Work, Knowledge and Identity
Implications for school learning of out-of-school mathematical knowledge

Arindam Bose

Synopsis of Ph.D. Thesis
submitted in partial fulfilment of the requirements for the degree
of
Doctor of Philosophy
in
Science Education

Homi Bhabha Centre for Science Education

Tata Institute of Fundamental Research
(*A Deemed University*)

02 April 2014

Thesis advisor: Prof K. Subramaniam

Chapter 1 : Introduction

1.1 Rationale and motivation for the study

The study was motivated by our observations during initial field visits and interaction with school students and non school going children from low-income settlements. We observed that children from such backgrounds often experience difficulties in learning mathematics in schools, face failure and in many cases drop out. We also observed that they gained considerable informal knowledge from outside of school which remains unacknowledged in the classroom. Researchers and philosophers have recommended the use of such informal knowledge to support classroom learning (NCERT, 2005; 2007). The present study explores the prevalence of out-of-school mathematical knowledge among students from the low SES backgrounds and the implications for teaching and learning mathematics in school.

Research studies on out-of-school mathematical knowledge of children and adults have been carried out in many cultures since the 1960s. These studies have explored the contours of out of school mathematical knowledge, the ways in which it is acquired, and how it is different from mathematics learnt at school. While such studies initially communicated a promise of reshaping school math education based on what was known about out of school knowledge, there is still a lack of clarity about the implications of such studies for school learning. In this study, we have explored the nature and extent of everyday mathematical knowledge possessed by middle grade school students living in an urban low-income settlement that has embedded in it a thriving micro-enterprise economy. Children living in this settlement either have exposure to the diverse work-contexts prevalent in the neighbourhood or participate in and contribute to the production and income generation right from an early age. In the course of our exploration of the nature and extent of the opportunities available to the middle graders to gather everyday mathematical knowledge, we have characterised the work-contexts from a mathematics learning perspective. Our purpose is to unpack and document the connections between students' mathematical knowledge, work practices and identity formation, and inquire into the implications of these connections for school learning.

1.2 Need for drawing on out-of-school math knowledge during formal math learning

In mathematics education research (henceforth, MER), it is increasingly felt that learning mathematics can be helpful for students if the classroom teaching involves familiar contexts and methods. The major educational policy document that is currently followed in India, the *National*

Curriculum Framework (NCERT, 2005) points out that “learning takes place both within school and outside school” and that “learning is enriched if the two arenas interact with each other” (p. 15). The Framework gives importance to connecting school learning with the child's lived experience, “not only because the local environment and the child’s own experiences are the best entry points into the study of disciplines of knowledge, but more so because the aim of knowledge is to connect with the world” (p. 30). Connecting with the child's environment also has a role to play in creating an educational culture that is equitable. “Our children need to feel that each one of them, their homes, communities, languages and cultures, are valuable as resources for experience to be analysed and enquired into at school; that their diverse capabilities are accepted” (p. 14). The position paper of the Focus Group on the teaching of Mathematics expresses the same concern and emphasises the use of “experience and prior knowledge” to construct new knowledge in school mathematics (NCERT, 2006, p. 8). It is therefore of importance to a community of mathematics educators to investigate the kind of mathematics children draw from the outside world and the possible bearing that such knowledge might have on their learning of school mathematics.

1.3 Organisation of thesis

The thesis is organised into eight chapters. Chapter 1 gives an introduction to the thesis, places the background and the context in which the research was undertaken by presenting the motivation and the rationale behind the study and its scope and limitations. Chapter 2 presents the relevant literature in the domain of out-of-school mathematics, mathematics in work-places and also takes a look at the curricular documents currently followed in India. Chapter 3 discusses the research questions and the research design, location of the study and its significance, sample, methods and ethical considerations that informed the study. Preliminary findings that function and set the context of the study are presented as well. Chapter 4 presents the work profiles of the sample students and their parents to give a picture of the diverse work-contexts that children are immersed in. Descriptive reports of four cases of students who participate in work contexts are presented, focusing on the mathematical elements embedded in their work practices. Chapter 5 analyses the opportunities and affordances available in diverse work-contexts and in everyday settings (drawn from case studies and supplementary data) for school learning of mathematics. Aspects of the participants' identities as learners in out-of-school contexts are analysed here. Chapter 6 unpacks the measurement knowledge embedded in various work-contexts emergent from the students' semi-structured interviews and discusses implications for the school learning of this topic. Chapter 7 presents an analysis of a teaching design experiment focusing on integrating students' out-of-school measurement knowledge. Chapter 8 concludes the thesis by discussing the results and findings of the study and presents possible curricular and pedagogic implications and future road-maps.

1.4 Limitations

The interaction with students and conducting interviews with them was a challenge. Though the researcher conducted the interviews after building a personal rapport with the students, there could be many invisible factors that might not have emerged in the responses. In such an eclectic exploration, there was also an ethical consideration about how deep one can probe about family and work details. At times, respondents might have felt uncomfortable to respond despite best of efforts by the researcher to create a comfortable environment. These affective factors could have limited the scope of the interviews. The vacation course following the teaching design experiment though conducted in the actual classroom set-up was a short, two-week long experiment. The implications drawn are thus indicative and need to be explored through more extensive, long-term work.

Chapter 2: Setting the Context

2.1 Out-of-school and School Math

Out-of-school mathematical knowledge (also termed as “everyday” math, “street” math and “informal” math) of children has been studied extensively beginning with the pioneering work of Nunes, Carraher and Schliemann (1985) and other authors in the Latin American and African countries (Lave, 1988; Saxe, 1988). Most of these studies referred to “out-of-school mathematics” as the form of mathematics that people make use of in everyday settings while engaging in contextually embedded practices, viz., work-contexts, shopping, house-hold activities, games and so on. These studies have focused on the work-place activities of street vendors, carpenters, fishermen, farmers, construction site foremen, tailors, carpet-weavers, grocery-shoppers, and provided a systematic comparison between the “everyday” and “school” mathematics primarily considering them as two forms of activities based on different cultural practices but on the same mathematical principles (Nunes, Schliemann & Carraher, 1993). Table 2.1 below summarises the distinction between school and everyday math that emerged in the literature.

Difference	Out-of-school Mathematics	School Mathematics
Basic feature	-Based on shared cognition (Resnick, 1987) -Manipulations are carried out using quantities -Use of group work and division of labour (Resnick, 1987) -Use of tool manipulations	-Based on individual cognition (Resnick, 1987) -Manipulations are carried out using symbols (Resnick, 1987) -Individual, independent work -Use of pure mentation
Goal	-Situation specific competencies	-Generalised learning, power of transfer (Resnick, 1987)
Difference in numeration/procedure	-Orality -Use of multiple units and operations (Saxe, 1988) -Use of contextualised reasoning (Resnick, 1987) -Use of decompostion and repeated groupings (Carraher et al., 1987) -Use of convenient numbers (Nunes, et al., 1985)	-Written -Use of symbols -Use of formal reasoning -Use of formal algorithms taught in schools
Mechanisms of acquiring knowledge	-Communication, Sharing, Legitimate Peripheral Participation (Lave and Wenger, 1991) -Learning from one-another, Circulates in communication, Role of artifacts and language (Carraher et al., 1987)	-Knowledge acquisition and knowledge building is textbook based -Based on individual thinking, group-work is not always encouraged
Meta-cognitive awareness	-Confidence in procedures, meaningfulness of obtained results (Nunes et. *al.* 1985; Saxe, 1988) -Continuous monitoring ('where they are' in the middle of calculations) (Carraher et al., 1987)	-Heavy use of algorithms, lack of meaningfulness and relevance -Continuous monitoring usually not possible
Test of the acquired knowledge	-No formal examination -Tested by seniors/experts through observations	-Use of formal examinations, consisting of mostly written tests

Table 2.1 Distinction between Everyday Math and School Math

Research on out-of-school mathematics has highlighted instances of meaning making and reasoning as embedded in work-contexts (Carraher & Schliemann, 2002; Nasir, Hand & Taylor, 2008). While solving problems in everyday contexts, participants operated meaningfully with quantities, made intermediate checks if the numbers obtained were reasonable, and used flexible procedures that were based on sound mathematical principles. Problem-solving strategies in everyday contexts were in stark contrast to the symbol pushing, mechanical implementation of procedures and tolerance of absurd solutions that characterised school mathematical performance (Khan, 2004; Nunes, Carraher & Schliemann, 1985; Saxe, 1988). This led to researchers exploring how to integrate out-of-school mathematical knowledge with school mathematics. Subsequently, researchers also raised doubts about the usefulness of everyday mathematics for school learning by pointing to the very different ways in which mathematical knowledge is acquired within and outside of school, and the very nature of the enterprise of school mathematics (Carraher & Schliemann, 2002; Dowling, 1998). It was argued that the goals of both the domains are different.

More recent studies have contested the distinction perspective adopted earlier between out-of-school and school math and claimed that they may not be mutually exclusive or dichotomous (Nasir, Hand & Taylor, 2008). It has been argued that there exists shared relationships between them hinting at the hybridised nature of mathematics that students gather. However, the nature of hybridity of mathematical knowledge has not emerged explicitly through empirical findings though there are claims about students constructing knowledge from their experience in different settings. From the cultural perspectives, school mathematical learning is also a cultural form. Pedagogical approaches informed by such perspectives seek to blur the boundary between culture and domain knowledge and allow multiple points of connection to form a body of knowledge that has overlaps of different forms of mathematics (Abreu, 2008). Further, school education and mathematics teaching is “not only about building on what students are familiar with... but also about introducing new ideas, concepts and sensibilities” (Nasir, Hand & Taylor, 2008, p. 220).

Indian studies of mathematics in diverse work-contexts

Studies done in India have highlighted that different procedures and strategies adopted in work-contexts were often governed by the situation-specific requirements depending upon the diversity of goods handled and requirement of varied calculation. Khan's (2004) study of the *paan* (betel leaf) vendors in Delhi or Naresh's study of bus conductors in Chennai (Naresh & Chahine, 2013) or Sitabkhan's (2009) study of child vendors in Mumbai's suburban trains indicated that diversity of goods handled helped the doers acquire greater proficiency with computations and often determined their problem-solving strategies that were distinctly different from the regular school procedures. Similarly, studies that looked at the development and use of measurement knowledge, viz., those by Mukhopadhyay (2013) and Saraswathi (1989) in their respective work on boat making and agricultural labour, emphasised that spatial visualisation and estimation skills often shaped the measurement knowledge and proportional reasoning in work-contexts. Though these studies underlined that diversity of work-contexts creates affordances for innovating newer, context specific problem-solving strategies, the possible pedagogic implications remained elusive. To our knowledge, there are few studies focused on children's out-of-school knowledge of mathematics in

Mumbai (other than Sitabkhan's), although it has a large population living in low-income settlements, which are often economically active centres of house-hold based micro-enterprise.

Role of Work and Education

The participation of children in work is a complex issue, enmeshed in questions about the notion of childhood, the role of education and the exploitation of children. In India, debates about child labour as a form of exploitation are a central part of the debate on the right to education. While it is undeniably the case that many children suffer economic and other forms of exploitation, it is important to recognize that conceptions of childhood can be different for different cultures and for different communities (NCERT, 2007; Vasanta, 2004). In particular, for children from low socio-economic background, work is a part of the experience of childhood and a site for learning. We feel that school education should not drive a wedge between such experiences and classroom learning, as is often the case. The recommendations of the NCF, that the knowledge children gain from work contexts should be seen as a means of connecting school learning with out-of-school experience, are hence an important corrective to the “bookish” knowledge dispensed in schools in India.

2.2 Analysing learning through the lens of identity

In the recent years, in MER, the notion of *identity* has emerged as an important construct in understanding how out-of-school experiences can influence classroom learning (Nasir, Hand & Taylor, 2008). The notion captures the growing belief in the MER community that interpersonal and affective relationships have a bearing on learning (for example, Boaler & Greeno, 2000; Cobb, Gresalfi & Hodge, 2009). However, there are not many studies in MER with analyses using this notion as a tool. None of the above studies adopted any particular operational definition of *identity,* instead they commonly drew the notion from the narratives of or about individuals (Heyd-Metzuyanim & Sfard, 2012). In MER, studies have typically used the lens of *identity* in two ways: constitution of normative identity as learners of mathematics have been explored in the classroom, and affective factors like emotional hue have been analysed by looking at the narratives of identifying and subjectifying the students themselves or between them or the teacher. Current educational discourse on *identity* seeks to replace the widely used motivational notions of *beliefs* and *attitudes* which are seen as discourse-independent (Sfard & Prusak, 2005).

2.3 Funds of Knowledge

It is widely seen that children in low-income conglomerations are often bound in social relationships and work practices from an early age and the broad features of their learning develop at their home as well as in their surroundings. Households and their surroundings contain resources of knowledge and cultural insights that anthropologists have termed as *funds of knowledge* (FoK) (Gonzalez, Moll & Amanti, 2005; Velez-Ibanez & Greenberg, 2005). The "funds of knowledge" perspective brings to mathematics education research insights that emphasise the hybridity of cultures and the notion of "practice" as "what people do and what they say about what they do" (Gonzalez, 2005, p. 40). The perspective also opens up possibilities of teachers drawing on such funds of knowledge and relating it to the work of the classroom (Moll *et. al*, 1992). When FoK are not readily available within households, then they are drawn from the networks in the community. The perspective thus emphasises social inter-dependence and shows children in households to be active participants, not passive by-standers.

We use the notion of "funds of knowledge" to inform the analysis of work contexts that students are exposed to, and in illuminating the nature and extent of everyday mathematical knowledge available within the community of the classroom. We look at FoK as a resource pool that emerges from people's life experiences and is available to the members of the group which could be households, communities or neighbourhoods. In a situation where people frequently change jobs and look for better wages and possibilities, community members need to possess a wide range of complex knowledge and skills to cope with and adapt to the changing circumstances and work contexts and to avoid reliance and dependence on experts or specialists.

2.4 Funds of knowledge and pedagogical implications

Socio-cultural studies in mathematics and science education have argued that cultural resources and *funds of knowledge* (Gonzalez, Andrade, Civil & Moll, 2001) of people from non-dominant and underprivileged backgrounds are often not leveraged (Barton & Tan, 2009) in school teaching and learning practices. Neither is their knowledge from everyday life experience valorised (Abreu, 2008) and built upon in the classrooms nor is their identity acknowledged.

Educational thinkers in the developing world, and particularly in India, have recognized the value of work experience for education conceived in a broad sense. Educational philosophers, such as Gandhi developed a vision of education centred around productive work and different from the traditional education in the crafts. The aim of his educational philosophy *Basic Education* or *Nai Talim* was not training in a particular craft, but a "well rounded education of the mind, the body and the heart” (Gandhi as quoted in Fagg, 2002). Gandhi argued that “the proposition of imparting the whole of education through the medium of trades (crafts) was not considered [in earlier days]. A trade (craft) was taught only from the standpoint of a trade (craft). We aim at *developing the intellect also with the aid of a trade or a handicraft...* we may… educate the children entirely through them” (NCERT, 2007, p. 4, italics in original).

In the context of a developing society like India, in contrast to societies with advanced economies, participation of school children from low socio-economic backgrounds in work either within the household or in the neighbourhood allows integration of children into social networks that generate *funds of knowledge*, and makes this knowledge present and available in the classroom. Taking on board this insight, the current policy document in India, the National Curriculum Framework urges educators to draw on work experiences as a resource for learning. It points out that “productive work can become an effective pedagogic medium for (a) connecting classroom knowledge to the life experiences of children; (b) allowing children from marginalised sections of society, having knowledge and skills related to work, to gain a definite edge and respect among their peers from privileged sections; and (c) facilitating an appreciation of cumulative human experience, knowledge and theories by building rationally upon the contextual experiences” (NCERT, 2005, p. 6).

Chapter 3: The Study, Setting and Style of the Research Study

3.1 Research questions

The main research objective of the study is to explore the implications of everyday mathematical knowledge prevalent among the low income students exposed to work contexts for learning school mathematics. This has been elaborated in the form of specific research questions as below.

Q.1 What is the nature and extent of out-of-school knowledge of mathematics prevalent among middle graders from urban, low SES backgrounds?

Q.2 What are the everyday contexts and situations in which school going children of 10-12 years of age have opportunity to gain and use mathematical knowledge?

Q.3 What are the overlap and differences between the out-of-school and school mathematical knowledge?

Q.4 In the topic of measurement specifically, what out-of-school knowledge do students gain and what are the implications for the school mathematics curriculum?

Q.5 How can mathematical knowledge gained from everyday and work-contexts be integrated with school learning so as to enhance students' conceptual understanding of mathematics?

3.2 Location of the study

The study was located in central Mumbai's large, densely populated low-income settlement which has a vibrant economy in the form of micro and small enterprises dispersed in house-hold based workshops and manufacturing, trade and service units with high economic output. The entire neighbourhood generates huge employment opportunities. Being an old and established settlement, this low-income area attracts skilled and unskilled workers from all parts of India who come to the financial hub of Mumbai in search of livelihood. Generally the single-room, small and low-height dwellings are used for dual purposes – as workshops and as living room for the family and the workers. The settlement is thus a co-location of workplace and home for most of its residents. Practically every house-hold here is involved in income-generating work and children start taking part in them when they are young. Even such children who do not participate in work also develop fair knowledge and reality perspective about the activities and diverse work-contexts around them by virtue of the high levels of social interaction prevalent in the neighbourhood. The settlement is multilingual, multi-religious and multi-ethnic. Common house-hold occupations include embroidery, *zari* (needle work with sequins), garment stitching, making plastic bags, leather goods, textile printing (dyeing), recycling work, pottery, food cooking and delivery and so on. The goods produced in this locality are not only sold in Mumbai but also exported.

Two government schools located in the settlement and run by the local civic body were chosen for the study - an English and an Urdu medium school co-located in a five-floor school building which also houses three other schools with different languages of instruction. All these schools draw students from the neighbourhood.

3.2.1 Significance of the location

The low-income settlement is economically active with resource-rich, diverse work places and communities of work practice which create varied opportunities for school going children to gather everyday mathematical knowledge. Learning sites for children of this settlement apart from the regular schools are house-holds engaged in work, diverse work-contexts, tuition classes, shopping and house-hold chores. People in this settlement maintain strong social connections and are well networked with their employers, middle-men, distributors, shopkeepers, friends and relatives. From our interactions with the community members, it appeared that the social relationships are mostly economy driven. The entire neighbourhood creates opportunities that expose children living here to the *funds of knowledge* available within the community.

3.2.2 Socio-cultural and socio-economic scenario

Participants in the study belonged to immigrant families living in low socio-economic conditions. Most women in the settlement are engaged in some house-hold based micro enterprise in the locality, while men either run their own workshops or small business, or are employed in one. We noted that parents of the students had varying years of schooling including no schooling. The settlement consists of heterogeneous groups of residents belonging to different ethnicity, language, religion and socio-cultural background.

3.3 Style of the research study

Establishing access to the field

The research study done over a period of two years and a half, was conducted in phases. Access to the field was established with the help of the key informant – a 74 year old resident of the settlement and a community leader, well regarded for his social work in the settlement. He guided the researcher in getting necessary permissions from the civic body (Municipal Corporation) for the study and introduced him to the English and the Urdu school authorities to begin classroom observation and interaction with the teachers. The researcher started visiting English and Urdu medium schools daily and began to observe mathematics lessons in Grades 5 and 7. Such visits helped in building rapport with the teachers, students and other people on the staff. The researcher held informal discussions with students, teachers and visited students' homes and workplaces. Discussions with the students helped to get a broad picture of the nature of their daily activities that have aspects of mathematics and the nature and extent of their everyday mathematical knowledge.

3.3.1 Research design and methodology

The research study followed a blend of ethnographic, case study and teaching design experiment methods in broadly three overlapping phases discussed in the following table:

Phase		Objective	Sample	Method	Data Sources
I (Ethnographic)		Exploration of the children's life-world and opportunities	Classroom observation: Grade 5 (Urdu); Grade 7 (English)	Building a rapport with the students, teachers and community members; classroom observation; visits to field, workshops, students' home, shops.	Field notes Photographs
II (Case-studies)	Part-I	To understand family background, SES, students' outdoor activities, engagement in work, parental occupation	Randomly selected representative sample of 31 students	Semi-structured interviews	Audio records Transcripts Students' worksheets Photocopies of students' “tuition” notebooks
	Part-II	To understand students' basic arithmetical knowledge	30 students from the previous sample	Interviews based on structured questionnaire	
	Part-III	To focus on students' knowledge about their work	Sub-sample of 10 students + 7 additional students who volunteered	Semi-structured interviews	
III (Teaching Design Experiment)		Connecting students' out-of-school math knowledge & school learning Exploring role of students' knowledge and identity	About 25 Grade 6 & 7 students of Urdu school and 3 Grade 6 students of English school	12-days' Teaching intervention camp	Classroom videos & logs Transcripts (first 2 lessons)

Table 3.1 Research Design & Methodology

Phase-I (ethnographic exploration) had an overlap with *Phase-II* (case studies) and *Phase-III* and continued till the end.

3.3.2 Strategies and Instruments for Data Collection and analysis

- Classroom observation
- Teaching as a participant observer
- Prolonged engagement of around three years with the field (visits & recordings)
- Exploratory interaction
- In-depth interview

Data transcripts & Coding scheme

All the students' interviews about work-contexts and the two lessons from the teaching intervention in Phase-III were fully transcribed for coding and further analysis. Written logs of the lessons in Phase-III supplemented the transcripts of the lessons. The interview transcripts were coded at first and second levels to review what they indicated about the nature of students' work and their knowledge about aspects of the work. The coding was done separately by the researcher and his colleague and coded for 10 different categories: work, learning, everyday math, affect, foreground, personal, and others, and the differences were reconciled through discussion. Lesson transcripts from Phase-III were read together with logs by three researchers. Segments of the transcripts relevant to the research questions were identified and carefully reviewed.

3.4 Ethical considerations

The researcher ensured that respondents' and informants' anonymity was protected. The researcher conveyed to the participants the purpose of the study in broad terms and did not engage in deception of any kind about the study. In this dissertation thesis, the researcher has not included such information that participants shared with him on "good faith" or to be kept "off the record". Such information has not been made part of the analysis while ensuring that the analysis is not inconsistent with such information.

3.5 Overview findings: Children's life-world

This section discusses findings based on visits and interaction with students and the community, from the classroom observations, from the records of students' work in the tuition classes and from other learning sites (not included in synopsis).

Chapter 4: Participation in work-contexts and mathematical knowledge: case studies

This chapter presents detailed findings from the interview data of Phase-II of the study concerning students' immersion in work contexts and their mathematical knowledge. A total of 31 students from the two grades were interviewed to obtain basic information about family background and participation in work. We observed that 30 of the 31 students were engaged in some work practice. We summarise the students' and their parents' work-profiles in Tables (not included in the synopsis). An in-depth semi-structured interview was conducted with a sub-sample of 10 students and 7 additional students about their knowledge of work-contexts and mathematical elements embedded in work-contexts. This is analysed through the case study approach and we discuss four case studies chosen from the reduced sample of 10 students in detail in this chapter. We use information from the remaining interviews to round-off the findings of the case studies to arrive at a reasonably comprehensive picture of the knowledge of work-contexts and of mathematics of our participants.

4.1 Case Studies

In this section, four representative cases are reported, two from the English medium and two from the Urdu medium schools (boys – E_5 and U_2, girls – E_{16} and U_{13}), with a focus on their exposure to work-contexts, their knowledge of arithmetic and measurement and the mathematics related to earning. The four cases described were chosen since together they reflected four different kinds of work-contexts with a range of opportunities for mathematics learning, viz., independent collection of material for recycling (E_5), an employee in tailoring work who goes through different learning stages (U_2), a girl with exposure to diverse kinds of stone-fixing work on jewellery done at home and of running a stationary shop (E_{16}), and a girl with relatively limited exposure to work-contexts (U_{13}) (Bose & Subramaniam, 2013). An embedded analysis of the cases has been preferred over a holistic analysis as the intent of conducting case study was to understand the extent of mathematical elements embedded in each work-context and students' knowledge of them. Hence, an analysis of themes (viz., knowledge of work context, mathematical knowledge, measurement knowledge, mathematics of earning) is presented focusing on the aspects of mathematics emergent from each work-context.

4.2 Drawing the cases together

Connections between out-of-school and school mathematics

From a viewpoint of looking at the overlaps between out-of-school and school mathematics, it was observed that use of inch scales and other standard and non-standard units/scales are more common in work practices than in schools. Some students showed more reliance over formal algorithms and used them in maintaining accounts or doing calculations. On the other hand, students like E_{16} preferred to use their own convenient and situation-specific strategies despite having learnt formal algorithms well. E_5 and U_2 had similar reliance on their own strategies for computations. Such characteristic features of students' work-contexts and everyday experience indicate the hybrid nature of mathematical knowledge prevalent among children and they draw from both school and everyday mathematical experience. Most mathematical procedures that the students used show inter-penetration of both school and out-of-school mathematics.

Most of the students that the researcher came across spoke about the work-contexts happening around them with confidence showing access to *funds of knowledge* which included not only the kinds of work that they themselves participated in, but also about other work-contexts that occur in the settlement. This phenomenon was interesting since within a single class, students had peers who were engaged in diverse work practices and created opportunities for learning about them.

Issues of fairness

Fairness is seldom taken into consideration in the world of work, which is governed far more by possibilities and bargains. For poor children in the metropolis, fairness is not easy to grasp. To cite an example, when the researcher discussed with U_{13} whether she was satisfied with the wage for making *Rakhi* (decorative wrist-bands) she answered in affirmative. On asking she could only tell the retail price of one dozen *Rakhi* – at least Rs 60 (one *rakhi* is sold for Rs 5; 1 USD = Rs 60 approx.), whereas for making one gross (12 dozen) *rakhi*, she gets Rs 15 or less. The researcher helped her calculate the retail price of one gross *rakhi* – Rs 720 and compared it with her wage (Rs 15 or less), but the discussion did not trigger any concern about fairness of wages in the student. Here is an occasion where knowledge of mathematics can possibly lend power to call for fairness and justice (Bose & Kantha, forthcoming).

Gender aspects of work-context

In some work-contexts, especially those which are typically done by women and girls at their home as in the case of U_{13} and E_{16}, the opportunities to use diverse goods or raw materials or awareness about the linkages that their work has with other tasks on the production network are largely constrained. Women in the community and school going girls like E_{16} or U_{13} are mostly involved in those kinds of work which require working at home. In the case of U_{13}, she did only a small chunk of the entire *rakhi* work or *garment manufacturing* work. Although her work was large in terms of quantity of output, there was little diversity in the work.

The examples emergent through case-studies underline our claims that the whole gamut of everyday experiences including diversity of cultural and work practices shape students' everyday mathematical knowledge and has structural difference with school mathematics. However, the inter-penetration between everyday and school mathematics indicates that learning in one domain has relevance for the other which remains to be unpacked.

Chapter 5: Learning, mathematical knowledge and identity in out-of-school contexts

In this chapter, we analyse the data from the case-studies to show how opportunities for learning in general and learning mathematics in particular arise in work contexts. We discuss aspects of the mathematical knowledge gained in out-of-school contexts and its relation to school mathematics. We also discuss how work-contexts shape the identities of participants in our study as learners. Finally we draw some implications from these analyses for the teaching and learning of school mathematics aimed at making connections with out-of-school knowledge.

5.1 How do work contexts create opportunities for learning?

Features of work contexts and the degree of students' engagement in them shape the learning experience of students who participate in the work-contexts and the richness of the knowledge that they acquire. From an analysis of the data, we discuss how opportunities arise for learning in work contexts under three rubrics (Bose & Subramaniam, 2013):

- Diversity
- Making decisions in relation to work; optimising resources and earnings
- Involvement in the work; awareness of linkages

Handling of diversity of goods and requirement of the tasks, control over and extent of decision making, need for optimisation, knowledge of backward and forward linkages are strongly related to the sense of ownership that participants have about their work. Study participants whose close relatives, friends or families own businesses have a stronger sense of ownership of the work, in comparison to those who work merely for wages. Such involvement creates greater opportunities to gather and use mathematical knowledge. For example, in the case of E_5 (garment recycling), U_{22} (mobile repairing), U_{23} (textile printing) and U_{24} (ready-made garment selling), we noticed that these students had a sense of ownership and were aware of diverse aspects of their work as well as the forward and backward linkages that the work had. Except for E_5, these students did not participate in the work primarily for the income, but rather also to learn something and to pick up useful skills which are valued as they are perceived as securing opportunities to get future employment. For example, U_{22} took pride in knowing about both kinds of work – mobile phone repairing as well as garment stitching work. His father runs a shirt stitching workshop where three other workers are employed and U_{22} does not particularly need to earn to support family as is the case with several other children. In the case of E_5, U_{22}, U_{23}, and U_{24}, where the sense of ownership and control over decisions was strong, frequent references were made to decisions over deals. In the case of some students, we noticed a reluctance to use mathematical calculation to engage with questions of fairness of income, and in some cases inappropriate use of calculation.

5.2 Features of participants' mathematical knowledge in relation to out-of-school context

We identify knowledge that students demonstrate as out-of-school knowledge on the basis of features identified in the literature. These include the form in which the task is presented and accompanying contextual details which elicit student knowledge, the presence of oral computation strategies and the reference to mathematical entities that do not appear in the school curriculum such as words for binary fractions. In the interview, arithmetical tasks that were both context-rich and presented in purely symbolic forms were used. Students' performance in context-rich forms was slightly better. Tasks which were formulated in rich contextual detail usually elicited oral computation strategies and factoring in of reality perspective. On several occasions participants used school math knowledge in the form of formal algorithms like unitary method as well as oral computation strategies. Although the tasks included prompts located in the out-of-school contexts, participants' responses often began with using school learnt method, subsequently falling back on their out-of-school math knowledge. Thus on multiple occasions, we found students using methods that they had learnt at school together with those that were likely not taught explicitly at school.

Students' response to arithmetical tasks allowed us to get a sense of the kinds of mathematical elements embedded in out of school contexts, and also the nature of such knowledge. We describe aspects of such out-of-school mathematical knowledge of the study participants.

Limited combinations and fragmented knowledge: A feature that we noticed about the mathematical aspects embedded in work contexts was that variation was limited to what the context itself included. Thus the mathematical experience of students was constrained and limited in terms of variations and possibilities, the exploration of which is an essential part of mathematical abstraction. For example, E_{16}'s stone-fixing work required her to make arrays of the finished jewellery pieces in only limited arrangements: 6 × 4 or 12 × 2 arrays on a card so that six such cards put together can make one gross (144 units). Though she knew about such combinations reflecting familiarity and ease of calculation, it was not clear from her responses whether she considered other combinations or total quantities other than 144.

Similarly, students displayed familiarity with inch and metre for length measurement, but were unaware of the connection. In the neighbourhood shops, small quantities of milk are sold by weight as well as by volume (interchanging of volume and weight units: “kilo” instead of “litre”), but our participants did not show awareness of the relation between these two measures. Thus everyday contexts give rise to pieces of mathematical knowledge that may be intimately familiar to students but may be unconnected to other mathematically related pieces of knowledge. The familiarity and confidence that students display about what they know suggests however that even such fragmented knowledge can be a potential resource for classroom learning.

Knowledge for use rather than conceptual knowledge: In the work-contexts, non-transparent mathematical artefacts are used which are familiar to the users in practice but the conceptual underpinnings are blurred. For example, inch tape is used for quantification of length, but the principles underlying its construction remains unclear to the users. Students may be aware that length can be measured (quantified) by iteratively covering with a unit, but may not be aware that this principle is the basis for constructing the length units (as was revealed in the teaching intervention discussed in Chapter 7). Similarly, construction of small weight measures like small stone markers or sub units marked on a scale remain unclear. Students may know about the use of length dimensions for designating “size” (area), such as in frame sizes, but the connection between length and area remains opaque. To take another example, it remains unclear as to how the numbers or labels designating the garment-sizes are arrived at and what they actually signify. In the teaching interaction phase of our study, we attempted to address this issue and the participants (middle graders) who took measures of different parts of the garments given to them as part of the activity, could not actually see the relation between the “size number” and any of the measures. We therefore argue that in most everyday or work contexts, knowledge and mathematical artefacts though frequently used, remain opaque and non-transparent when it comes to the conceptual knowledge associated with those artefacts.

The features discussed above have implications for school teaching and learning which aims at making connections with out-of-school knowledge of mathematics. The implications are explored in the context of a particular topic area, that of measurement, in chapter 7.

5.3 How does out-of-school knowledge shape learners' identities?

Our interaction with the community members indicated that learning of work skills as well as learning at school – both are valued in the community. Though we came across many children during the study who dropped out of school for different reasons, most parents and elders we came across seemed to be concerned about their children's school learning while at the same time wished their children learnt hand skills in their spare time.

Value of learning hand skills (*haath ka kaam sikhna*)

Learning *haath ka kaam* (hand skill) is seen as laying a foundation and making it easier to get *acchha kaam* (good job) in the future. This view was frequently echoed by students. Getting involved in "any kind of work" (*koi bhi kaam*) is valorised in the community since it builds networks with people including *seth* (workshop owners who provide jobs), helps in learning hand skill and utilising time in a better way. It is believed in the community that learning hand skill early would be "useful later on" (*aage kaam aayega)* to "learn something different" (*kuchh alag seekhne ko milega*) and also to earn more. "*Time barbad nahin karna"* (not to idle away time), "*khali nahin baithta"* (not to sit idle) and "*samay ka sahi istemal"* (proper use of time) are other phrases that students frequently used. This could be a reason why such children also work whose families do not need to supplement their income.

Value of school learning

As described before, school learning is valorised in the community and seen as a gateway to future opportunities. Graduating from school is taken as a benchmark and parents often urge their children to complete schooling. The excerpt presented below highlights how E_5's parents and relatives suggest that he continue his studies so that he can be placed in a job:

456	S	meri ammi boli barawi padh le, mera babora bola chaudha padh lega to tere ko police ki naukri mein daal dunga/	My *ammi* (mother) is asking me to study till twelfth, my uncle said if you study till fourteenth then I will put you in the police's job/

In the excerpt below, U_2 reiterates his interest in learning mathematics. On earlier occasions too, he expressed his concern to the researcher that he is not good at math and he wants to learn it well.

573	S	Main bus sir math padhna chahta hoon/ math achhi se koi padha de na...	Sir, I just want to study math/ if only someone can teach me math well...

Identities in work-contexts

Students identify themselves as knowledgeable, with skills and capabilities in work contexts. They often described their work with pride. For example, E_5 mentioned his record collection, when he once collected 95 kilos of *chindhi* and earned Rs 640. This functioned as a benchmark and helped him to decide how much time to spend on a given visit. Most students described their work arithmetically and with many precise details. Students often invited the researcher to their workplaces/workshops and explained their work and the related work processes. The researcher was able to visit U_{21}'s garment stitching workshop and U_{25}'s *zari* workshop several times on their invitations. During these visits, the researcher was invited to learn a few skills like holding the needle used in *zari* work and picking the sequins. In the process, the researcher's action was evaluated and often corrected. The researcher came across many students outside the sample who were keen to talk about their work-contexts. This included the seven students who volunteered to take part in the in-depth interviews as well as other students.

5.4 Implications

Work-contexts have rich resources and opportunities for mathematical learning though such contexts often extend only limited possibilities of using mathematics in the form of “use knowledge” devoid of the requirement of conceptual understanding. However, the affordances created by the work-contexts can scaffold school learning towards conceptual development. The case-studies indicated the formation of identities among the participants through their out-of-school knowledge. These implications are also important for school experience which as of now tends to reinforce the disconnect between out-of-school and school learning.

Chapter 6: Opportunities and Affordances for Measurement Learning

This chapter elaborates on the implications for school mathematics learning of out-of-school knowledge of our participants by focusing on the topic of measurement. The discussion begins with the research literature on measurement knowledge in out-of-school and in school learning contexts.

6.1 Measurement in the everyday context

Previous research on measurement within work-contexts or in other everyday settings was carried out alongside or within the research on out-of-school mathematics, with a particular focus on the alternative ways of thinking in different everyday contexts. Such research provided evidence of how mathematical ideas were developed and framed within work-contexts.

Most of these research studies focused on participants' measurement knowledge involving adults in their singular work-contexts. We have not come across studies that looked at the varied contexts in the everyday settings that students from low socio-economic backgrounds are exposed to and the affordances of these settings for school learning about measurement. The literature mentioned above has led to a cumulative understanding of the skills, procedures and strategies based on mathematical principles that are acquired in out of school work contexts. The focus has been on oral computation strategies, proportional reasoning strategies, visuo-spatial and geometric reasoning and estimation skills and strategies. In our study, we restrict focus to the topic of measurement, but take a broader view of not only what our participants know or can do, but also what they have observed and are familiar with even if the mathematical knowledge associated with these aspects is partial and fragmented. Our perspective is to explore what aspects can serve as starting points or building blocks for mathematical exploration in the classroom. We are also interested in how mathematical learning can strengthen the understanding of measurement practices in the real world.

6.2 Research on the learning of measurement as a school curriculum topic

Current Indian mathematics textbooks do not adequately cover many of the key ideas underlying measurement learning such as conservation, transitivity, equi-partitioning, iterative covering, additivity and role of scales. A look at textbooks prescribed by the central and state governments (followed by the vast majority of students in India) reveals that the dominant emphasis is on acquiring measurement skills and on knowledge of the international system of units (for example, Maharashtra Math Textbooks 5, 6, 7, 2006; NCERT Math textbooks 5, 6, 7, 2006). Conceptual issues are dealt with briefly under the rubrics of "use of non-standard units" and "need for standard units", before the treatment moves over wholly to the development of skills. These include familiarity with common measurement instruments, use of standard measurement procedures, inter-converting between smaller and larger international units and computing with units. Observations of the classroom teaching in the schools that formed part of the study revealed that there is even greater emphasis on paper and pencil computation skills with very little treatment of either conceptual matters or even of practical measurement.

6.3 Measurement related experience

The diversity of everyday and work settings discussed in the case studies (chapter 4) give rise to diverse experiences of measurement. In this section, an analysis of diversity is presented highlighting the inherent richness of concepts implicated in such experiences, which can help connecting such knowledge with classroom learning. These aspects are discussed under two rubrics:

6.3.1 Comparison, estimation, quantification and construction in relation to measurement

Measurement in everyday contexts including work and domestic settings is different from measurement in the scientific world. Precision and accuracy are not as important as convenience. In many situations approximate measurements suffice. Comparison at times draws on embodied kinesthetic knowledge, as in *zari* or *latkan* making work, while judging the suitability of a decorative piece within a defined space. Bag or purse-making commonly use congruence and similarity of shapes and designs. These features lead to a diversity of measurement modes that are used in everyday contexts. In contrast, the school curriculum emphasises scientific measurement based on full quantification using a system of units, with well-defined relationships between sub-units and between fundamental and derived units.

Comparison between school and everyday measurement experience

In our study, it was observed that measurement experience in the everyday context is richer and more sophisticated than measurement experience that arise in the classroom context. This is due to the diversity of measurement modes and aspects of construction of units and tools that are often encountered in everyday contexts. In work-contexts, construction of convenient units or templates derived from standard units are conceptually rich actions since they involve partitioning, combining or manipulating a standard unit and quantification. Students learn to use inch and foot in their everyday contexts and learn to carry out measurement of acceptable accuracy by reading off the lengths from the tape. Such measurement is fully quantified, but the quantification is opaque and the measurement itself depends critically on the integrity of the artefact. However, most children are unaware of how weight or length is quantified. Table 6.1 below presents a summary of the diversity of measurement modes, units, processes and attributes in a few illustrative contexts related to work, school and shopping.

Contexts	***Objects & Attributes measured***	***Measuring instruments***	***Measurement units***	***Measurement modes: Quantification, Estimation, Construction***
Tailoring, leather work	Length of cloth, area of leather pieces	24” steel scale, 1m steel bar, 60” plastic tape *tedha* scale (bent scale used in tailoring)	Old British units (inch, foot, *guj* or yard) International units (metre, centimetre) Non-standard units (cubit, finger-band) Indigenous units (*desi, waar, kattha, bigha*)	Construction of Templates (*farma*), iterating to measure length and area Estimation Comparison
Recycling	Grading of plastic sheets (recycling work) Weight of *chindhi*	Weighing hook/beam scale (*kaanta*)	Standard units (kg)	Ordinal numbers (grades of plastic sheets) Estimation Comparison
Shopping	Weight of provisions, goods length volume counts of discrete objects sizes of garments, shoes (denoted by number)	Balances of different kinds, meter scales, volume measures	Old British units (dozen, gross, ream) International units (metre, centimetre) *Mutthi* (fistful) *cutting* (tea)	Construction of standard units by partitioning construction of convenient units
School	Length Weight	Standard ruler (6” or 12”) None	International units (metre, cm, mm) (Kg, g)	Measurement by reading from a scale

Table 6.1 Diversity of measurement related experience

6.3.2 Diversity of object, measurement units and tools

As summarised in Table 6.1, this section discusses the variety of objects that are measured and the variety of measurement units, tools and modes that students encounter in their everyday settings.

- Objects and attributes measured
- Use of different measuring instruments
- Use of different measurement modes
- Use of different measurement units

6.3.3 School and work-context math: different requirements

We noted that the abstraction entailed in work-contexts vary and are different from abstractions handled at schools. For example, diverse measurement work-contexts implicitly use abstract notions like construction of units and sub-units, chunking of measures, partitioning, unit iteration, covering, use of convenient units and modes (like templates) which are available to students as part of the everyday mathematical knowledge. The school curriculum, in contrast, treats learning of measurement as a skill development and then moves towards abstraction without building on the knowledge resource already available to the children from the work-contexts. The abstractions available to students are in implicit form through their exposure and experience in work-contexts are potentially rich resources for building on measurement knowledge in the classrooms. Similarly, conservation of attributes, transitivity and seriation that are the foundation of comparison thence backbone for developing critical understanding of measurement are not sufficiently emphasised while handling abstractions in the school context. Thus, although experiences in work-contexts or in the cultural practices can help in broadening children's learning, they are not leveraged in the formal learning situation.

We highlight below some of the ways in which the measurement knowledge gathered from out-of-school contexts discussed in this chapter may be used as learning resources in the mathematics classroom.

Construction of templates and units:

Familiarity of students to construction of templates for purposes of comparison and measurement by iteration (e.g., *farma* in leather work for required size or for iteration to optimize use large leather piece) is valuable for measurement learning. Templates have fixed measures, while units can be chunked or partitioned to obtain larger or smaller units and generalisable beyond the immediate context of application. Participants' familiarity with the construction of templates to measure length, area and weight can give rise to questions that can lead to fruitful mathematical work in the classroom: why is the construction of units or templates needed; how do we construct new templates or units from given templates; in what contexts are units partitioned to yield smaller units; what quantities can be measured with a given combination of templates.

Measurement of area:

Area as seen in the examples above was frequently specified using a rectangular template. Templates can serve as a unit for iteration and give rise to discussion about the variations in the area measurement, use of different shaped units to measure area, their relationships and equivalences.

Opaque quantification & Archaeology of measurement tools:

Our study-participants were familiar with some quantifications but their origin was obscure, as in garment sizes. Similarly, they used common measuring tools such as the inch tape, but were unclear about the meaning and construction of the markings on the tape. This can gain be a powerful starting point for archaeological exploration that can lead to learning about length measurement and its uses.

Prevalence of different units and systems:

Students in our study used different kinds of units: international units, old Indian units, old British units and non-standard units. Besides the idea that units are purely conventional creations and are embedded in cultural and political histories, such diverse knowledge is useful in exploring the relation and differences between different systems. Questions that can be fruitfully explored for example are, why do we need unit systems rather than just units; what are the different principles of subdivision and the advantages and disadvantages of the binary and decimal systems.

Quantification of various attributes:

Drawing on their familiarity with the range of objects and attributes that are quantified, students can explore questions such as what is common and what is different in how we quantify different attributes; how is an abstract attribute like monetary (exchange) value quantified; how do we quantify different aspects of labour such as time, effort and expertise.

Chapter 7: The teaching intervention

Phase III of the study consisted of a teaching design experiment (Cobb, *et al*., 2003) in the form of a two-week long summer vacation course conducted for sixth and seventh graders, aimed at drawing curricular and pedagogic implications of connecting everyday and school math knowledge. The classes were conducted by the researcher's senior colleague from HBCSE for one hour and a half everyday for 12 days. All the lessons were recorded on video.

This chapter reports the analysis of the teaching design experiment that was focused on integrating students' out-of-school measurement knowledge with formal teaching aimed at building conceptual understanding among the participants. The design experiment was intended to explore the possibilities and limits of connecting everyday mathematical knowledge with school learning. The emphasis was on exploration and establishing feasibility, rather than effectiveness of instruction in terms of learning outcomes.

A major purpose of the analysis was to elaborate the enacted goals of the teaching design experiment. This analysis is important to answer the question “what should be the goals of an approach to teaching that attempts to connect out of school knowledge with school mathematical learning?” The analysis focuses broadly on two aspects (i) conceptual connections between everyday mathematical knowledge and school mathematical knowledge with a focus on the topic of measurement and (ii) agency and identity formation in the classroom in relation to the connection between out of school and school learning.

7.1 Goals and objectives of the vacation course

The broad goals of the vacation course were to

1. make connections between out-of-school mathematical knowledge of school children and learning of school mathematics
   1.1. By using their out-of-school knowledge to organise and build conceptual learning of school mathematical topics,
   1.2. By using school mathematical learning to illuminate aspects of out-of-school knowledge.
2. foster identities that allow connections to be made between out-of-school and school math knowledge and to align students’ identities as learners of mathematics and as experienced and knowledgeable persons in everyday contexts
   2.1. By legitimising the sharing of everyday knowledge in the classroom,
   2.2. By encouraging explanations that connect everyday and school knowledge,
   2.3. By building a culture of shared learning in the classroom.

The specific instructional objectives of the vacation course were formulated in the light of the broad goals. The enacted objectives focused on two mathematical topics (i) length measurement and (ii) fractions and decimals. On the topic of measurement, the objectives included drawing on students' out-of-school knowledge of length measurement to deepen conceptual understanding of units and sub-units in measurement by connecting them with out-of-school contexts known to children such as tailoring. The second objective was to connect students' out-of-school knowledge of fractions and proportional reasoning to the school topic of fractions and decimals by (i) strengthening and extending students' understanding of binary fractions gained from everyday contexts, (ii) making connections between binary and decimal fractions, and (iii) building students' understanding of decimal fractions.

The instructional goals described above spell out how classroom teaching might draw on and connect with out of school knowledge that students bring to the classroom. However, school learning is not the same as out-of-school learning and the goals need to acknowledge the complementary dimension of the differences between out-of-school and school learning. This complementary dimension of the instructional objectives could be viewed through the lens of enabling a series of shifts:

- Shift from oral to written math
- Shift from knowledge about use (tool/artefact) to understanding the tool (e.g., measuring tape, numeral sizes of garments)
- Shift from individual expression in private to shared, public expression
- Shift from co-operation to a mathematically focused discourse community (e.g., shift from making assertions to providing clarifications, justifications, explanations; moving from "helpful" interactions to a discursive culture)
- Shift from identities that are disconnected (or reinforce disconnection) to identities that are connected

7.2 Making conceptual connections

A second focus of the analysis of the teaching design experiment was on the conceptual connections between out of school and school mathematical knowledge with specific reference to the topics covered in the lessons of measurement and fractions. Classroom interaction in the course of the design experiment supported some findings concerning the mathematical knowledge of students from earlier phases of the study and elaborated on others. The fragmented nature of knowledge from out of school contexts was reinforced on multiple occasions.

Although students were familiar with words for binary fractions and made connections between them, the written notation was elusive. This indicates the fragmentary and tenuous connections that exist in students' minds about the fraction notation for binary fractions or the equivalent decimal fractions. Students often used proportional reasoning and convenient decompositions in computing answers when they were encouraged to do so in the classroom. An example was the task of finding out ½, ¼, 1½, 2½ and 3½ times a given number. Students explained the various strategies that they used to compute these multiples to their peers, which indicated that they had a robust and confident awareness about decompositions of fractions (twice 1½ is the same as three; half of 1½ is equal to ¾, half of 2 ½ is equal to 1¼, etc.)

7.3 Student participation: constructions of identity

A third major purpse of the analysis was to arrive at an understanding of students' receptivity to the instructional goals from the nature of participation in the classroom. The analysis draws on the notion of "normative identity" as a construct illuminating the participation structure in the classroom (Cobb, Gresalfi & Hodge 2009, see chapter 2). This refers to the set of norms co-constructed by teacher and students in the classroom that determine expectations about how students should ideally participate in the classroom. With regard to mathematics, the normative identity refers to what is considered appropriate mathematical engagement on the part of a student. Individual students may accept the normative identity, merely co-operate without accepting the identity, or actively resist the identity.

From the transcript of the lessons, we reviewed episodes that revealed the teacher's and learners' engagement in (i) requesting and sharing out-of-school knowledge, and (ii) asking for and providing explanations, clarifications and justifications. The teacher's invitations to students to share what they knew about work contexts was a striking feature of the initial teaching episodes, where setting up of norms is a primary goal. Students readily participated in such interaction suggesting acceptance of the norm of sharing knowledge about work and other out of school contexts. Another device implemented by the teacher was to bring artefacts from work contexts into the classroom, setting up a difference from a typical school classroom. For example, shirts and kurtas of different sizes as well as the measuring tape were introduced in the classroom followed by a non-standard but fixed unit (a paper-strip made from A4 paper). These moves by the teacher elicited enthusiastic participation from the students. The students worked in groups and quickly taught each other the correct ways of taking the measurements of a shirt. Simple calculator commonly used by shopkeepers and familiar to students was introduced eliciting similar enthusiastic participation from the students.

The teachers' questioning frequently focused on providing explanations, clarifications and justifications. Three kinds of sources of justification were accepted in resolving "how do we know" questions. One was the invoking of authority which was done in cases where information was to be shared, or conventions about symbols needed to be cited. A common source of authority was the teacher himself. Another source of authority was an artefact (e.g., calculator). Computations done on the calculator were frequently invoked to judge the correct decimal representation of a known fraction.

A second source of justification was prior knowledge of mathematics. For example, a relation between fractions may be justified using a computation procedure. A specific example is to explain that 1/10 is the same as 0.1 because adding 0.1 ten times using the vertical addition algorithm gives 1.0. A justification of this kind is lengthy and not mathematically elegant. For a justification such as this to be accepted, such procedures needed to be part of the shared knowledge of several students in the classroom. Prior knowledge was typically restricted to math procedures learnt at school.

A third important source of justification was experiential knowledge, which typically was from out-of-school contexts. The use of binary units and fractions was frequently invoked, as was proportionality reasoning and convenient decompositions. Knowledge about units and relations between units were sometimes cited. One student frequently justified his oral computation strategies using money as a convenient representation for quantity. That such justifications were accepted by other students indicated a level of shared knowledge drawn from out-of-school contexts.

Chapter 8

Conclusions, implications and future directions

This chapter attempts to bring together the results and findings of the study in terms of out-of-school knowledge of children immersed in work-contexts and its curricular and pedagogic implications under a unifying perspective. Future directions and road-maps are also discussed. The research study proposes that learning of mathematics is aimed at acquiring conceptual understanding and insight and not at practical training.

The funds of knowledge perspective illuminates how the connectedness of social networks gives rise to diverse and rich knowledge and experience that can be drawn on for the purposes of school learning. In our study, which is set in an urban, developing world context, we found that students often directly participate in work, or are closely aware of work contexts and practices. Experiences and knowledge of measurement drawn from such contexts are intimately familiar and present in the classroom. Such diversity of experience, within a school community hence presents potentially rich opportunity for learning that has been largely ignored in formal school education.

The findings

Overlapping school and out-of-school (everyday) math knowledge

In concurrence with the recent studies done in the areas of out-of-school or everyday mathematics, our study indicates the overlapping nature of students' school and out-of-school math knowledge, i.e., the forms of mathematical knowledge were not distinct but drew from each other as well as from other nodes on students' social and work network. Our data from from students' interviews on arithmetic tasks support this claim (discussed in Chap. 4). This is despite the prevalent classroom culture and the beliefs that many children hold, which tend to reinforce the separation of two forms of knowledge.

Diverse work contexts: potentially rich resources for learning

The diversity of work practices prevalent in the low-income settlement presents potentially rich resources and opportunities for gathering mathematical knowledge. For example, using funds of knowledge perspective, we observed that our student participants have varying degree of measurement knowledge derived from the work-contexts around them. Student participants are likely to know some elementary notions such as transitivity, conservation, partitioning and unit iteration through their exposure to work-contexts, although they may be unclear about how these notions form the basis for common measurement procedures, tools and conventions.

School & out-of-school math: difference in structure, goals & requirements

The experiences of measurement in out-of-school contexts are characterised by diversity as well as structural differences from the school mathematical treatment of measurement. A central aspect of school knowledge is its generality, of its not being tightly bound to particular contexts. Specialized knowledge is context-bound, well-practiced and embodied in individuals, and leads to expertise and efficient action in limited domains and situations in contrast to generalisability and wide applicability. From the standpoint of valuing such generality as an aspect of school learning, it is the diversity of out-of-school experiences that creates the context for school learning. Thus, from our perspective, it is incorrect to claim that work practice already reflects mathematical thinking or understanding. Mathematical aspects are only present in hybridised and opaque embeddings. It is also incorrect to expect school learning to illuminate or strengthen a single kind of practice in a particular work context. It is the diversity of practices taken together that formal mathematical learning can illuminate. It strengthens understanding, not practice.

Learning skills and acquiring knowledge through participation in work is valorised in the community that we studied, although some families discourage their children from participating in work because they think it would affect their studies. School learning too is valued, although for different reasons and as a different kind of learning. It has aspirational value, and the community believes that education is the route to social and economic mobility. However, it is self-defeating for an education system to merely aim to produce the trappings of social class, while depriving learners of knowledge that has power because it illuminates aspects of life. Students from deprived backgrounds enter the classroom with their own rich complement of experiences. Our perspective is that education that shuts this rich resource out of the classroom is a recipe for failure.

Conceptual underpinnings

The thesis focuses on the topic of measurement in the school curriculum to draw specific implications for curriculum and pedagogy. Post-Piagetian research studies have highlighted the importance of concepts such as conservation, transitivity, equi-partitioning, displacement, iterative covering, accumulation of distance and additivity and the role of the origin on scales. From the point of view of the diversity of out-of-school experience, we need to go beyond these critical concepts to include construction of units and templates, equi-partitioning and chunking of measures and unit, construction of measuring scales, design of convenient measuring instruments and units. Further aspects critical to the understanding of measurement that have not been adequately addressed in the curriculum include the extensive use of comparison and estimation in real life contexts, the use of the body as a measuring instrument, the trade offs between convenience and accuracy, the variety of purposes of measurement, the variety of modes of quantification and the limits of informal quantification, and the cultural-historical origins of units and systems of units.

These aspects, with the exception of estimation, have also not received the attention of mathematics education researchers. The diversity of measurement experiences in out-of-school work contexts can be drawn upon to illustrate each of these concepts and ideas, and also for understanding the difference between comparison, estimation and measurement and their purposes.

Archaeology of artefacts

A second aspect of out-of-school knowledge that makes for potentially powerful connections with school learning is the fact that artefacts and practices from everyday settings represent a sedimented and embodied form of mathematics (Chevellard, 2007). The measuring tape embodies the processes of unit construction, unit iteration and counting and partitioning of units into sub-units. These processes are however hidden from view and are opaque. The redundant inclusion of a second system of units in the form of inches and feet on the measuring tape captures an aspect of history and highlights the arbitrariness of the choice of the basic unit of length. The purpose of such embodiment is precisely to make the mathematical thought and processes behind the construction of the measuring scale unnecessary, and to reduce the practice of measurement to the simple act of reading off the scale. This is the general phenomenon of dematHematisation described by Chevellard (2007) and Gellert and Jablonka (2007) where material artefacts embed increasingly sophisticated mathematical ideas, while rendering the user's knowledge of such mathematics unnecessary. As long as we treat the learning of measurement as merely the learning of a skill, unpacking the mathematical ideas that are embodied in artefacts will remain unnecessary. However, if we view the learning of measurement as conceptual understanding, then such material artefacts present an opportunity for archaeological investigation. Such “archaeology” or “unpacking” aims to uncover the generally hidden and “black boxed” aspects of mathematics sedimented in artefacts and practices (Subramaniam, 2012). Archaeology as a pedagogical mode may have an important place in providing opportunities to learn powerful mathematics that illuminates the diverse aspects of everyday experience.

Pedagogical Implications

One of the challenges before the teacher or the instructional designer is to imagine connections between school and out-of-school knowledge that can produce powerful learning. What should be the goals of a pedagogical intervention that aims at building connections between out-of-school knowledge and school learning? What forms of participation could one expect to see in a classroom implementing these goals? These questions are addressed in the pedagogical intervention discussed in chapter 7. The goals must include conceptual aspects as well as the setting up of a classroom culture that values making connections. It must also explicitly attend to the shifts that are needed in bringing out-of-school knowledge with school learning in terms of shifts *oral* to *written* mathematics, from *knowledge about use* of tools and artefacts to *understanding* and building the identities of participants as a mathematically focused discursive community. The latter shift involves moving from helping interactions to a more discursive culture in which reasoning is central, where statements are listened to with attention, are challenged, elaborated and justified.

8.2 Personal postscript

The study entailed handling of sensitive issues of social and ethical nature, for example, the issues of child labour, difficult and oppressive work conditions, unfair wage pattern, social stereotypes (gender, caste and others), and so on. As a researcher, it was challenging to tackle and address such issues during interviews or during social interactions. The dissertation journey has brought to me a platter of learning and training to prepare myself to carry forward similar research and also to embark upon new research on other social issues. As a researcher, I feel better able now. The social relationship with the community developed during the prolonged engagement with the field will remain as an asset for me. Reflecting over the data and over myself since I undertook this research study, I realise that the study has given me tools to see things which I was unaware of or at the most vaguely aware of. The research study was hugely benefited by the students and teachers' support and their participation in it, but I am sceptical whether this study gave them back some tools to judge and tackle the equity, fairness and other social issues through learning of mathematics.

Revisiting the study in its entirety indicates to me that as a researcher, I felt, perhaps more sensitivity is required towards handling social issues arising out of low SES, work requirement, aspirations and child labour than what I had. Researchers embarking upon similar studies need to be more cautious with such issues.

References

Abreu, G. de (2008). From mathematics learning out-of-school to multicultural classrooms: A cultural psychology perspective. In L. D. English (Ed.). *Handbook of International Research in Mathematics Education*. 2nd ed., 352-384. NY: Routledge.

Barton, A. C. & Tan, E. (2009). Funds of knowledge and discourses and hybrid spaces. *Journal of Research in Science Teaching, 46*(1), 50-73.

Bishop, A. J. (1988). Mathematical Enculturation: A Cultural Perspective on Mathematics Education. Dordrecht: Kluwer Academic Publishers.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. *Multiple perspectives on mathematics teaching and learning*, 171-200.

Bose, A. & Kantha, V. K. (Accepted). Influence of socio-economic background and cultural practices on mathematics education in India: A contemporary overview in historical perspective. *ZDM – The International Journal on Mathematics Education*, *46*(7).

Bose, A. & Subramaniam, K. (2013). Characterising work-contexts from a mathematics learning perspective. In Nagarjuna, G., A. Jamakhandi & E. M. Sam (Eds.). *Proceedings of epiSTEME-5: Third International Conference to Review Research on Science, Technology and Mathematics Education.* pp. 173-179, Margao, India: CinnamonTeal Publishing.

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1987). Written and Oral Mathematics. *Journal for Research in Mathematics Education*, *18*(2), 83-97.

Carraher, D. W. & Schliemann, A. D. (2002). Is everyday mathematics truly relevant to mathematics education? *Journal for Research in Mathematics Education. Monograph, 11 (Everyday and Academic Mathematics in the Classroom)*, 131-153.

Chevellard, Y. (2007). Implicit mathematics. In U. Gellert & E. Jablonka (Eds.). *Mathematisation and demathematisation: Social, philosophical and educational ramifications.*Rotterdam: Sense Publishers.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schuble, L. (2003). Design experiment in educational research. *Educational Researcher*, 32(1), pp. 9-13.

Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education*, *40*(1), 40-68.

Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths/pedagogic texts*. London: The Falmer Press.

Fagg, H. (2002). *Back to the Sources: A Study of Gandhi's Basic Education*. New Delhi: National Book Trust.

Gellert, U. & Jablonka, E. (2007). *Mathematisation and demathematisation: Social, philosophical and educational ramifications.*Rotterdam: Sense Publishers.

González, N. (2005). Beyond culture: The hybridity of funds of knowledge. In N. González, L. C. Moll & C. Amanti (Eds.). *Funds of knowledge: Theorizing practices in households, communities, and classrooms*, pp. 29-46. Mahwah, NJ: Lawrence Erlbaum.

González, N., Moll, L. C., & Amanti, C. (Eds.). (2005). *Funds of knowledge: Theorizing practices in households, communities, and classrooms*. Mahwah, NJ: Lawrence Erlbaum.

González, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk*, *6*(1-2), 115-132.

Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom: on learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research*, *51*, 128-145.

Khan, F. A. (2004). Living, Learning and Doing Mathematics: A Study of Working Class Children in Delhi. *Contemporary Education Dialogue*, *2*, 199-227.

Lave, J. (1988). *Cognition in Practice*. Cambridge, UK: Cambridge University Press.

Lave, J., & Wenger, E. (1991). Situated Learning: Legitimate Peripheral Participation. Cambridge: Cambridge University Press.

Maharashtra Math Textbook 5, 6, 7. (2006). *Mathematics Standard 5, 6, 7*. Pune: Maharashtra State Bureau

of Textbook Production and Curriculum Research.
Moll, L. C., Amanti, C., Neff, D., & Gonzalez, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory into practice*, *31*(2), 132-141.
Mukhopadhyay, S. (2013). The mathematical practices of those without power. In *Mathematics Education and Society 7th International Conference.* Plenary talk. Cape Town: South Africa.
NCERT (2005). *The National Curriculum Framework*. New Delhi: National Council of Educational Research and Training.
NCERT (2006). *National Curriculum Framework 2005 Position Paper of National Focus Group on Teaching of Mathematics*. New Delhi: National Council of Educational Research and Training.
NCERT (2007). *Position Paper National Focus Group on Work and Education.* New Delhi: National Council of Educational Research and Training.
NCERT Math Textbook 5. (2006). *Math-Magic 5*. New Delhi: NCERT.
NCERT Math Textbook 6, 7. (2006). *Mathematics*. New Delhi: NCERT.
Naresh, N., & Chahine, I. (2013). Reconceptualizing Research on Workplace mathematics: Negotiations Grounded in Personal Practical Experiences. *REDIMAT-Journal of Research in Mathematics Education*, *2*(3), 316-342.
Nasir, N. S., Hand, V. & Taylor, E. (2008). Culture and Mathematics in School: Boundaries between "Cultural" and "Domain" Knowledge in the Mathematics Classroom and Beyond. *Review of Research in Education, 32*, 187-240.
Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). *Street Mathematics and School Mathematics.* New York: Cambridge University Press.
Nunes, T. C., Carraher, D.W., & Schliemann, A. D. (1985). Mathematics in the Streets and in Schools. British Journal of Developmental Psychology, 3, 21-29.
Resnick, L. B. (1987). Learning In School and Out. *Educational Researcher, 16*(9), 13-20.
Saraswathi, L. S. (1989). Practices in linear measurements in rural Tamil Nadu: implications for adult education programmes. *Journal of Education and Social Change, 3*(1), 29-46.
Saxe, G. B. (1988). Candy Selling and Math Learning. Educational Researcher, 17(6), 14-21.
Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the society for research in child development*.
Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14-22.
Sitabkhan, Y. (2009). The Use of Convenient Value Strategies among Young Train Vendors in Mumbai, India. In K. Subramaniam & A. Mazumdar (Eds.). *Proceedings of epiSTEME-3: An International Conference to Review Research on Science, Technology and Mathematics Education*, pp. 114-118. Mumbai, India: Macmillan.
Subramaniam, K. (2012). Does participation in household based work create opportunities for learning mathematics? In *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, (Vol. 1, pp. 107 – 112). Taipei, Taiwan: PME.
Vasanta, D. (2004). Childhood, Work and Schooling: Some Reflections. *Contemporary Education Dialogue*, *2*(1), 5-29.
Vélez-Ibáñez, C., & Greenberg, J. (2005). Formation and transformation of funds of knowledge. *Funds of knowledge: Theorizing practices in households, communities, and classrooms*, 47-70.