefs held by teachers. Core beliefs are expressed strongly in teacher responses. Peripheral beliefs are inferred by considering the tensions between the beliefs and practices reported by the teachers.

Questionnaire responses indicated alignment towards a student-centered view but the interview responses revealed a closer alignment to the transmissionist view. This is because the teachers elaborated on examples and gave explanations of the terms used during the interviews, while their responses to the questionnaires do not reveal the underlying interpretation. The teachers incorporated some student-centered practices into their repertoire as peripheral practices, while core practices were transmission based. The interviews also revealed the gaps in teachers' thinking about the purpose of student-centered practices and showed that they had limited knowledge of why and how procedures work. Thus there is need for triangulating teachers' expressions across questionnaire and interview, along with analysis of causes for why certain beliefs are espoused.

8.3.2 Core and peripheral practices

The analysis of data across questionnaire and interview, showed four core practices that were preferred among the teachers in the group and were reportedly used regularly by most teachers. These were teaching by showing procedures or solved examples, giving students repeated practice of solving problems, focus on speedy solutions through teaching shortcuts and close following of the textbook by doing exercises and problems. Peripheral practices were used less regularly and were given less priority by teachers. They included use of activities for introducing a topic or to help remember a procedure, focusing on explanation and justification, and connecting students' everyday experience with mathematics done in classroom.

8.3.3 Core and peripheral beliefs

The findings about practices indicated that the core beliefs held by teachers about mathematics, and about teaching and students, showed greater consistency with the transmission view of teaching and procedural view of mathematics. The teachers' insistence on practice for learning mathematics also pointed towards the belief that learning mathematics calls for memorization. Most believed that it is not possible for students to come up with mathematical ideas on their own without being taught (Kumar & Subramaniam, 2013).

The teachers recognized the role of justification and reasoning in school maths but still considered maths as restricted to learning the four operations. Tensions were evident in their talk concerning a focus on procedures versus a focus on reasoning. The teachers considered mathematics as difficult for students and tried to make it easy and interesting using concrete materials and activities. However topics in the higher grades like algebra and geometry were considered as difficult to represent through concrete material. The teachers showed a positive attitude towards using contexts from daily life but used them as descriptions of the problem and rarely focused on the mathematical meanings within contexts. They gave more socially appropriate responses to questionnaire items on class and gender bias. However, in the interview, the teachers' responses indicated that they have lower expectations from poor or girl students and that they focus on repeated practice and memorization of problems likely to appear in the exams to make weak students pass. Teachers who had positive experience with maths in their school education were critical of the lack of practice exercises in the new textbooks. Some of the teachers, especially primary teachers who had had unpleasant experiences, were critical of the widely prevalent practice of rote memorization in learning mathematics and appreciated the reform approach reflected in the textbook. The teachers spoke about the pressure to get 100% pass percentage results in examinations.

8.3.4 Interaction between practice and beliefs

I found that the core beliefs together form a coherent stable structure, as these beliefs are in alignment with each other and support the adoption of related practices. For e.g., the core belief of viewing mathematics as consisting of procedures and learning as memorization of these procedures is reinforced by the practice of teaching procedures and of repeated practice for memorization. Further, such practices and beliefs are at the core of the teachers' identity as they construct their sense of self from their students' performance on the tests and exams which evaluate their capacity to remember the procedure to solve a particular problem. Teachers' years of experience of learning and teaching mathematics focused on procedures supports the transmissionist view further and adds stability to this core belief structure. It makes the belief structure resistant to educational reform efforts where change is sought through the change in textbooks and issuing of circulars by authorities. Strong beliefs about procedures and memorization constrain the change while belief about maths as abstract made teachers integrate a few practices only superficially.

The reported practices are cognitive images of how teachers view their practice rather than objective descriptions of their practice. Therefore these are indicative of beliefs held by teachers since it involves some generalization and reflection by the teacher to report their teaching. Core beliefs are reflected in the core practices, while articulated beliefs, which are not reflected in practice or were not given due importance by teachers, might be more peripheral in nature. The findings indicate that much of the inconsistency, conflict and tension between beliefs can be inferred even from reports of practice, and not only from observations of actual practice.

8.3.5 Relation between knowledge, beliefs and practice

The interview also revealed that teachers have limited knowledge of alternative methods for the division operation. They believed that alternative methods might be confusing for students and favoured teaching the standard algorithm. The middle school teachers who discussed the Pythagoras theorem elaborated on how they would discuss its verification using different materials with students or use it for calculation but did not speak about developing an understanding of proof and justification. It is hypothesized that the teachers' limited knowledge of procedures and how and why they work may constrain them from trying out newer methods and practices for teaching mathematics.

8.4 Sub-study 2: Principles of design for the workshop

In-service teacher development in India has been driven largely through frequent orientation workshops held for teachers to make them familiar with the expected curriculum and pedagogy as visualized in the curriculum reform documents. Although there has been criticism of such a mode of teacher development, workshops continue to be primary intervention site for TPD. Hence, it is important to identify and analyze the design and enactment aspects of a workshop in the light of its goals to identify opportunities provided for teachers' reflection and learning (Kumar, Subramaniam & Naik, 2013).

In Sub-study 2 reported in Chapter 5 of the thesis, the data is analyzed from a ten day professional development workshop for the study participants. The goal of the workshop was to strengthen teachers' professional knowledge and to provide opportunities to reflect on their underlying beliefs. The analysis of the workshop was undertaken to identify elements that were aligned with this goal. The research questions addressed in this sub-study are:

1. What aspects of the workshop design and enactment are important from a TPD perspective?

2. How did the workshop tasks encode the design principles?
3. How was teachers' agency enabled in the course of the enactment of the workshop?
4. What aspects of the teacher educators' enactment of the task and interaction facilitated engagement by the teachers?
5. What were the learning gains from the PD workshop as perceived by the teachers?

Sub-section 8.4.1 addresses research question 1 by identifying aspects of the workshop design and enactment in the form of a framework that is subsequently used for analysis. Research question 2 is answered in Sub-section 8.4.2, question 3 is answered in Sub-section 8.4.3 and question 4 is answered in Sub-section 8.4.4. Research question 5 is answered in Sub-section 8.4.5, while Sub-section 8.4.6 comments on the usefulness of the framework developed in this sub-study.

8.4.1 Framework for design and enactment of the workshop

The framework consists of principles informing the design and enactment of a professional development workshop that addresses teachers' knowledge and beliefs. The framework, guided by prior research and the research group's experience of supporting TPD, was reconstructed from reflection on the workshop data. The goals of the workshop were to strengthen teachers' professional knowledge for teaching mathematics, provide opportunities for reflection on beliefs and foster the development of a professional learning community. The principles of design of the workshop that were identified were (i) situatedness in the work of teaching, (ii) offering challenges to teachers to revisit their knowledge and beliefs, and (iii) developing a sense of belonging to a professional community. The interaction aspects identified by the framework were (i) task structure (ii) teachers' agency and (iii) teacher educators' agency.

8.4.2 Principles embedded in task design and enactment for the workshop

The tasks used in the sessions drew on artifacts and contexts of teaching mathematics and were focused on mathematical or pedagogical issues that arise in the context of teaching or have implications for teaching. As the teachers were able to relate the artifacts to their practice, they shared their knowledge and beliefs about teaching mathematics in engaging with the tasks. The challenges embedded in the tasks like analyzing student work to explain why a student responded in a particular manner, identifying what a student knew and did not know, led to teachers revisiting the taken-as-granted practices in every day teaching like repeating the procedure again in case of error. The discussions on interesting examples of student thinking and the complexity of teaching helped in establishing the fact that developing knowledge of teaching is one of the main objectives of the community of which teachers, teacher educators as well as researchers are an important part.

8.4.3 Teachers' agency in workshop interactions

The teachers' role in the sessions involved not only sharing the practices adapted by them in their teaching but also bringing to bear their own professional knowledge through conjectures, assertions, counter-arguments, etc. Further, the discussion frequently led to articulation and reflection on beliefs held by the teachers. They were able to reflect on their own teaching as one of the sources for creating misconceptions among the students. However, their identification of conceptual gaps in the students' thinking was constrained by their limited knowledge of concepts.

The analysis of the types of teacher engagement that occurred during the episodes throw light on the kind of opportunities that arose for teacher learning. Teachers' engagement took the form of anticipating and predicting students' responses, identifying key knowledge pieces, conjecturing underlying causes, articulating and contesting beliefs and assessing a teaching resource or a teaching approach. Such engagement was crucial in building shared understanding not only among teachers, who rarely get opportunities to reflect collectively about teaching in their schools, but was also helpful to the teacher educators by providing windows into teacher thinking. Teachers' assertions, counterarguments, alternative explanations and assessments were also a resource, which deepened fellow teachers' and teacher educators' understanding about mathematics teaching as it takes place in classrooms. Teachers' learning from the workshop was reflected in the written feedback that they gave about the workshop as well as the revisions that the teachers made when they were asked once again to respond to the questionnaire items at the end of the workshop.

8.4.4 Teacher educators' agency in workshop interactions

The teacher educators' agency was reflected in the way the teacher educators engaged teachers in discussion. Looking at two contrasting episodes by two different teacher educators, it isfound that re-voicing and making arguments using teachers' assertions indicated high inter-animation in the session and showed increased participation by the teachers. Another factor that supported high inter-animation was the manner in which questions were posed and responses were evaluated by the teacher educator, which shifted the authority of evaluating the opinions to teachers. The teachers educators' beliefs and goals for the workshop as well as knowledge of the teacher's context were visible in the framing of the questions and moves to engage teachers in the discussion. Alternative viewpoints presented by the teacher educator were able to initiate cognitive conflict to make teachers think and reconsider alternative views in their thinking. The teacher educator's efforts to connect the specific discussion in the sessions with the broad goals of teaching of mathematics or about mathematics education were important. Thus, an analysis of the teacher educator's moves show alignment with the goals of the workshop and led to teachers' participation in sharing their knowledge and revisiting their beliefs.

8.4.5 Teachers' learning from the workshop

Teachers were expressive about the gains from the workshop in their written as well as oral feedback given at the end of the workshop. They explicitly appreciated the sessions of live teaching, research readings and maths activity sessions. However, the analysis reveals their engagement in the other sessions as well, and ways in which the engagement served to meet the workshop goals. The main learnings that the teachers spoke of were the realization of the complex and subtle aspects of student thinking, the importance of taking this into account while teaching, awareness about research in the field of mathematics education and its usefulness in improving classroom teaching. Teachers also responded to the beliefs and practices part of the questionnaire again at the end of the workshop, marking some changes in their responses. Although only a few items showed significant difference in the pre- and post-workshop means, the overall pattern in the shifts in responses to questionnaire items indicate that some dissonance occurred as a result of workshop. The changes suggested that teachers were reflecting about their beliefs and had possibly become more sensitive to students' thinking.

8.4.6 Usefulness of the framework for design and enactment of workshops

A framework has been presented that can be applied to an analysis of the components of a TPD workshop and interaction episodes and can identify design as well as enactment aspects. The framework does not describe what constitutes knowledge for teaching mathematics, nor does it elaborate on the nature of beliefs conducive to teaching for understanding. A framework that elaborates on the specifics of knowledge and beliefs relevant to teaching mathematics will need to be contextualized with regard to topics and to teacher communities. The framework presented here, in contrast, identifies certain principles that are important for the design of tasks and their enactment in workshop sessions. This framework would be useful in identifying and providing rich descriptions of elements that are important in a TPD intervention (Kumar, Subramaniam & Naik, 2015a).

8.5 Sub-study 3: Case study of Nupur[1]

Nupur was a mathematics teacher teaching the primary Grades 4 and 5 in a government school at the time of the study. She was a participant in the PD workshop and believed that teaching should be focused on mathematical concepts and that mathematics done in the classroom should be connected to the students' everyday experience. The case study of Nupur was considered as potentially useful since it could provide insights about the extent to which holding positive beliefs for student-centered teaching can motivate a teacher to explore new practices and the kinds of challenges such a teacher might face. This chapter discusses the analysis of lessons taught by Nupur on the topic of fractions post the professional development workshop. The researcher participated in her teaching as an observer and a collaborator.

The research questions addressed in this sub-study are:

1. What changes did Nupur try to bring in her classroom practice specifically with regard to the way in which tasks for students were framed and implemented? How did her beliefs and knowledge support and constrain the changes that she tried to implement?
2. What textbook resources were available to her to support her teaching and how did she make use of these resources?
3. What was the role of the researcher as a collaborator in Nupur's teaching?

A strong belief held by Nupur was that the mathematics that the children learn in school must be connected to their real life experiences. The topic of focus in Nupur's lessons that were selected for analysis was fractions. The theory of fraction sub-constructs (Kieren, 1976; 1988) deals with the variety of fraction interpretations as applied in real life contexts and was thus appropriate to analyze Nupur's teaching in the light of her beliefs and goals.

In the following sub-section (8.5.1), the framework used for analyzing Nupur's teaching is described. Research questions 1 and 2 are answered in three sub-sections below: Sub-section 8.5.2 and 8.5.3 focus on what the findings are from the analysis of task framing and task implementa-

1 Name changed

tion in Nupur's lessons. Sub-section 8.5.4 provides more explicit answers concerning the challenges she faced in providing meaningful interpretations and justifications of students' problem solving strategies. Sub-section 8.5.5 answers research question 3.

8.5.1 Framework for analysis of teaching

Kieren (1988, 1993) identified five sub-constructs of fractions namely part-whole, share, measure, operator and ratio which can be illustrated in different contexts and representations. He argued that children develop an impoverished concept of fractions as a result of being exposed to only those contexts and representations, which exclusively use the part-whole meaning of fractions. In this study, analysis shows how and which sub-constructs of fractions were used in the task framing and task implementation that occurred in selected lessons focusing on equivalent fractions across two years of the study.

The sub-construct framework for fractions was used to analyze the textbook chapter and it was found that the problems in the chapter foregrounded a variety of sub-constructs. The problems also focused on developing reasoning and understanding of fractions among students by using problems connected with daily life and different from the typical problems used for fractions using area representation.

8.5.2 Task framing

The tasks that were used by Nupur in the lessons analyzed were mostly from the textbooks, followed by similar tasks created by her. In the initial lessons these tasks were mostly calculation based while later she used tasks for reasoning and to address students' misconceptions using representations. Although the textbook tasks foregrounded a variety of sub-constructs of fraction using different contexts, Nupur was not able to fully utilize the potential of the tasks and her use of sub-constructs was limited using mostly area representations for discussing fractions.

8.5.3 Task Implementation

How the teacher implemented the task changed across the lessons as she introduced new practices in her repertoire. Nupur's efforts as well as her interaction with the researcher as collaborator led to her becoming more sensitive to students' thinking. She began to ask questions about whether the students really understood a concept, which led to her making changes in the tasks that she introduced in the classroom, as well as the manner in which she orchestrated classroom interaction around the tasks. The task implementation reveals the use of practices such as asking why questions, discussing different and wrong answers of students, building explanations based on students' responses, using representations and concrete materials to develop reasoning, pursuing students' responses and conjectures further to develop understanding of mathematics, giving students more autonomy by asking them to evaluate the answers and establishing equity in the classroom participation.

The examples given in this chapter illustrate how conducive beliefs towards student-centered practices set the stage for the teacher to appropriate these strategies. Her belief that students should engage in reasoning and understand why an answer is correct or incorrect, that repeated practice of similar/same problems leads to rote memorization led her to give more time to discussing reasoning and explanation in class rather than practicing problems. Her belief that understanding concepts takes time also helped as she devoted time for students to come up with their own reasoning and allowing different students to share their answers ensuring equity.

8.5.4 Challenges faced in implementing new practices

While implementing these new practices, Nupur struggled in moving from a focus on procedures to a focus on meaning. In the discussion of different solutions to classroom tasks proposed by students, she struggled to provide interpretations of fractions and operations on fractions meaningfully in contexts. She was only partially successful in eliciting students' reasoning and spontaneous problem-solving strategies. She faced a challenge especially in coherent explanations and justifications based on meaning for the procedures that students presented. This may be due to her inadequate knowledge of fraction sub-constructs, i.e., interpretations of fractions in contexts. Nupur had not explored fraction representations and interpretations in various contexts in depth and she often fell back on earlier practices such as leading to the correct answers or emphasizing procedures.

8.5.5 Role of the researcher as collaborator

In all these efforts at changing questioning, evaluation and explanation based practices, the researcher played the role of a collaborator. The discussions between teacher and the researcher usually focused on students' thinking, planning for teaching and modeling certain practices within the classroom (Kumar & Subramaniam, 2012b). Nupur's efforts as well as her interaction with the researcher as collaborator led to her becoming more sensitive to students' thinking. She began to ask questions about whether students really understood a concept, which led to her making changes in the tasks that she introduced in the classroom, as well as the manner in which she orchestrated classroom interaction around the tasks. This indicates that when a teacher starts to incorporate student-centered practices because of some conducive beliefs, support is needed to sustain the change in practices. For teaching to be determined by the student's thinking, a teacher has to be empowered to take decisions like selecting appropriate examples for teaching, recognizing opportunities from student's responses to develop important concepts, constructing questions for assessment of understanding and also determining which responses of students should and should not be considered as indications of understanding. I infer that the development of specialized professional knowledge of instructional topics is necessary for such development.

8.6 Sub-study 4: Topic focused professional development – The case of integers

Sub-study 4, reported in chapter 7 of the thesis, identifies and analyses the topic specific specialized knowledge of a group of 4 middle school in-service mathematics teachers (Rajni, Swati, Anita and Ajay[2]) on the topic of integers. It tracks the growth of such knowledge as a result of participation in "Topic focused workshops" and its impact on teaching. The 6 one-day workshops were held over a period of five months. The research questions addressed in this sub-study are:

1. What were the teachers' concerns about the teaching of integers and how are they related to issues of meaning of integers?
2. How did teachers construct Specialized Content knowledge (SCK) for teaching integers using the framework of integer meanings through the exploration of contexts?
3. How did the criteria used by teachers for judging adequacy of representations evolve in the course of the topic study workshops?
4. What was the impact of the learnings from the topic study workshops on teaching of integers as reported and as observed?

In Sub-section 8.6.1, the framework used for integer meanings and representational adequacy is described. Research question 1 is answered in Sub-section 8.6.2, question 2 in Sub-section 8.6.3, question 3 in Sub-section 8.6.4 and question 4 in Sub-section 8.6.5.

2 All names changed

8.6.1 Integer meanings and representational adequacy

A framework of specialized content knowledge (SCK) for the topic of integers was developed to analyse the growth of teachers' knowledge in the workshop. Elements of the framework were also used to design the resources used in the workshop. The framework takes into consideration the meanings of the negative sign used in symbolic expressions, the meanings attributed to integers in contexts as well as the models used for ordering integers and operations on integers. The negative sign in symbolic expressions can denote unary (–7), binary (2 – (+7)) and symmetric functions ($-x$) (Vlassis, 2004; 2008). In contexts of application, integers can represent state, change or relation between quantities (Vergnaud, 1982; Kumar, Subramaniam, & Naik, 2015b). Models like the number line and the neutralization model are used by teachers to help make sense of the comparison of integers and operation on integers (Stephan & Akyuz, 2012). We have used this framework to analyze teachers' discourse concerning teaching of integers in the workshop.

The framework was also used to analyze the textbook chapter on integers for Grade 6 (which was the focus of the study). The analysis revealed that the symbolic and the number line representations were predominant in the textbook. Further, integers were used to represent 'state' in most of the textbook tasks, and the integer meanings of 'change' and 'relation' did not emerge adequately.

In addition to the above framework on integer meaning, a framework to analyze the criteria evident in teachers' talk for selecting and using representations for teaching integers has been used. I have identified three dimensions of what teachers consider as "representational adequacy" (diSessa, 2002), namely, translatability, consistency-coherence and meaningfulness of representations. It was found that the criteria teachers applied shifted from surface level concerns to deeper concerns for representational adequacy by applying all the three criteria in a coordinated manner.

8.6.2 Teachers' initial concerns and knowledge of integer meanings and representations

Initial teacher talk in the workshop revealed the concerns that they had about the teaching and learning of the topic of integers. However teachers related students' difficulties to forgetting the rules that they had been taught. The did not think that student difficulties were related to issues of meaning.

The initial talk also revealed their limited repertoire of representations, ways of connecting and using representations and meanings of integers in contexts. When teachers were asked initially to think of contexts for representing integers, they proposed several examples which did not really need integers and could be represented using whole numbers or were restricted to the 'state' meaning for integers.

8.6.3 Extending the range of integer meanings and contexts using SCK framework

In the Topic Focus Group workshops, the teachers were exposed to the integer meanings of 'state', 'change' and 'relation' through worksheet tasks. A large number of contexts were discussed, integer meanings explored and a judgment was made about their pedagogical usefulness. Teachers engaged in discussions of contexts that use change and relation meaning of integers like positive and negative scores in a quiz competition and change in baby's weight. They were able to distinguish the two meanings of the minus sign: indicating subtraction and as a sign for negative integers. They also recognized that the distinction posed a challenge for students to develop an understanding of operations with integers. The exploration of meanings in relation to contexts using the integer meanings framework led to teachers describing an increased variety of situations that can be represented by integers. This contributes to increase in the richness of the example space (Watson and Mason, 2005) that teachers can access for generating tasks, guiding classroom interactions and assessing learners' understanding. Not only more contexts but context features other than 'state' were also represented by integers within the same context. Teachers thus began to move from initial beliefs that symbolic representations are more efficient than other more concrete representations like models or contexts. Initially teachers used contexts only to introduce integers and the need for integers, but not for integer operations. In the workshops, teachers began to explore the use of contexts to explore and learn integer operations.

In a variety of contexts, the teachers used integers to represent *derived quantities* that were different from the salient quantities – for example, change in temperature as opposed to temperature, change in baby's weight as opposed to weight, and relative position in the mall as opposed to floor number. This led the teachers to design and adopt such contexts where integers represent change and hence could be added meaningfully – hourly change in temperature, weekly change in a baby's weight and movement of a lift in an "integer mall". Interpreting an integer as representing a "static relation" allowed further exploration of contexts and the possibility of modeling the subtraction operation using contexts. In the integer mall, for example, subtraction was used to find the movement required to reach a target floor from a given floor. The teachers' shift from using integers to represent only states to representing transformation and relation is an important move, whose significance and challenge has been identified by other researchers (Thompson & Dreyfus, 1988). Teachers identified features and processes associated with representations, especially contexts, that corresponded to one or the other meaning of integers. Teachers were able to use the idea of reference point to distinguish the various meanings of integers and appreciated the importance of the distinction between the two meanings of the minus sign (integer and subtraction operation) for teaching and learning. They were able to use the framework of meanings to interpret student errors, to offer explanations using representations and finding new ways of modeling procedures for addition and subtraction using representations (Kumar, Subramaniam & Naik, 2015b).

8.6.4 Refining the criteria for representational adequacy

Analysis of the teachers' talk in the workshop indicated a shift in the criteria applied to evaluate the adequacy of representation used to teach integers. Initially, the teachers preferred the symbolic mode and the criteria for evaluating representations were based solely on translatability but did not show much concern for meaningfulness or consistency of meaning. The teachers' discourse shifted from attributing students' errors to students' failure to memorize, recognizing that instruction too can lead to errors, and that the meanings that students associate with symbols change as they learn new topics like integers. This indicated that the teachers began to value the consistency in meaning across different representations used for purposes of explanation. The discourse around representations thus deepened to establish connection with meanings and even leading to revisions in teachers' explanations.

Teachers were able to make connections between different representations like number line and an "integer mall" with a lift, using the framework of integer meanings. They were able to interpret movement on the number line as increase or decrease, and to provide a more meaningful explanation of why additive inverses sum to zero (Bajaj & Kumar, 2012). Thus meanings helped in bringing coherence among different representations that could be used for teaching. The discourse of teachers after engagement with the framework of meanings showed deeper concerns for translatability, meaningfulness and consistency of representation and their use.

This shift to a deeper level of concern for representational adequacy could be due to the type of interactions in a collaborative setting. Teachers' explanations of using representations were critiqued, challenged and subjected to analysis by peers as to how they represent different facets of integers as well as discussions about why the particular representation works. Teachers were also exposed to a variety of representations and their explanation and justification given by their colleagues and the teacher educators. This may have pushed the need to consider their choice of representation more deeply, and the capability to critique and design representations. So the collaborative discussions helped build both knowledge of representations and knowledge about representations.

8.6.5 Impact of topic focused professional development

Teachers' learning from participation in these workshops was visible across two dimensions. One was the way in which there was a shift in teachers' goals and beliefs and the other was the extent to which teachers' went beyond the textbook and used the problems designed during the workshop. In their lesson plans and report of teaching experiences, the three teachers (Swati, Anita and Rajni) acknowledged a change in their approach from telling rules in the beginning to exploring contexts first with students and then inducing rules. Another shift in the goals was from avoiding student mistakes to understanding the thinking behind student errors (Kumar & Subramaniam, 2012c). The shift in beliefs about teaching integers through rules versus engaging students in reasoning involved teachers experiencing the struggle of managing classroom interactions to develop understanding while developing their own knowledge in order to support students' engagement in reasoning. The teachers' reflections indicated that they went beyond the textbook by using tasks constructed in the workshop to engage students in thinking and reasoning rather than having them solve the tasks mechanically using a known procedure (Kumar & Subramaniam, 2015).

From the observation of teaching, it was evident that the teachers did use the contexts and key ideas discussed in the workshops, although in different ways and with different degrees of integration with the textbook exercises and teaching of rules. Among the range of representations discussed in the workshop, the teachers used the vertical (integer mall context) number line together with the integer mall context to discuss the meaning of integers, ordering and comparison as well as for addition and subtraction of integers. They found subtraction the most challenging to discuss using the number line. Anita also used the context of team scores (both +ve and –ve) in a quiz competition to discuss addition.

Although the teachers used different representations and meanings in their teaching, there was back and forth movement between discussing meanings using representations and contexts, and using rules to find answer. There was also pressure from students to use rules since they were already familiar with them from their tuition (coaching) classes. The main reason that the teachers cited for not being able to implement the lesson as they had planned was that they faced the pressure of syllabus completion and resistance on part of students to engage in meaning based discussion using contexts. Teachers too find it challenging to engage and respond to students in discussions about meanings of integers and to reason using representations and contexts. However, they identified students' responses which indicated the need to understand meanings and shared them in the workshop discussion and presented them to other teachers.

All the teachers in the study were highly experienced, knowledgeable and resourceful and had many years of teaching experience. They were aware of student errors and were familiar with the textbook and the curriculum. Given this background, the lack of detailed attention paid by the teachers to issues of meaning in the initial phase of the workshop was remarkable. It suggests that the knowledge encoded by the framework of integer meanings is an important part of SCK that is not gained directly through the practice of teaching alone. One reason for this might be that developing such distinctions and frameworks needs deep engagement with issues connected with both content and with the learning of content. Hence SCK elements such as integer meanings may be important bridges between the knowledge acquired through mathematics education research and the knowledge that is essential for effective teaching.

8.7 Implications for teacher professional development

The sub-studies indicate the features in the professional development design that worked in promoting professional growth of teachers as well as aspects that can constrain teachers from adopting practices that support understanding mathematics. In-service professional development has suffered in India due to fragmented efforts which are neither perceived to be useful in the classroom, nor have been able to address teachers' concern for developing students' understanding of key and foundational ideas in mathematics. The sub-studies have several implications for designing of professional development opportunities which are listed below.

1. In all the sub-studies there are indications of interactions between the beliefs held by the

teachers and the practices preferred or adopted. Sub-study 1 showed how without change in core beliefs, the practices advocated by reform documents get incorporated as peripheral practices. However, beliefs about the improvement of reasoning and the use of contexts to support mathematize learning can serve as stepping stones towards adoption of practices that support understanding mathematics as indicated in Sub-studies 3 and 4. These practices may become central to teachers' practice when they develop the requisite knowledge to support the practices. Teachers will thus need opportunities to reflect on the beliefs held by them and the development of specialized knowledge for teaching mathematics in both professional development contexts and in classrooms.

2. The sub-studies also indicate that teachers' knowledge of content, meanings, students' thinking, and representations influences the interaction between beliefs and practice. In Sub-study 1, when teachers exhibited limited knowledge of representations and why algorithms work, they had beliefs more aligned towards mathematics as procedures and teaching as a transmission based activity. However Sub-studies 2, 3 and 4 show instances of teachers expressing and developing their knowledge of meanings and representations through engagement in tasks in workshop and through efforts to focus on reasoning and development of meanings in their teaching.

3. It is important for teacher educators designing teacher professional development interventions to anticipate the teachers' common beliefs and practices and to create opportunities for teachers to reflect on them. The framework developed in Sub-study 2 of situatedness, challenge and community building can be used to design the sessions and also manage the interactions within the workshop. This can be supported through use of artifacts from teaching, making teachers articulate their beliefs and knowledge and asking teachers to explain student thinking as done in Sub-study 2. Another possibility is to encourage teachers' to explicate the criteria for evaluating the resources of teaching; these shared criteria can be established through negotiation, as was done in Sub-study 4.

4. Even when a teacher is open to bringing about change in practice and is reflective, he/she might face challenges in actually changing teaching practice. These challenges can vary from identifying the gaps in the teaching and use of representations, to the knowledge of meanings, contexts or representations, to negotiations of the norms of the classrooms, to making students participate actively as reflected in Sub-studies 3 and 4.

5. The theoretical framework used in Sub-study 4 was derived from the research literature. There are few studies that try to establish connection between the research done in the field of mathematics education with the design and enactment of teacher education. Teachers at large remain unaware of the research findings and even when exposed to it may find it irrelevant for their own teaching needs. To establish a meaningful connection between theory and practice on one hand and between research and practice on the other, there is need to use theoretical ideas and empirical findings from research literature in teacher education in such a way that they are accessible to teachers and they get opportunities to relate it to their practice.

6. The sub-studies have illustrated the knowledge that the participating in-service teachers have about the common mistakes and knowledge of key ideas and explanations. Some of these ideas are already documented in the research literature. However, the research studies with in-service teachers have mostly adopted a deficit view of the teachers rather than contributing to the development of the knowledge of teaching mathematics derived from teachers' experience of teaching. Therefore, the sub-studies would hopefully contribute towards building a balanced vision of status of teachers' knowledge which need to be elicited and built upon in professional development space rather than merely emphasizing the deficits.

7. The teachers' learning in Sub-study 4 indicates that topic focused professional development is needed to develop in-depth knowledge through activities like textbook analysis, analysis of students' errors, analysis of meanings, representations and explanations, and collaborative lesson planning and sharing. While this kind of exercise cannot be expected from teachers for all the lessons, focusing on one topic each year in this integrated manner will help in developing teachers' specialized content knowledge in integration with other knowledge, while also contributing to the knowledge of teaching within the community of mathematics educators.

8. The two specific topics of mathematics that have been focused in this thesis are fractions and integers in Sub-studies 3 and 4 respectively. There are implications for designing PD for these topics for mathematics teachers. Firstly, teachers need to be aware of the different meanings that these concepts may have in various contexts. Secondly, teachers need to make the connections between these meanings and the representations

used in teaching and the tasks used in order to support meaningful discussion using them. Thirdly, the understanding of meanings and representations need to be connected with the standard symbolic procedures, so that teachers are able to unpack it for the students and engage those students who know and perform procedures mechanically without understanding them. Fourthly, the teachers also need to understand why rules work and how to make shifts from teaching using contexts and representations to developing an understanding of rules and other generalizations that can be made about mathematical concepts. Lastly, teachers need to integrate specialized content knowledge with the knowledge of students, applying both to designing tasks to develop understanding.

8.8 Limitations of the study

I briefly recall the limitations that this exploratory research study has as was described in Chapter 1. The study had components that addressed issues in-depth. However, since the sample selected for the study was small, the findings of the studies are indicative and not generalizable across the population of teachers. The rich descriptions of teachers' struggles in adopting new practices in classroom will, I feel still be useful in throwing light on similar struggles in other contexts. A large volume of data was collected for the study in the form of audio recordings which required transcribing. Practical considerations entailed that only some of this data could be analyzed. It was not possible to observe the teaching of participating teachers before Sub-studies 1 and 2, which took place at the first point of contact, namely, the TPD workshop. Data about the initial preferred practices of teachers was thus collected through questionnaire and interviews rather than through lesson observation. The claims for teachers' learning and changes in practice have been made from observations by the researcher and the self-reports of the teachers, rather than any standardized tool to assess the teachers' knowledge or quality of instruction. However, in-depth analysis of teachers' discourse in workshops and their teaching provide evidence for their learning.

8.9 The way forward

The four sub-studies have provided rich insights into the beliefs held and practices preferred by the teachers and the shifts that may take place when teachers engage in different practice-based professional development activities. However, further studies can be done on the following lines:

1. The interview prompts and questionnaire used in Sub-study 1 could be modified using the analysis in the thesis and could be used for teachers from other geographical areas and contexts like rural and semi-urban areas and teachers of different types of schools. A possible variation is to assess domain specificity of the beliefs by assessing beliefs of teachers about respective topics which can provide further help in identifying the need for professional development.

2. The framework of analysis of the task, the agency of the teacher and the teacher educator can be analysed to throw light on the types of opportunities for teachers' professional development in other PD initiatives. Looking at professional development initiatives from a research perspective will help in building the corpus of knowledge for the design of effective professional development in India to cater to the diversity of contexts and participants.

3. The study has illustrated the challenge that teachers face when implementing change in their classroom, after becoming motivated by a professional development initiative to bring about change. I contend that such studies that identify challenges faced by the teachers in teaching specific topics will contribute immensely to strengthening the professional development initiatives and will contribute towards actual change in learning opportunities in the classroom for students.

4. Sub-studies 3 and 4 highlighted the issues related to the teaching of topics like fractions and integers as a result of interference of whole number learning and teachers' limited knowledge of the meanings related to these concepts as well as the use of representations. Thus studies illustrating connections between the topics of whole numbers, fractions and integers and their various meanings in contexts would support the teaching of these topics in a way that attends to the meanings held by students.

5. Lesson planning might be important for researchers to study because it can provide a window into the teachers' thinking and decision making which takes place in the classroom. The study has implications for considering collaborative lesson planning as a tool for in-service teacher professional development by enhancing their awareness and knowledge about aspects otherwise ignored. It also points to the process of how negotiation of beliefs can occur as a community when teachers as a Topic Study Group engage with tasks to challenge their thinking.

8.10 My journey from being a teacher to a researcher

When I joined the PhD program at HBCSE, I brought the perspective of being a teacher concerned about developing understanding of mathematics among students. The challenges that I had faced in teaching mathematics and the teaching approach taken in summer camps conducted for students by the mathematics education group in HBCSE inspired me to take up research in the field of mathematics education. My own school experience of learning mathematics and the difficulties and challenges faced in understanding mathematics have guided me to explore, together with other teachers, ideas and methods in this study that can make mathematics accessible to students. In the course of the research study, I was able to integrate the perspective of the teacher by appreciating the kind of challenges a teacher faces in implementing the changes advocated by reform efforts. The interactions with other researchers and teacher educators and close study and analysis of their practice provided me an opportunity to understand the aspects that contribute towards teachers' growth. Further, interactions with the teachers in the workshops as well as collaborating with them in their classrooms for teaching challenging topics helped in developing an appreciation of the aspects of practice and knowledge that contribute towards development of students' understanding. As a result of being engaged in these studies, I got an opportunity for growth in my own knowledge, and beliefs and practice about mathematics and its teaching. As a person not having a Bachelor's degree in mathematics, it was challenging at times, but it was also an opportunity to learn while being actively engaged in the practice of studying the teaching of mathematics. I believe that my background helped me in being sensitive to the difficulties faced by the students and teachers.

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List of Appendices

1. Questionnaire:
    a) Part 1: Background information
    b) Part 2: Frequency of the practices for teaching mathematics
    c) Part 3: Beliefs about nature of mathematics
    d) Part 4: Beliefs about teaching and learning of mathematics
    e) Part 5: Beliefs about students' capability
    f) Part 6: Beliefs about self as a mathematics teacher
2. Semi- structured Interview schedule about beliefs and practices
3. Time table of the Professional development workshop
4. List of research readings used in the professional development workshop at HBCSE
5. Worksheet on Integers used in Topic focused professional development workshop
6. Lesson plan by Anita teacher for teaching integers
7. List of codes used in the Sub-Study 1, 2 and 4
8. Consent form for teachers' participation in the study

Appendix 1

Teachers' Questionnaire

We would like to know what your views are about mathematics teaching and learning. This will help us in planning the interactions in this workshop and also in designing better workshops for teachers in the future. We request you to freely express your views. We do not believe that there are right or wrong answers to any of the questions in this questionnaire. We value your honest opinion. We know that opinions and views can be different and that we have much to learn from teachers such as you who have a wealth of experience.

By analysing your responses we hope to get a better understanding of the inputs and support that teachers require. We assure you that all the names of respondents and their schools will be kept confidential and will be known only to the research team. Whenever data is presented to any person other than the research team, all references to personal identity will be removed.

Appendix 1a

Part I: Background information

1. Name ***(optional)***:
2. Age:
3. Sex:
4. Educational and professional qualifications (Mention subject specialization and qualifying year):
5. Teaching experience (mention which classes and subjects you have taught and for how many years):
6. Which classes have you taught in the past three years?______________.
7. In which schools have you taught in the past three years? ***(Optional)***
8. Average number of students in the classes you have taught for the past three years:
9. How many other teachers in your school teach mathematics for the same class level as you teach?
10. Percentage (roughly) of the students in the class that you taught in the current / previous academic year
    i. Who have a computer at home _____
    ii. Who have a TV at home _____
    iii. Who have a two-wheeler at home _____
    iv. Who have a car at home _____
    v. Who have one parent who is educated upto class 12 and above _____
    vi. Who have both parents educated upto class 12 and above _____
11. Have you attended any training workshops before? Please specify.
12. If you would like to mention any specific accomplishments related to your teaching, or any project that you have done, please do so here:

Appendix 1b

Part II

Name___________________________

Given below are some statements that describe the practices used in the mathematics classroom. Think of ***your own classroom*** and ***your own teaching*** when you respond to these questions. Read the statements carefully and choose the option based on how often you use these practices ***in your own classroom***.

The acronyms in the column correspond to following:

AN – Almost never

S – Sometimes

H – Half of the time

M – most of the time

AA – Almost always

If you have any additional comments about any of the questions, please write it on the sheet or ask for a blank sheet.

1.	In the beginning of the class, I show students how to solve a particular problem and then give similar problems to practice from the textbook.	AN	S	H	M	AA
2.	If any student does not understand what was taught, I explain to the student once or twice by repeating the steps in detail slowly.	AN	S	H	M	AA
3.	I use only English language (Hindi if the medium of instruction is Hindi) to explain mathematics to students.	AN	S	H	M	AA
4.	Students come up with interesting ideas of their own to solve a problem.	AN	S	H	M	AA
5.	I use knowledge about students' daily life and culture for teaching mathematics.	AN	S	H	M	AA
6.	Students in my class give examples of application of	AN	S	H	M	AA

	mathematical concept being taught from their daily life.					
7.	I plan minimally and build the lesson based on students' responses.	AN	S	H	M	AA
8.	I ask students to practice the problems very similar to the one done in class as home work.	AN	S	H	M	AA
9.	I teach only one method for each question.	AN	S	H	M	AA
10.	Students express their ideas or reply to questions using their mother tongue.	AN	S	H	M	AA
11.	I ask my students to write explanations and justifications through words or pictures in their notebook.	AN	S	H	M	AA
12.	After students have solved the problem, I give them correct solution so that they can copy it down.	AN	S	H	M	AA
13.	I discuss my students' responses in the class with my colleagues.	AN	S	H	M	AA
14.	In my mathematics class I stick to the plan I have made before.	AN	S	H	M	AA
15.	I use commonly available objects from daily life while teaching.	AN	S	H	M	AA
16.	I tell students not to talk among themselves but ask their questions to me.	AN	S	H	M	AA
17.	I avoid students making mistakes by explaining things carefully first.	AN	S	H	M	AA
18.	When students get different answers then I ask them to discuss and compare their methods.	AN	S	H	M	AA
19.	I follow the sequence given in the textbook in my teaching.	AN	S	H	M	AA
20.	I ask students to discuss problems in groups.	AN	S	H	M	AA

21.	I give challenging problems/tasks to students where one problem takes the whole period of 30 minutes.	AN	S	H	M	AA
22.	I ask students the reason or justification for using a particular method to solve a problem.	AN	S	H	M	AA
23.	To explain mathematical concept, I show the steps of the procedure to solve the problem.	AN	S	H	M	AA
24.	I ask students to come up to the blackboard to show their solution to the whole class.	AN	S	H	M	AA
25.	When students get different answers then I tell them at once which one is correct.	AN	S	H	M	AA
26.	I am speaking most of the time in my mathematics class.	AN	S	H	M	AA
27.	I ask students to work more quickly on their problems.	AN	S	H	M	AA
28.	I draw links between topics and move back and forth between topics.	AN	S	H	M	AA
29.	I allow students to make mistakes and then discuss them.	AN	S	H	M	AA
30.	I use manipulatives and models for teaching mathematics.	AN	S	H	M	AA
31.	I give problems only from the textbook to my students.	AN	S	H	M	AA

Appendix 1c

Part -III

Name____________________________

Given below are some statements that express views about mathematics. Read the statements carefully and tick one box in each row to show your degree of agreement/disagreement. The acronyms in the columns stand for:

SA = Strongly agree

A = Agree

U = unsure

D = Disagree

SD= Strongly disagree

If you have any additional comments about any of the questions, please write it on the sheet or ask for a blank sheet.

1	It is important for students to not only know procedures (methods) for calculation but also why the procedures work.	SA	A	U	D	SD
2	Everything in mathematics is already known, and nothing new remains to be found.	SA	A	U	D	SD
3	In mathematics, we can give proper reason or justification for all statements and procedures.	SA	A	U	D	SD
4	Mathematics has strong connections with real world applications and these connections must be emphasized whenever we teach mathematics.	SA	A	U	D	SD
5	Given a chance most students can discover correct procedures (methods) for calculation although these may be different from standard procedures.	SA	A	U	D	SD
6	We should not emphasize real life examples too much in mathematics because they distract the students.	SA	A	U	D	SD
7	Being good at mathematics means being able to perform cal-	SA	A	U	D	SD

	culation quickly and accurately.					
8	Mathematics is abstract; there is not much connection between mathematics and the real world.	SA	A	U	D	SD
9	If they have a doubt about a mathematical statement or procedure, then most students can check and justify it on their own.	SA	A	U	D	SD
10	Mathematics is basically the four number operations (addition, subtraction, multiplication and division) and application of these.	SA	A	U	D	SD
11	Mathematics is a creative subject in which most students can create their own concepts by discovering patterns.	SA	A	U	D	SD
12	Mathematics problems, which have more than one correct solution, are a very good resource for developing a deeper understanding among students.	SA	A	U	D	SD
13	Mathematical concepts have to be taught one at a time, there is not much interconnection between mathematical concepts.	SA	A	U	D	SD
14	In school mathematics, students must not only learn mathematical tools, but also understand and be able to justify why the tools work.	SA	A	U	D	SD
15	It is important to teach students "shortcuts" or "thumb rules" for solving mathematical problems.	SA	A	U	D	SD
16	It is only very rarely that students can discover procedures (methods) for calculation on their own. They need to be taught these procedures.	SA	A	U	D	SD
17	Mathematics is useful in understanding and coping with daily life. It helps us lead a better life.	SA	A	U	D	SD

Appendix 1d

Part IV

Name____

Given below are some statements that express views about how mathematics is best taught and learned. Read the statements carefully and tick one box in each row to show your degree of agreement/disagreement. The acronyms in the columns stand for:

SA = Strongly agree

A = Agree

U = unsure

D = Disagree

SD= Strongly disagree

If you have any additional comments about any of the questions, please write it on the sheet or ask for a blank sheet.

1	The best way to teach mathematics is to clearly show the procedures (methods) to solve the mathematics problems.	SA	A	U	D	SD
2	While teaching its better to change the sequence given in the textbook as needed than to follow it.	SA	A	U	D	SD
3	Listening carefully to the teacher explain the mathematics lesson is the most effective way to learn mathematics.	SA	A	U	D	SD
4	If a student practices solving all the problems in the textbook two or three times, that is the best way to learn mathematics.	SA	A	U	D	SD
5	By asking probing questions that create uncertainty and confusion in the students' minds, we can help them learn better.	SA	A	U	D	SD
6	It is essential that students express their ideas in classrooms to help them learn mathematics better.	SA	A	U	D	SD
7	A teacher should teach each topic from the beginning assuming that the students know nothing.	SA	A	U	D	SD
8	Students learn maths best if the teacher organizes the work clearly for them.	SA	A	U	D	SD

9	The best way to teach mathematics is to show students how to solve some example problems.	SA	A	U	D	SD
10	Students learn better when they can work at their own pace.	SA	A	U	D	SD
11	Students learn best if they figure things out for themselves instead of getting explanations from the teacher.	SA	A	U	D	SD
12	We should not give difficult and challenging exercises to the students while beginning a topic.	SA	A	U	D	SD
13	A teacher should explain things carefully in the beginning so that students can avoid mistakes.	SA	A	U	D	SD
14	Only one method should be taught to students for solving otherwise they get confused.	SA	A	U	D	SD
15	Students should be allowed to make mistakes and then discuss them.	SA	A	U	D	SD
16	Students learn better by discussing their ideas in the classroom.	SA	A	U	D	SD
17	Sometimes students surprise teachers by coming up with interesting new ideas about mathematics on their own.	SA	A	U	D	SD
18	The key to learning mathematics well is to repeat the textbook exercises two or three times (or more).	SA	A	U	D	SD
19	The teacher need not explain how to solve all the different problem types or problem variations.	SA	A	U	D	SD
20	The best way to teach mathematics is to explain one procedure at a time on the blackboard and then to make students practice it.	SA	A	U	D	SD
21	Students should be encouraged to find different methods to solve a problem.	SA	A	U	D	SD
22	When students make errors, the best remedy is to make them repeatedly practice these types of problems.	SA	A	U	D	SD
23	Even if a student's question takes the teacher away from what she has planned for the lesson, it should be taken up for discussion.	SA	A	U	D	SD

Appendix 1e

Part V

Name____

Given below are some statements that express how you may think and feel about mathematics. Read the statements carefully and tick one box in each row to show your degree of agreement or disagreement. The acronyms in the columns stand for:

SA = Strongly agree

A = Agree

U = unsure

D = Disagree

SD= Strongly disagree

If you have any additional comments about any of the questions, please write it on the sheet or ask for a blank sheet.

1	When I teach mathematics, I generally follow whatever is given in the textbook.	SA	A	U	D	SD
2	Whenever I have to teach a difficult topic in mathematics I feel nervous.	SA	A	U	D	SD
3	If I could permanently give away one subject that would be mathematics.	SA	A	U	D	SD
4	Generally I feel comfortable about teaching mathematics.	SA	A	U	D	SD
5	If I were to chose one subject to teach as a substitute in my colleague's class it would be mathematics.	SA	A	U	D	SD
6	I don't feel confident about teaching difficult topics in mathematics.	SA	A	U	D	SD
7	I am not sure if I can really help children who are weak in mathematics.	SA	A	U	D	SD
8	Sometimes even if I have planned my teaching, I feel worried before the mathematics period.	SA	A	U	D	SD
9	I enjoy the challenge of teaching new and difficult topics in	SA	A	U	D	SD

	mathematics.					
10	I don't feel worried about taking the mathematics period.	SA	A	U	D	SD
11	If the maths textbook does not explain something clearly, it creates a serious problem for the teachers.	SA	A	U	D	SD
12	If something is not clear in the mathematics textbook, I am confident that I can work it out on my own.	SA	A	U	D	SD
13	I am confident about handling students' doubts in mathematics.	SA	A	U	D	SD
14	Compared to other subjects, teaching mathematics does not worry me.	SA	A	U	D	SD
15	At school, my friends generally came to me for help in mathematics.	SA	A	U	D	SD
16	I go through the textbook but prefer to teach in my own way.	SA	A	U	D	SD
17	Mathematics is a subject that I don't like to teach.	SA	A	U	D	SD
18	I find many mathematics problems interesting and challenging.	SA	A	U	D	SD
19	Sometimes, I feel anxious because I am afraid that a student might ask me a question that I do not know how to answer or I cannot explain.	SA	A	U	D	SD
20	In my view, maths textbook authors should give all the steps in a problem without skipping any step.	SA	A	U	D	SD
21	I am not the type of person who can teach mathematics well.	SA	A	U	D	SD
22	Time passes quickly when I am teaching mathematics.	SA	A	U	D	SD
23	I don't depend much on the textbook.	SA	A	U	D	SD
24	During my education, whenever I had an option, I avoided taking mathematics.	SA	A	U	D	SD
25	There is no point in trying to doubt what is given in the mathematics textbook.	SA	A	U	D	SD

Appendix 1f

Part VI

Name______________________________________

Given below are some statements about students in the mathematics class. Think of ***your own classroom*** and ***your own students*** when you respond to these questions.

Read the statements carefully and tick one box in each row to show your degree of agreement or disagreement. The acronyms in the columns stand for:

SA = Strongly agree

A = Agree

U = unsure

D = Disagree

SD= Strongly disagre

If you have any additional comments about any of the questions, please write it on the sheet or ask for a blank sheet.

1	Even primary school students are capable of coming up with mathematical arguments.	SA	A	U	D	SD
2	Every one has the ability to learn mathematics.	SA	A	U	D	SD
3	For some reason boys are better at doing mathematics than girls.	SA	A	U	D	SD
4	Mathematics is usually one among the students' favorite subjects.	SA	A	U	D	SD
5	Teachers need to tell students the answers, because students expect them to do so.	SA	A	U	D	SD
6	Boys are more interested in mathematics than girls.	SA	A	U	D	SD
7	Most of my students will not be needing all the mathematics that they learn once they finish school.	SA	A	U	D	SD
8	Some students face so many difficulties in mathematics that	SA	A	U	D	SD

	teachers are unable to help them.					
9	With the teacher's help students can overcome the negative influence of their home environment and do well in mathematics.	SA	A	U	D	SD
10	Students whose parents are well educated and students whose parents are not well educated face the same level of difficulty in learning mathematics.	SA	A	U	D	SD
11	Students from poor homes tend to struggle in mathematics.	SA	A	U	D	SD
12	Boys and girls are equally fast in grasping mathematical concepts.	SA	A	U	D	SD
13	Students from poor homes can perform well in mathematics because they are used to buying things and doing other such work in their daily life.	SA	A	U	D	SD
14	Girls find it difficult to understand mathematics and so they rote learn it.	SA	A	U	D	SD
15	Most students in my class do not have the talent to do well in maths.	SA	A	U	D	SD
16	Most students in my class are likely to study up to the degree level.	SA	A	U	D	SD

Appendix 2

Teachers' interview schedule

The interview schedule used for conducting semi- structured interview with the 11 teachers who participated in the professional development workshop in Year 1.

	Prompt
1.	I would like to know what made you interested in the teaching profession?
2.	How do you describe yourself as a mathematics teacher? How do you think your students see you as a maths teacher?
3	How was your experience with mathematics during your education? Can you describe your school experience/ college. Do you remember: your teachers; how you felt about maths class?
4	What does being good in mathematics mean to you? Describe a person who is good in mathematics.
5	Have your views about mathematics changed in any way from your education days as compared to your views about mathematics today? If yes, how are your views different from before. If no, what are the views that you still strongly adhere to since your education days. If yes: your change in views brought about any change in your teaching of mathematics in classroom? If so, in what ways
6	Do you find any major difference in the textbooks that are being used now and those that were in use three to four years before? In what ways do you find it different? Did you have to change your teaching in any way to adjust with the content given in the textbook? If so how? If not, then how do you teach these textbooks.
7	In what ways do you find the textbook helpful/unhelpful in your teaching mathematics?/ Wwhat do you think is the relationship between a mathematics teacher and a textbook?
8	Describe a typical mathematics period? What do you do in the beginning, how do you develop your lesson in class and how do you end it?

9	What was your response to statement "I use knowledge about daily life and culture for teaching mathematics". Can you elaborate or give example of how you exactly do it in the classroom?
10	How do you deal with the student who has got a wrong answer to a problem?
11	What do we mean by "justification" and "explanation" in mathematics? Do you think students can arrive at them on their own without any teacher's help? How would you justify the procedure of division in the problem 36036 divided by 9.
12	Do you think teaching short cuts are helpful? How does it help a student in learning mathematics?
13	What kind of expectations to you have from your students in your mathematics class?
14	What do you think is the role of practice in learning mathematics?
15	Do you have to deal with students from poor uneducated homes in your class? If yes, what kind of problems do you face in dealing with them and how do you tackle it?
16	Do you find differences among gender in your class in performance in mathematics? If yes, What do you think could be reason? Can it be balanced in some way?

Appendix 3

Time table – Teacher Professional development workshop, May, 2009

Day 1: 25th May 2009, Monday

09.00 am to 10.00 am	Registration
10.00 am to 10.30 pm	Welcome and introduction
10.30 am to 11.30 am	Questionnaire about Mathematics
11.30 am to 11.45 am	Tea Break
11.45 am to 01.15 pm	Learning from Problems I: Numbers and operations
01.15 pm to 02.00 pm	Lunch Break
02.00 pm to 03.30 pm	Understanding basic operations
03.30 pm to 03.45 pm	Tea Break
03.45 pm to 05.20 pm	Teaching episodes: videos and discussion
05.20 pm to 05.30 pm	Readings for Discussion

Day 2: 26th May 2009, Tuesday

9.30 am to 10.00 am	Using matchstick bundles for subtraction
10.00 am to 11.00 am	Fractions: Part-whole, measure and share
11.00 am to 11.15 am	Tea Break

11.15 am to 12.45 pm	Fractions (contd.)
12.45 pm to 01.30 pm	Lunch Break
1.30 pm to 03.00 pm	Understanding students' errors
03.00 pm to 03.15 pm	Tea Break
03.15 pm to 04.45 pm	Lesson study: planning a fraction lesson
04.45 pm to 05.00 pm	Teachers' resources – wiki

Day 3: 27th May 2009, Wednesday

9.30 am to 10.45 am	Lesson study: Mock presentations
10.45 am to 11.00 am	Tea Break
11.00 am to 1.00 pm	Lectures on mathematics
01.00 pm to 02.00 pm	Lunch Break
02.00 pm to 04.00 pm	Classroom Observation
04.00 pm to 04.15 pm	Tea Break
04.15 pm to 5.00 pm	Discussion

Day 4: 28th May 2009, Thursday

9.30 am to 10.00 am	Teachers' resources – wiki
10.00 am to 11.00 am	Learning from problems II: Proportion problems
11.00 am to 11.15 am	Tea Break
11.15 am to 12.45 pm	Library work / interviews
12.45 pm to 1.30 pm	Lunch Break
01.30 pm to 03.00 pm	Library work / interviews
03.00 pm to 03.15 pm	Tea Break

03.15 pm to 05.00 pm	Mathematics Laboratory

Day 5: 29th May 2009, Friday

09.30 am to 10.45 am	Narratives questionnaire
10.45 am to 11.00 am	Tea Break
11.00 am to 12.00 noon	Narratives questionnaire - discussion
12.00 to 1.00 pm	Textbook analysis and discussion
1.00 pm to 2.00 pm	Lunch Break
02.00 pm to 04.00 pm	Classroom Observation
04.00 pm to 05.00 pm	Discussion

Day 6: 30th May 2009, Saturday

09.30 am to 10.45 am	Multiplication and Division
10.45 am to 11.00 am	Tea Break
11.15 am to 12.45 pm	Learning from problems III – Algebra
12.45 pm to 1.30 pm	Lunch Break
01.30 pm to 03.00 pm	Teachers Presentations
03.00 pm to 03.15 pm	Tea Break
03.15 pm to 05.00 pm	Teachers Presentations

Day 7: 1st June 2009, Monday

10.00 am to 01.15 pm	Classroom Observation and Discussion
01.15 pm to 02.00 pm	Lunch Break

02.00 pm to 03.30 pm	Teaching episodes: videos and discussion
03.30 pm to 03.45 pm	Tea Break
03.45 pm to 05.30 pm	Textbook analysis

Day 8: 2nd June 2009, Tuesday

10.00 am to 11.30 am	Invited Lecture/Learning from problems – Proportions
11.30 am to 11.45 am	Tea Break
11.45 am to 01.15 pm	Questionnaire about Mathematics
01.15 pm to 02.00 pm	Lunch Break
02.00 pm to 03.30 pm	Dynamic Geometry software
03.30 pm to 03.45 pm	Tea Break
03.45 pm to 05.30 pm	Mathematics Laboratory

Day 9: 3rd June 2009, Wednesday

10.00 am to 11.30 am	Discussion and Planning
11.30 am to 11.45 am	Tea Break
11.45 am to 12.00 pm	Closing

Appendix 4

List of readings used in HBCSE teacher professional development workshops*

1. An instance of teaching Practice (Lampert, M. (Ed.)(2003). An Instance of teaching practice, In *Teaching problems and the problems of teaching.* (pp. 9- 27). Yale University Press.)
2. A day in the life of one cognitively guided instruction classroom (Hiebert, J. (1997). *Making sense: Teaching and learning mathematics with understanding*. Heinemann, 361 Hanover Street, Portsmouth, NH 03801-3912.)
3. Subtraction with regrouping: Approaches to teaching a topic (Ma, L. (1999). Subtraction with regrouping: Approaches to teaching a topic, In *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* (pp. 1-22), Mahwah, NJ: Lawrence Erlbaum Associates Inc.)
4. Multidigit Number Multiplication: Dealing With Students' Mistakes (Ma, L. (1999). Multidigit Number Multiplication: Dealing With Students' Mistakes, In *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* (pp. 24-46), Mahwah, NJ: Lawrence Erlbaum Associates Inc.)
5. Generating Representations: Division by Fractions (Ma, L. (1999). Generating Representations: Division by Fractions, In *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* (pp.47-71), Mahwah, NJ: Lawrence Erlbaum Associates Inc.)
6. Images of teaching (Stigler, J. W., & Hiebert, J. (2009). Images of teaching, In *The teaching gap: Best ideas from the world's teachers for improving education in the classroom* (pp 25-54). Simon and Schuster.

* This is an indicative list of the readings used in the workshops held at HBCSE

Appendix 5
WORKSHEET ON INTEGERS

One of the difficulties that children face is in interpreting negative numbers. What does '−2' exactly mean? There are broad senses in which negative numbers or more generally integers (positive, negative numbers and zero) are interpreted.

1.**As a change:** Change includes increase or decrease, movement up or down (or forward and backward) or positive or negative growth (for eg: total annual sales of a company).

Think of situations which involve change and can therefore be described using integers. The situations should be meaningful and interesting. Some suggested examples are given below. Think of more such examples.

- increase/decrease: Make a table of the weight gained by a baby every week (may be negative, what does it indicate?),
- movement forward/backward or up/down: Change in tennis ranking of a tennis player, change in run rate from over to over.

Make a presentation of such data in a way that would be interesting to students.

2.**As a state**: We can specify the state of something we are interested in using integers but only when it is meaningful to talk about positive and negative states.

Think of such situations where integers represent sate. Some suggested examples are given below. Think of more such examples.

- Position of a lift in a building which also has basement floors
- Temperature of water in a freezer

Again think of ways in which such situations can be presented in an interesting way to students.

3.**As relation between numbers and quantities**: An important point here is that this is a directed relation. The relation makes sense if we distinguish the direction of the relationship and use positive and negative numbers to indicate it.

Consider these two examples:

- Me and my sister are standing in a queue to buy ice-cream. How far is my sister from me?
- Me and my sister are on different floors of a tall building with several basement floor levels for parking. How far away is my sister from me?

Why is it meaningful to give the answers to these questions using integers? Is there any difference between the two examples? Think of more examples where relations can be represented using integers.

Appendix 6

Lesson plan by Anita for teaching integers

Daily Lesson Plan

Day 1:
The chapter will be introduced using the DREAM MALL figure.
The child learns that to move upward there is a '+' button and to move downwards '-' button is to be used. Using this idea he can number the floors accordingly.

Movement problems:
Suppose Sapna and Kiran are in the ice-cream parlour. Sapna wants to go to the movie hall and Kiran wants to go for shopping. How many steps each would move?
Their attention can be brought to the point that each would move same number of steps but in different directions, they can answer using appropriate signs.
Discussion regarding the importance of using the correct sign will be done at this moment.
In the figure, the boat is on the sea level. The aeroplane is flying 2000 km above the sea level. The submarine is at 800km below sea level. Express their distances from the sea level.

Have you seen numbers with '-' sign earlier?
Every day we see the weather report in a newspaper or a T.V. Do you know there are places where the temperature is less than 0°C?
(Refer Text Book page 154) for the list of temperatures of 5 places in India.

Appendix 7

List of codes used in Sub-studies 1, 2 and 4

Sub-study 1:

Following categories were used to code the transcript of interviews of the participating teachers:

1. Codes for practices preferred by teachers:
   - 1.1. Showing procedures/Examples/ Steps of the solution/ Shortcuts/ Formulas/ Alternative procedures/ All types of problems/ students to copy solutions
   - 1.2. Student solutions: After showing procedure/ Students' methods/ Alternative solutions/ Appreciate speedy solutions/ Appreciate accuracy
   - 1.3. Practice – frequency –amount:
     - 1.3.1. Nature of practice: Similar problems after showing procedure/ Practice problems after activity/ Repeat in homework/ Repeat easy problems for weak students
     - 1.3.2. Source of problems: Textbook/ Constructing similar problems
     - 1.3.3. Amount of practice: Lot of problems/ typical problems/ type of problems
   - 1.4. Textbook use:
     - 1.4.1. Purpose: Planning teaching/ Using problems/ Methods
     - 1.4.2. Old vs new textbook: Appreciate practice problems/ Appreciate activities/ Appreciate daily life contexts/ Appreciate colourfulness/ Appreciate reasoning
   - 1.5. Dealing with student error:
     - 1.5.1. Avoiding mistakes: Showing procedure before students solve/ Giving correct solutions/ Identifying typical words
     - 1.5.2. Classroom interaction: Focusing on correct answer/ discussing mistakes/identifying the operation
     - 1.5.3. Remedial steps: Practicing typical questions/ Repeating the problem
   - 1.6. Activity:

1.6.1. Nature: Demonstrating/ Using manipulatives/ Using pictures or visuals

1.6.2. Student engagement- Repeating the method or showing solutions/ Single vs. Multiple methods/ Practice problems after activity

1.6.3. Source of activity: Textbook or constructed

1.7. Explanation: Steps of the solution/ Use of manipulatives/ Use of pictures or visuals/ Reasoning/ Justification of procedure or method

1.8. Connection to daily life: Using daily life contexts to pose problems/ As examples/ Objects and pictures from daily life/ Applications/ Asking students to give examples/ Eliciting students' knowledge

1.9. Equity in classroom participation:

1.9.1. Opportunity to share students' ideas: Students' ideas/ Use of mother tongue/use of pictures or words by students to justify their solutions

1.9.2. Opportunity for all students to participate: Discussion among students/ Strategies for poor students/ Asking question row-wise/ Asking different students to respond

2. Beliefs about teaching and learning mathematics:

2.1. Process of teaching: Transmitting procedures through showing procedures or steps or solutions/ Expecting students to listen carefully/ Solving all types of problems/ Avoiding mistakes

2.2. Process of learning: Memorisation/ Practising similar or same problems/ Practising many problems of same type/ Practising problems from the textbook/ Practising as rote memorisation/ Learning through discussions

2.3. Indicators of learning: Understanding how and why procedures work/ Quick and correct solutions/ Solving application problems

2.4. Type and sequence of tasks: Simple to complex tasks/ Challenging tasks/ Confusion among students

2.5. Good teacher: Students' marks / Making math interesting or simple/ Students performance in higher classes.

3. Beliefs about nature of mathematics:

3.1. Math as fixed vs. Discoverable

3.2. Math as body of procedures vs. Interconnected knowledge

3.3. Math as abstract vs. Connected to daily life

3.4. Math as easy vs. Difficult

4. Beliefs about students

4.1. Poor students: Difficulties / Teachers' role/ Parental help/ Daily life contexts

4.2. Gender: Difference between boys and girls/ No difference

5. Beliefs about self as a Math teacher

5.1. Experience of mathematics in past: school/ college/ teaching: Negative/ positive experience, Role of textbooks- old/new, Change in teaching

5.2. Confidence as maths teacher: Dependence on textbook/ Love for mathematics/ Relationship with students/ Inspection results/ Pass percentage of students

5.3. Support by administrators

5.4. Sources of learning: Higher education/ Textbooks/ Teaching

Sub-study 2:

Following categories were used to code the transcript from the workshop sessions:

1. The design features:

1.1. Type of artifact used: Student work/ A mathematical problem/ Textbook/ Blackboard work/ Teaching aid/ Manipulative/ Video recording/ Lesson plan

1.2. Task demands: Articulate student thinking/ Analyse student error/ Predict student responses/ Propose teaching approach/ Identify conceptual gap/ Explanation of the procedure/ Identify key concepts/ Analysing remedial strategy/ Anticipating student response/ Eliciting beliefs about mathematics and its teaching

2. Facilitation features: Probing/ Asking for elaboration/ Supporting/ Revoicing/ Connecting with other ideas/ Raising questions/ Raising larger issues of education/ soliciting teachers' responses/ Comparing responses/ Proposing teaching approach/ Making counter-argument, Probing meaning/ Reviving discussion/ Making inferences/ Making connections/ Sharing protocol

3. Teachers explorations and reflections: Affirming/ Challenging/ Making conjectures/ Identifying student errors/ Inferences/ Assertions/ Challenging assertion/ Disagreement/ Agreement/ Initiation/ Supporting/ Giving example/ Giving explanation/ Reflection on

teaching/ Explanation of student errors

Sub-study 4:

Following categories were used to code the transcript from the workshop sessions:

1. Speaker: M1, M2, M3, M4, TE1, TE2, TE3
2. Mathematical purpose: Integer meaning, sign meaning, Integer order, Arithmetic expression, distributive property, addition, addition-subtraction, subtraction, Multiplication-division
3. Pedagogical purpose: Description, connection, explanation, evaluating accessibility, evaluating connection, evaluating consistency, Student understanding, sharing teaching experience, teaching plan, textbook analysis
4. Integer sense: State, change, relation, Symbol
5. Operation sense: Combine, compare, change
6. representation used: Number line, neutralisation model, arithmetic expression, Integer mall context, other context
7. Contexts used: Integer mall, temperature, marks/ score, baby's weight, profit-loss, mixing water at different temperature, ticket reservation, loan taking/ giving, family size, queue, length of the shadow at different times, water level, journey by train, steps, altitude, speed of the car

Appendix 8

Consent form for teachers participation in study

Consent Form for Interview

Homi Bhabha Centre for Science Education

(Tata Institute for Fundamental Research)

V.N. Purav Marg Mankhurd, Mumbai 400088

The purpose of the research study has been explained to me and I am willing to participate in the study. I am aware that my interview has been audio recorded since it is necessary for the research study. The interview would be later analysed to arrive at research findings. I understand that the interview would be heard only by the research team and will be kept strictly confidential. The interview data will be used strictly for the research purposes. All results reported from the study will keep the identity of the participants strictly confidential. I give my consent for the audio recording of the interview and for the use of the recording for research purposes.

Signature:_______________

Name: _________________

Date:___________________

Teachers' written consent form

As you are aware, many of the sessions in the workshop are being recorded. This will help us analyze how the workshops have gone and to improve our planning and presentation for future workshops. It will also help us understand more deeply teachers' needs from workshops such as this. To use the video recording for analysis, we need your consent.

There are several statements below. You may agree to any or all or none of them depending on what you feel comfortable with. Please feel free to tick any or none.

Consent for use in research

I am aware that the workshop sessions are video recorded for the purpose of research and and analysis. It has been explained to me that for research purpose only the research team at HBCSE will be viewing the recordings.

1. I agree to the recording of sessions in which I have participated used for purpose of analysis and research. □

Consent for use in teacher and researchers workshops

2. I agree to the recordings to the recordings of sessions in which I have participated being shown as illustrations before other teachers and researchers during workshops. □

Consent for uploading on website

3. I agree to the recordings of the sessions in which I have participated being uploaded on a website. □

I give my consent with respect to the statements numbered ___________ above and do not give consent with respect to the statements numbered _____________above.

Signature with date

Name ______________

Consent form for classroom data

Homi Bhabha Centre For Science Education (Tata Institute for Fundamental Research)

V.N. Purav Marg Mankhurd, Mumbai 400088

*Homi Bhabha Centre for Science Education is undertaking a research project in Mathematics education at ******(Name of the school), Mumbai. Its focus is on analyzing classroom interactions to promote mathematical understanding. As part of this research study, it is necessary to make video/ audio recording of your interview/classroom interactions. This recording would be later analyzed to arrive at the research findings. The videos/audio will only be viewed/ listened by the team of researchers and will be kept strictly confidential. All results reported from the study will keep the identity of the participants strictly confidential. They will be used strictly for the* ***research purposes*** *only.*

By generally accepted international norms, all video recording for research purposes should be done only after obtaining a written consent from the participant in the study. This is also the policy of HBCSE. We therefore request you to sign the consent form below.

The purpose of the research study has been explained to me and I am willing to participate in the study. I understand that my interview/classroom interactions will be recorded on the video/audio since it is necessary for the research study and that the video/audio will be viewed/listened only by the research team and will be kept strictly confidential. I consent to the video/audio recording.

Signature:

Name:

Date:

*It is also helpful for the work done at HBCSE for some of the video/audio excerpts to be shared with students and other researchers for **educational purposes** such as training or sharing research results. If you agree to this further use of your video/audio recording, we request you to sign this additional consent form. Only if you give your consent, then excerpts from the recordings may be used for above purposes.*

I have no objection to excerpts from my video/audio recording being shared with students, teachers, researchers, etc., for **educational purposes**.

Signature:

Name:

Date: