Cognitive Studies in Relativity

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Chapter 1 : Introduction

1.1 Research aim and scope

This work belongs to the emerging new field of Physics Education Research (PER). Its aim is to probe systematically students' understanding of some of the basic ideas in the realm of relativity, a cornerstone of modern physics. Though we begin with Galilean relativity, most of the themes that we discuss pertain to general relativity[1] (GR). The work does <u>not</u> aim to study conceptual change after instruction; nor do we propose or test any specific new pedagogic strategy. Our aim is simply to study in depth students' cognition of the subject, and diagnose and interpret their conceptual vulnerabilities when they engage with its ideas in a first course on the subject.

The thesis consists of five different cognitive studies. In Study 1, we explore students' understanding of the meaning of the relativity principle; this study is carried out in the context of Newtonian mechanics. In Study 2, we begin to probe students' qualitative understanding of the broad conceptual themes of general relativity. Study 3 is devoted to probing in more detail students' understanding of the Principle of Equivalence. In Study 4, we explore their understanding of the meaning of covariance of a physical theory in different contexts: rotational, Galilean, Lorentz and general covariance. In Study 5, we explore to a modest extent how students engage with the more technical notions of the subject.

The entire programme is premised on the possibility that students' understanding of the qualitative conceptual core of the subject <u>can</u> be probed to a considerable extent, if not fully,

[1] The work is part of a more comprehensive programme in which students' ideas in the domain of 'special relativity' are being investigated by another researcher.

without testing their competence in its advanced mathematical technicalities. Accordingly, all the five studies including the slightly more technical Study 5 do not demand mathematical prerequisites beyond the undergraduate level. The thesis consists of nine chapters which are summarized below.

1.2 The field of Physics Education Research (PER)

In Chapter 1 of the thesis, we briefly review the new field of physics education research, an endeavour in which one carries out systematic and focused enquiry aimed at characterizing students' learning of physics and explores the possible factors that affect it. This is what is sometimes referred to as 'Basic PER' (Beichner, 2009). 'Applied PER' consists in using the insights on students' learning so gained to improving instructional practice. Basic PER typically involves collection of data on students' learning in different domains of physics in diverse contexts through a variety of tools: free response and forced-option tests, case studies, interviews, etc. Applied PER involves development of curricula, instructional designs and teaching methodologies, and testing their effectiveness. The two aspects, howsoever we name them, are thus somewhat distinct, though some PER work may involve both. Jenkins (2001) calls these twin traditions in science education 'the empirical' and 'the pedagogical'. Our work is entirely in the empirical tradition.

There are two dominant theoretical perspectives in PER, the cognitive and the socio-cultural. In the cognitive perspective that informs our work, there are a number of different standpoints [see, for example, di Sessa (1993), Vosniadou (1994) and Carey (1987)]. Our work is guided by the alternative conception research programme (see below). The notion of 'alternative conception' or 'alternative framework' has, however, been debated since students' knowledge is often found to be fragmented and not coherent enough to be termed as such; indeed the constructivist paradigm itself has been critiqued (Matthews, 1996). In this thesis, we adopt a more agnostic term, namely 'conceptual vulnerabilities', to indicate the domain-specific points where students are prone to depart from the standard conceptions of a physicist.

Though it took a few decades to emerge as a new discipline, the beginnings of PER may be traced to the post-war curriculum development efforts in the US, UK and other countries. A realisation grew that for a majority of college students learning introductory physics, there is a wide gap that separates what is learnt from what is taught and that there is something basic to the difficulty (see, for example, McDermott, 1984). Around the same time, science education researchers were discovering that young students at the middle and secondary school levels learn science not with a 'clean slate' but with a baggage of domain specific conceptions that they bring to the classroom, regarding matter, motion, heat, light, electricity and so on. In time this simple insight evolved into a major programme – the 'alternative conceptions' programme of science education research, particularly in the domain of physics. See, for example, Driver (1985), Viennot (2001), McCloskey (1983) and Clement (1993).

The term 'alternative conceptions' refers to the student conceptions in different domains of science, which are incongruent to the standard conceptions accepted by scientists. They are not dismissed pejoratively as 'misconceptions' because they seem to have a certain degree of organization, coherence and 'reasonableness' about them, and they may well have some functional use in describing a limited range of experience. Indeed they seem to be constructs of knowledge from daily experience, and this is likely to be the reason why they are found universally, across different languages and cultures. Moreover, some of the widely held 'alternative conceptions' are known to be robust and students continue to hold them with conviction even after instruction. A useful bibliography of alternative conceptions of students' ideas in science was prepared by Pfundt & Duit (1994) and updated from time to time.

There are numerous groups and individuals involved in Physics Education Research at the college level. The group at the University of Washington, a pioneer in PER, has researched on students' ideas in several areas like mechanics, thermodynamics, optics, electricity and relativity. As an example, in their early studies on the understanding of kinematics among undergraduates, Trowbridge & McDermott (1980, 1981) discovered that students tend to consider the position of a body as a criterion to determine relative velocity. A body lagging behind another body is believed to have less velocity than the other. Similarly, values of velocity play a role in decisions concerning the relative accelerations of two bodies. The Washington group has carried out considerable pedagogic research also. This has culminated in a set of books titled *Physics By Enquiry* [McDermott (1995), McDermott *et* al. (1996)] and *Tutorials in Introductory Physics* [McDermott & Shaffer (2001)]

The group at Maryland uses insights from cognitive science for their PER work. Students do not bring to the classroom only their conceptions regarding the content of physics; they also bring their epistemological beliefs about science and learning, and their expectations about what they will learn. These aspects, among other things, have been carefully studied by the group. *The Physics Suite* by Edward Redish (Redish, 2003) is a book for physics teachers aimed at improving their instruction by developing an overall understanding of students' beliefs and alternative conceptions.

The cognitive science underpinnings for PER have also been advocated by Reif (2010). This is a phenomenological approach that begins with characterizing good performance in a complex domain by indicators like usability, effectiveness, flexibility, efficiency and reliability. The cognitive needs for each are then identified and the implications for instruction explored. To interpret a concept in science, we generally need not only its declarative content but also procedural knowledge. For example, Reif & Allen (1992) found that few students can handle some unconventional but straightforward problem tasks on acceleration, because they do not interpret the concept (familiar to them otherwise) procedurally.

An important trend in this field is the development of assessment instruments using multiple choice questions on some basic notion. These can be used on a large scale and the quantitative data can yield statistically valid inferences. The *Force Concept Inventory* developed by Hestenes *et* al. (1992) is a well-known example. A number of such concept inventories are now available [see, for example, Thornton & Sokoloff (1998)]. Such validated instruments can be used in a pre-post research design to evaluate instructional techniques. This has spurred the growth of PER based instructional materials. See, for example, Mazur (1996).

A group at the Hebrew University of Jerusalem has carried out numerous studies in PER. Some of their work is quoted in Sec. 6.3. In recent years, the efforts of the group have been aimed at using narratives from the history of physics to aid physics learning by students (Galili, 2008).

Research in problem solving has been another major trend in PER. For a recent review of this trend, see Mason & Singh (2010). In an early work, Chi *et* al. (1981) studied the difference between experts and novices in problem solving. The group at Minnesota has specialized on students' and teachers' approach to problem solving in physics. In India, PER work is carried out at HBCSE, Mumbai, University of Delhi, University of Bangalore and RIE, Mysore, to name a few. HBCSE group's early work on Galilean relativity is described in chapter 2.

A useful reference in the field is the Resource Letter on PER by McDermott & Redish (1999); see also a recent review by Beichner (2009) and the website http://www.compadre.org/per/ .

Chapter 2 : Physics Education Research in relativity - survey of literature

In Chapter 2 of the thesis we survey in some detail the literature on physics education research in the domain of relativity. Student conception studies in this area are rather limited, while there is enormous pedagogic and expository literature, too large to be adequately summarized. Our interest is mainly in the diagnostic cognitive studies on students' ideas in relativity.

2.1 Students' conceptions in Galilean relativity

Aguirre (1988) observed that students tend to look at speed as an intrinsic property of a moving body, independent of any frame of reference. A similar finding reported earlier (Saltiel & Malgrange, 1980) shows that many students view speed of a body unrelated to a reference frame because they connect speed to driving forces. The ground or earth is implicitly assumed to be the standard reference frame; students consider only those motions that appear in the earth's frame as real. Similarly students tend to regard the path of a moving body devoid of a reference frame. The authors call it the "geometrization of a dynamically defined motion", where the path of motion is a frozen entity from which the time variable is eliminated. Also, velocity addition of Galilean relativity is not regarded in a kinematic sense, but more in the dynamical sense of a 'physical drag'.

A comprehensive study on alternative conceptions in different aspects of Galilean relativity was carried out by the HBCSE group in Mumbai more than a decade ago [Panse *et* al. 1994; Ramadas *et* al. 1996(a) & 1996(b)]. A few of the key student conceptions found in this work are: (i) frames have 'boundaries' defined by the spatial extension of the associated objects; (ii) phenomena belong to frames (e.g. a ball thrown up inside a train's compartment 'belongs' to the train's frame); (iii) inertial or non-inertial character of a frame is a relative property of two frames. An interesting question in this work referred to a ball traversing the length of a fixed tube in a moving tram with a certain speed. Students were asked to state the speed of the ball, the distance travelled and the time taken relative to the outside ground observer. While most undergraduates correctly used Galilean velocity addition, several of them did not hesitate to violate time interval absoluteness to 'save' the supposed distance invariance (since length of the tube is invariant), not realizing that the latter is not true for non-simultaneous events even in Galilean relativity.

2.2 Students' conceptions in Einstein's relativity

These have been mainly researched in the area of special relativistic kinematics. Villani & Pacca

(1987) found that several first year college students had difficulties in coming to terms with the invariance of light speed and that it is also a limiting speed for all bodies. Several students tended to think that crossing the limit of light speed is a technological problem rather than an intrinsic property of the physical world. Studies conducted on first and fourth year undergraduates by Pietrocola & Zylbersztajn (1999) and by Dimitriadi *et* al. (2005) found that most students pay lip service to the relativity principle, failing to use it as a conceptual tool for problem tasks. Recent studies by Hosson *et* al. (2010) reveal that students often fail to separate the notions of reference frames and events. They also regard the time of an event to vary between observers in the same frame.

Studies by Scherr *et* al. (2001, 2002) and Scherr (2007) of the Washington group have uncovered a significant cache of students' conceptions in relativistic kinematics. Their area of focus was the relativity of simultaneity. They found that most college students fail to understand that the relativity of simultaneity is an inherent feature of inertial frames in (uniform) relative motion. Instead, students believe that simultaneity is an absolute feature of events, and tend to accept its relativity only in the limited sense of different times that signals take to reach observers. Thus, students argue that if observers could account for the time of travel, the relativity of simultaneity would disappear. (Interestingly, these students saw the 'desynchronization term' $-v\,\delta x / c^2$ in the Lorentz transformations as 'evidence' of the observer's position dependence.) Not surprisingly, therefore, these students believed in a 'relativity of simultaneity' even for observers in a given frame, depending on where they were located. The authors found that students' misconceptions of simultaneity stem from a poor understanding of what is meant by a reference frame. Several college students believe that every observer constitutes a distinct reference frame, unless they are at the same location. Observers at the same location, however, are regarded to belong to the same reference frame, even if they are in relative motion.

The Washington group has also carried out some studies on relativistic dynamics (Boudreaux, 2005) and on general relativity (Vokos, 2006). The latter uses problem tasks involving the motion of a ball in a Schwarzschild geometry to see if students can deal with measurements and relate these between local and distant observers. However, to the best of our knowledge, detailed diagnostic studies in general relativity like ours have not been undertaken elsewhere.

2.3 Pedagogic literature in relativity

In contrast to the limited literature on student conception studies in relativity, there are numerous papers in such journals as American Journal of Physics, European Journal Physics, and others that deal with pedagogic elaborations and clarifications of the ideas of the subject, particularly the counterintuitive ideas of special relativity and the subtle technicalities of general relativity. We have briefly reviewed this in the thesis, but omit it here, except bringing to attention a few pedagogic suggestions.

One interesting suggestion has been due to Bell (1976) in which a case is made to teach special relativity from the dynamic perspective. Called a 'Lorentzian pedagogy', this unorthodox proposal suggests that we should motivate the kinematic consequences of special relativity like length contraction and time dilation using electrodynamics, following “very much the approach of H.A. Lorentz”. Villani and Arruda (1998) and later Arriassceq and Greca (2007) make a case for the teaching of special relativity by focusing on the history of its conceptual development.

There is now an increasing trend to teach general relativity at the undergraduate level. A strong advocate of this move is Hartle (2006), who makes a case for a "physics first" approach before introducing the advanced mathematics needed for the subject. Rindler (1994) has put forward the idea that undergraduate students can be given a glimpse of general relativity before, or independently of, special relativity. A paper by Wald (2006) discusses the teaching of the mathematics of general relativity both for the graduate and undergraduate levels, with particular reference to the techniques of differential geometry. Several papers on the topic of teaching general relativity to undergraduates can be found in the Proceedings of the AAPT Workshop held at the Syracuse University in 2006 [Proceedings of AAPT, 2006].

Chapter 3 : Methodology

Given the aims of the study clarified at the start of Chapter 1, a non-experimental research design seemed most suited for our work. Further within this design, a fully quantitative approach was neither feasible nor even desirable. There are simply not enough senior undergraduates or beginning graduate students taking general relativity courses in the various institutes in the city. But even if we could somehow manage bigger samples, a quantitative large scale survey was unlikely to give insights into students' thinking on the various aspects of this subject. A detailed qualitative study of students' ideas seemed to us a more meaningful exercise.

3.1 Samples, instructional courses and tools

In the non-experimental qualitative design, the specific methodology had to be tailored for the kind of study at hand. Methodologically, the five studies reported here belong to two classes.

Study 2 addresses the broad conceptual issues of general relativity as dwelt on in the well-known expository book by Einstein (1920). This was an in depth study on how a small sample of 5-6 students engaged with those issues. These were beginning third year undergraduates of Institute 1 who had just completed a one semester course on special relativity (Jan.-April, 2009). At the start of a three-month long vacation course on general relativity (June-Aug, 2009) these students were administered text comprehension exercises based on the GR part of Einstein's text referred to above. It took two long sessions, each about four to five hours, on two consecutive days to administer the questionnaire on the twelve articles on general relativity in this book. The standard course that followed was based on the book *Introducing Einstein's Relativity* by Ray d'Inverno (1992). The course entailed 70 hours of instruction that also included solving some exercises of the book (see sec.5.2 for the content of the course). During the course, three specially designed technical diagnostic questionnaires were administered on (a) the principle of equivalence, (b) the principle of general covariance and (c) space-time curvature after these topics were taught. At the end of the course, we conducted interviews of the students, each recorded interview lasting for two hours or so in which we essentially revisited Einstein's text and asked students similar (but not identical) questions as in the text comprehension exercises at the beginning of the course. Additionally we asked them questions based on some technical points concerning GR that they had learnt in the course. Study 2 is based on the combined analysis of data on the text comprehension exercises and interviews. The three technical questionnaires mentioned above were the basis of the pilot versions of Studies 3, 4 and 5 (see below).

The other class of studies (Studies 1, 3, 4 and 5 in roughly increasing order of complexity) adopted a different approach. In each of these, we first carried out a pilot study on a small sample and used its findings to develop a focused diagnostic questionnaire as a tool for the main study carried out subsequently on a moderately-sized sample (26-30). However, Study 1 differed somewhat in detail from the other studies. In this study the preliminary version itself was quite elaborate. The sample for the preliminary version of Study 1 was a group of 20 second year physics undergraduates at the Institute 1 who had enrolled for a course on special relativity (Jan.-April, 2009). Two questionnaires were administered on Galilean relativity (a topic that precedes special relativity in any standard course); a pre-test on the first day of the term and a post-test after about one week of instruction and tutorial on the topic. The idea was to look for some robust conceptions that persist among students even after instruction. Analysis of the scripts resulted in several qualitative insights on students' thinking on the topic, which were published (Bandyopadhyay, 2009). The main version of Study 1 utilized these insights. The earlier questionnaire was modified considerably and made more focused. The sample for the main study was a different group of 26 first year physics undergraduates taking a course on classical physics (August- Nov., 2010) at Institute 1. Two weeks of instruction (8 hours of teaching and tutorial) were devoted to Galilean invariance and related topics. After this instruction, the improved questionnaire was administered and the written answers analyzed.

Studies 3, 4 and 5 had a common methodology. The samples for their pilot versions consisted of 9, 6 and 5 students respectively undergoing the vacation course on GR at Institute 1 (June-Aug. 2009) described earlier in connection with Study 2. The pilot version data were analysed in detail, which led to improved and more focused questionnaires for the main versions of Studies 3, 4 and 5, which were carried out around an instructional course on GR for a different group at Institute 2 (Jan.- April, 2010). This was again a standard course but based on a different book (Weinberg, 1972) and entailed 70 hours of teaching and tutorials. The samples for these main versions consisted of 30, 27 and 26 students who could be regarded as senior undergraduates (See 6.2). The improved questionnaires were administered at different times in this course after the instruction on the respective topics was completed. The written scripts were then analysed.

As a general remark, instruction including tutorials for all the courses was along standard lines and covered every theme on which the items of the post-tests were based. Students were informed in advance that the test items would not be traditional and would require conceptual understanding of the topics covered in the class. Thus students could not answer the test items by recall and had to reveal their own conceptions of the themes taught in the course.

Some further details on the sample background, course content, and the questionnaires appear in respective chapters. Table 1 gives the time line of the different instructional courses and their relation to the pilot and main versions of the five studies. All instructional courses were given by the former research supervisor, as acknowledged at the end. Sec. 3.3 below details the role of the present researcher in this work. Table 2 summarizes the methodology of different studies explained in this chapter.

Table 1 : Time line of courses and studies

S. No	Topic / Course	Venue	Period	Duration (Instruction+Tutorials)	Sample	Study
1.	Galilean relativity[1]	Institute 1[5]	January 2009	6 hours	20 second-year undergraduates	Study 1 (Pilot)
2.	General relativity[2]	Institute 1	June – August 2009	70 hours	5-6 third-year undergraduates	Study 2
					9 third year undergraduates	Study 3 (Pilot)
					6 third year undergraduates	Study 4 (Pilot)
					5 third year undergraduates	Study 5 (Pilot)
3.	General relativity[3]	Institute 2 [5]	January – April 2010	70 hours	30 senior undergraduates	Study 3 (Main)
					27 senior undergraduates	Study 4 (Main)
					26 senior undergraduates	Study 5 (Main)
4.	Galilean relativity[4]	Institute 1	August 2010	8 hrs	26 first year undergraduates	Study 1 (Main)

Notes:

(1) This topic was covered in the first week of a one-semester course on special relativity in January – April, 2009.

(2) This was a vacation course with optional attendance. It began with text-comprehension exercises and ended with interviews. Three technical questionnaires for Studies 3, 4 and 5 were administered after the topics were taught in the course. 9 students initially opted for the course of which 6 continued throughout till the interviews. Of these 6, 1 did not write the text-comprehension exercises and 1 did not participate in Study 5.

(3) This was a regular course. The sample decreased slightly as the course progressed. The three main technical questionnaires were administered after the topics were taught.

(4) This topic was covered in the initial part of a one-semester course on classical physics (August – November, 2010).

(5) Institute 1: UM-DAE Centre for Basic Sciences, Mumbai. Institute 2: Indian Institute of Technology Bombay

Table 2 : Summary of Methodology

Study	Topic	Sample size #	Tool +	No. of test items	Raw data	Coding	Presentation
Study 1 (Pilot)*	Galilean relativity	20	Pre-test, post-test	12 12	Written responses	Pointer statements	Impressions from pointer statements.
Study 1 (Main)	Galilean relativity	26	Post-test	17	Written responses	Pointer statements	Response categories.
Study 2*	Conceptual themes of GR	5 – 6	Text-comprehen sion exercises and interviews	52 text items and 2 hr interviews for each student	Written responses to text + transcripts of interviews	No coding. Summaries for facilitation	Narrative with extensive quotes from text scripts and interviews.
Study 3 (Pilot)	Principle of Equivalence (PoE)	9	Post-test	30	Written responses	Pointer statements	Summary of impressions from pointer statements.
Study 3 (Main)*	PoE	30	Post-test	20	Written responses	Pointer statements	Response categories
Study 4 (Pilot)	Ideas of covariance	6	Post-test	23	Written responses	Pointer statements	Summary of impressions from pointer statements.
Study 4 (Main)*	Ideas of covariance	27	Post-test	20	Written responses	Pointer statements	Response categories
Study 5 (Pilot)	Technical notions of GR	5	Post-test	28	Written responses	Pointer statements	No presentation. Pointer statements fed into the Study 5 (Main).
Study 5 (Main)	Technical notions of GR	26	Post-test	8	Written responses	Pointer statements	Response categories

* These studies have been published. See the list of publications at the end.

For other details of the samples, see Table 1 and text.

+ In this column, pre-test and post-test mean tests before and after instruction of the topic respectively.

3.2 Data processing and analysis

These studies yielded a substantial amount of raw qualitative data in the form of (a) written responses to the large number of items in (i) text comprehension exercises, and (ii) questionnaires for different studies (more than 50 in (i) and more than 150 in (ii)), and (b) oral responses in interviews (about 100 pages of interview transcripts transcribed from the recordings). The processing of the data was carried out in the following way.

For the Studies (1, 3, 4 and 5), students' written responses (raw data) to the questionnaires, both for the pilot and main versions, were codified in terms of 'pointer statements'. A 'pointer statement' is one that in the researcher's view best captured the intended meaning of the student's response. Most questions were short and specific aimed at eliciting student's reasoning, so this exercise basically amounted to understanding the offered reasoning or explanation and summarizing it using proper English syntax; it usually did not need any involved interpretation of the student's response. Our interest was not confined only to the majority view or a view that was significant in some way. Every variety of response was captured in these pointer statements. Only those responses which were muddled and uninterpretable were set aside and their frequencies noted. Usually a considerable number of pointer statements would thus emerge from responses to each question or sub-question.

To make the analysis manageable and meaningful, the large number of pointer statements was then clubbed into fewer interpretative response categories. Since some detail is lost in this way, we add nuances to our interpretative categories, wherever this is necessary. At the end of it all, we have an empirically supported repertoire of student conceptions, which highlight their conceptual vulnerabilities in the topic.[2]

This approach seemed appropriate for the more technical studies above. However, it did not seem proper for Study 2 involving the broad conceptual themes of general relativity. The richness of qualitative data on student responses to text comprehension exercises based on Einstein's text and their interview responses could not be captured through such response categories. Since, for Study 2, the number of students was rather small (5-6), our analysis used the raw data directly. For convenience, however, the significant phrases or lines of the written answers to different text comprehension questions were marked and summary impressions noted under each theme for

[2] The pilot version of Study 1 has only pointer statements, which were not reduced to response categories; the diagnosis of conceptual vulnerabilities followed directly from the pointer statements.

different students. Likewise, each interview script was segmented thematically, the segments labelled and a summary impression noted for each segment. (The parts of the interview, usually at the end, that went beyond the text themes were not analysed.) All this was, however, only for facilitation and not for reducing data to another level; this should be contrasted with the method of 'pointer statements'. By juxtaposing the two sources of data for each student under various themes, a detailed narrative of our qualitative impressions of students' ideas was then constructed, with extensive quotes from the raw data to substantiate our impressions. The narrative also includes a commentary at the end of each theme that encapsulates our over-all impressions of students' ideas. These commentaries represent a summary of our empirical findings of Study 2. These appear in a still more condensed form in Chapter 5 of this synopsis.

3.3 Role of the 'researcher'

In the following, the 'researcher' means the present candidate, and the 'research collaborator' stands for the former research supervisor, acknowledged at the end. The work as a whole is a joint work of the two, with their respective roles in different studies as follows. The research collaborator was the instructor for all the courses around which the five studies were formulated; instruction included lectures and tutorials. The pilot version of Study 1 (that included designing pre-test and post-test questionnaires and their analysis in terms of pointer statements and reporting of findings) was carried out independently by the researcher; the written report was vetted by the research collaborator. All the remaining studies were carried out jointly. For the main version of Study 1 and the pilot and main versions of Studies 3, 4 and 5, the questionnaires were initially developed by the researcher and modified where necessary by the research collaborator. The written data for all these studies were codified into pointer statements by the researcher independently. Their clubbing into response categories and final reporting were done jointly. For Study 2, both the text comprehension exercises and the interview questions were devised jointly. The slow reading of the Einstein's text for Study 2, as explained in Chapter 5, was done by the research collaborator. The interviews were conducted jointly, with the research collaborator doing the 'speaking' and the researcher occasionally 'prompting' him with the questions on different themes. The recorded interviews were transcribed into written scripts by the researcher. The preparation of the narrative of qualitative impressions from the two sources of data for Study 2 was done jointly.

3.4 Validation

No formal validation of the questionnaires was carried out. The content validity of the text comprehension exercises for Study 2 seems to be on secure grounds, as they involved no more than clarifying the key lines of Einstein's text. For the same reason, but to a lesser extent, this is true also of the interview questions since they revisited the same text. Content validity of the questionnaires of the remaining four studies can only be gauged by experts. As mentioned before, all the questionnaires were vetted by the research collaborator. These questionnaires together constitute 65 items in the main versions and more than 100 items in the pilot versions, all requiring short reasoning-based answers. Further validation of this substantial number of items by local experts was possible but did not seem practical. Accordingly, all the studies were submitted to peer-reviewed journals and published (except Study 5 which is to be submitted after some expansion of its scope). We should add that the published papers detail nearly all of the test items of the corresponding studies.

The coding scheme of pointer statements and response categories for data analysis in Studies 1, 3, 4 and 5 has not been validated. This is certainly a limitation of our work, as noted also at the end. The only point we wish to state is that in all these studies, the conceptual vulnerabilities arising from the pilot and main versions were very similar. This seems to us a reasonable indicator of validity, though we did not probe this similarity systematically.

For Study 2, no real coding of data was involved. The summary impressions of text scripts and interviews were made merely for convenience, as already mentioned; the final narrative dealt directly with raw data facilitated by these summaries. This seemed not very problematic for the text scripts, which had short answers to each question and were generally straightforward to interpret. However, our interpretation of the various segments of interviews certainly requires content validation. This is one reason why our narrative of Study 2 extensively quotes from the original scripts of text and interviews to substantiate our impressions.

The question of validation should be seen relative to the modest claim of this entire work, which is the <u>identification</u> of content specific vulnerabilities in a fairly large area of a complex discipline. Our methodological limitations do not allow us to claim that the results on frequencies of different vulnerabilities can be generalized to other populations. However, the numerous conceptual vulnerabilities we have identified seem to us significant in terms of their basic content, whatever their frequency. Our feeling is that they are not likely to be entirely the idiosyncrasies of our student samples. Replication studies will be needed to see to what extent is this feeling correct.

Chapter 4 : Students' ideas on the meaning of the relativity principle (Study 1)

The thesis focuses mainly on students' cognition of some of the key ideas relevant to general relativity. However, the overarching physical idea in relativity in all its versions – Galilean, special and general – is the relativity principle itself. What do students understand by this principle? To probe the meanings that students ascribe to the Relativity Principle (RP), without getting into the more advanced aspects of general relativity, our starting investigation (Study 1) is confined to the domain of Newtonian mechanics which satisfies the principle of Galilean relativity. Subsequent studies deal mainly with aspects of general relativity.

4.1 Three 'meanings' of the relativity principle

For a diagnostic study, we identified three 'meanings' (apparently distinct but in fact entirely equivalent) of the RP. We know that an inertial frame of reference is one relative to which the law of inertia (Newton's I law) holds good. The laws of Newtonian mechanics are found to be the same for all inertial frames. This implies that two inertial observers in uniform relative motion would have no way to say as to which of them is 'really at rest' or 'really moving'. Only their relative velocity has an objective significance. <u>There is no way in Newtonian mechanics to privilege one inertial frame over another or to identify a 'frame of absolute rest'</u>. We call this well-known fact of physics the 'first meaning of the RP'.

Another way of expressing the RP is that by measurements with respect to a frame of reference, the frame's 'intrinsic' velocity (absolute velocity) cannot be ascertained. This should be contrasted with the fact that the frame's 'intrinsic' acceleration (absolute acceleration) $\vec{\alpha}$ or angular velocity $\vec{\omega}$ <u>can</u> be determined by measurements with respect to the frame itself. This inability of an observer to determine the absolute velocity of a frame is nothing but the RP itself, for if observers could determine their absolute velocities, they could determine who among them was 'really at rest' or 'really moving' which would violate the equivalence of inertial frames - the first meaning of RP. Let us call this <u>inability to determine the velocity of a frame from measurements made with respect to the frame itself</u> the 'second meaning of the RP'.

The RP (and the lack of it for inertial versus non-inertial frames) can also be expressed a little more technically in a form that is well within the reach of undergraduate students. It can be 'read off' so to say from the second law of Newton for a general non-inertial frame, which is proved in any standard textbook on mechanics (Thornton & Marion, 2004, p.392):

$$m\vec{a} = \vec{F} - m\vec{\alpha} - 2m(\vec{\omega}\times\vec{u}) - m\vec{\omega}\times(\vec{\omega}\times\vec{r}), \quad (1)$$

here m is the (inertial) mass of the body and $\vec{F}$ is the force on the body due to some external physical agency, to be distinguished from the next three terms on the right side (pseudo forces) which have a kinematic origin, i.e. they arise only due to the absolute translational acceleration $\vec{\alpha}$ and the angular velocity $\vec{\omega}$ of the given non-inertial frame of reference with respect to any inertial frame. The quantities $\vec{r}$, $\vec{u}$ and $\vec{a}$ refer respectively to the position, velocity and acceleration of the body with respect to the frame in question. [An additional angular acceleration term appears on the right hand side if $\vec{\omega}$ is not constant, but we ignore it here for simplicity.]

The RP follows from the fact that the right hand side of Eq. (1) has $\vec{\alpha}$ and $\vec{\omega}$ of the frame but not its translational velocity $\vec{v}_{frame}$. The translational velocity of the frame simply does not appear in the Second Law of Motion in any frame, inertial or non-inertial. We call this fact: the absence of 'velocity of frame' term in the second law of Newton for any general frame the 'third meaning of the RP'. If $\vec{\alpha}$ = $\vec{\omega}$ = 0 the frame is inertial. Clearly we can determine the $\vec{\alpha}$ and $\vec{\omega}$ of a frame by measurements with respect to it and making use of the above equation, but whether the frame is inertial or non-inertial, we cannot determine its translational velocity, since the term $\vec{v}_{frame}$ is just not there in Eq. (1). The third meaning is evidently a slightly technical way of saying the same thing as the first two.

In Chapter 2, we cited several studies related to Galilean relativity. However, these studies have not focussed on the students' understanding of the underlying equivalent meanings of the RP as explained above.

4.2 Methodology

As explained in Chapter 3 (p. 9), the investigation consisted of two parts: a preliminary study carried out in 2009 and its sequel, the main study carried out after a gap of a year in 2010 on a different group of students. The samples were 20 second year physics undergraduates and 26 first year physics undergraduates respectively, both groups from Institute 1. Both these groups were admitted to the Institute 1 (after completion of Higher Secondary School) on the basis of a national test whose content level is roughly comparable to the book *Fundamentals of Physics* by

Halliday, Resnick and Walker (1996). No test was carried out on their general prerequisites before the study.

Instruction including tutorial entailed about 6 hours for the preliminary study and 8 hours for the main study. Instruction on Galilean relativity covered the usual topics of Newton' laws, Galilean transformations and Galilean invariance, inertial and non-inertial frames, pseudo forces, etc. The main study included a more detailed treatment of non-inertial frames (derivation of the expressions of Coriolis and centrifugal forces, etc.). It must be emphasized that instruction was along standard lines and it tacitly covered all the three 'meanings of RP' indicated earlier, but care was taken not to project these out in a form that would enable students to answer the test items by mere recall.

The questionnaire of the main version had in all 17 items, a few of them with 2 or 3 related sub-parts. It was insisted in each question that students give the reasoning behind the response. The qualitative data of students' responses were analyzed using the coding method involving pointer statements and interpretive response categories, as explained in Sec.3.2 and illustrated in 4.3A below. We summarize some of the salient findings around a few broad themes, omitting details of data.

4.3 Students' conceptions

A. Kinematic reciprocity and the relativity principle

The study confirmed what the preliminary study had indicated – students understand/justify the RP using kinematic reasoning. They were offered the following interpretation of the RP and asked to comment if it is true or false giving reasons: *Consider two inertial frames S and S'. If S' moves relative to S with velocity $\vec{v}$, S moves relative to S' with velocity $-\vec{v}$. There is thus no way to privilege any one of the two frames – hence S and S' are equivalent.*
To illustrate our method of analysis, we quote the response categories and their frequencies.

Table 3 : Response categories

S.No.	Conception	Frequency	Prevalence (%)
1.	Clear statement of standard conception	0/26	0%
2.	Kinematic reciprocity accepted as an argument for equivalence of frames	16/26	~ 62%

3.	Kinematic reciprocity <u>and</u> identity of laws advanced as an argument for equivalence of frames	7/26	~ 27%
4.	Equivalence not accepted, neutral observer invoked to decide which is 'really moving'	1/26	~ 4%
5.	Invariance of physical quantities as an argument for equivalence	1/26	~ 4%
6.	Insignificant/ nil responses	1/26	~ 4%

Clearly, a great majority of students are aware of the equivalence of inertial frames (first meaning) but most of them equate it to kinematic reciprocity. The next question changed the situation to relative acceleration between S and S'. Few students articulated the basic insight we were looking for, namely that while kinematic reciprocity is true even in the second case, the frames in that case are not equivalent, for the essence of RP is the identity of <u>laws</u> in the two frames.

The trivialization of the RP in the notion of kinematic reciprocity emerges even more clearly in the next question in which students were asked to comment on the following statement: *S is an inertial frame. S' is a non-inertial frame moving relative to S with a uniform acceleration $\vec{\alpha}$. Clearly S moves relative to S' with uniform acceleration $-\vec{\alpha}$. Thus frame S is non-inertial with respect to frame S'.*

In this question, 19/26 students express the view that S and S' are non-inertial with respect to each other. This deeply flawed view of regarding inertial or non-inertial character as a relative property between two frames was found in an earlier study also [Ramadas *et.*al 1996 (a)]. Our study reconfirmed it.

B. Impossibility of determining the absolute velocity of a frame of reference

This 'second meaning of the RP' revealed an interesting conception among students. Students attribute the impossibility of determining the 'absolute velocity' of a frame to the fact that an observer associated with a frame (e.g. an observer in a train), is at rest with respect to the frame, so how can velocity (or other motion parameters) of the frame show up in the measurements relative to the frame? This tempting reasoning that we have termed as the 'at-rest syndrome' is revealed repeatedly in different questions. Students do not realize that an observer at rest in a frame <u>can</u> determine the frame's absolute acceleration/rotation; so the inability to determine the absolute velocity of the frame is not obvious—it is a principle of nature (RP).

C. The absence of velocity of frame term (v_{frame}) in the second law of motion for a non-inertial frame

The following three statements were posed to the students for comments : (a) the absolute velocity of the frame v_{frame} does not appear in the law because the observer is at rest in the frame, (b) the fact that the absolute velocity of the frame v_{frame} does not appear in the law is because Galilean relativity forbids the possibility of determining the absolute velocity of a frame, (c) the fact that the absolute velocity of the frame v_{frame} does not appear in the law is consistent with the fact that a frame at absolute rest cannot be defined in Galilean relativity.

Several students (10/26) agreed with the (incorrect) statement (a) revealing the 'at-rest syndrome'. Overall, our analysis of students' responses shows that students are not able to connect the absence of the velocity of frame term in the equation of motion in non-inertial frames ('third meaning of the RP') to its first two meanings.

D. Space and time intervals

The time interval between a given pair of events is an invariant in Galilean relativity for all reference frames, inertial and non-inertial. In contrast, the spatial separation between a given pair of events is not invariant, unless the events are simultaneous. Our questions regarding these elementary notions consisted of two parts. The first part referred to a phenomenon (a ball dropped from the top of a tower reaching the ground) viewed relative to three different frames. The three frames were (i) a lift at rest on the ground, (ii) an identical lift moving uniformly up and (iii) another identical lift accelerating upwards relative to the ground. The second part referred to analogous phenomena (a ball dropped from the ceiling of the lift reaching its bottom) in the three lifts.

Analysis indicated that many students did not realize the basic distinction between the two parts of the question: the first referring to the same pair of events, the second to three different but analogous pairs of events. It also seemed that 'distance invariance' is appealing to many students even when events are not simultaneous and that students prefer procedural kinematic explanations to RP-based ones, even when the use of RP is more efficient.

Chapter 5 : Students' understanding of some conceptual themes in general relativity (Study 2)

5.1 Introduction

In Chapter 4, we probed in detail how students engage with the meaning of the principle of relativity. Though the discussion was confined to the domain of Newtonian mechanics, the RP as such continues to have the same meaning in special relativity, except that it is taken to be valid beyond mechanics to all of physics, and combined with the constancy of speed of light postulate, has profoundly new ramifications. The restriction to inertial frames, however, remains unchanged in this extension. Thus, several of the vulnerabilities regarding the RP found in the last chapter are relevant for learning special relativity as well.

But when the relativity principle is sought to be extended to include all frames, inertial and non-inertial (actually, arbitrary co-ordinate systems), the problem is altogether different. For Einstein, the quest for a principle of general relativity was inseparable from his search for a relativistic theory of gravity, the two being linked by the Principle of Equivalence (see Sec.6D). The mathematical translation of this insight led him to a theory that required the use of tensor analysis in curved space-time.

Physics Education Research in advanced domains such as this has a basic methodological issue. It is that the conceptual and the technical are so much intertwined here that it is virtually impossible to separate conceptual understanding (which is generally the focus of basic PER work) from technical /mathematical understanding of a topic. This is perhaps why PER, for the most part, deals with more conceptual topics which do not require advanced mathematical prerequisites. Is Einstein's relativity then a suitable domain for PER work?

Basically, in our view, the suitability of this subject for PER lies in the very nature of what it deals with. Relativity involves a refinement of some of the most primitive notions that everybody has about the physical world—the notions of space, time, motion, mass, energy, light, gravity and geometry. This refinement is undoubtedly very subtle and its complexity increases as we go from Galilean to special to general relativity. Yet at its core is a qualitative conceptual structure that can be separated from its intricate technicalities—to a considerable extent in special relativity and smaller yet significant extent in general relativity. Our entire programme is premised on this possibility. The emphasis on the qualitative does not mean that we neglect the essential mathematical parts of the subject during instruction. The idea was to teach relativity in all its technical detail appropriate at the senior undergraduate/beginning graduate level, but probe students' understanding of mainly its conceptual ideas and also some technical ideas that do not demand high degree of mathematical preparation.

5.2 Methodology

This study aimed at an in-depth investigation of the nuances of students' understanding of some of the broad conceptual themes of general relativity (GR). We wished to probe their thinking as they began to engage with the domain. The methodology that suggested itself for the study was to use text comprehension exercises as a tool to begin with. This was followed by a standard teaching course on GR wherein special questionnaires on some key notions were administered. The study concluded with detailed clinical interviews of the students. Since the sample was small, we studied each case - the written and interview responses - in detail.

- Text comprehension

The requirements of the text for our purpose were clear: the text had to be authentic without being technically sophisticated, lucidly written (in simple English as far as possible) and reasonably short with a 'volume' of matter that was manageable for our study. Fortunately, a little reflection suggested that the ideal text for our cognitive study of relativity was among the best known books of the world, written by none other than the originator of the subject himself (Einstein A., *Relativity : The Special and General Theory*, 1920). That this book is not a 'popular' exposition of relativity (in the sense of compromising the conceptual rigour of the subject) has been clearly stated by Einstein in the preface to the book:

"The present book is intended, as far as possible, to give an <u>exact</u> insight into the Theory of Relativity to those readers who…… are not conversant with the mathematical apparatus of theoretical physics" [our emphasis]

The book, written in a clear and simple style, is eminently suitable for reading sessions and comprehension exercises. In about a hundred pages it covers both special and general relativity. Accordingly, the first step in our methodology was as follows: the instructor would read out an article of the book slowly, without explaining its 'physics'. Occasionally, the meaning of an English word was clarified. Students were then asked to write in their own words their responses to some questions (usually three or four) based on the article. The questions mainly required the student to interpret or clarify some key lines of the article. They did <u>not</u> involve going beyond the text or applying the ideas of the article to new situations. In short, it was a plain text comprehension questionnaire. The GR part of the book has 12 articles. It took two consecutive days entailing a total of about 9 hours to administer the questionnaire.

- Student sample

The sample for the study consisted essentially of six third year undergraduate students of which one did not attend the text comprehension sessions but participated in the rest of the course including the end-course interviews.[3] At the Institute 1, in the last two years, students had gone through the usual undergraduate courses in physics which included a full semester course on special relativity.

Since general relativity is not part of the syllabus, a special three-month long vacation course (June-August, 2009) was offered on a voluntary basis. Being a vacation, the course was short but intensive, entailing about 70 hours in all. The course began with the text comprehension exercise on the GR part of Einstein's text as explained earlier. This was followed by a standard teaching course on GR based on the book *Introducing Einstein's Relativity* by Ray d'Inverno (1992). Parts B and C of this book (chapters 5 to 15) covering tensor formalism, the principles of general relativity, field equations, the energy momentum tensor, the Schwarzschild solution, experimental tests, etc. were worked out nearly completely, except for a few sections. Additionally, as an application of GR, chapter 22 and part of chapter 23 on Cosmology from the same book were also taught. Students went through the exercises aimed at filling in the gaps in the text but did not solve other exercises. However, three special technical questionnaires were administered on (a) the principle of equivalence, (b) the principle of general covariance and (c) space-time curvature[4]. All of the themes of Einstein' text were thus covered by the course in much greater technical detail. The study ended with interviews of all the six students. The interviews, each of about 2 hours' duration, were audio recorded and later transcribed into written scripts. The interviews revisited Einstein's text and students engaged with similar (but not identical) questions as in the text comprehension exercises. Additionally, they dealt with some technical points concerning GR taught in the course. This study is based on the two sources of data: text comprehension exercises and interviews.

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[3] Three more students attended the sessions initially but dropped out subsequently – they are not included in our analysis.

[4] The pilot versions of the studies (3, 4 and 5) were based on the analysis of the three special questionnaires.

5.3 Analysis of students' responses

Students' ideas are discussed below around the major themes given in Einstein' text. Under each theme, we first indicate what students were asked to clarify - usually Einstein's quotes in the book. The analysis is not focused on looking for conceptual change but on identifying conceptual vulnerabilities, which usually survive technical exposure to the subject. This is why we have juxtaposed the text comprehension data with the interviews data to look for students' notions that stay through and after the standard course on the subject. Not all vulnerabilities are, however, common to both sources of data; some exist only in text data, some others emerge only from the interviews. Our published work [Bandyopadhyay & Kumar, 2010 (a)] and the thesis give the evidential basis of our conclusions, quoting extensively from the written scripts and the interviews. For brevity, we omit this in the synopsis and only give the Einstein text summary (in small font), followed by our questions, and our findings in a very condensed form. The actual findings are far more nuanced.

A. The principle of relativity ''need not of necessity hold a priori".

Einstein begins his exposition of general relativity by recalling the special principle of relativity i.e. relativity of uniform motion. Taking the familiar carriage-embankment example, he says that either of the two could be taken as a reference body and motion referred to it but this self evident assertion "must not be confused with the more comprehensive statement called the principle of relativity". "Unlike the first, this latter statement need not of necessity hold a priori" – only experience can decide if it is correct or incorrect.

Students were asked to explain the underlined quotes above.

Though all students could readily recite the principle of special relativity (laws have the same form in different inertial frames), only three of them clearly grasped the important point that this principle is not a priori true and requires experimental verification. This point is true even for Galilean relativity and students' analogous notions have been discussed in detail in Chapter 4.

B. Experience with non-uniformly moving bodies of reference seems to go against the general principle of relativity.

Now, Einstein says, the temptation to generalise the special principle of relativity to the general principle of relativity (the laws of nature hold for all bodies of reference, whatever be their state of motion) is natural. But there is a problem. For example, when a carriage moves non-uniformly, the behaviour of bodies is different from the uniformly moving case. "Because of this we feel compelled at the present juncture to grant a kind of absolute physical reality to non uniform motion, in opposition to the general principle of relativity".

Students were asked to state what Einstein meant by the general principle of relativity and explain the underlined quote above.

Our impression from the data is that students do not clearly distinguish between the 'description of phenomena' (measurement of the concerned physical quantities) and the 'laws of nature' (general relations between the measurements). They readily understand that the laws are different in non-inertial frames, but some of them entertain anthropomorphic notions of the 'ease' (or the lack of it) in discovering the laws of physics in different frames.

C. Experiment shows that gravitational field produces the same acceleration for all bodies, in contrast to electric and magnetic fields. This implies that the gravitational mass (m_G) can be taken to be equal to the inertial mass (m_I).

Einstein points out the remarkable property of the gravitational field (not shared by electric or magnetic fields) namely that "bodies which are moving under the sole influence of a gravitational field receive an acceleration, which does not in the least depend either on the material or the physical state of the body". From this experimental fact he concludes that the inertial mass (m_I) appearing in the law of motion is proportional to (equal to, by choice) the gravitational mass (m_G) that appears in the gravitational law. A satisfactory understanding of this equality is possible, he says, if we recognize that "the same quality of a body manifests itself according to circumstances as 'inertia' or 'weight'."

Students were asked: (a) what was remarkable about the gravitational field in contrast to electric and magnetic fields, (b) whether the equality $m_I = m_G$ was a priori obvious, and (c) to explain the underlined statement above.

Most students appreciate that the universality of acceleration is characteristic of a gravitational field, in contrast to electric and magnetic fields. However, a flawed notion of gravitational mass is shared by some of them, namely that it is the quantity that 'resists' gravitational force. The correct conceptual schema of a physicist (gravitational mass is to gravitational force what charge is to electric force; both, a priori, unconnected to inertial mass that resists any force via the law of motion; but experimentally $m_I \propto m_G$) is not as easy as one might think. See the following quote of a student:

"An inertial mass responds to any force. If some force acts on a body, its inertial mass (m_I) will resist it. In contrast, gravitational mass responds to gravitational force only. If a body experiences a gravitational force due to some massive object around, its gravitational mass (m_G) will resist it. But it is not at all obvious why these two things should be identical."

The conceptual vulnerability of a priori viewing gravitational mass as instantiation of inertial mass in a gravitational context is an unexpected and significant finding.

D. The Principle of Equivalence (PoE) based on the equality $m_I = m_G$ is an argument for the general principle of relativity.

Einstein now introduces the Principle of Equivalence from his well known thought experiment. A man in a spacious chest in empty space, which is being accelerated 'upwards' will have the same experience as when the chest is at rest in a gravitational field acting 'downwards'. The absoluteness of acceleration no longer holds and the conflict with the general principle of relativity (theme B) disappears. "We have thus good grounds for extending the principle of relativity to include bodies of reference which are accelerated with respect to each other, and as a result we have gained a powerful argument for the general principle of relativity." (a) Next, he turns the argument around, considers a body suspended in the chest 'vertically' and explains the tension in the rope in two equivalent ways to arrive at the equality $m_I = m_G$. "Guided by this example, we see that our extension of the principle of relativity implies the necessity of the law of equality of inertial and gravitational mass. " (b) Equipped with the new insight, he says that a deaccelerating carriage could be equally well regarded as at rest with respect to which there exists (during the period of application of brakes) a time-varying gravitational field in the forward direction. Einstein cautions that all gravitational fields (e.g. the earth's field) are not of the type for which you can find another reference body with respect to which the field disappears.

Students were asked to explain the underlined quotes (a) and (b) above, (c) to explain the alternative interpretation of the 'jerk' experienced by the observer in the deaccelerated carriage, and (d) to explain Einstein's caution above.

Data suggest that students do understand that the Principle of Equivalence arises from the equality $m_I = m_G$ and some even appreciate that this helped Einstein overcome the absoluteness of non-uniform motion, and thus generalize the principle of relativity. They regard the gravitational fields that can be transformed away as apparent. However, one significant vulnerability emerged, namely of restricting the equivalent gravitational field to 'inside the reference body'—the deaccelerating train. Also, Einstein's reversal of reasoning in (b) – using the PoE to arrive at the equality of the masses – was missed out.

E. Newtonian mechanics and special relativity give no reason why some reference bodies (inertial frames) happen to have preference over other reference bodies (non-inertial frames).

Einstein asks: "how does it come that certain reference bodies (or their states of motion) are given priority over other reference bodies (or their states of motion)? What is the reason for this preference?"(a) Einstein goes on to state that "the objection is of importance more especially when the state of motion of the reference body does not require any external agency for maintenance, e.g. in the case when the reference body is rotating uniformly."(b) The objection is resolved only by a theory that treats all reference bodies on the same footing.

Students were asked to explain the quotes (a) and (b) above and state how the general principle of relativity resolves the problem.

Students clearly knew that classical mechanics privileged inertial frames over non-inertial frames since the latter had different laws in them. But they only vaguely appreciated Einstein's unease with the absence of any a priori reason for this privilege. Only one student realized that for Newton, there did exist an explanation, one that involved absolute space. (Inertial frames have uniform motion with respect to the absolute space, frames with non-uniform motion with respect to the same are non-inertial). But Einstein had banished absolute space (ether) in special relativity and so the absence of any reason for preference was a most unsatisfactory situation to him. The objection is even more glaring in the case for a uniformly rotating body; it needs no external force or torque to maintain its state of motion. Fundamentally, there are two things: (a) in classical mechanics and special relativity, some frames (inertial frames) are singled out, and, more importantly, (b) there is no a priori way to tell how to single out these frames. To Einstein (a) by itself is not problematic; the real difficulty was (b). The two issues (a) and (b) were not clearly separated in the students' responses.

F. The Principle of Equivalence extended to all of physics (i.e. domains not restricted to mechanics alone) predicts bending of light under gravity.

Einstein considers two frames of reference: an inertial frame K and a frame K' that is uniformly accelerated with respect to K. Light travelling rectilinearly with respect to K will be bent in general with respect to the accelerated frame K'. If the PoE is regarded as valid for all phenomena, K' may be regarded at rest with a gravitational field in a direction opposite to acceleration, and we can conclude that "in general, rays of light are propagated curvilinearly in gravitational fields."

Students were asked to explain Einstein's argument above for the bending of light under gravity.

We found that students tend to prefer an explanation based on the notion - light has mass, so it will bend - to the one based on the Principle of Equivalence. A standard course may equip them superficially with additional explanations based on the idea of light following curved paths - null geodesics in curved space-time. However, technical exposure does not seem to guard them against flawed conceptions regarding the 'velocity of light in a gravitational field', the point that we probed in the interviews.

The question may be strictly incommensurable in this theory; yet the 'constancy of the speed of light' is such a strong hang-over from special relativity that students tend to regard the phenomenon of 'bending of light' as one in which the speed of light remains fixed while its direction changes due to gravity. Five of the six students held this view, and even the sixth student giving the correct conception became guarded on further probing. This is a significant finding.

G. A differentially heated marble table can be a simple illustration of a non-Euclidean continuum.

Einstein next elucidates the notion of a non-Euclidean continuum by a simple example. Consider the surface of a plane marble table. Take little rods of equal length and begin by making a square on the surface with four of these rods. Go on adding squares one on another until the whole slab is covered. Einstein emphasizes that it is not logically obvious that the construction would succeed. The fact that it does is a property of the slab and the rods: For this situation (Case I), Euclidean geometry is valid.

Next (Case II), suppose we heat the table more at the centre than at the periphery, so the temperature is non-uniform over the slab. Suppose the rods expand, the increase in length being proportional to the increase in temperature. The 'square mesh construction' of Case I will now fail. But there might exist rods of a special material which are not influenced by temperature, using which the 'square mesh construction' would still succeed. Thus the failure of the 'square mesh construction' with the expandable little rods is attributed to the varying lengths of those rods at different positions, and not to any basically different geometrical property of the continuum itself.

But now imagine the situation (Case III) where the rods of every material expand identically with temperature and there is no other way of detecting the effect of temperature. In this case, the 'square mesh construction' would fail for every kind of rod. We must naturally assign the same length (say unit length) to every little rod no matter where it is placed on the differentially heated marble surface. With this assignment, Euclidean geometry would be violated on the slab. In this case then, the slab is a non-Euclidean continuum.

Students were asked why in Case II can we continue to regard the surface as a Euclidean continuum and why in Case III should we regard it as a non-Euclidean continuum.

The written responses and interviews on related questions uncovered some interesting alternative conceptions of students regarding Euclidean and non-Euclidean geometry. First, they do not seem to have internalized the notion that the geometry of a continuum can only be decided by measurements. Second, their visualization that 'straight' rods cannot be used to measure distances on a curved surface (e.g. a sphere) leads them to doubt the ability to measure distances in a non-Euclidean continuum. Third, they find it hard to absorb the view of the observer 'in' the surface, but instead go more readily for the view of the 'outside' observer in the higher dimensional space in which the surface is embedded. This is what makes them uncomfortable with assigning equal lengths (unit lengths) to all rods on the differentially heated surface (Case III above), since it seems to conflict with the 'outside neutral' view. The embedding picture persists in that the 'unity ' in 'unit length' is regarded as not being well-defined or fixed. Fourth, the continuum itself, besides the measuring rods, is regarded as a physical entity which 'expands' or 'bulges'.

H. The propositions of Euclidean geometry do not hold for a rotating disc observer.

From this theme onwards, the analysis given in the thesis and our published work [Bandyopadhyay & Kumar, 2010 (a)] is much more involved. Here we omit the text summary and the questions, and only indicate some findings.

Students are able to see that the gravitational field corresponding to a rotating disc is not like an ordinary Newtonian field. They appreciate that it is not possible to define a unique time in the rotating frame as a whole, since clocks at rest in the frame run at different rates. Uncritical use of time dilation of special relativity, however, leads them to conclude that synchronization of clocks is possible along the rim since they all run at the same rates. The rotating disc geometry presents the expected difficulty, namely, that students do not see why the circumference of the disc should not contract like the measuring rods used to measure its length. Students' difficulties in this theme partly arise from the fact that it is the outside inertial observer who infers about the problems (lack of synchronization of clocks, non-Euclidean character, etc.) of the rotating disc observer.

I. The space-time continuum of general relativity is non-Euclidean.

This last theme is rather technical and is detailed in the thesis. Among other things, we aimed at probing if students have grasped (i) Einstein's resolution of the problem of loss of metrical meaning of coordinates in general relativity and (ii) what is involved in his abandoning description using bodies of reference in favour of Gaussian coordinate systems. Our analysis suggests that students continue to think in terms of space-time meanings of coordinates, though they can recite the correct text assertions in this regard. Further, they understand that the description in terms of Gaussian coordinate systems is needed to overcome the problems of space-time coordinates of a general body of reference but do not seem to appreciate the critical extension involved in this step, namely going from the equivalence of all reference bodies to the equivalence of all Gaussian coordinate systems.

Chapter 6 : Students' ideas of the Principle of Equivalence (Study 3)

6.1 Introduction

We saw in Study 2 (Chapter 5) how Einstein advanced the Principle of Equivalence (PoE) as an argument for his extension of special relativity to general relativity; and how students in our case study engaged with these notions dealt with in Einstein's text. In Chapter 6 of the thesis we investigate in greater technical detail students' understanding of this principle in a variety of contexts.

There is an equivalent way of stating the PoE, one that is more natural in general relativity and, though touched upon already, has not been scrutinized so far regarding its cognition. It is that a freely falling non-rotating frame in a uniform gravitational field is an inertial frame. The reasoning is as follows: all bodies including the body of reference have the same acceleration in a uniform gravitational field, so relative to the frame, bodies without any (non-gravitational) external forces have uniform motion; i.e. for the freely falling frame, the gravity has disappeared. The frame is no different from a non-rotating unaccelerated frame in free space and hence is inertial. In a non-uniform gravitational field, the same reasoning shows that a freely falling frame is inertial only locally, i.e. in a small space-time neighbourhood of the centre of mass of the frame over which the gravitational field does not vary appreciably.

To the best of our knowledge, there has been no detailed diagnostic research on students' ideas of the PoE. Pedagogic studies on the PoE clarify and elaborate on its various aspects. For example, an interesting study by Ohanian (1977) questions the idea of local validity of this principle by showing that gravitational fields do not vanish in a liquid drop even when it is made to shrink to an arbitrarily small size. On the whole, the exact meaning of the PoE and its place in general relativity are matters of considerable scholarly debate (Norton, 1989). We steer clear of this debate in this study and adopt a consensual view of the PoE as reflected in most introductory books on the subject.

6.2 Methodology

In contrast to Study 2 which involved a small sample and employed a purely narrative presentation of data, this study employs a sample of moderately large size and thus follows the same technique of analysis and presentation as Study 1. As explained in Chapter 3.1, a pilot study was carried out on a small group of 9 undergraduates during a vacation course on GR at Institute

1. A diagnostic questionnaire on the PoE consisting of about 30 items was administered after the topic was taught, roughly in the middle of the course. Its detailed analysis gave clues to several significant conceptual vulnerabilities among the students. The questionnaire was then substantially improved in terms of its content as well as design.

The student sample of the main study consisted of 30 students who could be regarded as being senior undergraduates. The group was a mix of first and second year students in the Masters programme in Physics and third and fourth year Engineering Physics students enrolled in Institute 2. All of them had been exposed to special relativity in their earlier courses and had also undergone (and were undergoing) the standard tertiary physics courses. A one-semester teaching course on GR was given to these students in the first half of 2010. The course entailed about 70 hours of teaching and tutorials, and was based on the first eight chapters of the well-known book on the subject by Weinberg (1972). These chapters cover with great rigour all the technical themes dealt with in our entire work. The improved questionnaire administered after the instruction of the topic consisted of 20 items on different aspects involving broadly 8 themes. In order not to cue the students, the themes were not spelt out; the test items were arranged around them without any order. The items, though technical, did not demand high mathematical skills needed otherwise for the course.

6.3 Students' conceptions

The questionnaire and the data analysis in terms of pointer statements and interpretive response categories (see 3.2) are detailed in the published paper [Bandyopadhyay & Kumar, 2010 (b)]. Here we only summarize the conclusions under the various themes surrounding the PoE that we set out to investigate.

A. Galileo's principle

The appearance of the same constant (m) in both Newton's II Law and the Law of Gravitation is so customary in high school physics that many students continue to be influenced by this prior knowledge and regard the equality of m_I and m_G as obvious. They do not regard it as something that needs to be verified experimentally. Indeed, some of them took the equality of the two masses for granted and 'proved' Galileo's principle – a common practice in school physics.

B. The universal proportionality of m_I and m_G

Do observations show that m_I is *equal* to m_G or only that their ratio is a constant for all bodies?

The study indicates that many students have not grasped the point that observations only show that the ratio is the same constant for all bodies, independent of their inertial mass and internal constitution. Observations do not determine this ratio; taking the ratio to be equal to 1 is a matter of choice by which we lose no generality.

Data show that many students tend to link the validity of the PoE with the *equality* of the two masses. The PoE uses the equality of pseudo force in an accelerated frame to gravitational force in an unaccelerated frame with gravity, i.e. $m_I\vec{a} = m_G\vec{g}$, so a different choice of the ratio would only amount to a different magnitude of the equivalent gravitational field. Note, the symbol g in the preceding equation stands for gravitational field strength; it equals 'acceleration due to gravity' only for the usual choice $m_I = m_G$. The prevalence of flawed thinking in this regard shows how much is uncritical recall of familiar knowledge responsible for students' vulnerabilities.

C. The equality $m_I = m_G$ and internal energy

In Newtonian physics, mass and energy are independent concepts, so this equality has nothing to do with internal energy. But what happens in relativity where mass and energy are interrelated? Many students are not explicitly conscious of the fact that the energy which contributes to inertial mass (via $E = mc^2$) also gravitates, i.e. it has a corresponding gravitational mass ($m_G = E/c^2$).

D. PoE and the notion of an inertial frame

As said before, in the PoE based view, a freely falling frame in uniform gravity (which is an accelerating non-inertial frame in Newtonian view) is inertial. Related to this is the altered view of gravitational force which is like any external force in the Newtonian view but is frame-dependent in the new view, becoming zero in a freely falling frame. Consider the following question:

Observer *E* is standing on the ground. Observer *F* is in a small cabin falling freely towards *E*. State how *F* explains the observed motion of *E* using the PoE-based view of regarding the frame F as inertial.

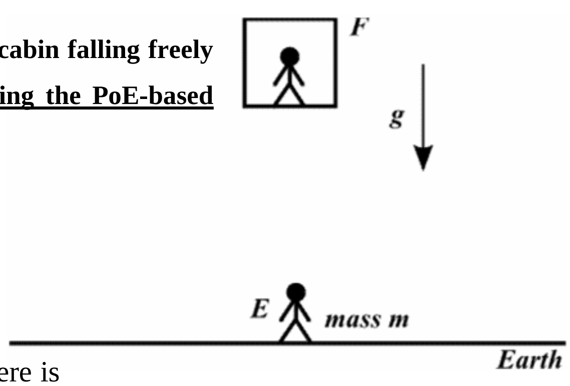

Fig. 1 : A freely falling cabin

The PoE-based reasoning is that for the inertial observer F, there is no gravity around, so E experiences only the non-gravitational force (normal contact force on E by the earth) which causes his upward acceleration. Contrast this with the Newtonian view: *F* is non-inertial, so *E* is under three forces, his weight *mg*, the pseudo force –*mg* and the normal contact force *mg*. The resultant is *mg* which causes a net acceleration of *E* relative to *F*.

Only 4/30 students answered the question using the correct PoE-based view. The remaining students answered in purely kinematic terms and did not address the question. The required cognitive change from the Newtonian to the PoE based view is evidently non-trivial.

E. Using the PoE in problem tasks

We distinguish two equivalent versions of the PoE.

Version 1: A uniformly accelerated frame in free space is equivalent to a frame 'at rest' in a gravitational field of the same magnitude as the acceleration but directed oppositely.

Version 2: A non-rotating frame falling freely in a uniform gravitational field is inertial.

In all the problem tasks we designed, we urged the students to use the PoE. Three of the tasks required version 1, while two tasks required version 2. Of course, all the tasks could be handled without invoking the PoE, using the usual Newtonian mechanics. Our motivation was to see to what extent students had assimilated the two versions of the PoE.

Our conclusion was that students find it easier to use version 1 in solving problems. Problem tasks which are a direct application of the second version are handled by students usually by Newtonian methods, even when they are urged to use the new view. This, of course, is related to the fact that students have not assimilated the notion of a freely falling frame as an inertial frame, as mentioned in the preceding theme.

F. The local nature of the PoE

The local validity of the PoE refers to small neighbourhoods both in space and time even if the field has no time dependence. Students' responses to the questions designed for this theme indicate that they grasp the spatial locality of the PoE better and ascribe the locality in time only to time varying fields.

G. The PoE and weightlessness

The topic of weightlessness of bodies has been investigated extensively in the literature (see Lehavi & Galili, 2009). In an important work, Galili (1995) observed that young students attributed weight to medium – *the state of weightlessness of a body in free space was due to the absence of air*. Older school students believed *weightlessness to be the outcome of the 'large distance' of the body from the earth*. These students were clearly making use of the notion of decreasing gravitational force with distance, following their instruction.

Our aim was twofold: (i) to discover the extent to which Galili's findings reappear in the understandings of college students, and (ii) to explore students' ideas regarding deviations from a state of total weightlessness due to the local nature of the PoE (presence of tidal forces due to non-uniformity in gravitational fields).

Students were asked to answer the following three questions in the given order. (a) According to the PoE, gravity disappears in a freely falling frame. The earth's frame is also freely falling under the gravity of external objects. Why then does a body on the earth have a weight? (b) For a body at rest on the surface of the earth, does the gravitational pull of the sun contribute to its weight? (c) If the earth were closer to the sun, would the contribution of the sun to the body's weight be different from that in (b) above? Explain.

In brief, we found that older college students generally know that free fall corresponds to weightlessness but the origin of weight when a body is not freely falling and the contribution of tidal forces are harder to assimilate.

H. The PoE and non-mechanical domains

We again omit the details. Some students tend to extend the strong form of the PoE to non-gravitational fields, and do not realize that the equivalence is between acceleration and gravitational field only, even if non-mechanical phenomena are under consideration. They do not exercise the caution that the term 'free fall' refers to the motion of a body in a pure gravitational field environment, and that a charged body accelerating 'down' in an electric field is not a freely falling inertial frame.

Chapter 7 : Students' ideas of the meaning of 'covariance' of a physical theory (Study 4)

7.1 Introduction

The principle of general covariance is a central conceptual theme of general relativity. It is a technical way of stating that all co-ordinate systems are on the same footing for the laws of physics. To capture this notion mathematically requires the full machinery of tensor calculus. Since our work mainly deals with topics that do not require high level of mathematical prerequisites, we do not focus exclusively on general covariance in the present study. Instead, we broaden the scope of the study and investigate students' ideas on the meaning of 'covariance' of a physical theory in a variety of contexts (rotational, Galilean, Lorentz and general covariance). This strategy of enlarging the scope of the study proves to be useful. We identify several conceptual vulnerabilities of students regarding such basic notions as invariance, covariance, manifest covariance, co-ordinate system-independence, conservation, absoluteness, etc. which permeate several domains of physics at the university level. We must add that in terms of content the study is pitched at about the senior undergraduate level. The more subtle matters regarding covariance of a physical theory and, in particular, the meaning and content of the principle of general covariance (whether or not it is physically vacuous) are issues of scholarly debate that lie outside the scope of this study [see Norton (1993)].

7.2 Methodology

The methodology of this study – a pilot study followed by the main study – is identical to that of the preceding study (Study 3) on the Principle of Equivalence. The samples were also from the same groups; the only difference is that the sample size had slightly decreased by the time of this study during the courses: 6 students of Institute 1 for the pilot study and 27 students of Institute 2 for the main study. The method of data analysis (pointer statements and response categories) was again similar. The pilot questionnaire consisted of 23 items, whose analysis led to an improved and more focused questionnaire consisting of 20 items on different aspects of the topic, involving about 10 conceptual themes. As before, the themes were not spelt out in the questionnaire to prevent cueing. The results of the study have been published [Bandyopadhyay & Kumar, 2010 (c)]. Here we only summarize our findings.

7.3 Students' conceptions

A. Covariance, invariance and transformations

Most students could recite the standard meaning of the term 'covariance', presumably because it was part of instruction. Yet the examples they cited showed poor discrimination between the terms: covariance, invariance and transformation. For example, velocity transformation was cited as an example of covariance.

B. Rotational covariance and 3-vector equations

Questions under this theme were designed to examine if students, accustomed to thinking of covariance in the context of relativity, understand the notion in the more elementary context of three-dimensional rotations. Figures in the questions showed rotation of axes; that is, the 'passive' view was adopted in which only the components of the vectors transform, the vectors themselves remain fixed. Yet several students in this study transformed vectors such as $\vec{E}$ and $\vec{B}$, to new vectors $\vec{E}'$ and $\vec{B}'$ under rotations even when the passive view was adopted, influenced as they were by the learnt schema during instruction regarding covariance: "quantities change, but forms of equations do not". This is an interesting example of how students 'over-learn' an idea.

C. Particularization and the apparent violation of covariance

The equations of a covariant theory may not show form invariance if they have been particularized for some situation. Thus a special choice of axes, initial conditions, gauge, etc. may result in the equations of a covariant theory lose form invariance. This does not mean that the theory is no longer covariant. Responses to simple questions in the context of rotational covariance, involving particular choice of axes, showed that very few students showed awareness of this point.

D. Covariance, manifest covariance and tensors

Students had picked up the standard conception from instruction that a theory may be covariant, though its equations may not show this manifestly. Many also knew that if the equations of a theory have been expressed using the language of tensors, the theory is manifestly covariant by the very defining transformation properties of tensors. However, the analysis revealed a stereotype: *associating tensor language only with general relativity*. This is clearly a fall-out of the curriculum. Physics students usually meet tensors for the first time only in Einstein's relativity.

E. Manifest covariance and co-ordinate independent 'geometrical objects'

When the equations of a theory are expressed in terms of coordinate independent geometrical objects, its covariance is manifest. Few students showed appreciation of this point, even though they are long familiar with vectors: a 3-vector equation is manifestly covariant under rotations since vectors are coordinate independent objects. For general relativity, this mode of manifest covariance takes us to the language of differential geometry that is outside the scope of undergraduate physics. In our study, students tended to associate manifest covariance only with the usual coordinate-dependent formulation of tensor equations they first encounter in relativity.

F. Covariance of a set of coupled equations

The equations of a theory as a whole may be covariant, and yet each equation by itself may not transform in a form invariant manner. This point was posed in connection with Lorentz covariance of Maxwell's equations of electrodynamics. In this case, not many realized that we need to take two equations together, the homogeneous and inhomogeneous pairs: a single equation of each pair by itself is not form invariant.

G. Covariance and conservation

Conservation of a quantity is a statement referred to a given frame pertaining to a process – it is invariance in time of some quantity in the process. The study indicates that many students do not regard conservation as a distinct notion from covariance, which is form invariance of laws for different frames. There can, of course, be interconnectedness between these notions. For example, the law of conservation of relativistic momentum combined with Lorentz covariance leads to the conservation of relativistic energy. However, the matter is rather subtle and could not be probed adequately.

H. Distinction between sub-set and limiting case

The study brought out a significant vulnerability—students are prone to regard a limiting case as a sub-set. This showed up in the questions regarding Galilean invariants and Lorentz invariants. The study revealed a tempting but flawed view that Galilean transformations are in some sense a subset of the Lorentz transformations (since the latter reduce to the former in the limit $c \to \infty$); so if a quantity is invariant under Lorentz transformations, it would be so in the "smaller" set of Galilean transformations.

I. 3-vectors and boosts

Do electric and magnetic field transform and mix in Galilean relativity? Some students carry the notion that the transformation and mixing of the fields is a feature of Lorentz boosts and that these fields remain invariant under Galilean boosts, not realizing that the 3-vectors ($\vec{E}$ and $\vec{B}$) are invariant geometrical objects only for spatial rotations—they are not invariant under 'boosts' whether Galilean or Lorentz.

J. Absoluteness and invariance

Absoluteness is a more general idea than invariance. Acceleration is an absolute notion both in Galilean and special relativity though it is invariant only in Galilean relativity. Few students showed a clear discrimination between the two concepts. Clearly the meaning in physics of something being absolute is not easily grasped, though used very frequently.

K. Terms in a tensor equation

This question came up in connection with the usual way of writing the geodesic equation of a particle. Several students did not realize the elementary point that though each term of the equation is non-tensorial, their sum is a tensor.

Chapter 8 : Probing students' ideas on some technical concepts in general relativity (Study 5)

8.1 Introduction

In this last study of the work (Chapter 8), we visit some technical notions which underlie the usual mathematical formulation of general relativity. Here too, our aim, as in the preceding studies, was not to test mathematical competence per se, but to probe students' qualitative 'feel' about the technical/mathematical notions of the subject. Even so, some measure of higher mathematical skills was admittedly necessary for the students participating in the study. The study in a sense lies at the border beyond which advanced mathematics of general relativity becomes inseparable from its concepts. The scope of the study is, therefore, very modest.

8.2 Methodology

The methodology of the study was identical to that described for our Studies 3 and 4. The sample of 5 students for the pilot study was also from the same group of students undergoing an optional vacation course on GR at Institute 1. The sample of 26 students for the main study was again from the same group at Institute 2 described earlier. The only difference was that the questionnaire for the pilot study contained a large number of short technical questions (about 40). The reason was that it was not clear in the beginning what kind of technical notions would be suitable for a study on a larger sample. A careful look at the written scripts of the pilot study led us to a much smaller but focused questionnaire for the main study, which was administered at the end of the course. The analysis of students' responses was carried out in the same manner as in the earlier studies. The themes discussed below refer both to the pilot and main studies. The details of data analysis are omitted below; we only summarize the findings.

8.3 Students' conceptions

A. Parallel transport

A tangent vector along a geodesic propagates itself parallely. It was found that this fact that often comes up in instruction, creates a stereotype. Parallel propagation is equated to taking the vector along the tangent to any arbitrary curve at every point, whether the space is curved or flat. The notion can lead to answers like a vector changing its direction when propagated parallely along a curve in a plane or a vector not returning to itself under parallel propagation around a closed curve on the surface of a cylinder (which has zero curvature).

B. Metric and metric connections in flat manifolds

The metric in special relativity is generally written as $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ where $\eta_{\mu\nu}$ are constants $(-1,1,1,1)$. Since this metric repeatedly comes up in a relativity course, the flat space-time is equated to it. Students do not seem to appreciate that this metric results from choosing a (pseudo) Cartesian coordinate system. A spherical polar coordinate system in the same space-time would yield a different non-constant metric, $g_{\mu\nu}(x)$. Most students do not show the conceptual discrimination between a manifold and the coordinate system and stereotypically associate the line element $ds^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu$ with general relativity.

A related stereotype is with regard to the appearance of metric connections in special relativity. Most students assert that the equation of motion of a free particle cannot be of the form $\left[\ddot{x}^\lambda + \Gamma^\lambda_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = 0\right]$ in special relativity. Few realize that such an equation can appear even in Galilean relativity, where in a non-inertial frame the second term would be related to the familiar inertial forces.

C. Local flatness in curved space

Most students are aware after a course on general relativity that the metric $g_{\mu\nu}$ can be transformed to the flat metric $\eta_{\mu\nu}$ and that the metric connections can be made to vanish locally in the so-called geodesic coordinate system. The term 'locally flat', however, leads to the expected vulnerability that the Riemann tensor at the given point can also be made to vanish. This is an example where language can cloud the mathematical fact that students are aware of, namely, that a null tensor remains null in all coordinate systems.

D. Curvature tensor and curved space-time

An important vulnerability noticed in our study is that students tend to equate curved space with non-zero values of not only the curvature tensor ($R^{\lambda}_{\mu\nu\rho}$) but also its contractions, the Ricci tensor $\left(R_{\mu\nu}\right)$ and the curvature scalar $\left(\Re\right)$. This showed up in questions about the values of these quantities in the matter free region outside a star. Several students, aware that the space-time in this case is curved, believed that all the quantities above would be non-zero. There seem to be two sources for this error. One, they do not realize the obvious mathematical point that a non-zero tensor can lead to zero values of its contractions. Two, though students can recite the field equations, their local nature and the importance of boundary condition are not fully appreciated.

Note: The empirical findings given above in Chapters 4 to 8 are in a greatly abbreviated form; they are much more nuanced in the thesis, which also gives the frequency of different vulnerabilities in our samples. Statements above regarding what students understand or do not understand, and phrases like 'most students' or 'few students', etc. should be viewed with this qualification.

Chapter 9 : Conclusion

9.1 Sources of conceptual vulnerabilities

In the concluding chapter, we comment on the possible sources of the large number of domain-specific vulnerabilities of students that were found in this work: (i) alternative conceptions of students in antecedent domains which survive instruction reappear in new domains based on them, an example is confining gravitational fields (in the context of PoE) to the supposed spatial boundaries of reference frames; (ii) the use of anthropomorphic notions about 'observer', 'difficulty in discovering laws in non-inertial frames', 'inability to infer laws for other observers', etc. is basic to many difficulties; (iii) the tendency to understand the abstract in concrete terms shows up repeatedly, a notable instance is seen in theme G in study 2 regarding the geometry of the continuum; (iv) in many situations, prior knowledge of an antecedent domain is correct but is being carried uncritically to a new domain (e.g. holding on to the constancy of speed of light during bending under gravity); (v) a new notion is assimilated in an existing notion, a striking example of this is to regard gravitational mass a priori as an inertial mass in gravitational context; (vi) the finer conceptual discriminations necessary in a complex domain are often ignored or subsumed under the broader conceptions of students, which may arise not merely because of inattention to the nuances and detail but may also be rooted in students' epistemologies, an aspect not probed in this paper; (vii) a general source is imprecise use of scientific language (e.g., not distinguishing 'laws of nature' from 'description of phenomena', ambiguous use of the word 'obvious', etc.), a problem perhaps compounded by the fact that for the students under the present study, the medium of instruction (English) is not their first language.

9.2 Methodological limitations

Some methodological limitations of this work arising mainly due to practical constraints are:

(1) The samples for our Studies 1, 3, 4 and 5 are moderately sized (26-30). A bigger sample size would have helped in greater generalizability of our conclusions.

(2) The Studies 1, 3, 4 and 5 give only the prevalence of the various response categories. A study of correlations between responses of a student between different related questions would have deepened the analysis. This, however, would need a major and separate undertaking.

(3) Detailed recorded interviews were a vital feature of Study 2. However, no interviews could be held for the Studies 1, 3, 4 and 5 due to reasons of logistics.

(4) The content validity of our technical questionnaires and of data interpretation for Study 2 needs to be established. Further, the coding method employed in Studies 1, 3, 4 and 5 in terms of pointer statements and interpretative response categories needs to be validated. See Sec.3.4

9.3 Outlook

The field of physics education research has matured a great deal in the introductory domains of physics, as reviewed briefly earlier. This work represents a modest effort in the direction of physics education research in a complex domain. The methodologies for investigation in such domains need to be developed further. Our work may be regarded as a coarse-grained cognitive study covering a fairly large content area; the identification of numerous significant vulnerabilities reported here can be a useful input for further research. Future studies can be more fine-grained and methodologically stronger, focusing perhaps on a small part of the content area covered here in greater detail. Perhaps one lesson from this work, if we may put it so, is that advanced level instruction of complex domains does not always insure against conceptual vulnerabilities in the same domains, even when students may develop reasonable technical competence in the subject through instruction. We believe studies of this kind could be useful in other advanced and basic domains of physics such as quantum mechanics, continuum mechanics, field theory, etc. The insights on students' cognition obtained from such studies would not only add to the growth of this field but should hopefully be of direct use to physics teachers and researchers at the college/university level.

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List of Publications

1. **Bandyopadhyay, A. (2009). Students' ideas of the meaning of the relativity principle. *European Journal of Physics , 30* (6), 1239-1256.**

2. **Bandyopadhyay, A., & Kumar, A. (2010). Probing students’ understanding of some conceptual themes in general relativity. *Physical Review Special Topics : Physics Education Research , 6* (2), 020104-1 to 020104-14.**

3. **Bandyopadhyay, A., & Kumar, A. (2010). Probing students' ideas of the principle of equivalence. *European Journal of Physics , 32* (1), 139-159.**

4. **Bandyopadhyay, A., & Kumar, A. (2010). Students' notions regarding 'covariance' of a physical theory. *European Journal of Physics , 31* (6), 1391 - 1413.**

5. **Bandyopadhyay, A. & Kumar, A. (2010). Galileo's Principle in early models of gravity. *Physics Education* (India), *27* (2), 97 – 110.**

References

Aguirre, J. M. (1988). Student preconceptions about vector kinematics. *The Physics Teacher , 26* (4), 212-216.

Arriassecq, I., & Greca, I. M. (2007). Approaches to the Teaching of Special Relativity Theory in High School and University Textbooks of Argentina. *Science and Education , 16* (1), 65-86.

Bandyopadhyay, A. (2009). Students' ideas of the meaning of the relativity principle. *European Journal of Physics , 30* (6), 1239-1256.

Bandyopadhyay, A., & Kumar, A. (2010a). Probing students' understanding of some conceptual themes in general relativity. *Physical Review Special Topics : Physics Education Research , 6* (2), 020104-1 to 020104-14.

Bandyopadhyay, A., & Kumar, A. (2010b). Probing students' ideas of the principle of equivalence. *European Journal of Physics , 32* (1), 139-159.

Bandyopadhyay, A., & Kumar, A. (2010c). Students' notions regarding 'covariance' of a physical theory. *European Journal of Physics , 31* (6), 1391 - 1413.

Beichner, R. J. (2009). *An Introduction to Physics Education Research.* Retrieved from Reviews in PER Vol. 2: http://www.per-central.org/document/ServeFile.cfm?ID=8806

Bell, J. S. (1976). 'How to teach special relativity' in *Speakable and unspeakable in quantum mechanics.* Cambridge, England: Cambridge University Press, pp. 67 – 80.

Boudreaux, A. (2005). Tracing Difficulties With Relativistically Invariant Mass To Difficulties With Vector Addition Of Momentum In Newtonian Contexts. *AIP Conference Proceedings,* (pp. 35-38). Sacramento.

Carey, S. (1987). *Conceptual Change in Childhood*. MIT Press.

Chi, M. T., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science , 5* (2), 121-152.

Clement, J. (1993). Using bridging analogies and anchoring intuitions to deal with students' preconceptions in physics. *Journal of Research in Science Teaching , 30* (10), 1241 - 1257.

d'Inverno, R.(1992). *Introducing Einstein's Relativity.* Oxford University Press.

Dimitriadi, K., Halkia, K., & Skordoulis, C. (2005). Basic Concepts of Special Theory of Relativity in Secondary Education: Do students understand them? *European Science Education Research Association Conference (ESERA) "Contributions of Research to Enhancing Students' Interest in Learning Science".* Barcelona.

di Sessa, A. A. (1993). Toward an Epistemology of Physics. *Cognition and Instruction , 10* (2), 105 - 225.

Driver, R. (1985). *Children's Ideas in Science* (1st ed.). Open University Press.

Einstein, A. (1920). *Relativity : the Special and General Theory*. Methuen & Co Ltd.

Galili, I. (2008). History of Physics as a tool for teaching. In M. Vicentini, & E. Sassi (Eds.), *Connecting Research in Physics Education with Teachers Education.* ICPE.

Galili, I. (1995). Interpretation of students' understanding of the concept of weightlessness. *Research in Science Education , 25* (1), 51 - 74.

Halliday, D., Resnick, R., & Walker, J. (1996). *Fundamentals of Physics*. Wiley, New York.

Hartle, J. B. (2006). General relativity in the undergraduate physics curriculum. *American Journal of Physics , 74* (1), 14 - 21.

Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force concept inventory. *The Physics Teacher , 30* (3), 141 - 158.

Hosson, C. D., Kermen, I., & Parizot, E. (2010). Exploring students' understanding of reference frames and time in Galilean and special relativity. *European Journal of Physics , 31* (6), 1527 -1538.

Jenkins, E. (2001). Research in science education in Europe. In H. Behrendt, H. Dahncke, R. Duit, W. Gräber, M. Komorek, A. Kross, et al. (Eds.), *Research in Science Education - Past, Present, and Future* (1 ed.). Kiel: Springer.

Lehavi, Y., & Galili, I. (2009). The status of Galileo's law of free fall and its implications for physics education. *American Journal of Physics , 77* (5), 417 - 423.

Mason, A., & Singh, C. (2010). Helping students learn effective problem solving strategies by reflecting with peers. *American Journal of Physics , 78* (7), 748 - 754.

Matthews, M. R. (Ed.). (1996). *Constructivism in Science Education.* Kluwer.

Mazur, E. (1996). *Peer Instruction: A User's Manual* (Pap/Dskt edition ed.). Benjamin Cummings.

McCloskey, M. (1983). Naive Theories of Motion. In D. Gentner, & A. Stevens (Eds.), *Mental Models (Cognitive Science Series)*. Lawrence Erlbaum Associates.

McDermott, L. C. (1984). Research on conceptual understanding in mechanics. *Physics Today , 37* (7), 24-32.

McDermott, L. C. (1995). *Physics by Inquiry: An Introduction to Physics and the Physical Sciences* (Vol. 1). John Wiley & Sons, Inc.

McDermott, L. C., Shaffer, P. S., & Rosenquist, M. L. (1996). *Physics by Inquiry: An Introduction to Physics and the Physical Sciences* (Vol. 2). John Wiley & Sons, Inc.

McDermott, L. C., & Redish, E. F. (1999). Resource Letter: PER-1: Physics Education Research. *American Journal of Physics , 67* (9), 755-767.

McDermott, L. C., & Shaffer, P. S. (2001). *Tutorials In Introductory Physics.* Benjamin Cummings.

Norton, J. D. (1989). What was Einstein's Principle of Equivalence? In *Einstein and the History of general relativity : Einstein Studies* (Vol. 1, pp. 5 - 47). Boston: Birkhauser.

Norton, J. D. (1993). General Covariance and the Foundations of general relativity: Eight Decades of Dispute. *Reports on Progress in Physics , 56* (7), 791 - 858.

Ohanian, H. C. (1977). What is the principle of equivalence? *American Journal of Physics , 45* (10), 903-909.

Panse, S., Ramadas, J., & Kumar, A. (1994). Alternative conceptions in Galilean relativity: frames of reference. *International Journal of Science Education , 16* (1), 63 - 82.

Pfundt, H., & Duit, R. (1994). *Bibliography* : *students' alternative frameworks and science education* (4^{th} ed.). Kiel: Institute for Science Education.

PER-Central. (n.d.). Retrieved from PER-Central: http://www.compadre.org/per/

Pietrocola, M., & Zylbersztajn, A. (1999). The use of the Principle of Relativity in the interpretation of phenomena by undergraduate physics students. *International Journal of Science Education , 21* (3), 261-276.

PROCEEDINGS OF THE AAPT TOPICAL WORKSHOP: TEACHING GENERAL RELATIVITY TO UNDERGRADUATES. (2006). Retrieved from http://www.aapt.org/doorway/TGRU/

Ramadas, J., Barve, S., & Kumar, A. (1996 (a)). Alternative conceptions in Galilean relativity: inertial and non-inertial observers. *International Journal of Science Education , 18* (5), 615 – 629.

Ramadas, J., Barve, S., & Kumar, A. (1996(b)). Alternative conceptions in Galilean relativity: distance, time, energy and laws. *International Journal of Science Education , 18* (4), 463 – 477.

Redish, E. F. (2003). *Teaching Physics with the Physics Suite.* Wiley.

Reif, F., & Allen, S. (1992). Cognition for Interpreting Scientific Concepts : A Study of Acceletation. *Cognition and Instruction , 9* (1), 1-44.

Reif, F. (2010). *Applying Cognitive Science to Education: Thinking and Learning in Scientific and Other Complex Domains* . The MIT Press.

Rindler, W. (1994). general relativity before special relativity: An unconventional overview of relativity theory. *American Journal of Physics , 62* (10), 887-893.

Saltiel, E., & Malgrange, J. L. (1980). 'Spontaneous' ways of reasoning in elementary kinematics. *European Journal of Physics , 1* (2), 73-80.

Scherr, R. E., Shaffer, P. S., & Vokos, S. (2001). Student understanding of time in special relativity: Simultaneity and reference frames. *American Journal of Physics , 69* (7), S24-S35.

Scherr, R. S., Shaffer, P. S., & Vokos, S. (2002). The challenge of changing deeply held student beliefs about the relativity of simultaneity. *American Journal of Physics , 70* (12), 1238 - 1248.

Scherr, R. S. (2007). Modeling student thinking: An example from special relativity. *American Journal of Physics* , 272 - 280.

Thornton, R. K., & Sokoloff, D. K. (1998). Assessing student learning of Newton’s laws: The Force and Motion Conceptual Evaluation and the Evaluation of Active Learning Laboratory and Lecture Curricula. *American Journal of Physics , 66* (4), 338 - 352.

Thornton, S. T., & Marion, J. B. (2004). *Classical Dynamics of Systems and Particles.* Belmont: Harcourt.

Trowbridge, D. E., & McDermott, L. C. (1980). Investigation of student understanding of the concept of velocity in one dimension. *American Journal of Physics , 48* (12), 1020 - 1028.

Trowbridge, D. E., & McDermott, L. C. (1981). Investigation of student understanding of the concept of acceleration in one dimension. *American Journal of Physics , 49* (3), 242 - 253.

Viennot, L. (2001). *Reasoning in Physics: The Part of Common Sense* (1 ed.). Springer.

Villani, A., & Pacca, J. (1987). Students' spontaneous ideas about the speed of light. *International Journal of Science Education , 9* (1), 55 - 66.

Villani, A., & Arruda, S. M. (1998). Special Theory of Relativity, Conceptual Change and History of Science. *Science and Education , 7* (1), 85-100.

Vokos, S. (2006). Student Understanding (and Misunderstanding) of Important Concepts in Relativity. *Teaching general relativity to Undergraduates.* Proceedings of AAPT Workshop, op. cit.

Vosniadou, S. (1994). Capturing and modeling the process of conceptual change. *Learning and Instruction, 4* (1), 45-69.

Wald, R. M. (2006). Resource Letter TMGR-1: Teaching the mathematics of general relativity. *American Journal of Physics , 74* (6), 471 - 477.

Weinberg, S. (1972). *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity.* John Wiley & Sons, Inc.