Work, Knowledge and Identity

Implications for school learning of out-of-school mathematical knowledge

A Thesis

Submitted to the
Tata Institute of Fundamental Research, Mumbai
for the degree of Doctor of Philosophy
in Science Education

by

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Mumbai
December, 2014

DECLARATION

This thesis is a presentation of my original research work. Wherever contributions of others are involved, every effort is made to indicate this clearly, with due reference to the literature, and acknowledgement of collaborative research and discussions.

The work was done under the guidance of Professor K. Subramaniam, at the Tata Institute of Fundamental Research, Mumbai.

[Arindam Bose]

In my capacity as supervisor of the candidate's thesis, I certify that the above statements are true to the best of my knowledge.

[K. Subramaniam]

Date:

This study is dedicated to the settlement children's perseverance, learning spirit and indomitable strength.

May we learn from different ways of learning.

Acknowledgements

About a decade ago I had my first tryst with mathematics education research first as a pursuit of pedagogic improvement in a rural school in a less advanced state of Bihar – my home state, followed by an exposure to a detailed and engaging school mathematics curriculum revision process in the same state. These two experiences triggered my interest in educational research. I was eager to embark on a socially meaningful research and the unstinted encouragement and efforts of one of my mentors, my parents and two of my school teachers helped me in deciding to shift from mathematics to mathematics education. I acknowledge with deep sense of gratitude all the good things that have contributed to the advancement of this research study and have made a difference to my life. I place my reverence and solemn appreciation of kindness to those special people, interactions with whom have changed my outlook towards some aspects of the world and how I experience them now.

My creator, my saviour and my benevolent Almighty – my parents – my "*Maa*" Mukul Bose and my "*Bauba*" Asoke Bose nurtured me sacrificing all their pleasure, happiness, financial savings and their own sweet little dreams for their two sons – my sibling Aniruddha and I. That we brothers should be able to grow up cultivating and achieving our own dreams had become their new dream. Despite several hindrances, difficulties and health-related problems, they endured my prolonged absence from home and attended to their needs themselves when as a son I should have been around. *Bauba* has been watching the last leg of my PhD journey from his new eternal abode, while *Maa* awaits a meaningful and a relevant piece of work back home. My PhD work owes them its existence. I owe every bit of myself to my *Bauba* and *Maa*. I can never thank them enough.

The participants of the study, especially the students of the English and Urdu medium schools – the field of study, have been the pivot of the study. The research team and I have been deeply indebted to their enthusiastic participation in the study, some of whom came for the interactions even on Sundays and on holidays sacrificing their time and work. Their love, affection and fond hugs have been instrumental in motivating me to visit "my children" frequently. My special thanks and hearty wishes to them and also to their parents and the community elders who welcomed me to their workshops and homes and also participated in the study.

A word or two about my mentors. Yes, I'm fortunate to have more than one mentor in my academic life. One, my PhD dissertation supervisor Professor K. Subramaniam (Ravi) and the other my godfather – my teacher and also a colleague Professor Vinay K. Kantha. They are my academic fathers. Ravi's fine-tuned art of critical and insightful questioning, intellectual and erudite engagement with scholarship was a training for me for ensuring rigour, quality and sense making in my work. More than being an exemplary mentor, Ravi has been a friend and a teacher whose biggest gift to me has been the unbiased and precise narration blend with critical thinking. When it came to sharing credits for ideas, it was Ravi's integrity and intellectual generosity that has been the hallmark of this work and our relationship as colleagues. He would always say, "the data is yours" and I had to counter back by saying, "but the idea was yours, it came from you"! This thesis also owes to Ravi its very existence. Kantha has been the inspiration and guiding force behind my initiation and switch to mathematics education from pure mathematics. His academic rigour, righteousness, social and academic activism, his upright nature and stimulating discussions helped me understand many facets of education in the socio-cultural historical milieu. I feel honoured being associated with both Kantha and Ravi. I owe a lot to them. I would dare not thank them, for I know, I cannot do that enough, it can never be enough.

I shall remain indebted to my second home and my abode of research – *Homi Bhabha Centre for Science Education* and my office room 218, a perfect place of work where I got the vibes for my thinking and felt at home. I am grateful to Profs. Arvind Kumar and Hemachandra Pradhan, the past directors of the Centre, to Prof. Jayashree Ramadas, the current director, and to Prof. Chitra Natarajan, the current dean and to other members on the Subject Board (Science Education) for providing me an opportunity to carry out this study and also for providing me accommodation and other facilities and privileges. I acknowledge the financial support that I received from the Centre on many occasions for attending international conferences overseas which helped me interact with peers, noted researchers in my area and opened further opportunities. My special thanks and sincere gratitude for Prof. Sugra Chunawala for her intellectual critique, suggestions and constructive discussions about the study. She examined some of my writings and gave insightful comments that were helpful.

I reserve special thanks for Mr Bhau Korde who guided me at the beginning of the study for obtaining necessary letters of permission from the authorities and took me around the field and to the school. He introduced me to the School Management Committee (SMC) and from time-to-time shared with me his personal experience of working in the same settlement being part of the school education system himself. It was Mr Korde’s guidance and support that ensured a smooth sail for my field-work. My sincere gratitude to the Education Officer, MCGM, Mr Abhasaheb Jadhav for granting me permission to carry out the study and visit the Municipal Corporation-run schools for the study. Special thanks to the headmistress and the deputy headmistress/master of English and Urdu Schools – Ruta Wankhede, Ujjwala Konda, Ramashray Dubey, Hakim Raeesa Iqbal and Naseema Khatoon Maqsood for facilitating this study. My special gratitude to Mr Sachin Chavan, who is on the faculty of the English School for his constant help and support. Chavan spent crucial time with me discussing about the issues related to the students, teachers and the policies. His insights helped me understand different aspects of the teaching process better. Other teachers, viz., Kiran Bharsat, Sarang Shinde, Sudam Durgade, Radheshyam Mhale, Balkrishna Patkar, Shahnaz Raza Khan, Farha Shaukat Ali, Nasir Yusuf, Iqbal Shaikh, Imran Ahmad helped me throughout the study. This study gave me good friends in them.

I acknowledge contributions of my three friends Varun Kumar and Aloke Jain – both PhD students in Tata Institute of Social Sciences, Bombay and Renato Marcone, a graduate student in UNESP at Rio Claro, Brazil for giving critical comments and feedback to my writings. Discussions with Varun, Aloke and Renato enriched my ideas. My interactions with Profs. Bill Atweh, Richard Barwell, Nadia Douek, Brian Greer, Farida Khan, Swapna Mukhopadhyay, Terezinha Nunes, Mamokgethi Setati Phakeng, Nuria Planas, Anita Rampal, Padma Sarangapani, Geoffrey Saxe, Ole Skovsmose, Jayasree Subramanian, and Paola Valero over different time were very insightful and helpful. Discussions with them always brought up new ideas, clarity and helped me fine-tune my thoughts.

My colleagues at HBCSE hold a very special place in this work. This research study cannot be called an individual’s work, it has been a team work where I was just the principal researcher. I acknowledge with deep sense of gratitude, the help and unstinting support that I and this study received from my colleagues in the Mathematics Education Cell – Arati Bapat, Rajendra Kadam, Aaloka Kanhere, Tuba Khan, Ruchi S. Kumar, Hari Mishra, Shweta Naik, Manoj Nair, Amol Parab, Smita Patil, Jeenath Rahaman, Shikha Takker, Arsheya Taskeen, Yogaraj, V., and Saritha V. N. Special thanks are for Tuba Khan who helped me with data coding and along with Hari Mishra assisted me in some parts of the transcription work. Hari helped me in checking the references. Manoj's was a big help with his computer graphics skill. Shweta, Jeenath, Shikha and Ravi's help has been immense right from the beginning of the study in formulating and refining the ideas, data analysis and overall support. Rajendra Kadam's assistance in the video recording during the vacation course was extremely useful. It has indeed been the entire team's effort.

My deep appreciation for my other colleagues at the Centre whose technical and academic help came when every time when I required them. My special thanks to Manojendu Choudhury, Shubhayan Kabir, Prajakt Pande, Aswathy Raveendran, Rafikh Shaikh, Saurav Shome, Anveshna Srivastava, Himanshu Srivastava. My colleague Mr Gajanan Mistry's logistics support has been immensely helpful and so was Ms Sumana Amin's and Ms Smita Burli's support with the technical and official matters. My sincere gratitude to them. I also thank Mr V. P. Raul and people at the accounts section, Mr R. S. Korgaonkar, Mr P. Balakrishnan and the Establishment Section team, Ms M. M. Mastakar and others at the Stores, Mr D. D. Pednekar and others at the HBCSE Library, Mr U. V. Shenoy and Mr S. D. Pardeshi for their help and support which made things easier for me. My special thanks to our canteen friends Mr Raju Mohanty and his team for taking care of food. Support from all these people ensured that I could focus on my work and not worry about anything else.

As a son, brother, uncle, friend, teacher and colleague, I have received love, guidance, and utmost care. My friends and relatives had to undertake immense hardships in my absence, made sacrifices and extended to me moral support and constant encouragement during my PhD journey. Special thanks to the large cohort of my friends with years of bonding for being there with me. My warm gratitude especially to Abhijit Krishna, Abhishek Ravi, Amit Sinha, Neel Mani Rangesh, Roshan Bharadwaj, Surojit Roy, and Vibhuti for their care, support and encouragement. At the hours of crisis they were beside me. Meeting them has always been a stress reliever which boosted my strength.

Thanks my *Almighty*, it's just a beginning …

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Introduction

The highest education is that which does not merely give us information but makes our life in harmony with all existence

– Rabindranath Tagore

Mathematics is often equated with precision and rigour manifested in symbols and proofs. As a school subject mathematics is also seen as a tool for moving towards abstraction and generalisation. Such is our belief about its esoteric position in human society that its origin in human ideas and human endeavours appears obscure and counter-intuitive. In its endeavour towards abstraction and generalisation, school mathematics gets detached from the routine everyday settings and requirements. At the same time, knowledge of mathematics is valorised in the society, and is seen as an integral part of the school curriculum for all children. However, we may argue that it is the picture of mathematics as an esoteric collection of rules and formulae, unconnected with reality, leads to it being a subject that causes the greatest fear and sense of failure (National Council for Educational Research and Training [NCERT], 2006a). It is ironical that many who study mathematics formally in schools find it difficult and frightening and also opt out midway, but those who are outside of schools or have dropped out, continue successful use of mathematics in different everyday settings and in work-contexts with or without being aware of it.

This dichotomy between the two situations can be addressed if both the domains draw on from each other. In other words, one way to address this tension is by enriching school mathematics learning by connecting it with the outside world where mathematics remains hidden in the cultural practices, in artefacts, in work-contexts that children are familiar with. It is therefore pertinent to explore the nature of mathematics that remains embedded in the everyday world and the ways in which such knowledge emerges or is gathered. It is with this idea of exploration of what school going children encounter in their outdoor activities and endeavours and whether or not such activities or artefacts bear elements of mathematics, that the researcher embarked on this study. He was particularly keen to understand the work-context construction of mathematics and its pedagogical significance for classroom learning. This research study sought to gain an understanding about the connection between the work-contexts that many young children are engaged in, their identities, and knowledge of mathematics that they draw from such experience. Ways in which connections between work, knowledge and identity can illuminate school learning and aspects of school learning that can facilitate building such connections, have been investigated. Opportunities and affordances availabile in the work-contexts and other everyday settings that are potentially resource-rich for dealing with elements of mathematics have been unpacked by what we (the researcher and his mentor) have termed as archaeological exploration. This thesis has also uncovered the rich resource of measurement knowledge that children possess from their exposure to the work-contexts which is a new addition to the existing literature. Pedagogical significance of this study lies in the teaching intervention which has sought to merge students' identities and exercise of shifts in the classroom norms to invite and connect out-of-school mathematics with school mathematics. In the process, the study has suggested that merging of students' identities can facilitate transfer of knowledge and hence build a connection between both the forms of knowledge.

This thesis is not the culmination of the research study but a modest beginning of such explorations. We begin this opening chapter with an overview of the thesis along with brief summaries of the chapters that set the scene for the study and the thesis.

1.1 Structure of the thesis

This thesis is organised into eight chapters. Chapter 1 gives an introduction to the thesis and places the background and the context in which the research was undertaken. It also presents the motivation and the rationale behind the study. The scope of the study is discussed along with a discussion about the limitations of the study. At the end of the first chapter, a set of key terminologies with their working meanings that are widely used in this thesis are given. The other technical terminologies have been defined and discussed at the place where they appear.

Chapter 2 presents the context of the study by reviewing the relevant literature in the domain of out-of-school mathematics and mathematics in work-places. This review includes the insights obtained from the previous studies including those done in India in terms of distinction between out-of-school and school mathematical knowledge. In the light of the "*funds of knowledge*" as a guiding notion of this study, the role of work in education has been explored. Debates around this issue are located in the context of curriculum policy documents in India. The relevance of Gandhian "*Nai Taleem*" ("New Education") in the present educational system has been noted by connecting it with the "*funds of knowledge*" perspective. This discussion provides a basis for the analysis in the subsequent chapters under different phases of the study and topics. Chapter 2 ends with an analysis of the "needs (for survival, to support family economically) versus rights (to get meaningful education)" debate in the context of the widely prevalent child labour in India. The analysis takes a critical look at the debates on the involvement of children in work practices in the light of the *Right to Free and Compulsory Education* as a fundamental right and underlines the need to look at the notion of childhood as is seen in the community.

Chapter 3 discusses the larger study, the research questions and the research design including the choice of the location of the study, the sample and methods. The significance of the location has been highlighted. The nature of work-contexts within the house-holds or in the neighbourhood in which the students are engaged are discussed. A broad sketch is drawn to introduce to the reader the location of the study, the community, the house-holds, the neighbourhood, and the schools. A visit map of the conduct of the study and ethical considerations are presented as well. This chapter includes the preliminary findings that set the context of the study.

Chapter 4 presents the work profiles of the sample students including their parental occupations to give a glimpse of the diverse work-contexts that children living in the field of the study get exposed to. This chapter further contains descriptive reports of four cases of students who participate in work contexts by focusing on their exposure and involvement in work practices and features of their mathematical knowledge. The four cases are then drawn together and supplemented with descriptions of other sample students' involvement in work-contexts. In this way, a more comprehensive sketch of the work-contexts that students in the sample are exposed to together with opportunities to acquire mathematical knowledge is drawn. The chapter also discusses students' arithmetical knowledge drawing on data from the first two phases of the study.

Chapter 5 analyses the opportunities and affordances available in diverse work-contexts and in everyday settings (drawn from case studies and supplementary data) for the learning of mathematics. This chapter discusses how handling of diverse goods or limited access to them, optimising resources and decision making processes create different possibilities of dealing with mathematical knowledge. In this backdrop, the chapter presents the features of participants' mathematical knowledge in relation to out-of-school contexts. The chapter further discusses demathematisation and transfer of knowledge and directs the reader towards the existence of hybrid forms of mathematical knowledge. It discusses how students' identities are shaped through participation in work contexts. This chapter then traces the pedagogical implications arising from the above discussion.

Chapter 6 unpacks the measurement knowledge embedded in various work-contexts emergent from the semi-structured interviews with the students and researcher's observations of the community practices. The chapter begins with the review of literature on previous studies on measurement in the everyday work contexts as well as measurement learning as a school curriculum topic. A characterisation of out-of-school measurement related experience is discussed in this chapter. A possible connection of measurement learning in both the domains (school and out-of-school) has been drawn which also traces the implications for classroom pedagogy.

Chapter 7 presents an analysis of a teaching design experiment focusing on integrating students' out-of-school measurement knowledge and school learning by focusing on conceptual connections and on students' identities and their possible implications for learning. One of the highlights of this chapter is the proposal of a classroom pedagogy that enables a series shifts in the classroom norms that might be needed to integrate out-of-school and school learning.

Chapter 8 is the concluding chapter of the thesis which discusses the results and findings of the study and presents possible curricular and pedagogic implications. Future road-maps and directions for future research are also discussed. The researcher at the end of this chapter presents his postscript for the readers.

1.2 Rationale and motivation for the study

The study was motivated by our observations during initial field visits and during interactions with school students and non school going children from low-income settlements both in rural and urban conglomerations. These observations concurred with the findings of previous studies that children living in disadvantaged conditions often experience difficulties in learning mathematics in schools and face failure and in many cases drop out from studies (Sarama and Clements, 2009). Informal knowledge gained in out-of-school contexts remains poorly built on in the classrooms, which may be the reason for knowledge gaps that are detrimental to their progress in formal mathematics learning (Saxe, Guberman, Gearhart, Gelman, Massey & Rogoff, 1987). However, children who drop out of formal studies as well as children from economically poor families often get into income generating practices, where they acquire and successfully use their informal knowledge of mathematics. It is expected that economically active surroundings offer diverse contexts to children to acquire and learn mathematical knowledge. Therefore, identifying the mathematical learning opportunities from economically active surroundings and from community based social networks and building on such knowledge was thought of as a way of making learning meaningful.

It is of interest to mathematics educators to explore whether students of a certain grade know a particular mathematical concept from their out-of-school experiences, ways in which their prior math knowledge help them in gaining new knowledge in the classroom by building on their out-of-school knowledge and whether their school math learning illuminates their everyday math knowledge. Also of interest are the questions about the knowledge organisation in everyday mathematics, for example, its forms of representations, different and flexible ways to arrive at solutions, and how they differ from the school mathematics in terms of abstractions and procedures. It is felt that the prevalence of everyday mathematics in the culture is a potential resource that can make school mathematics learning more effective. There are not many studies on how this can in fact be realised. Criticisms of efforts to use the insights from out-of-school mathematics in the school contexts are usually based on a clear separation between out-of-school and school knowledge with different goals. But, researchers adopting cultural perspective assume mathematical activities as embedded in culture and consider school mathematics as one cultural practice of mathematics among others (de Abreu, 2008). Recent studies however do not deny the distinctiveness of school and everyday forms of mathematics, but seek to blur the boundary between the two by allowing multiple points of connection to form a more hybrid form of knowledge (Nasir, Hand & Taylor, 2008). It is with this idea that we have explored what our participants know or can do, and what they have observed or are familiar with even if the associated mathematical knowledge is partial and fragmented. Our perspective is to explore what aspects can serve as starting points or building blocks for mathematical exploration in the classroom.

Research studies on out-of-school mathematical knowledge of children and adults have been carried out in many cultures since the 1960s. These studies have explored the contours of out of school mathematical knowledge, the ways in which it is acquired, and how it is different from mathematics learnt at school. While such studies initially communicated a promise of reshaping school math education based on what was known about out of school knowledge, there is still a lack of clarity about the implications of such studies for school learning. In this study, we have explored the nature and extent of everyday mathematical knowledge possessed by middle grade school students living in an urban low-income settlement that has embedded in it a thriving micro-enterprise economy. Children living in this settlement either have exposure to the diverse work-contexts prevalent in their neighbourhood or participate in and contribute to the production and income generation right from an early age. We also observed that they gained considerable informal knowledge from outside of school which remains unacknowledged in the classroom. Use of such informal knowledge to support classroom learning have been highlighted by several curricular documents as well as by researchers and philosophers (NCERT, 2005; 2007). The present study thus explores the prevalence of out-of-school mathematical knowledge among students from the low SES backgrounds and the implications for teaching and learning mathematics in school.

In the course of our exploration of the nature and extent of the opportunities available to the middle graders to gather everyday mathematical knowledge, we have characterised the work-contexts from a mathematics learning perspective. Our purpose is to unpack and document the connections between students' mathematical knowledge, work practices and identities, and inquire into the implications of these connections for school learning.

1.3 Need for drawing on out-of-school mathematical knowledge during formal mathematics learning

With education becoming politicised throughout the world, mathematics education has been in the focus of the politics of education (Valero, 2004). For educational researchers, it is important to understand the influence of low socio-economic conditions on mathematics learning. Researchers in the past have underlined the need to look at mathematics education in the socio-cultural-political dimensions including conceptual, cognitive, historical, epistemological, and ethical (D'Ambrosio, 2010; Skovsmose, 2009). In mathematics education research (henceforth, MER), it is increasingly being felt that learning mathematics can become helpful for the students if the classroom teaching involves familiar contexts and methods. The major educational policy document that is currently followed in India, the *National Curriculum Framework* (NCERT, 2005), points out that "learning takes place both within school and outside school" and that "learning is enriched if the two arenas interact with each other" (p. 15). The Framework gives importance to connecting school learning with the child's lived experience, "not only because the local environment and the child's own experiences are the best entry points into the study of disciplines of knowledge, but more so because the aim of knowledge is to connect with the world" (p. 30). Connecting with the child's environment also has a role to play in creating an educational culture that is equitable. "Our children need to feel that each one of them, their homes, communities, languages and cultures, are valuable as resources for experience to be analysed and enquired into at school; that their diverse capabilities are accepted" (p. 14). The position paper of the Focus Group on the teaching of Mathematics expresses the same concern and emphasises the use of "experience and prior knowledge" to construct new knowledge in school mathematics (NCERT, 2006a, p. 8).

Mathematics is an important school subject taught right from the kindergarten level across most of the contemporary cultures. However, it is one of the least understood subjects and not many people feel comfortable with it (Bishop, 1988a). Post-Piagetian constructivism has emphasised the fact that a child's mind is not a *tabula rasa*, that children studying in elementary classes enter their schools with prior knowledge drawn from their environment and everyday experiences. Hence, it is of importance to a community of mathematics educators to investigate the kind of mathematics children draw from the outside world and in what ways, and the possible bearing that such knowledge might have on their learning of school mathematics.

1.4 Scope of the study

The findings from this study can be valuable for teachers, curriculum planners, educational thinkers and the research community. The study builds insights about potential resources available in the everyday settings where mathematical elements remain embedded in hybridised and opaque forms, but available to community members including students and children. It is important that curriculum planners and educational thinkers recognise such potential resources of knowledge and the value of work-context knowledge for making meaningful connections with school learning. Gandhi's vision of connecting education with work related knowledge and experience calls for such integration. Gandhi felt that such "education alone is of value which draws out the faculties of a student so as to enable him or her to solve correctly the problems of life in every department" (Gandhi, 1937, *Complete Works, Vol. 69, p. 32 as cited in Govt. of India, 1999*). He shared,

> "I hold that true education of intellect can only come through a proper exercise and training of the bodily organs, e.g., hands, feet, eyes, ears, nose, etc. In other words, an intelligent use of the bodily organs in a child provides the best and quickest way of developing his intellect" (Gandhi, 1937, Complete Works, Vol. 71, p. 122, as cited in Govt. of India, 1999).

Gandhi argued that a child who is given education in manual vocation would develop a "sound, vigorous academic intellect … rooted in … day-to-day experience" (p. 123). He advocated that,

> "a proper and harmonious combination of all the three [vocation, mathematics and sciences, literature] is required for the making of the whole man and constitutes the true economics of education" (p. 124).

The learning of mathematics in school must therefore be framed in broad terms. It is aimed at acquiring understanding and insight and not at practical training. However, the existing curricula and teaching practices, in contrast to policy documents, serve to reinforce the separation of the everyday from formal school learning. Students too believe in the apparent disconnect between school and work-context knowledge and do not expect what they learn in school to be related to the knowledge that they acquire from out-of-school contexts. This study argues that experiences from everyday work-contexts make students familiar with artefacts and practices that represent a crystallised and embodied form of mathematics, and that such experiences are resources to make potentially powerful connections with school mathematics. It also argues that school mathematical knowledge represents a form of generalisation or abstraction consisting of ideas or constructs that illuminates diverse instances. From a standpoint of school learning, focusing on such generalisation can make school learning more meaningful. The present study unpacks the affordances created by the awareness or experience in diverse work-contexts for acquiring measurement knowledge and its possible connection and disconnect with school mathematics is presented.

This thesis also highlights aspects of the work-contexts that went beyond the researcher's expectations. For example, the work contexts were found to be richer and more diverse in potential and opportunities for uncovering and learning mathematics than was expected. They were multiple sources of gathering knowledge and about different kinds of work, perhaps linked to the special feature of the settlement. Furthermore, the rich knowledge of measurement available to children through diverse work-contexts that has affordances for classroom learning goes beyond what is currently available in the literature.

In the current economic scenario where knowledge is produced by the market demands and the markets determine what knowledge is, knowledge of mathematics is seen as necessary for producing economic goods. In the process of such knowledge production, the community’s knowledge resources and social skills often make way for a wider process of demathematisation (discussed in Chapter 5). This study shows that work practices in the community resist the processes of demathematisation. One of the contributions of this study has been the unpacking of mathematical knowledge of different forms which remains embedded in work practices and the different ways such knowledge is gathered by the children. This unpacking has curricular as well as pedagogical underpinnings.

This study points to the disconnection between students’ identities formed in out-of-school contexts as well as those formed during formal classroom learning. Formal mathematics learning in school facilitates shaping of students’ identity as learners. However, their exposure and experience in the out-of-school work-contexts help build their identities as knowers and learners as well as doers. Students draw out-of-school mathematical knowledge from their own work practices or by observing others work, which helps them look at themselves as “knowers” of some body of knowledge that is valued in the community. The identity of a “doer” is shaped when children reflect on themselves as doing certain tasks in the out-of-school contexts, even if those tasks are fragmented, piece-meal or routine house-hold chores. The present study has argued that when classroom teaching practices acknowledge students’ knowledge resource and allow merging of their identities then such practices facilitate building powerful connections between out-of-school and school knowledge (of mathematics) and enhances understanding. The present study thus goes beyond the contemporary research on connecting students' identities with classroom norms and argues for bringing together students' identities and funds of knowledge.

1.5 Limitations

The intent of the study was to understand school going middle grade students' out-of-school mathematical exposure and knowledge and the opportunities and affordances that came from their everyday and work settings to gather such knowledge. The researcher thus needed to understand the patterns of the social networks and their ideas, beliefs and students' immersion in them. The study was limited to Grade 6 students to carry out in-depth study of their engagement in the work-practices. Some constraints and limitations of the study that arose due to various reasons are discussed below.

The interaction with students and conducting interviews with them was a challenge. Though the researcher tried to conduct the interviews after building a personal rapport with the students, there could be many invisible factors that might not have emerged in the responses. The researcher's (male) gender and class location must have had an impact in the beginning particularly during interactions with the students, teachers and adults in the community. However, the researcher felt that such constraints had been overcome to quite an extent as the study progressed and as the researcher mingled with people in the field location. However, in such an eclectic exploration as was adopted in the study, there was also an ethical consideration about how deep one can probe about the family and work details or their income and other practices. At times, respondents might have felt uncomfortable to respond despite the best of efforts by the researcher to create a comfortable environment or the respondent might have been too tired to respond. These affective factors could have limited the scope of the interviews.

The researcher also noted a different kind of limitation that arose due to the prevalent cultural norms which did not allow some of the girl students to freely mix with the researcher as much as boys could do. Though there were some enriching and engaging interactions with the girl students, it was easier for the researcher to venture out with the boys in the locality compared to girls. Similarly, it was easier to visit the work places or homes of the male students than female students. There was no restriction or social binding, but the researcher felt that in some cases, a sense of inhibition worked among the girl students (barring a few exceptions) interacting with a male researcher like himself. For example, the boys were more forthcoming in sharing and talking about their work-contexts and were keen to be interviewed, or to take the researcher to their workplaces or to show him around. The researcher tried to follow the shared patterns of beliefs, values and language of the community to the extent possible.

Although the overall study was ethnographic in nature, the researcher did not live in the community as one of its members. However, the researcher made regular visits to the school and to the locality for more than two and a half years spending rich amount of time in the field to understand the situations, peoples' life-worlds and perceptions, and allowed room for issues to emerge over time. This being a prolonged study, the researcher was able to interact with a large number of people in this period, such as, many teachers working in the schools involved in the study, other members on the staff, several cohorts of students, many residents in the locality and community elders and social activists. Hence, the researcher could carry out extended observations of the students, teachers and community members in the study and could also immerse himself in the everyday settings of the community. In order to understand the issues that teachers need to grapple with as part of their work and "duties", the researcher did however travel more than once from one of the teachers' home to the school catching the early morning suburban train along with other teachers of different municipal corporation-run schools who were known to each other, and spent the entire "school-day" at school.

The vacation camp following the teaching design experiment though conducted in the school classroom set-up was a short, two-week long experiment with a long duration of 1.5 hours per day unlike actual classroom period lengths. The implications drawn are thus indicative and require more extensive, long-term work to acquire a firm footing. The caste, religion and ethnic dimensions operating in work contexts have not been looked at in this study. Similarly, the use of language in terms of technical and mathematical registers and their overlap with home languages have not been explored barring a focus on the thematic categories that are prominently spoken by the community members.

1.6 Key terminologies and their meanings

We discuss below key terminologies used in this thesis and their broad meanings that were adopted and used in the study:

Affordance: Characteristics of a context or a setting that hold possibilities of potential actions for gathering knowledge or learning.

Classroom observation: A qualitative data collection technique by making notes about the goings-on inside a classroom through the senses of the researcher often by employing an instrument for recording (notebook, recorder, etc.) for the research study purposes.

Community network: Groups of families, adults and children in a community who keep in contact with each other and exchange information, knowledge and experience; inter-connections within and across communities.

Ethnographic exploration: Inquiry into and discussion in detail about an unfamiliar domain in order to learn about it by examining the shared pattern of behaviour, beliefs, languages, community practices and activities. It involved an extensive fieldwork in terms of a prolonged period of observation and attempts to understand how the group functions.

Gross domestic product: An economic statistic that reveals the state of the economy in terms of the values of goods and services produced taking into account measures of expenditure and income. It is an important indicator of the economic conditions.

Low-income settlement: Habitat of people belonging to low-income groups; the settlement studied was old and established neighbourhood in central Mumbai with distinct social norms and cultural practices.

Middle grades: Grades 5-7 in schools in the state of Maharashtra are deemed as middle grades (or upper primary grades) which together with the (lower) primary Grades 1-4 are known as elementary grades. In many other Indian states, Grades 1-5 are deemed as primary and Grades 6-8 are taken as middle grades.

Work: Activity involving physical and mental labour. Work is often a means of earning money but such a premise is not always correct.

Work-context: Settings and practices involved in a particular work in which a group of people is engaged.

Socio-cultural practices: Practices embedded in the contexts of religious, ethnic, linguistic groups and practices associated with gender roles.

Thematic categories: Classification into chunks or units of information or instructions based on certain themes. These are also the typologies frequently used by the community members which reflect parental aspirations, foreground and thoughts. We have focused only on the thematic categories about children's learning and education.

2

Setting the Context

The real voyage of discovery consists not in seeking new landscapes but in having new eyes.

– Marcel Proust

The late sixties and early seventies witnessed a spurt in research focusing on alternate ways of learning outside of school, which drew on cultural anthropology to analyse various contexts that created the ground for learning in general. Cultural contexts of thinking and learning soon became part of the main focus of the mathematico-anthropological studies. For example, Michael Cole and John Gay's study in Liberia in western Africa in the sixties (Gay & Cole, 1967) looked at Kpelle children's learning of mathematics embedded in their cultural practices. Around the same time, Claudia Zaslavsky's study in Nigeria and East Africa focused on numeracy learning and use of patterns and shapes as part of the culture and work practices (Zaslavsky, 1973). Zaslavsky's work on "sociomathematics of Africa" as she called it, revolved mainly around the "applications of mathematics in the lives of African people" (p. 7). This was also the time when Sylvia Scribner explored literacy and numeracy development among Vai people in Liberia in their everyday practices (Cole & Scribner, 1974; Scribner & Cole, 1978). Though there have been earlier studies and texts on numeracy development (for example, Karl Menninger's *Number Words and Number Symbols* in the fifties) or on mathematical development; but in mathematics education research (MER), the studies done by Zaslavsky, Cole and Gay, and Scribner in many ways set the direction for future work that looked at mathematics learning as embedded in both cultural as well as work practices. Interestingly, it was also the time when there was a shift in the focus of the studies in cognitive psychology from individual psychological to socio-cultural aspects of one's cognitive development. There was a shift in research from using Piagetian developmental psychology framework to Vygotskian socio-cultural psychology and social learning theory. It was also the time when research in MER began to use tools from cultural anthropology and drew on other socio-cultural tools looking for alternate ways of development of mathematical thinking and reasoning in individuals and this new trend bore a parallel to the shift towards cross-cultural studies that was already underway in developmental psychology. It was the time when studies in MER increasingly started looking at one's cultural resources as well as at work practices as possible locations of math learning and development of mathematical cognition. These studies provided evidence to the growing belief and claim that school is not the only site of math learning but there exist other alternative sites of learning. These alternate sites are interesting for educational researchers to look at for they entail everyday contexts and situations which create opportunities for children to acquire mathematical knowledge.

This chapter looks at the broad areas of research that has investigated, as Saxe (1988b) contends, the critical questions of how children formed their mathematical understanding in out-of-school contexts such as work-contexts, everyday shopping and looked at the implications for mathematics learning in school (formal) settings. This chapter presents a review of the previous studies on out-of-school and school mathematics to acquire an understanding about the nature of the claims and counter-claims made about the overlaps, connections and disconnections between the two forms of mathematics. An attempt has been made to locate the direction in which the major studies in this area progressed and the broader themes of research that emerged. In the last three decades there have been many cross-cultural studies in this area particularly located in Latin American, African, Asia-Pacific countries that began with the description of non-Western, indigenous forms of mathematics and mathematics in work practices (Brenner, 1985, 1998; Knijnik, 1998; Lave, 1977, 1988; Lipka, 1991; Mukhopadhyay, 2013; Nunes, 1992; Nunes & Schliemann, 1985; Rogoff, 1997; Saxe, 1988a, 1988b; Scribner, 1984 and others). There have been studies in the US and UK and other other countries looking at out-of-school math in the Western setting and also comparative studies (viz., Civil, 1995; de Abreu, 2008; Guberman, 2004;

Masingila, 1993; Masingila & De Silva, 2001; Taylor, 2009 and others).

The review presented here further focuses on how the notion of *Funds of knowledge* have been used as a framework to examine the potential resource available to the community in the form of embedded mathematical practices in the work-contexts. There are not many studies in MER other than those by Civil (1995), de Abreu (2008), Moll, Amanti, Neff and Gonzalez (1992) and a few others (viz., Andrews *et al*., 2005; Velez-Ibanez & Greenberg, 2005) who have used this framework to explore the resources available in the community that can potentially support school math learning of the children. We have not come across many studies that have looked at middle graders' varied exposure to work-contexts, diverse handling of goods and availability of rich cultural resources and *funds of knowledge* in the community in the literature on out-of-school mathematics. We wish to argue in the thesis that such opportunities available to the children create affordances for them to gather mathematical knowledge and to build a connection to their school mathematics learning. Our argument is based on the analysis of the students' identities as they are shaped in their work-contexts, through their interaction with the economically active everyday settings, and funds of knowledge that they have access to. The research literature that forms the background for the analysis consists of studies that have focused on students' identities in out-of-school contexts and in the mathematics classroom settings. There are not many studies in MER that have looked at pedagogic approaches focused on building on students' identities particularly for connecting out-of-school mathematical knowledge with school math learning and the pedagogical implications of analyses that use identity as a lens.

An important topic area of school mathematics that makes strong connections with out-of-school knowledge is the topic of measurement. A review of the literature on measurement learning is presented in Chapter 6 to situate our discussion on the pedagogical implications of out-of-school knowledge for the learning of this topic in that chapter.

One may also pose the question of the relation between out-of-school and school mathematics learning as a question of the transfer of learning. Hence, we briefly examine previous studies in MER which have addressed the problem of transfer as a goal of learning. The triad of work, knowledge and identity is thus important to be explored in MER which is the broad objective of the research study discussed in this thesis.

The next section situates the present study in the Indian context of the debates on work and education and the notion of childhood. An exploration of this debate described in the section on “work and education” locates the present study in the historical backdrop of Gandhi’s *Nai Taleem* (Basic Education) and Tagore’s *Sriniketan* experiment of developing individual's self-reliance (Das Gupta, 2008). It presents a review of the regulations on child labour currently enacted in the Indian context and also in the backdrop of the UN’s Convention on the Rights of the Children (CRC). This section looks at the broader issues of child labour in the context of the “needs vs rights” debate.

2.1 Out-of-school Mathematics and School Mathematics

Out-of-school mathematical knowledge (also termed as “everyday” math, “street” math and “informal” math) of children has been studied extensively beginning with the pioneering work of Nunes, Carraher and Schliemann (1985) and other authors in the Latin American and African countries. Notable among these studies were those carried out in Liberia in the mid-seventies (Lave, 1988), in Papua New Guinea in the late seventies and in the early nineties (Saxe, 2002, 1992, 1988a, 1988b), and in Brazil in the eighties (Carraher, Carraher & Schliemann, 1985, 1987; Nunes, Schliemann & Carraher, 1993). These studies have looked into the different aspects of everyday mathematics developed in out-of-school settings and have compared this with school mathematics. Most of the studies referred to “out-of-school mathematics” or “everyday mathematics” as the form of mathematics that people use in everyday settings while engaging in contextually embedded practices, viz., work-contexts, everyday shopping, house-hold activities, games and so on. These studies focused on the work-place activities of street vendors, carpenters, fishermen, farmers, construction site foremen, tailors, carpet-weavers, grocery-shoppers, and provided a systematic comparison between the “everyday” and “school” mathematics primarily considering them as two forms of activities based on different cultural practices but on the same mathematical principles (Nunes, Schliemann & Carraher, 1993). These studies on out-of-school mathematics have also indicated the differences in the cognitive features, in mechanisms of knowledge acquisition and procedural differences between out-of-school and school mathematics.

To take an example, Carraher and Schliemann (1985) and Carraher, Carraher and Schliemann (1985) presented a contrast between the problem-solving strategies adopted by the participants trained and untrained in school mathematics. Their study showed that some school students did not realise that they had arrived at an absurd solution in a simple subtraction problem. As an answer to 21 – 6 some students arrived at 25 and in another case while "solving 22 – 8, a child operated in the following way: 8 – 2 = 6; 8 – 2 = 6; 22 – 8 = 66 " (Carraher & Schliemann, 1985, p. 42). Among school students, Carraher *et al*. found a propensity of "counting as the preferred procedure", "limited use of school-taught algorithms" and that "children rarely referred to previously obtained results when doing related exercises" (p. 37). In contrast, Bose and Subramaniam (2011) observed that a school student (middle grader) of 11 years (same age-group as in Carraher *et al*.'s study) who had diverse work-context knowledge and experience could not only realise the flaw in his answer obtained through school taught long-division algorithm but he quickly corrected the answer using his out-of-school strategies of decomposition and sharing (discussed in Chapter 5).

Some other differences between school and out-of-school mathematics emerged in the later studies. For example, Brenner's study (1998) on everyday shopping practices of Hawaiian school students highlighted the mismatch between the way school mathematics curriculum dealt with numbers and their place values and the corresponding use of place values by the students during economic transactions in the shopping contexts. She showed that students made less use of pennies as compared to dollars or cents (bigger quantities) whereas in schools the place values are taught in the reverse order, beginning with units, tens, hundreds and so on with an assumption that pennies as currency units would be the simplest quantities to deal with at the outset (Taylor, 2012). In the everyday shopping context, it is a common practice to begin with higher denominations first and then gradually narrow down to the smaller parts or changes. Brenner's study about how school children made sense of the currency denominations reflected children's knowledge of currency related experience in their everyday contexts. Recent studies by Taylor (2009, 2012) have further shown that children handled contextual arithmetical computations easily when dealing with numbers as money and currency denominations as compared to when similar problems were presented in decontextualised situations.

2.1.1 Mathematics in diverse work-contexts

Among the studies that focused on the work-contexts for understanding the development of individual's mathematical cognition are Sylvia Scribner's study (1984) of the US dairy workers in late seventies and early eighties, Jean Lave's study (1977) of the tailoring apprenticeship in Liberia in the seventies, Terezinha Nunes' study (1985, 1993) with the Brazilian construction foremen in the eighties, and Geoffrey Saxe's study (1988, 1992) with Brazilian candy sellers around the same time. These studies pioneered and influenced the emergent field of research in cultural psychology as well as in ethnomathematics.

Scribner's study (1984) provided a connection of the individual thinking in workplace and in schools. She looked at dairy workers' different kinds of actions with varied number of dairy products (around “220 items” of varied products – milk, cheese, yogurt, fruit drinks and others) and the way they assembled and priced orders. Items under each kind varied by size and qualitative characteristics such as flavour or fat content (p. 200). Her study showed the purposive nature of the workers' cognitive strategies who looked for ways that were flexible and which minimised their effort to accomplish a task. Furthermore, the workers’ actions reflected less application of formalisms learnt at schools while engaging in complex mental processes for their actions (Rogoff, 1997). In her work, Scribner illustrated the environment's functional role in practical problem solving situations where settings function as symbols. For example, delivery drivers who transport dairy products often use “case” as a quantification of a variable in an arithmetic problem task. For her, “environment” meant all the social, symbolic and material resources around the individual doer. She argued that “social knowledge is incorporated in the way dairy products are stacked in the warehouse: milk, cheese, and fruit drinks are not distributed at random” but stacked at particular locations depending upon the “proximity to where they are packed, proximity to similar items and floor space” (Scribner, 1984, p. 204). She highlighted that workers often sought such solution modes that were economical, required lesser effort and steps, and with least complex procedures. Different sets of workers, viz., assemblers, inventory staff and delivery drivers had different ways of accomplishing their tasks and problems but they all indicated use of “effort-saving strategies” and “procedures for simplifying and shortening solutions in different problem settings” (Scribner, 1986, pp. 25, 26). One of the main implications of Scribner's dairy study has been the unpacking of the multiple ways in which the complexities of the working knowledge and illustrations of the actions were guided by the social knowledge (physical environment and symbolic forms, i.e., the case size and the available physical space) that helps to shape the task according to the human needs.

Lave (1977, 1988) in her study with Liberian tailors learning the traditional forms of tailoring task highlighted the prevalence of a “curriculum” that follows different stages of task learning through which a novice (or a beginner) turns into an expert. In the beginning at the first stage, novices do mere observation of the ongoing tasks and assist co-workers and slowly graduate through apprenticeship and eventually learn to become a task-expert. Lave illustrated that a sequence of training follows and an evaluation by experts or senior co-workers by observations is routinely carried out by providing immediate feedback. Opportunities for self-correction was another feature that Lave observed. Lave's study presented an overview of a work-site training as an alternative to the school based job training or vocational training. In the conventional everyday setting, it is customary for a beginner in the field to follow a trajectory of apprenticeship and move ahead to acquire expertise in the task (Resnick, 1987).

Nunes' study with construction foremen suggested that familiar contexts ease the work even if the task is entirely new (Nunes, Schliemann and Carraher, 1993). In the study, the researchers gave the foremen (participants in the study) a task based on proportionality of wall measurements. The blueprint of the plan did not contain the scale of measurement unlike the routine practice and instead contained three measures of the walls of a room and the foremen were required to find the measure for the fourth wall by finding the scale using a pair of measures. The task was just the inverse of their usual practice but the context was familiar. The foremen could reverse the operations in order to solve the task. Strategies used by them preserved the meaning of their routine tasks. But for the control group of seventh grade students who knew the “rule of three” for solving proportional problems, the task was not familiar as they had not seen scale-drawings in their texts. Interestingly, students used their everyday knowledge to find solutions and not the strategies learned at school. Carraher, Carraher and Schliemann (1987) suggested that situational variables influence such tendencies as was evident in Scribner's study as well. They further suggested that concrete problem situations like in this problem tempt the doer to use oral computation procedures drawn from everyday knowledge, whereas, routine computations encourage the use of school-taught computation algorithms.

Saxe's study (1988b) with child candy sellers on the streets in Brazil noted propensities of the doers similar to those revealed by Nunes *et al.*, and Scribner in their respective studies. Saxe highlighted the prevalence of a practice among the sellers who with little or no schooling were able to handle and practice complex mathematics that contrasted with school mathematics. In his study, Saxe compared the mathematical understanding of two groups of non-vendors matched for age and schooling (one group each from rural and urban setting) with vendors (candy-sellers). Saxe's study revealed an interplay between school and out-of-school mathematics where sellers (some of who also attended schools) interchangeably used strategies learnt at work-contexts as well as those learnt in schools. He reports differences in the procedures of vendors and non-vendors. Sellers, while solving formal mathematical problems, used knowledge of their “street mathematics” and schooled vendors could use only limited aspects of school mathematics. His study showed transfer of learning from work context to school context and vice versa when sellers used aspects of school learnt strategies during work and out-of-school strategies to solve school math problems. Such a claim is in contrast to the view from Lave's and Nunes *et al.*'s work that broadly claimed situatedness of learning within tasks (discussed in Section 2.1.6).

All the above-mentioned studies illustrate the variety of arithmetic procedures that are part of out-of-school mathematics and at the same time discard the idea that arithmetical competencies can only be obtained in schools (Greiffenhagen & Sharrock, 2008). In fact, one of the major contributions of the studies done on out-of-school (or everyday or street) mathematics in MER and other areas of anthropology or cognitive science has been to challenge the prevalent notion that considers “western” school as the only platform for acquiring mathematical competency. These studies have illustrated the diverse arithmetical competencies and procedures that arise in such out-of-school locations which have strong potential to inform school math learning. As Lave (1988) puts it, these studies “discovered mathematics in both non-Western traditional societies as well as the academic hinterland of the West” (p. 3).

Some of these studies also looked at the transfer of learning from one context to the other – whether alternative methods used in one context can be transferred to other contexts even if the contexts are unfamiliar? All the studies concluded that people in their workplaces were far more successful in their everyday mathematics calculations than in their attempts to solve the tasks using school algorithms. As evidence of situatedness of learning within tasks, Nunes, Schliemann, and Carraher (1993) presented examples from their study showing that participants who were untrained in school mathematics could competently perform the calculations needed in their workplace activities, whereas in contrast, the school students, trained in school mathematics when presented with such problems came up with incorrect solutions or even absurd solutions. School students “concentrated more on the numbers” given in the problems and paid little attention to the “meanings of the problems”. (Nunes, Schliemann, and Carraher, 1993, p. 75; Carraher, Carraher & Schliemann, 1985; Lave, 1988). On the other hand, street vendors who with 'impressive ease' solved their routine problems in everyday settings, could not solve the same types of problems which they had earlier solved in their workplace contexts when presented to them as formal word problems without any contexts. Sometimes they gave insensible solutions, for example, getting as an answer a number in a subtraction problem that is bigger than the minuend (Brown & Burton, 1978; Nunes, Schliemann & Carraher, 1993; Resnick, 1987).

These studies also show how people think in different situations. For example, Lave (1988) claims that everyday cognition is usually drawn from cultural practices and is not individualistic. The meanings attached to common practices are often collective (Lave, 1988). Lave's study made it possible to compare performance across situations and concluded that the process of moving the enterprise into the 'lived-in world' requires disentangling of the alternative approaches to practice. In everyday activities, the social process of inter-subjective reliability for accomplishing mathematical tasks is often a common practice that is usually not seen in the classrooms, which, according to Resnick (1987), are more individualistic in nature.

Other studies have shown that while using mathematics in everyday contexts, the doer has a continuous engagement with the objects and the situation and she does not burden herself with the extra effort to remember the algorithms, calculation-techniques and the reasoning used. In contrast, school mathematics aims at mastering computational proficiency and symbol manipulation following the correct steps in the algorithms without much freedom of using alternate techniques (Resnick, 1987). While socially shared knowledge and situation-specific competencies are the hallmarks of everyday mathematics, school mathematics emphasises improving individual's performances and skills. Situation-specific competencies developed in everyday contexts are often linked with “physical referents” and with “socio-cultural meanings” that the contexts carry (Schliemann, 1998). Such competencies reflect consistent logical/mathematical principles across different work-contexts.

More recent studies have attempted to uncover the embedded mathematics in different practices under different labels, viz., *academic, school, workplace and everyday* *mathematics*. However, some researchers like Moschkovich (2002) have contested these labels and claim that they may not be mutually exclusive or dichotomous. It has been argued that there exists shared relationships between everyday and academic mathematics (of which school math is a part) hinting at the hybridised nature of mathematics that students gather. However, the nature or form of hybridity of mathematical knowledge emergent in the above four kinds of mathematical practices or in students' approaches have not emerged explicitly through empirical findings. In the present study, one of our goals is to unpack the nature of students' mathematical knowledge. From the viewpoints of researchers adopting cultural perspectives, mathematical activity remains embedded in culture and school mathematics, and is seen as one cultural practice of mathematics among others (de Abreu, 2008). According to this perspective, culture is seen as a hybrid and a dynamic notion and not as a fixed and static concept. Hence, approaches to school mathematics education drawing from the cultural perspective take into account out-of-school knowledge and practices but do not deny the distinctiveness of the two forms – the school and everyday (or out-of-school), rather they seek to blur the boundary between culture and domain knowledge and allow multiple points of connection to form a body of knowledge that has overlaps of different forms of mathematics (Moschkovich, 2002; Taylor, 2012). As such, school education and mathematics teaching is “not only about building on what students are familiar with... but also about introducing new ideas, concepts and sensibilities” (Nasir, Hand & Taylor, 2008, p. 220).

The last several decades have also seen increased emphasis on meaning making in school mathematical learning. Research on out-of-school mathematics has similarly looked at instances of meaning making and reasoning as embedded in work-contexts (Carraher & Schliemann, 2002; Nasir, Hand & Taylor, 2008). While solving problems in everyday contexts, participants operated meaningfully with quantities, made intermediate checks if the numbers obtained were reasonable, and used flexible procedures that were based on sound mathematical principles. As mentioned earlier, the problem-solving strategies in everyday contexts were in stark contrast to the symbol pushing, mechanical implementation of procedures and tolerance of absurd solutions that characterised school mathematical performance (Khan, 2004; Carraher, Carraher & Schliemann, 1985; Saxe, 1988a & b). One of the outcomes of such findings was to develop school mathematical tasks that were framed in more authentic real world contexts (Nasir, Hand & Taylor, 2008). Contextualising school mathematics problems was seen as a way to support meaning making, and increasing complexity and depth of mathematics that students experience (Arcavi, 2002). The challenge however was to balance the details of context with the level of mathematical sophistication that the curriculum expects, and to grapple with the tendency for contextual details to distract learners from mathematical aspects. Subsequently, researchers also raised doubts about the usefulness of everyday mathematics for school learning by pointing to the very different ways in which mathematical knowledge is acquired within and outside of school, and the very nature of the enterprise of school mathematics (Carraher & Schliemann, 2002; Dowling, 1998). It was argued that the goals of both the domains are different. However, the present study seeks to deviate from this stance by arguing that the familiar everyday and work contexts entail potentially strong resource that can scaffold math learning in schools.

Table 2.1 (next page) summarises the distinction between school and everyday math that emerged in the literature. This table also serves to present a framework to distinguish out-of-school and school mathematics in the analysis of student responses to arithmetical tasks in the present study.

Table 2.1 Distinction between Out-of-school Math and School Math

Difference	Out-of-school Mathematics	School Mathematics
Cognitive features	-Based on shared cognition (Resnick, 1987) -Manipulations are carried out using quantities -Use of group work and division of labour (Resnick, 1987) -Use of tool manipulations	-Based on individual cognition (Resnick, 1987) -Manipulations are carried out using symbols (Resnick, 1987) -Individual, independent work -Use of pure mentation
Intended outcomes of learning	-Situation specific competencies	-Generalised learning, power of transfer (Resnick, 1987)
Difference in numeration/ procedures	-Oral -Use of multiple units and operations (Saxe, 1988) -Use of contextualised reasoning (Resnick, 1987) -Use of decompostion and repeated groupings (Carraher et al., 1987) -Use of convenient numbers (Carraher, et al., 1985)	-Written -Use of symbols -Use of formal reasoning -Use of formal algorithms taught in schools
Mechanisms of acquiring knowledge	-Communication, Sharing, Legitimate Peripheral Participation (Lave and Wenger, 1991) -Learning from one-another, Circulates in communication, Role of artefacts and language (Carraher et al., 1987)	-Knowledge acquisition and knowledge building is textbook based -Based on individual thinking, group-work is not always encouraged
Meta-cognitive awareness	-Meaningfulness, Confidence in procedures and obtained results (Nunes et. *al.* 1993; Saxe, 1988a) -Continuous monitoring ('where they are' in the middle of calculations) (Carraher et al., 1987)	-Heavy use of algorithms, lack of meaningfulness and relevance -Continuous monitoring usually not possible
Test of the acquired knowledge	-No formal examination -Tested by seniors/experts through observations	-Use of formal examinations, consisting of mostly written tests

2.1.2 Funds of Knowledge

It is widely seen that children in low-income conglomerations are often bound in social relationships and work practices from an early age and the broad features of their learning develop at their home as well as in their surroundings. Households and their surroundings contain resources of knowledge and cultural insights that anthropologists have termed as *funds of knowledge* (Gonzalez, Moll & Amanti, 2005; Moll, Amanti, Neff & Gonzalez, 1992; Velez-Ibanez & Greenberg, 2005). The “funds of knowledge” perspective brings to mathematics education research insights that are related to, but different from the perspectives embedded in studies of “culture and mathematics”. In contrast to restrictive and sometimes reified notions of “culture”, “funds of knowledge” emphasise the hybridity of cultures and the notion of “practice” as “what people do and what they say about what they do” (Gonzalez, 2005, p. 40). The perspective also opens up possibilities of teachers drawing on such funds of knowledge and relating it to the work of the classroom (Moll *et. al*, 1992).

Funds of knowledge (FoK) are acknowledged to be broad and diverse. They are embedded in networks of relationships that are often thick and multi-stranded, in the sense that one may be related to the same person in multiple ways, and that one may interact with the same person for different kinds of knowledge. In other words, FoK points to the diversity of contexts and settings from which knowledge is acquired. FoK are also connected and reciprocal. When they are not readily available within households, then they are drawn from outside the household from the networks in the community. The concept thus emphasises social inter-dependence. Further, from the funds of knowledge perspective, children in households are active participants, not passive by-standers.

The funds of knowledge perspective has been developed largely in studies among immigrant communities in the US. In the context of a developing society, in contrast to societies with advanced economies, school children from low socio-economic backgrounds often directly participate in work either within the household or in the neighbourhood. Such participation allows a closer integration of children into the social networks that generate funds of knowledge, and makes this knowledge present and available in the classroom. For the purposes of our study, the notion of “funds of knowledge” guides our understanding of the nature and extent of knowledge gained in out-of-school contexts, and therefore available within the community of the classroom. Such knowledge is closely tied to practice and to use in specific contexts. It may be hybridized in the sense of including elements of domain knowledge (i.e., formal mathematical knowledge) as well as contextual details. It emerges in and through interaction with members of the community. We use the notion of “funds of knowledge” as a guiding notion in analysing the work contexts that students are exposed to, and in illuminating the nature and extent of everyday mathematical knowledge available within the community of the classroom. We look at “funds of knowledge” as a resource pool that emerges from people's life experiences and is available to the members of the group which could be households, communities or neighbourhoods. In a situation where people frequently change jobs and look for better wages and possibilities, members of the household need to possess a wide range of complex knowledge and skills to cope and adapt with the changing circumstances and work contexts. Such a knowledge base becomes necessary to avoid reliance and dependence on experts or specialists, particularly in jobs that require maintenance of machines and equipment.

Socio-cultural studies in mathematics and science education have argued that cultural resources and *funds of knowledge* (Gonzalez, Andrade, Civil & Moll, 2001) of people from non dominant and underprivileged backgrounds are often not leveraged (Barton & Tan, 2009) in school teaching and learning practices. Neither is their knowledge from everyday life experience valorised (Abreu, 2008) and built upon in the classrooms nor is their identity acknowledged. Access to such school education that is seen as meaningful and relevant by the underprivileged communities and connected to their life settings has remained elusive. What is offered in schools at present is a structured educational package detached from most students' everyday life experiences, yet accepted as “legitimate” knowledge since it acts as the “gate-keeper” (Skovsmose, 2005) to different kinds of opportunities and future social well-being. The legitimacy and necessity of the “formal” school mathematics renders all other forms of mathematical knowledge not only insignificant but also ineffective. This “package” of formal school mathematics either repels or attracts people depending largely on their socio-economic status. In this backdrop, it is widely accepted that hierarchical social structure (for example, caste and class division in the Indian society) has bearings on academic achievements including mathematics learning (Kantha, 2009; Weiner, Burra & Bajpai, 2006).

2.1.3 Analysing learning through the lens of identity

In the recent years, in MER, use of the notion of identity has been operationalised with the hope of getting an analytic tool for investigating mathematics learning since it is widely believed that interpersonal and affective relationships have a bearing on learning (for example, Boaler & Greeno, 2000; Cobb, Gresalfi & Hodge, 2009). Learning has come to be seen through “activities, tasks, functions and understandings” as “part of broader system of relations … developed within social communities”. Learning is also viewed as involving “construction of identities” (Lave & Wenger, 1991, p. 53). Researchers have argued that learning evolves in terms of membership and participation in the communities of practice entailing a social network and with the sharing of *funds of knowledge*.

The role of students’ identities in the development of their learning has been examined through different lenses. Most of such studies in MER have focused on classroom pedagogy and students’ involvement in them. There are not many studies in MER that looked at the co-construction of students’ identities drawing from their experience in out-of-school contexts and whether or not such identities were merged or kept isolated during mathematics lessons in schools. The notion of identity is still evolving and there are not many studies in MER with analyses using this notion as a tool. The current trend of educational discourse on identity has been to replace the widely used motivational notions of beliefs and attitudes which were now seen as discourse-independent with the notion of identity (Sfard & Prusak, 2005). Some researchers have attempted to operationalize the notion of identity through the narratives of or about individuals (Heyd-Metzuyanim & Sfard, 2012). Affective factors like emotional hue were sought to be analysed by looking at the narratives of identifying and subjectifying the students themselves or between them and the teacher. Some researchers have explored the constitution of normative identity of learners of mathematics in classrooms, and have contrasted it with the notion of personal identity as signifying an individual learner’s extent of participation in the normative identity (Cobb, Gresalfi & Hodge, 2009)..

Nasir and Saxe (2003) have analysed how minority African-American students’ identities were shaped in the US classrooms by drawing from their ethnicities as well as academic compulsions. The construction and negotiation of the ethnic and academic identities, according to Nasir and Saxe, came across while such students managed the emerging tensions both in the contexts of everyday activities as well as in the classrooms. Instead of adopting the prevalent dominant psychological approaches by considering ethnic identity, school identity and school achievement as distinct variables, Nasir and Saxe used a multi-stranded framework and used correlational methods to determine the contribution of ethnic identity towards academic outcomes and formation of academic identity. Their study looked at how social interactions during (or otherwise) local playing activity (dominoes) mostly restricted to the ethnic minority African-American people in an educational institution, constrain or enable the negotiations between ethnic and academic identities in such marginalised practices. Drawing a parallel, we have explored negotiations and conflicts between learner identities formed in out-of-school and in school contexts in the course of a teaching design intervention.

Researchers have also raised the role of agency in students' identity formation. Boaler and Greeno (2000) have described agency as disciplinary and conceptual in nature, that emerges from participation in the academic discourse and the understanding gained from community practices. Boaler (2002) viewed the relation between knowledge, learning and practices as cyclic and interconnected. Agency that comes from practices and knowledge then determined the broad or narrow nature of learning. Taking a socio-political stance, Gutierrez (2013) argued that an individual's identity is formed by her doing and not her being. She saw learning and knowledge as "situated in social interaction" (p. 9) and a bearing of others' interpretations on shaping of one's identity.

Lave (1988) and Rogoff (1994) both have viewed learning as (transformation of) participation in socially situated practices. Students' involvement in the work-contexts within the communities of practice came through participation, acquistion and assimilation of hand skills by becoming a member of the group of doers of the task and knowers of the diverse processes. Participation and exposure to work-contexts shape participants' identities. But in mathematics classrooms, students' identities are also linked with what Harris (1997, cited in Civil & Andrade, 2002) refers to as social imposition of what mathematics is and what counts as mathematics. Millroy (1992) asked if we can deviate from our own experience of learning of mathematics and appreciate other forms of mathematics that may appear different to us. These researchers suggest that negotiation between different identities is a part of meaning making processes as students engage with the curriculum content in mathematics. Such negotiation reshapes students' identities in both the domains of work and school. Meaning making and identity in practices have been seen as co-constitutive (Lave

& Wenger, 1991). In addition, membership of the social communities supports constitution of one's identity. However, membership alone is not sufficient but as Wenger (1998) argues negotiated experience and different forms of competence entailed in such experience collectively translate into identities. Thus participation, competence and learning are the markers of one's identity in the domains of school and out-of-school settings.

2.1.4 Transfer of mathematical knowledge

Previous studies that have examined transfer of knowledge with respect to mathematical learning have investigated and analysed a broad range of learning situations and have applied different theoretical perspectives to understand transfer. In the recent studies fresh perspectives were applied for examining learning transfer by incorporating the influence of social interaction, use of artefacts and cultural practices (Lobato, Rhodehamel & Hohensee, 2012). Some studies have examined the development or non-development of knowledge and understanding during the process of transfer, tracing the shifts and mediating factors that influence transfer (Triantafillou & Potari, 2014). However, there have been conflicting views among researchers not only about the occurrence of transfer but also about the need to look at the transfer phenomenon and its contribution to theory of learning. Some researchers have argued that transfer phenomenon can be seen as "direct carrying over of procedures from one situation to the other", whereas others have argued that learning happens by making adjustments to knowledge and by reconciling conflicting interpretations (Carraher and Schliemann, 2002, p. 21). Following these criticisms, suggestions were made to broaden the outlook on transfer of learning. Researchers underscored the need to gather clear evidence that learners "rely upon former knowledge and experience" (p. 4) in diverse ways as they encounter new situations. However, researchers are still unclear about the factors that influence (facilitate or constrain) transfer of mathematical knowledge (Triantafillou & Potari, 2014). More prominent is the lack of literature about how school going middle graders transfer their mathematical knowledge gathered from work-contexts, everyday practices, and school.

Beginning with Thorndike's associationism, transfer of learning has been viewed as an application of previously learnt knowledge and/or strategies to new problem-solving situations or contents. Thorndike's work emphasised similarity between "identical elements" from the original learning situations to the target situations (Thorndike & Woodworth, 1901, as cited in Carraher & Schliemann, 2002a, p. 2). In the process however, structural transformation that the body of knowledge undergoes in both the learning situations had not been focused upon (Carraher & Schliemann, 2002). In later studies, Lave (1988) and Nunes, Schlieman and Carraher (1993) believed in the distinct situated character of mathematical learning. Greeno, Smith and Moore (1993) looked at transfer of learning as dependent on how tasks or activities get transformed in new situations as compared to the previous situations. Carraher and Schliemann (2002b) viewed transfer as a "carrying over" phenomenon in which learning from one task situation is deployed in another situation "once learners recognise the similarity between the situations" (p. 19). There were studies that looked at transfer as "continuities between activities" in the form of inter-penetration and combination of learning at different domains, say, at school and at workplaces (for example, Beach, 1992 as cited in Carraher & Schliemann, 2002b, p. 4). Such continuities were linked with the notions of "situated generalisations" or "situated abstraction" as suggested by Hoyles and Noss (1992, cited in Carraher & Schliemann, 2002b, p. 5) and Carraher, Nemirovsky and Schliemann (1995) respectively, which dealt with the ideas of inseparability and interpenetration of learning from previously learnt experience. Triantafillou & Potari (2014) adopted Radford's notion of "objectification" as a process of "noticing" something by individuals mediating the goals of the actions while generating composite units using "denary" numbers (base 10) in a physical material representation (telecommunication closets [TC]) by apprentice engineering students. They reported that those who could use their prior knowledge to modify the mathematical objects at hand reported "immediate" and "developmental" transfer whereas in other cases, students brought in their "personal dispositions" such as beliefs about formulas which "constrained their meaning-making processes" resulting in, as the researchers called, "non-developmental transfer" (p. 356).

Earlier debates on transfer of learning indicate a growing feeling within cognition researchers about too many instances of transfer failure (instances where transfer between the domains did not occur) and lack of evidence that can challenge Thorndike's assertion that transfer is rare and occurs only between two similar situations (Bransford & Schwartz, 2001). The transfer literature is based on claims and counterclaims that looked at different notions of transfer. Most debates came within the paradigm of "direct application" (DA) of learning to new problem situations (Bransford & Schwartz, 2001). The direct application (DA) paradigm was based on the notion of "initial learning followed by problem solving" often tested with individually administered tasks. Bransford and Schwartz (2001) however moved from direct application of knowledge to the “perspective of preparation for future learning”. This notion is in opposition to those that adopted by Lave, Anderson et al. or Greeno. Lave and Wenger (1991) and Greeno, Smith and Moore (1993) broadly claimed that learning is situated within tasks at hand and that knowledge is not transferable between different tasks. But, Anderson, Reder and Simon (1996) contested this claim by arguing that such claims are “sometimes inaccurate and exaggerated”, and the “implications drawn are mistaken” (p. 5, 6). Anderson et al.'s contention (ibid) was further challenged by Greeno's (1997) objections of the generality and presuppositions about the levels of analysis that Anderson et al. had adopted. Greeno argued that the counterclaims of Anderson et al. addressed different questions by focusing on “knowledge and contexts of performance” and not the “activities and situations in which activities occurred and learned” (p. 6) and therefore they answered wrong questions.

Table 2.2 below highlights the transfer notions that prominent researchers adopted. Claims for both successes and failures in achieving learning transfer came up due to inconsistencies prevalent in the way transfer was defined. It is pertinent therefore for educational researchers to revisit the definition and make claims that can be used for drawing larger pedagogic pointers for effective mathematics learning. Lave and Wenger (1991) looked at learning in the processes of co-participation as a situated activity, focusing on skill acquisition through engagement in tasks and claimed that situated perspective demonstrated that skills (or action) grounded in tasks often did not “generalise to school situations”. In contrast, Anderson et al. argued that closer analyses of the tasks were required to make tenable claims of non-transferability of learning (1996, p. 6) and demonstrated situations where learning transfer occurred across contexts by showing transfer of mathematical competence from classroom situations to laboratory situations.

Table 2.2: Notion of transfer used by prominent cognition researchers

Thorndike's definition	Whether people can apply their knowledge to new a problem or situation (1901, as cited in Bransford & Schwartz, 2001
Lave's definition	Transferring one's knowledge and skills from one problem-solving situation to another (1988)
Anderson et al.'s & Greeno's definition	Not explicitly defined, used prevalent notion of direct application

Bransford & Schwartz's definition	Moved from Direct application of knowledge (DA) to Preparation for future learning (PFL) (2001)

In this thesis (Chapter 5), it is argued that sticking to the rigid boundaries of DA paradigm could be one reason for many instances of transfer failure and that we need a broader perspective to look at the transfer phenomenon. The broader perspective on transfer can help address educators' concern for ensuring effective learning among students and enhance their ability to carry forward such learning. DA characterisations detect transfers with a “yes-no”, or “either-or” result and fail to indicate occurrences of partial transfer which can actually prepare ground for future learning. Transfer in everyday settings seldom leads to black and white conclusions. In this thesis, we have attempted to revisit the transfer problem and reject the previously held characterisation of transfer as only direct application (DA) and also Lave's claim of non-transferability of knowledge. Extending and partially revisiting Bransford and Schwartz' notions of transfer, we address the transfer problem by considering conceptual understanding as the central goal followed by learning to apply or relate to the underlying principles implicitly or explicitly.

Carraher and Schliemann (2002b) have suggested that arguments and counter-arguments about transfer indicate a situation of dilemma where refuting and endorsing the notion of transfer of learning becomes problematic where neither of the option is acceptable to the researchers. They in fact called for abandonment of viewing transfer as a theory of learning. However, it would be interesting to note from a pedagogic viewpoint what one can say about the features of transfer of mathematical knowledge drawing from both classrooms learning situations and workplaces and the nature of such learning. It is interesting for math educators to revisit the issue of transfer. In our analysis, we also examine the relation between learning transfer and identity formation by examining the students' responses and looking at their roles in the respective work-contexts.

2.1.5 Bringing together Out-of-school and School Math

Children from low-income families often experience difficulties in mathematics and are at the risk of failure at schools and drop out from schools (Sarama and Clements, 2009). Saxe, Guberman, Gearhart, Gelman, Massey and Rogoff (1987) point out that there is a pronounced knowledge gap among the children of different socio-economic groups. Some researchers, like Resnick (1987), have emphasised that knowledge gaps appear because of lack of connection between informal, intuitive knowledge and formal, school knowledge. It was therefore argued that mathematics learning becomes helpful for the students if the classroom teaching involves familiar contexts and methods. Hence, these studies indicate towards the need to build upon informal knowledge and also meet the equity demands for quality mathematics education for all children right from the early stage.

Several curricular documents have also highlighted the requirement of a pedagogical approach for bringing together students' out-of-school mathematical knowledge and school math learning. A prominent example is The National Council for Teachers of Mathematics's (NCTM) *Principles and Standards for School Mathematics* that has argued that a connecting instructional approach is necessary "because students learn by connecting new ideas to prior knowledge, teachers must understand what their students already know " (NCTM, 2000, p. 18). On a similar note, India's primary curricular document on school education, *The National Curriculum Framework – 2005* has emphasised building on the "conceptual elements" embedded in the "experience and prior knowledge" of the students for constructing new knowledge in the classroom (NCERT, 2006a, p. 8). Call for building such connections are not new and we find acknowledgement of the role of social interaction and use of artefacts in children's knowledge construction in Vygotsky's learning theory (1978) and Leont'ev's activity theory (1978). As discussed before, it is commonly observed that low-income neighbourhoods are often socially bound and the communities' *funds of knowledge* are socially available to the members. Children's access to such knowledge has also been noted in several research studies including the present one. It is therefore pertinent that any attempt to bring together both the forms of knowledge would look at the available resources to the students in the out-of-school contexts.

Initial studies of everyday or out-of-school mathematics gave rise to a positive outlook concerning its role in school learning. The last several decades have seen increased emphasis on meaning making in school mathematical learning. Research on out-of-school mathematics has similarly looked at instances of meaning making and reasoning as embedded in work-contexts (Carraher & Schliemann, 2002a; Nasir, Hand & Taylor, 2008). Carraher and Schliemann (2002a) have argued that it is the meaningfulness of tasks embedded in everyday contexts that makes everyday mathematics more powerful. It is in contrast to the mathematics learning that is common in formal setups like schools where the main focus remains on “mastering the techniques” even at the cost of “foregoing meaning” (Carraher & Schliemann, 2002a).

Some researchers (for example, Masingila *et al.*, 1996) have advocated that it is important to identify and capitalise on mathematics learned outside of school to enhance learning school mathematics. For doing this, it is necessary that students see a connection between the two forms of knowledge but many studies have indicated the opposite. For example, Resnick (1987) has argued that children often treat arithmetic class as “a setting in which to learn rules” and are discouraged from bringing knowledge acquired outside to the school (p. 16). This, she claims, is isolating schooling from “the rest of what children do”. Therefore, if school taught mathematics remains detached from the child's daily-life experiences then it becomes difficult for her to make connections between the two and make sense of the relevance of learning mathematics as a subject.

A few studies such as those by Masingila and DeSilva (2001) have explored how mathematically meaningful contexts can be used in order to learn mathematics in school. In this study the researchers examined school students' strategies and the use of mathematical ideas in a variety of out-of-school situations, for example, while playing games like soccer and miniature golf. The responses of students to tasks requiring determination of the angles of attack on soccer-balls or tasks centred around making a miniature golf-hole were analysed. The framework of their study integrated “realistic mathematics education” which is an instructional theory that builds formal mathematical knowledge on the foundation of “everyday mathematics”, with Saxe's (1992) “emergent goal framework” which is a learning theory and looks at the interplay between the four parameters (prior understandings, social interactions, activity structures, and conventions/ artefacts) that influence the emergent goals.

Mathematics educators would be interested in knowing whether students of a certain grade know a particular mathematics concept from their out-of-school experiences, whether their prior knowledge helps them in gaining new knowledge in the classroom and further builds upon their knowledge outside school and whether classroom teaching for the novices and those involved in out-of-school economic activities should be different. Also of interest are the questions about the knowledge organisation in everyday mathematics, for example, its forms of representations, its power to arrive at solutions, and how they differ from the school mathematics of the children having knowledge of everyday mathematics when they come to the classrooms.

The prevalence of out-of-school mathematics in the culture is thus a potential resource that could make school mathematics learning more effective. There are not many studies on how this can in fact be realised. The studies described above focusing on children have largely studied the differences between the mathematical practices of out-of-school children and school children, studying them as separate groups. Some studies have presented problems familiar to one group to the other group and have studied their responses: for example, presenting school mathematical problems to out-of-school children, which elicited responses completely different from problems in everyday settings. There are also reports about secondary students solving proportion problems using their everyday knowledge and not the school procedures which they did not learn well and quickly forgot (Nunes et al., 1993). These findings suggest that bringing together out-of-school mathematical knowledge and school mathematics in a productive manner is not a trivial or easy problem.

Subsequently, researchers also raised doubts about the usefulness of everyday mathematics for school learning by pointing to the very different ways in which mathematical knowledge is acquired within and outside of school, and the very nature of the enterprise of school mathematics (Carraher & Schliemann, 2002a, Dowling, 1998). The goals of both the domains, it was pointed out, are different. Such criticism of efforts to use the insights from out of school mathematics in the school contexts usually assume that a clear separation exists between out-of-school and school knowledge, framing this separation in terms of the distinction between culture and domain knowledge. Researchers adopting cultural perspectives have however, pointed out that mathematical activity is always embedded in culture, and school mathematics is one cultural practice of mathematics among others (de Abreu, 2008). Culture is not a fixed and static concept, but is hybrid and dynamic. Thus more recent approaches to school mathematics education that take into account out-of-school knowledge and practice do not deny the distinctiveness of these two forms, but seek to blur the boundary between culture and domain knowledge and allow multiple points of connection to form a hybrid culture. School education and mathematics teaching is “not only about building on what students are familiar with... but also about introducing new ideas, concepts and sensibilities” (Nasir, Hand & Taylor, 2008, p. 220). The FoK perspective as discussed above, advances a more fluid conception of knowledge that opens up the potential of out-of-school knowledge for school knowledge and a richer vision of how the two are connected.

The MER community focusing on everyday mathematics and academic mathematics have evaluated the earlier attempts of bringing together the two domains and addressed the already existing “tensions” and polarisation of the two domains of knowledge. A JRME Monograph (Brenner & Moschkovich, 2002) on everyday mathematics and academic mathematics compiled such attempts in the form of a set of research findings which sought to evaluate “teachers' effectiveness in providing opportunities for their students to use everyday and informal knowledge” and empowering students to make the connections between two domains of knowledge (p. ix). Researchers have called for precise use of contextualisation as a means to ensure meaning making which was increasingly being seen as an important tool for effective mathematics instruction (Brenner, 2002; Guberman, 2004). Contextualising school mathematics problems was also seen as a way to support meaning making, and increasing complexity and depth of mathematics that students experience (Arcavi, 2002).

The last two decades have seen increasing attempts in MER in framing instructional tools and methods that build on children's understanding and learning from their exposure in the out-of-school settings (for example, Brenner, 1998; Civil, 2002; Masingila, Davidenko & Prus-Wisniowska, 1996; Taylor, 2009, 2012; and others). Some researchers thought that despite the inherent differences in the ways mathematics was learnt in school and out-of-school contexts, there existed complementary dimensions between the two kinds of math knowledge (Masingila, Davidenko & Prus-Wisniowska, 1996). Researchers have shown concern about the need to examine the “opportunities and supports” available for the teachers to bring together out-of-school and school mathematics and the ways to use those supports for effective teacher instruction (Taylor, 2012, p. 273). There are a few studies in MER that looked at ways to enrich classroom pedagogy by building on students’ out-of-school mathematical knowledge. However, there are not many studies that attempted to connect out-of-school mathematics with school math in a particular topic domain and also looked at the implications on such connections of the interplay between students’ identities.

Researchers have adopted different methods of using out-of-school knowledge as a resource for classroom math teaching. Civil (2002) worked with parents and enlisted their help directly in classroom learning as "intellectual resources" from a "dialogic learning perspective" (p. 133). They framed their effort as a "two-way dialogue" drawing on different forms of discourses in both the forms of knowledge (school and out-of-school). Masingila on the other hand, looked for the elements of school mathematics that middle graders drew on in their out-of-school activity of play. Her study showed that concrete cues such as games (in this case, making of mini-golf course) require use of mathematical knowledge and make powerful links to building the desired connections. Neither of these studies explored how students' identities are shaped through their exposure and interaction in the out-of-school settings and how such identities can help build the connections between school and out-of-school learning.

Some researchers like Knijnik (1993) and Civil (2002) have expressed concerns that descriptions of funds of knowledge or calls to integrate them with school learning do not necessarily ameliorate the prevailing conditions or help in the "process of social change" (p. 146). We take the view however that education is concerned with projecting futures for learners and believe that school mathematics education, as education in general, has a role to play in securing such futures.

2.2 Debates on the relation between work and education – The Indian context

Educational thinkers in the developing world, and particularly in India, have recognized the value of work experience for education conceived in a broad sense. Policy documents on education have taken on board this insight. Educational philosophers, such as Gandhi, thought of productive work as central to education, and developed a vision of education centred around work. Gandhi emphasised that modern education centred around work is different from the traditional education in the crafts. The aim of his educational philosophy of *Basic Education* or *Nai Taleem* was not training in a particular craft, but a well rounded education of the mind, the body and the heart (Fagg, 2002). At an education conference in India in 1937, Gandhi argued that "the proposition of imparting the whole of education through the medium of trades (crafts) was not considered [in earlier days]. A trade (craft) was taught only from the standpoint of a trade (craft). We aim at *developing the intellect also with the aid of a trade or a handicraft...* we may… educate the children entirely through them" (quoted in NCERT, 2007, p. 4, italics as in the quote).

In the present Indian context, this perspective has had an influence on the new National Curriculum Framework (NCF) formulated in 2005. The NCF 2005 urges educators to draw on work experiences as a resource for learning. It points out that “productive work can become an effective pedagogic medium for (a) connecting classroom knowledge to the life experiences of children; (b) allowing children from marginalised sections of society, having knowledge and skills related to work, to gain a definite edge and respect among their peers from privileged sections; and (c) facilitating a growing appreciation of cumulative human experience, knowledge and theories by building rationally upon the contextual experiences” (NCERT, 2005, p. 6).

Studies of out-of-school mathematical knowledge in the Indian context have highlighted that different procedures and strategies adopted in work-contexts were often governed by the situation-specific requirements depending upon the diversity of goods handled and the requirement of varied calculations. Many of these empirical studies have documented the use of mathematics in everyday practices and different strategies adopted to solve problems and compared them with school students' techniques. The methodologies adopted in these studies were comparative methods to compare the strategies of the school students and participants at their workplaces. Farida Khan's study (2004) highlighted that diversity of goods handled helped the vendors acquire and greater proficiency skills. She compared the strategies used by three different groups of children: school students, *paan* (betel leaf) sellers and newspaper vendors belonging to the same age-group and concluded that the vendors had better understanding of mathematical principles and computations than school students although the vendors were constrained by the lack of formal mathematical knowledge. Nirmala Naresh in her study of mathematical strategies used by bus conductors in Chennai found their problem solving strategies used quick mental schema (Naresh & Chahine, 2013, p. 327). Sitabkhan (2009) in her study on child vendors in Mumbai's suburban trains found vendors' mathematical problem-solving strategies were distinctly different from the regular school learnt procedures and depended on the diverse goods the vendors sold. She compared her sample with the students (of the same age) from the kind of schools which the vendors would have attended to had they been going to schools. She found distinction between the solution strategies of the unschooled vendors and school children. Vendors commonly used convenient value strategies. Similar account of strategy-use was observed in Bhadke's study (2011) with adult vegetable vendors in a small Indian town. These vendors although had no formal learning of mathematics but showed use of accurate mental computations using common strategies like decomposition, use of convenient grouping, proportions and so on. All these studies in the Indian context had primary focus on arithmetical computations and explored problem-solving strategies in different sites.

There have been a few studies in India that looked at mathematics used in work-contexts and illustrated development and use of measurement knowledge. Mukhopadhyay (2013) and Saraswathi (1989) in their respective work on boat making and agricultural labour, emphasised that spatial visualisation and estimation skills often shaped the measurement knowledge and proportional reasoning in work-contexts (discussed in detail in Section 2.8).

The studies described above have addressed the diverse ways in which work-related experience creates opportunities for mathematical learning. To our knowledge, although big cities like Mumbai have a large population living in low-income settlements (slums), which are often economically active centres of house-hold based micro-enterprise, there are no studies focused on children's out-of-school knowledge of mathematics in a particular locality that is a hub of economic activity. Further, though the studies referred to above underlined that diversity of work-contexts creates affordances for innovating newer, context specific problem-solving strategies, they have not explicitly addressed the pedagogical implications of out-of-school knowledge. In this study, we seek to explore the contours of out-of-school knowledge of children immersed in economic activity in an urban location and its potential implications for school learning of mathematics.

2.2.1 Role of Work and Education in Gandhi's *Nai Taleem*

In many important educational experiments in India right from the era of struggle for independence from the British rule, social concerns, indigenous traditions of learning or knowledge formation and equity issues have been addressed in many different ways (Bose & Kantha, 2014). The most notable example was the scheme of "*Nai Taleem*" or "New Education"[1] designed under the inspiration of Gandhi (1951). Under Nai Taleem, education, including learning of mathematics, was to be given through the medium of crafts, which implied productive work, since Gandhi was advocating a self-supporting system of education (NCERT 2007). However, this must not be equated with child labour (discussed

1 The idea of 'New or Basic Education' was formulated under Gandhi's inspiration and guidance at an Educational Conference in Wardha, India, in 1937.

in the next section), since work in Gandhi’s view is a means of education in the broadest sense of the term, including character-building and preparation for life. On similar lines, the eminent educationist Tagore’s vision of education was more culture-oriented, and in his social action and Sriniketan experiment he proposed a system of education related with life and society focusing on education for developing individual self-reliance (Dasgupta, 2008). Nai Taleem and Sriniketan were educational experiments with distinct philosophical perspectives and a clearly designed pedagogy drawing on cultural resources and community practices which often entailed work practices as part of community resources or *funds of knowledge* available to the community members and called for building self-reliance by supporting work practices.

Gandhi’s advocacy was for “new education” that was “deeply rooted in culture and in student’s life” (Gandhi, 1953, p. 28). He favoured integration of school education with “useful manual vocation” (Gandhi, 1927, p. 280). Gandhi in his book “*An autobiography or the story of my experiments with truth*” (1927), published before he presented his formulation of *Nai Taleem* (which came later in 1937), has presented a sketch of his experiments with children’s basic education. In his *Tolstoy Farm*, children's basic education was clubbed with manual vocation such as gardening (“digging pits, felling timber and lifting loads”, p. 279), which according to him, subsumed physical exercise and games. Almost all youngsters knew cooking as well. He did not believe in the “existing system of education” and was firm in his resolve to “find out by experience and experiment the true system” (Gandhi, 1953, p. 14). *Tolstoy Farm* was his experimental system. Mr Kallenbach, who helped Gandhi in this *farm,* had learnt shoe-making and carpentry, and Gandhi learnt the art and they had “a small class in carpenty” in the *farm* (p. 16). Gandhi insisted that he did not feel the need for textbooks and did not use the ones that were available. He accorded importance to imparting of knowledge and skills. He envisioned education through practice or work, which was a means of connecting with life.

The participation of children in work is a complex issue in today’s world in a developing country context. It is enmeshed in questions about the notion of childhood, the role of education and the exploitation of children. In India, debates about child labour as a form of exploitation are a central part of the debate on the right to education. While it is undeniably the case that many children suffer economic and other forms of exploitation, it is important to recognize that conceptions of childhood can be different for different cultures and for different communities (NCERT, 2007; Vasanta, 2004). In particular, for children from low socio-economic background, work is a part of the experience of childhood and a site for learning. We feel that school education should not drive a wedge between such experiences and classroom learning, as is often the case. The recommendations of the NCF, that the knowledge children gain from work contexts should be seen as a means of connecting school learning with out-of-school experience, are hence an important corrective to the "bookish" knowledge dispensed in schools in India.

Post-independence, despite considerable success of the *basic schools* in connecting school education through the use of crafts, subsequent education policies did not accord much importance to Gandhi's idea of *Nai Taleem* (Kumar, 2008). Noted educationist Krishna Kumar (2008) has argued that handicrafts (which also subsumes the vast and diverse micro-enterprise linked work-contexts) signify India's cultural plurality.

> They signify the integration of work and values, in a context which recognizes the presence of the artist in every human being … we ought to remember that traditionally the artisan was an ordinary member of the village community. Indeed the practice of a craft was an aspect of ordinary life, and a craft product was meant to be used in the course of everyday living."(Kumar, 2008, p. 99).

Education, according to Kumar, "represents a space where a society can regenerate itself if it uses the space judiciously – the heart of education is reflection in the course of relating" (p. 100). Unfortunately, India's educational system continues to drive a wedge between literary and intellectual practices or learning on the one hand, and crafts, manual work and dexterity on the other (Kumar, 2008). Such segregation not only has a damaging effect on creativity, self-confidence, and entrepreneurship among the young, it also alienates and presents an educational package seen as meaningless to a large population. Gandhi's idea of *Nai Taleem* which charted a new kind of early and basic education, different from the colonial staple of school education, was aimed at narrowing this segregation by connecting work and education. The following quote from Kumar summarizes the argument:

> "There is no reason why we cannot revisit Gandhi's idea of introducing crafts into school curriculum, not as an extra-curricular activity, but rather as an experience which will give greater meaning and depth to the rest of the

> curriculum. If we think about this matter afresh and work on it with imagination and hindsight, we might reform the system of education in a manner which only crafts can help us reform it, and in the process, we might also provide to our heritage of crafts a major institutional space where new designs, techniques, relationships, and visions can flourish. Like much else in a caste-ridden social order, both the knowledge and skill aspects of crafts have suffered from the effects of isolation and stagnation. Linking formal education with crafts could help foster creativity in both". (p. 101)

2.2.2 Child labour in India: Debates on "*needs versus rights*"

Sociologists often suggest primacy of family's sustenance capacity as a determinant of child labour while others question the validity of rendering child labour illegal under such circumstances where sending children at an early age for work arises out of "household decision-making" mainly for "reasons of survival" (Basu & Van, 1998, p. 413; Nambissan & Rao, 2013). In some communities, particularly in Mumbai's low-income settlement where this study was conducted, children's engagement in work practices are seen as a part of childhood and growing up much in the manner articulated by Vasanta (2004). Vasanta emphasised that in certain life-worlds child labour cannot be equated with the work practices that children are engaged in, rather the diversity of childhood practices in such communities is valued. In some practices, even children of low to moderate income families, who otherwise do not need to work to supplement their family income, are seen to be working or assisting their family elders, as we observed in our study.. Children's engagement in work is primarily seen as a way to pick up skills thought to be useful in future by drawing on the "funds of knowledge" present in the community. As discussed earlier, India's current educational policy calls for "institutionalising the pedagogic role of work in education" and considers the funds of knowledge of the "vast productive sections of society as a powerful means to transform the education system (NCERT, 2007; p. iii). Eminent educationists have raised concerns about the "alien symbolic forms and values" prevalent in the current pedagogic practices (Kumar, 2006, p. 4033). Kumar has argued that such forms and values have metamorphosed communities' tacit practices of transferring (funds of) knowledge "from generation to generation" into adopting the currently available explicit knowledge for the sake of survival.

It is well regarded that protection of children's rights for their survival and development is required since childhood needs “special care and assistance” (Nambissan, 2003, p. 1). Activism towards the need of children's welfare and well being has therefore assumed significance across the world. Ensuring children's “best interests” is seen as the primary condition in any resolution taken in this direction. Quality education during the formative years of growth is recognised as crucial for the intellectual development of the children. Therefore, any attempt taken towards ensuring and protecting children's rights also involves availability of quality education to them. Historically, in many countries, making education compulsory till a certain age was seen as a way to curb violation of children's rights which happens through various means, prominent among them are: child labour that may be voluntary or forced, child trafficking, bonded labour, some forms of slavery, and so on (Bissell, 2003; Burra, 1995). In India, activism towards curbing child rights violation got a boost with the adoption of the “Right of a child to free and compulsory education” (RTE) as a fundamental right through a constitutional amendment in 2002, later promulgated as an act by the Parliament in August 2009 and which subsequently came into force in April 2010.

According to many sociologists, anthropologists, economists and child rights activists, a huge population of children in India is still forced to work right from an early age at the cost of their education and holistic development (for example, Basu, 2006; Burra, 1995; Nambissan & Rao, 2013; Weiner, 1991; Weiner, Burra & Bajpai, 2006; etc.). Non-enrolment, discontinuation and early drop out from schools remain maximum among children from low income families (NSSO, n.d.; PROBE, 1999). According to the 66th round of survey of National Sample Survey Organisation (NSSO) conducted in 2009-10 under the Ministry of Statistics and Programme Implementation, Govt. of India, there is an estimate of 49.84 lakh children engaged in child labour in the age-group of 5-14 years in the country which is down from 90.75 lakh recorded in 2004-05 – a drop or around 45% in 5 years (NSSO, n.d.). India's census 2011 data showed that the population of children in the same age-group is 25.31 crores or 253.16 million (Census India, n.d.) which implies that about 2% of the total children in the age-group of 5-14 years, that is, about 50 lakh or 5 million children were engaged as child labour in India.

2.2.3 What is child labour?

There have been several international treaties, declarations and documents over history that have outlined the Child's Rights. The adoption of an integrated framework for ameliorating children's living conditions by the United Nation's Convention on the Rights of the Child (UNCRC) is the first legally binding international instrument incorporating a full range of protective rights of the children and is currently in practice. This human rights treaty came into force in September 1990 and makes it obligatory for all the participating national countries towards protecting and ensuring children's rights and makes them accountable before the international community. India became a participating nation in 1992 and committed “to take measures to progressively implement the provisions” (United Nations Treaty Collection, n.d.)

The UN policy defines a child as “a human being below the age of 18 years unless, under the law applicable to the child, majority is attained earlier” (United Nations International Children’s Emergency Fund [UNICEF], 2002). Child labour, according to the UNCRC, is a work practice that engages children below the age of 18 years and who are involved in activities that detract them from leisure, play and education. The International Labour Organisation (ILO) has defined child labour as “work that deprives children of their access to education and the acquisition of skills, and which is performed under deplorable conditions harmful to their health and their development” (Burra, 2005, p. 5199).

The Govt. of India's Ministry of Statistics and Programme Implementation (MoSPI) defines child labour as the “practice of engaging children in economic activity, on part-time or full-time basis” (MoSPI, 2012). By this interpretation, a child employed in income-generating practices whether earning a wage or not, who is below the minimum age of working as may be considered under the act of law, is deemed as a child labourer. The minimum age of working varies in different countries. In India, the “Child Labour (Prohibition and Regulation) Act 1986” that is currently in practice, defines a child as “a person who has not completed his fourteenth year of age” (Act no. 61, 1986), while South Africa after coming out of the apartheid regime has kept 15 years as minimum working age for all groups of children. ILO's Convention No. 138 accepted the minimum age for participating in economic activities as fifteen years under normal circumstances (Basu, 1999, p. 1085). However, India, having ratified the UNCRC (discussed above), stands obligated to raise the minimum working age from 14 years to 18 years.

The regulations in force at present in India do not prohibit child labour outright. Currently in India, children under the age of 14 are prohibited from employment in "hazardous occupations and processes" while their conditions of work in non-hazardous occupations and processes are merely regulated. This means that at present in India, a child below the age of 14 years can work in regulated non-hazardous occupations and processes. Moreover, there is no specific minimum age below which a child cannot be engaged in any kind of work. The proposed amendments to the existing "Child Labour (Prohibition and Regulation) Act 1986" include raising the age of prohibition for employment of children and adolescents in hazardous occupations from 14 to 18 years.

There are other provisions that have been invoked to curtail or call for a ban on child labour. The Indian constitution makes a special provision for the protection of children in Article 24 which reads as, "no child below the age of fourteen years, shall be employed to work in any factory or mine or engaged in any other hazardous employment" (Basu, 2008). Eminent jurist Durga Das Basu interprets this Article by claiming that the prohibition imposed herein is "absolute and does not provide any exception for the employment of a child..." (p. 118). The Indian constitution has also laid down provisions to prevent "exploitation of the weaker sections of the society by unscrupulous individuals or even by the state" in Article 23 as "Right against Exploitation" (p. 118). Furthermore, Article 39 of the constitution mandates every state to frame policies that can ensure protection of children from abuses and from entering into avocations "unsuited to their age and strength" (Labour & Employment Ministry, Govt. of India, p. 5). Article 45 provides right to early childhood care and education to all children until they complete the age of six years while Article 15(3) empowers the State to make special provisions for children.

The recent "Right of children to free and compulsory education Act 2009" has a mandate of ensuring 8 years of free and compulsory elementary education for the children between 6-14 years of age in age appropriate classrooms in the vicinity of the child. The issue of child labour has often been framed within the "needs versus rights" debate over what is necessary – need of income for survival or right to education for better future prospects? Neera Burra (Kabeer *et al*., 2003) has emphasised the need for a blanket ban on child labour and therefore the affected families may be compensated to address their needs. Economist

Kaushik Basu, on the other hand, has suggested that the debate is not simple and the existence of many dimensions to the problem that needs to be taken care of (Basu, 2003). Basu’s concerns were more about the family’s economic conditions than looking at children's involvement in work practices as part of the communities’ socio-cultural practice. From the perspective of education itself, it is important to move beyond the frame of the “needs vs rights” debate. Some scholars viewing child work from an educational perspective, such as Talib (2003), have been emphatic in advocating schooling with a mix of work experiences.

Criticism of policies allowing children to work

The policy documents of the UNICEF recognize that the involvement of children in work falls along a broad spectrum from the exploitative and harmful practices of child labour to work which enhances skill and is beneficial (Unicef, 1997). Thus “child work” is a generic term that is considered to subsume all kinds of work that a child does (Raman, 2000). Child labour, on the other hand, is generally equated with exploitation and child abuse. Critics have pointed out that the avarice of entrepreneurs makes them seek cheap labour, which has a causal relationship with forced labour (Burra, 1995, 2005; Bissell, 2003). Child labour is seen as a major workforce that is used as domestic help in most Indian cities (Weiner, 1991; Weiner, Burra & Bajpai, 2006). Hence critics of child labour regulations have argued that no clear line can be drawn between work that is exploitative and work that is not. Burra (2005) claims that conventionally, child labour has been defined as all such work that involves children who are “economically active” and fall in the age-group of 5-14 years. “Economically active” stands for “work on a regular basis” for which the child receives remuneration. Child rights activists have pointed at the political use of the term “child work” to make the issue of child labour look smaller and reduce the problem size to make it manageable. Burra (2005) has argued that this way the definition of child labour is narrowed down and many children who are engaged in wage or non-wage earning economic practices or involved in managing household chores so that adults can take on more wage earning jobs become invisible to the government policies and their benefits. According to Mishra (2000), as per the 1991 Census, around 100 million children were out of school out of a total of 203 million children in the age-group of 5-14 years, whereas the official statistics described only 11.28 million as child labourers in the same age bracket (Childline, n.d.; Mishra, 2000 cited in Burra, 2005, p. 5199). Evidently, based on the above statistics, around

89 million children (5-14 years) who are out of school remain unaccounted from the purview of child labour as per the official statistics because of the narrow definition. Child rights activists therefore conclude that the Govt. figures are not accurate and hence the prevalence of child labour is much bigger than the estimate of around 50 lakh declared in the NSSO data of 2009-2010 (66th round of survey). Thus, activists have therefore argued that the distinction between child labour and child work is superficial and flawed. According to many activists against child labour like Burra, such narrow and unnecessarily technical interpretation of a statutory provision defeats the purpose of curbing child labour altogether.

In a major deviation from the child's rights activists' stand, India's Supreme Court in June 1997 while acknowledging "the structural roots" of child labour as a phenomenon has suggested to involve child labourer's family in finding a solution (Raman, 2000, p. 4058). This judgement by India's apex court has been landmark in the sense that it has not only acknowledged but has called for viewing family's decision in the ambit of social reality. In other words, engagement of children in work practices (wage or non-wage) needs to be seen in the light of the notion of childhood as is prevalent in the child's family or in her community. The present research study has contended that children's engagement in work practices is a purposive activity and a part of the community's cultural representation of childhood. The researcher while calling for banishment of all forms of exploitative and forceful work practices, calls for taking on board the social construction of childhood prevailing in the community while locating children's engagement in the work practices. Similar to Vasanta's (2004) and Takei's (2003) stance, the researcher emphasises that students' participation in work practices is an important part of the community's socialisation process and therefore to characterise such participation as "child labour" in connection to the Supreme Court's judgement and the above debates, is not tenable.

2.2.4 Notion of childhood

Some researchers have communicated that the issue of child labour or child work needs to be viewed in the light of the notion of childhood as is perceived by the community (Raman, 2000; Vasanta, 2004; Talib, 2003). Different communities view childhood with different expectation and requirement. Many a times, "child labour is premised on a fixed meaning of labour" (2003, p. 161), whereas, lives of children in many communities may be vastly different from the lives of middle class children. Therefore, any policy formulation that directs the place of children in society must be community specific. Vasanta (2004) points to how practices through which children from poorer homes acquire community knowledge may shape an identity very different from those of middle-class children.

> The vast amount of popular (local) knowledge that is so central to the constitution of social identity is rarely discussed as being an important component of childhood. For instance, Ferguson (2001) demonstrates effectively that the children of the poor develop popular knowledge through experience, observation and practice within a specific material and social milieu, and that this learning, because it is used and elaborated in concrete situations, seems more relevant to these children than school knowledge. It is this knowledge and the structures in which it is embedded that serve as interpretive frames through which children from marginalised backgrounds try to make sense of encounters, practices, school rituals, curriculum and authority. (p. 17)

One of the striking differences between middle class children and children of different disadvantaged groups and communities is that only the children in the latter cohort get involved in work. The present study shows that in low-income communities “learning hand skills” and “learning of work” may be seen as important components of a child’s growth and development. The premise underlying the call for a complete prohibition of child work carries a deficit perspective of the children and bears a notion of childhood that is vulnerable and which needs to be protected. Such a premise looks at children across the board with the same lens and assumes that children can only be consumers and not producers (Vasanta, 2004). The notion of global childhood in contrast to a competent childhood has always influenced the mainstream ideology of the childhood (Nieuwenhuys, 1999, cited in Vasanta, 2004).

Many ethnographic studies that sought to understand the connection between child’s work and education have pointed out children's work is often embedded in their community’s life structure and often a child is seen as part of an adult’s space in the everyday practices (Viruru, 2001; Takei, 1999 as cited in Vasanta, 2004). These studies have argued against characterising all work practices of such children as child labour, since such practices conform to the norms, structure and life world of the community they belong to. According to Vasanta,

> … one needs to study the lifestyles [of the working class] more closely in order to assess the impact of such [children's] work on schooling. …the life-world of the adult or the child [from the working class] has no relationship with that of the school, not even in terms of lessons taught in the school. On the other hand, children from middle-class, urban locations do see some aspects of their lives reflected in the textbooks used in the schools. (Vasanta, 2004, p. 19)

In our study, we came across varied instances of children's engagement in work practices. Some of the practices were indeed exploitative and were severely violative of child's rights – where the child did not go to school at all and was working (either for family or for others) full time. In some instances, the choice of leaving school to take up work was forced on a child due to the loss of an earning member in the family and consequent extreme poverty. At the same time, many adults in the family wanted children to acquire skills and experience through participation in work. From the perspective of the community, we need to view children's participation in work in terms of (i) space that children can create for themselves in the adult world, and (ii) value of gaining foundational learning through skill development. Such experience and requirement is seen in the community as over and above school education. Therefore, for educationists and curriculum thinkers, it comes as a challenge to formulate education centred around work. Talib (2003) articulates this challenge.

> … while the majority of policy propositions in ameliorating child labour through education remain restricted to the province of discursive resources, little attention is paid to the generation of alternative practical skills chosen as part of free choice. It needs to be clarified that practical skills in themselves may be as alienating. But the real challenge lies in transforming the purely discursive package of education into one where skilled work provides the foundation to the learning programme in an atmosphere of democratic choice. (p. 160)

Hence discussions about the relationship between work and children's education need to probe deeper into underlying notions of childhood. Childhood in Kumar's (2006) notion is an opportunity for socialisation into a way of life while for Vasanta (2004) there exists differences in the experience of childhood and the “sense of worth instilled in children” (p. 21). Families that have had exposure to education for multi-generations would possess a different outlook about the notion of childhood as compared to the families living in the low-income settlement that we interacted with. These families from the settlement, who did not have much acquaintance with school education, have all along been viewing childhood as a preparatory period for training in different avocations and also to slowly train them to take up adult responsibilities. For such families, the practice of child work or child labour is not about child’s exploitation or health hazards, but also to socialise children into the community and into adulthood. Blanket ban on all kinds of work practices that involve children below 14 years of age, will lead to the decline of cultural practices and avenues of learning from the opportunities created in the diverse work-contexts. It is of course imperative that the child’d right to education be fully protected and all forms of exploitation be ended. However, while it is essential to ameliorate the work conditions and ensuring of non-exploitative environment, it is imortant to value the prevalence of community’s *funds of knowledge* and students’ access to them and also connecting their formal education with resources drawn from the work practices.

3

The Study, Setting and Style

The Journey of a thousand miles begins with one step

– Lao Tzu

This chapter discusses the research study, the research questions, the setting (location, neighbourhood, school, workplaces), the community and the work practices, students' life-world and the style (research design and methodology) adopted for the study. The motivation for the study came from a prior study carried out by the researcher among low SES communities that explored their out-of-school knowledge of mathematics. This study as well as the researcher's informal interaction with students from economically poor localities suggested that they are immersed in economic activities to varying extents. Economically active low-income settlements dotted with micro enterprises are usually rich in the occurrence of work-contexts involving dealing with quantities of different kinds, the use of multiple units, problems involving proportions and a cluster of related mathematical concepts such as fractions, proportions, multiplicative reasoning, division and measurement. Some of the strategies used by children to solve proportion problems appear to arise spontaneously in the context of everyday mathematics (Nunes and Bryant, 1996). Initial observations suggested that even children who are in the primary and upper primary grades from low socio-economic backgrounds are often exposed to such contexts.

As discussed in the previous chapter, although the research literature examining out-of-school mathematical knowledge is vast, there are relatively few studies that explore the implications of such knowledge for school learning. Further, the existing literature does not throw light on the nature and extent of out-of-school knowledge of children in a single classroom, who are immersed in an environment rich with work related opportunities. Looking at implications for school teaching will also entail obtaining a picture of such knowledge as it relates to core topics in the school curriculum. The existing literature also does not inquire into children's identities as related to their participation in work practices, and how these might have a bearing on their identities as learners of mathematics in school.

3.1 The study

The research study explores the nature and extent of everyday mathematical knowledge possessed by middle grade school students living in an urban low-income settlement that has embedded in it a thriving micro-enterprise economy. Children living in this settlement either have exposure to the diverse work-contexts prevalent in their neighbourhood or participate in and contribute to the production and income generation right from an early age. In the course of exploration of the potentially rich opportunities available to the middle graders to gather everyday mathematical knowledge, the work-contexts have been characterised from a mathematics learning perspective. The objective of the study is to unpack and document the connections between students' mathematical knowledge, work practices and identity formation, and inquire into the implications of these connections for school learning.

3.2 Research questions

The main research objective of the study is to explore the implications of everyday mathematical knowledge prevalent among the low income students exposed to work contexts for learning school mathematics. This has been elaborated in the form of specific research questions as below.

Q.1 What is the nature and extent of out-of-school knowledge of mathematics prevalent among middle graders from urban, low SES backgrounds?

Q.2 What are the everyday contexts and situations in which school going children of 10-12 years of age have opportunity to gain and use mathematical knowledge?

Q.3 What are the overlap and differences between the out-of-school and school mathematical knowledge?

Q.4 In the topic of measurement specifically, what out-of-school knowledge do students gain and what are the implications for the school mathematics curriculum?

Q.5 How can mathematical knowledge gained from everyday and work-contexts be integrated with school learning so as to enhance students' conceptual understanding of mathematics?

3.3 The setting

3.3.1 Location of the study: *The neighbourhood*

The study was located in a large, densely populated low-income settlement in central Mumbai with high economic viability but where the residents are economically poor. This economically active low-income neighbourhood is spread over a 2 square kilometre area beside locations that fetch some of the highest property values (real estate) in the world (Campana, 2013). The population of the settlememt is estimated to be around one million which indicates to the high population density of the locality (Campana, 2013).

As a characteristic feature, the settlement has a vibrant economy in the form of micro and small enterprises dispersed among house-holds, which include manufacturing, trade and service units with high economic output. The entire neighbourhood generates huge employment opportunities and being an old and established settlement, this low-income neighbourhood attracts skilled and unskilled workers from all parts of India who come to the financial hub of Mumbai in search of livelihood. The immigrant unskilled workers find jobs in the workshops and some of them become apprentices in the small factories.

Generally the single-room, small and low-height dwellings are used for dual purposes – as workshops and as living room for the family and the workers. The settlement is thus a co-location of workplace and home for most of its residents. Practically every house-hold here is involved in income-generating work and children start taking part in them right from an early age. The settlement is multilingual and people from different ethnic, caste and language groups such as Hindi, Urdu, Bhojpuri, Gujarati, Marathi, Konkani, Tamil, Telugu coexist here. In recent years, there is a move to relocate the population from here for redevelopment work, which is a great source of concern among its residents (Patel & Arputham, 2007).

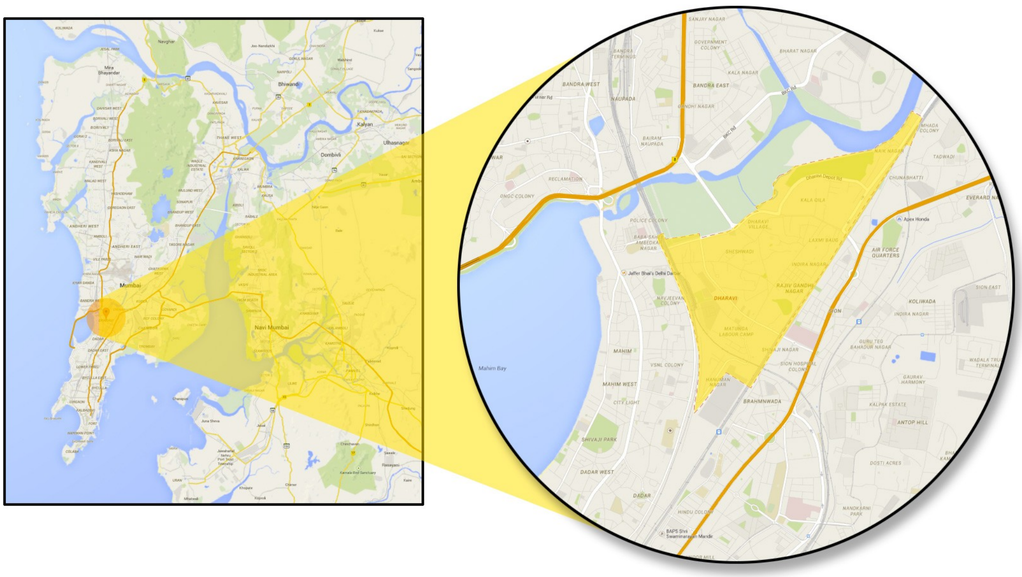


Fig. 3.1 Location of the low-income settlement in Mumbai city

The researcher observed that almost every child in the settlement is involved in house-hold based economic activities as well as in micro enterprise in the neighbourhood in a variety of ways. Common house-hold occupations include embroidery, *zari* (needle work with sequins), garment stitching, making plastic bags, leather goods (bags, wallets, purses, files, folders, belts, briefcases, waist and hand pouches, shoes), textile printing (dyeing), recycling work, pottery making, and so on. There are a few big and small tanneries in the neighbourhood where leather pieces are cleaned and processed. The goods produced in this locality are not only sold in Mumbai but also exported. There are many *bissi* – catering services where food is prepared and delivered to different nearby localities. Children are involved in delivering food (“tiffin”) boxes. Children living in the area get engaged in some of these economic activities from an early age. However, not only because of economic compulsions but seen in the community as a part of the childhood, children's upbringing and for learning purposes (discussed later), there are parents who do not let their children work until they finish their studies. But, even such children who do not participate in work develop a fair knowledge and reality perspective about the activities and diverse work-contexts around them by virtue of the high levels of social interaction prevalent in the neighbourhood. The researcher noted that in many cases, families (often along with the extended members) worked as a production unit where many family members were engaged in the house-hold based workshops or manufacturing units.

Fig. 3.2 The Neighbourhood (southern view)

Fig. 3.3 The Neighbourhood (northern view)

3.3.2 Work contexts within the house-hold: The fragmented tasks

The kinds of work discussed above are carried out in workshops where as a usual practice men are employed (except for “*bissi*” which is also popular among many families). However, there are other small pieces of work that are typically done by women and girls in their homes. Such house-hold based work is often fragmented and disconnected from the production network; it is perceived as less “technical”. Women in the neighbourhood take up such work as “side business” for supplementing family income, though there are families that completely depend on income from such work.

Examples of house-hold based work are making *rakhis* (decorative wrist bands exchanged as gifts during the Hindu festival of *Raksha bandhan*), fixing stones on ear-rings, pendants and buckles, making decorative door and wall hangings called *latkan*; removing thread from newly stitched garments (called “*fees* cutting”), and food production and delivery (“*bissi”).* In the case of these kinds of house-hold based work, excepting *bissi*, goods are delivered to the houses to be worked. Once the work is completed, the “contractor” or “middle-man” collects the goods and the payment is made. Such work may involve maintaining accounts of the goods produced and delivered and the payment made. Goods are typically measured in counts where sometimes older British units like dozen or gross are used. Stone fixing work for example, involves fixing definite numbers of decorative stones on ornaments such as earrings, pendants, buckles, etc. The worker requires to keep a count of the number of goods she has completed. Payment is based on the number of stones fixed as well as the numbers of pieces made.

Fig. 3.4 Making of bag-straps

Fig. 3.5 Pasting work in bag making

Fig. 3.6 *Zari* needle with sequins

Rakhi making is another example of work done in households that makes little demands on skills and knowledge. Here paper pieces cut to the size of the *rakhi* are used as the base on top of which the *rakhi* is glued. This paper base is cut using a template that comes with a sample *rakhi*. *Latkans* are decorative door hangings usually made by looking at a sample and following the sequence in which the sequins, bells and other objects are tied together on a thread of certain length. Lengths of the threads are taken by iterating the length of the sample design. The sequins and other decorative elements are counted and placed in precise order to maintain the sequence. Both *rakhi* and *latkan* are “piece-work” and the payment is based on the number of pieces made.

Another common house-hold based occupation is the thread removing work popularly called *fees cutting*. This work involves removing the extra threads from the newly-stitched garments. It is a routinised activity that does not demand skills or knowledge from the workers. Women and girls from an early age do this work. Payment is made per-piece of work which varies from 0.25 rupee (locally referred to as *char* anna or 4 anna, anna being an older unit no longer in use) to one rupee depending on the garment size. As mentioned before, some families or groups of people are involved in *bissi* or catering work which involves preparing and delivering lunch-boxes (called *tiffin-box*) to clients. Some *bissi* units are also eateries where workers from nearby workshops come and eat. During the time of interview[1], *bissi* units charged between Rs 220-250 per week per lunch box. Some students who were engaged in lunch box delivery work mentioned that they collect the money and maintain an account of the payments received. While *bissi* work is a family based occupation, the other kinds of work described here are solicited through contacts with the middlemen as well as with the nearby factory units. “*Bissi*” is also the term used for a fund deposit scheme where a group (specified number) of people enter into an agreement to deposit a fixed amount of money periodically (usually on a monthly basis) for a certain period of time. Based on a lottery system, by turn, every month one person from the group gets to keep the whole collected amount. This is a popular saving scheme among the residents of the settlement who claim that under this scheme they get access to a lump-sum amount at a regular interval.

1 The interviews were part of different phases of the study (discussed in the later sections). The whole study was conducted between 2010-2013.

3.3.3 Economic generation in the neighbourhood

Mumbai is the industrial and commercial centre of India and the low-income settlement is located in the geographical centre of the urban Mumbai agglomeration. The strong economic generation and output of this settlement, according to some estimates, contribute around one-third to the gross domestic product (GDP) of the entire Mumbai city (Sharma, 2000). The large-scale immigration and strong labour mobility seen in this economically active neighbourhood over the years is now witnessing increased inflow of immigrant workforce from other Indian states. Such labour mobility, according to some recent studies (for example, Pais, 2006), is attributed to post economic liberalisation in India (1991) when several big and small industries experienced unprecedented growth in terms of product output, product export and employment generation. Prominent among them was the leather industry. This low-income settlement is a hub for the manufacturing of leather accessories and leather products which provides employment to a large population living in the neighbourhood. However, Pais points out that the leather industry which is “a booming industry with a large growth in output, exports and employment, does not necessarily ensure enhanced quality of jobs” (p. 697). This may well be true of other sectors of production in the neighbourhood where the researcher observed similar “issues of fairness” which is discussed in Chapter 5 of this thesis.

This low-income settlement has garnered much attention particularly from the policy makers, media personnel, urban planners and researchers alike. There has been wide range of research focusing on a variety of issues emerging from the broader domains of social, economic, and educational viewpoints in the contexts of the low-income settlement. The researcher found that it is a common phenomenon to see foreign and local tourists/researchers visiting the place and taking photographs or writing notes of their observations. We noted that even children do not get amused by such visitors. The key informant (introduced in Section 3.3) on a few occasions pointed out sarcastically that “researchers working on the low-income settlement have produced more *kachda* (garbage) of their notes and papers than the actual garbage produced in the entire low-income settlement”. He often indicated that such research and studies could not bring much of a change in the lives of the people living in the settlement.

Nature of enterprises

Micro, small, medium enterprise (MSME) sector is a critical segment of the Indian economy. The low-income settlement's economy falls in the micro and small enterprises category. The Fourth All India Census of MSME conducted in 2008-09 highlighted the large employment opportunities that this sector generates and the significant contribution that this sector makes to the gross domestic product (GDP), manufacturing output and exports of the country. The enterprise in the unregistered service units is classified as –

- Micro enterprise, if investment in equipment does not exceed ten lakh (1 million) rupees;
- Small enterprise, if investment in equipment is more than ten lakh rupees but does not exceed two crore (20 million) rupees; or
- Medium enterprise, if investment in equipment is more than two crore but does not exceed five crore (50 million) rupees (p. 1).

The All India figure for the proportion of micro and small enterprises in the unregistered sector has been reported as 99.83% and 0.17% respectively (p. vii). The MSME sector also provides opportunities to people to create their entrepreneurial base. More than 94% of this sector fall under the unregistered enterprise category generating around 83.4% of the total employment in the MSME sector. More than 87% of both registered and unregistered MSME enterprises are self-financed or non-financed. The structure of the unregistered MSME sector is distributed more or less equally between rural (52.18%) and urban (47.82%) set ups with a major domination of the service units (73.85%) over manufacturing units (26.15%). While service units are much more in number in the unregistered sector unlike the registered sector, the unregistered manufacturing units generate around similar employment opportunities: around 22.4 million people work in the manufacturing units while around 27.8 million people in the service units (fourth Census, 2009, p. 29). This shows that the manufacturing units under the unregistered sector draws biggest proportion of labour than any other unit/sector. In addition to this, more than 95% of the enterprises under unregistered MSMEs are proprietary units – a characteristic that we observed in the low-income settlement in our study.

The participants (students) in our sample (discussed below) belonged to immigrant families living in low socio-economic conditions in the settlement. Most women members of the families residing in the settlement are engaged in some house-hold based micro enterprise in the locality, while their male counterparts either run their own workshops, or are employed in one; get work orders from middlemen and work at home or run a small business. In other words, nearly every adult (of any age group) in the settlement is engaged in some income-generating practice and children too start working (part time if attending schools) at a young age. We noted that some parents of students we came across had had no schooling, while some others had a few years of school education. There were only a few parents who had completed school. As mentioned before, the settlement consists of heterogeneous groups of residents belonging to different caste and ethnicity, language, and religion. But, the social networks in the settlement have developed a cultural system in terms of work practices which makes the settlement appear homogeneous at the outset.

3.3.4 Workers and working conditions

The working conditions and the plight of workers depend on the physical conditions of the work and workplaces, the terms of service, and the wages and payments (Pais, 2006). In relation to the physical conditions, we (the researcher and his colleagues) observed that most of the workplaces lacked illumination, had poor ventilation and were crowded working spaces. Many workplaces (workshops that we visited) used exhaust fans in lieu of proper ventilation but that was often not enough. Due to low-height ceilings, most workplaces do not have ceiling fans but use pedestal fans for air circulation. Mumbai being a coastal town generally remains warm and humid throughout the year. The working hours are long and vary between 10 and 12 hours on any normal work day and go beyond 16 hours when production demand soars. Most workplaces have a work schedule of six days a week with a weekly off on Sundays. Recycling work or collection of the recyclable materials are done on Sundays as well. In relation to service terms and conditions, we observed that no such conditions existed or were followed. There was no job security or severance pay, health, maternity or other kinds of benefits available to the workers. In fact, some students from our sample reported that their mothers lost the current job when they went on maternity leave while in some other cases, mothers of newly born babies had to report back to their work soon after the child delivery lest they were replaced by their employers (*seth*), and during their absence from home, elder siblings take care of the newly born children and attend to household chores (discussed later). We observed that wage is given either on a monthly basis or on a weekly basis. Borrowing money or taking advance from the monthly salary is a common occurrence and many workers that we came across did not have bank accounts. Being part of the unorganised sector, there was neither any workers' union nor was there existence of any safety and security regulation.

3.3.5 The House-holds and the neighbourhood

As mentioned before, there have been many research studies on this particular low-income settlement focusing on economic generation and poverty issues, deprivation and disadvantaged conditions of the residents, general housing and hygienic conditions, work practices and so on (Pais, 2006; Patel & Arputham, 2007; Sharma, 2000; Swaminathan, 1995). Most of these studies have shown the deficient living conditions and habitations, and inadequate availability of housing facilities, waste disposal mechanisms, access to clean water, non-degraded environment conditions and so on (for example, Swaminathan, 1995). However, we felt there is perseverance and grit among the people with whom we interacted while facing financial insecurity or while depending on the limited and scarce resources. The small tenements clustered together in narrow alleys are generally dark without sunlight and consist usually of two floors. There are tenements with three or four floors as well having one room in each floor. Along the lanes which are wider than the narrow alleys, the front rooms facing the lane or parts of them often serve as shops or make-shift places for selling goods. Most of these shops deal in stationary goods, grocery and other food items, meat, freshly cooked snacks, small eating joints called “restaurants” or “hotels”, shops selling mobile phones and their parts, dress materials for bulk selling (produced locally in the neighbourhood), fruits and so on. Hand-pulled carts are also seen on the lanes selling fruits, small stationary goods, ice-creams or juice-balls. The whole neighbourhood remains crowded through most of the day bustling with activities and with loud music. There are only a few open spaces in the densely populated settlement which “epitomises the crises of all fast-growing Indian cities, not just Mumbai” (Sharma, 2000, p. xvi). The only open spaces are cemented open platforms near a few mosques and temples, which are generally occupied by people or with vendors' carts. There are a few unmaintained grounds which according to our participants are occupied by “older kids” (adolescents) and are not accessible to younger children for play or other recreational activities.

3.3.6 The schools

For the purpose of the study, the researcher selected two government schools in the settlement run by the local civic body – Municipal Corporation of Greater Mumbai (MCGM). Following the key informant's suggestion and based on the convenience of the researcher's language proficiency, an English and an Urdu medium school were identified. The five-floor school building located in the middle of the low-income settlement has five different schools with different languages of instruction (viz. Telugu, Marathi, English, Tamil and Urdu) co-located in the same building each on a separate floor. Two more Urdu schools run in two long old barracks in the school compound. All these schools draw students from the neighbourhood. The schools which draw a big chunk of the student population, viz., English, Urdu and Marathi run in two shifts – morning and afternoon sessions for a duration of around five hours each, while Tamil and Telugu schools run only in the afternoon shift. For the sake of convenience, the researcher visited the morning-shift schools. All these schools run one or two divisions/sections (at times six or seven) per class based on the number of students enrolled in a grade. According to Teacher S (who teaches in the English school, henceforth, “Tr S”), in the early grades, the enrollment remains high. For example, Grade 1 generally has six to seven divisions which gradually go down to three to four divisions beyond Grade 4. Tr S elaborated that there is a high drop out after Grade 4 when students shift to other schools (mostly privately-run). Many schools run by the local civic body are up to Grade 4 (primary schools) while there are private schools from Grades 5 to 10. Because of the high drop out in the early grades, there are fewer divisions in the higher grades.

Fig. 3.7 School-building that houses five different language schools one on each floor

All the schools in the compound are from Grades 1 through 7. There is only one Urdu school in the same building which has Grades 8-10 but it is a separate school with a different set of administrative staff. Grade 10 is the school completion year in the Indian school education system up to which all the school-taught subjects are compulsory for every student. The streaming of subjects happens in Grade 11 when a student gets to opt for one from among the three streams of science, arts (humanities and social science) and commerce. The low-income settlement has other private schools, mostly English medium, which have grades up to 10. In addition, there are some privately-run Urdu and Tamil schools as well with grades up to 10. However, there is no government-run school in the settlement other than the above mentioned schools (researcher's field of study) that has Grades 8, 9 and 10. Therefore, students who complete Grade 7 from a government-run school in the neighbourhood look for private schools in the locality for taking admissions in the higher grades or opt for a government-run school outside the settlement. As the researcher gathered from conversations, this hurdle of changing schools coupled with financial constraints are the primary reasons behind high school drop out rate which is more common among girls from poor families who can neither afford a private school, nor can send their daughters to a faraway school. Recent data (national level rural India figure), indicates the extent of drop out – the number of children “not in school” which includes school drop outs and never enrolled children between the age-groups 11-14 years and 15-16 years from 5.1% to 16.8% (Annual Status of Education Report [ASER], 2013, p. 69). Similar figure for rural Maharashtra (state where the study was conducted) shows a jump from 2.3% to 9.1% between the same age-groups (p. 155). Although the the trend over years suggests a decline in the percentage of children “not in school”, the age-group beyond 14 years shows a large number of children opting out of studies. Although this is rural data, the scenario in urban low-income settlements is similar.

3.3.7 Significance of the location

The economically active low-income settlement is dotted with diverse work places and communities of work practice which are resource-rich for creating varied opportunities for school going children to gather everyday mathematical knowledge. There are many learning sites for children of this settlement apart from the regular schools, viz., house-holds engaged in work, diverse work-contexts, tuition classes, shopping and house-hold chores. People in this settlement maintain strong social connections and they are well networked with their employers, middle-men, distributors, shopkeepers, friends, relatives and significant others. From our interactions with the community members, it appeared that the social relationships are mostly economy driven. The entire neighbourhood creates opportunities that expose children living here to the *funds of knowledge* available within the community. There are many nodes on the social network through which one gathers knowledge. The resource-rich characteristics of this settlement make it significant for the study.

Tr S further explained that as a unique feature this settlement does not have any vocational training institutions or training classes for learning any kind of skill, rather the “hand skill” (*haath ka hunar*) and knowledge resource are available within the community which anyone willing to learn can visit, observe and pick up. Therefore, the entire neighbourhood provides an opportunity not just for income generation but also for such learning which is open for anybody without having any prerequisite of formal training or past experience whatsoever.

3.4 Style of the research study

The style of the study – methodology adopted, the research design, the choice of sample and its selection, the various phases of the study are discussed in this section.

3.4.1 Establishing access to the field

The larger ethnographic part of the research study done over a period of two years and a half, was conducted in phases. In the beginning, the researcher met the key informant (henceforth KI) who lives in the low-income settlement and had worked in a government aided middle school in the locality as a member of the administrative staff. This 74 year old man (age at the beginning of the study in the year 2010) is a community leader and well regarded for his social work in the settlement. He guided the researcher in getting access to the community, and in getting the necessary permissions from the Municipal Corporation, MCGM (local civic body). Based on KI’s suggestion, the researcher met the Education Officer, MCGM and sought permission for classroom lesson observation and for working with students and in two schools (one each of English and Urdu) in the settlememt. The researcher carried a letter seeking permission (see Appendix E) and also submitted a short proposal of the study. The Education Office soon issued a letter of permission and also directed the respective departments and the concerned school authorities about the study and the researcher’s visit.

On the first day of the school visit, KI accompanied the researcher to the school and introduced him to the English and the Urdu school authorities, which helped researcher to get permission from the Head Master/Mistress (HM) to enter into the classrooms and for classroom observation and interaction with the teachers. Thereafter, the researcher started visiting English and Urdu medium schools and began to observe mathematics lessons in Grades 5 and 7 of both the schools on a regular basis. Teachers S, K and D then the class-teachers of Grades 7, 6 and 5 respectively in the English school helped the researcher by talking to the respective teachers of the Grades which he wished to observe, introduced him to the teachers and to the students. During the first week, the other Grade 6 teacher of the English school was on leave and the class teacher of Grade 7 invited the researcher to his class. Similarly, the Grade 5 class teacher of the Urdu school gave access to the researcher to his class. The research study focused on the children of the age-group of 10-12 years who are generally in Grades 5 to 7. The researcher's earlier informal interaction with students from poor localities, suggested that they are immersed in economic activities to varying extents. They can relate to quantities of various kinds and are familiar with various units used in everyday transactions, and have an idea of the quantity specified by these units. Therefore, it was hypothesised that such knowledge can be used in the initial learning of the core concepts connected with multiplicative thinking and hence the choice of the topic-area and the grades at the outset.

During the lesson observation, the researcher would sit at the back of the classrooms, observe and take notes. Regular visits to the schools as a non-participant observer helped in building rapport with the authorities, teachers, students and other people on the staff. During school visits, the researcher informally discussed with students and teachers during the recess, at other free times or after school hours. After a month and a half of the regular school visit came the month-long Diwali and Eid vacation during which time the researcher started visiting other places in the settlement including students' homes and workplaces/workshops. Informal discussions with the students helped to get a broad picture of the nature of their daily activities that have aspects of mathematics and the nature and extent of their everyday mathematical knowledge. This helped in getting an initial understanding of the variation in children's out-of-school mathematical knowledge, opportunities available to gather such knowledge as well as the extent of their involvement in economic activities. The school visit and visit to the workshops continued for the next two years and a half.

3.4.2 Research design and methodology

In the beginning, an ethnographic qualitative research design was adopted to explore students' everyday mathematical knowledge and also to explore the work-contexts that they had exposure to. The student participants and other members of the community that the researcher interacted with had a shared pattern of language and belief system. Case studies methodology was subsequently adopted to enquire into the connections between work practices and opportunities available for gathering everyday mathematical knowledge. In the subsequent third phase, a teaching design experiment was implemented in the form of a 2-weeks long vacation course that the research team from HBCSE conducted for the sixth and seventh graders of the Urdu and English schools.

In this research study, the researcher did not become part of the observed field, but an approach was adopted to assimilate as a “non-participant observer” (at times as “observer as a participant”), an understanding of the insider's knowledge of the field (Creswell, 2013, p. 167; Flick, 2009, p. 222). Thus, the research study followed a blend of ethnographic, case studies and action research methodology in three broad overlapping phases. The first two phases of the study assessed research questions 1 to 4 and Phase III addresses research question 5.

Phase-I:

The first phase of the study was ethnographic in nature involving eclectic exploration of the children's life-world and opportunities available to them to gather everyday mathematical knowledge. The focus was on:

- building a rapport with the students, teachers and community members,
- observing formal classroom teaching learning processes and noting the pedagogic underpinnings,

- making exploratory visits to the field, viz., workshops, manufacturing units, students' home, shops, and so on.

Phase-I (ethnographic exploration) continued even when *Phase-II* (case studies) and *Phase-III* was on. Hence, *Phase-I* had an overlap with the other two phases and continued till the end.

Phase-II:

The second phase of the study was in the form of case studies. The sample for the study was drawn from two Grade 6 classes each from the English and the Urdu medium school. A representative sample of 31 students was chosen (every third student from the attendance register) to form the original sample which comprised of 16 students from the English school (11 boys and 6 girls) and 15 students from the Urdu school (7 boys and 8 girls). A sub-sample of 10 students, five each from English (3 boys + 2 girls) and Urdu school (2 boys + 3 girls) was chosen for a further round of interviews based on the varying extent of their engagement with work, everyday mathematical practices and also willingness to take part in the in-depth interviews/discussions. Seven additional students from the same grade of the Urdu school, who were keen to participate in the study were recruited for the interviews about work-contexts. These students had high exposure to work contexts and were all male students. No girl student, like those seven boys came up to talk about their work-context and therefore the additional sample had all boys. Hence, in depth interviews about work contexts were conducted with 17 students.

Why the age-group of 10-12 years?

The decision of choosing students of 10-12 years for the study sample was made since in this age-group in the corresponding grades, the mathematical course content goes through a number of transitions – viz., from integers to fractions, from arithmetic to algebra, and following the Piagetian stage theory of cognitive development, from concrete operational stage to formal operational stage when children learn to handle abstractions (Flavell, Miller & Miller, 2002). One can also expect that children of 10-12 years of age will be able to use multiplicative thinking and be able to connect their learning from different sources.

Furthermore, this is the beginning of teenage when children are given responsibilities during their work-contexts and they start building their own peer group. It is therefore a crucial transition phase in one's life.

Table 3.1 Sample size

	No. of students	English school		Urdu school	
		Boys	Girls	Boys	Girls
Original sample	31	11	5	7	8
Sub-sample (for work-context interviews)	10	3	2	2	3
Additional sample (for work-context interviews)	7	-	-	7	-

Data Collection in Phase II

Data for Phase II was collected through interviews in three parts. The first two parts were conducted together and the third part was conducted after a gap of one year:

- Part-I: data was collected through semi-structured interviews of 31 students to understand their family-background, socio-economic status, parental occupations, productive work done at home/elsewhere and student's involvement in them,
- Part-II: interviews of 30 students (1 student had left the school by then) based on a structured questionnaire to understand students' basic arithmetical knowledge,
- Part-III: semi-structured interviews of 17 students focusing on their knowledge about their work.

All the interviews were audio recorded with prior permission from the respondents, the school authorities and from the parents. The main data sources are the audio records, audio transcripts, field-notes, students' worksheets, photocopies of their “tuition” notebooks and photographs. Tables 4.1, 4.2 and 4.3 in Chapter 4 summarise the work-profile of the sample including the occupations of their parents.

The interviews in Part-I focused on information about students' family background, parental occupations, family's SES (socio-economic status), house-hold based work practices, students' outdoor, leisure activities and engagement in work, savings and their modes, and so on (See Appendices A and B). The interviews of Part-II on the arithmetical knowledge had items on “number knowledge”, “currency knowledge”, “count-on strategies (number enumeration)”, “computation using arithmetical operations” and “proportional reasoning” (See Appendix C). The first item in the “number knowledge” section was on reading the numbers from the number cards that were shown to the students one-by-one. The numbers (viz., 279, 607, 1010, 2303, 4800 and 10010) were written in standard numerals each on one number-card. Each student was asked to read these numbers and their answers were recorded. In every item the students were asked whether they were sure and whether they wanted to make any changes. The changes or corrections made were recorded. The second item was on writing the numbers called out by the researcher. These numbers were called out in English and Hindi/Urdu both and the students were given a paper-sheet to write down the numbers. In the “currency knowledge” task item, the students were given two boxes of currency notes and coins that are used in money games. For this task, three different numbers were called out (one each in hundreds, thousands and ten thousands) corresponding to each of which an equivalent amount of currency was to be chosen from those boxes. The numbers were called out in English as well as in Hindi/Urdu. In the “count on” task items number-cards were presented and the task was to read the number and count on. The next item was similar, but students had to first count the amount kept in an envelope and then count on beyond that number (the amount of money). The other items were contextualised word problems depicting everyday shopping scenario involving all the arithmetic operations and one task on proportional thinking.

Phase-III:

The third and final phase of the study involved a teaching intervention. This intervention involved a vacation course of 2 weeks duration in the form of a teaching design experiment. This course was conducted by the research team for sixth and seventh graders of English and Urdu medium students. The seventh graders included a few students from the sample and some non-sample students from the same grade. This phase of the study addressed Research Question 5. The objective of this phase was two-fold:

- to make connection between students' everyday math knowledge and school learning with a focus on length measurement,
- to explore ways in which students' knowledge and identity played out in a classroom setting.

Complete research design and adopted methods are summarised in Tables 3.2 & 3.3 below:

Table. 3.2 Different phases of the study

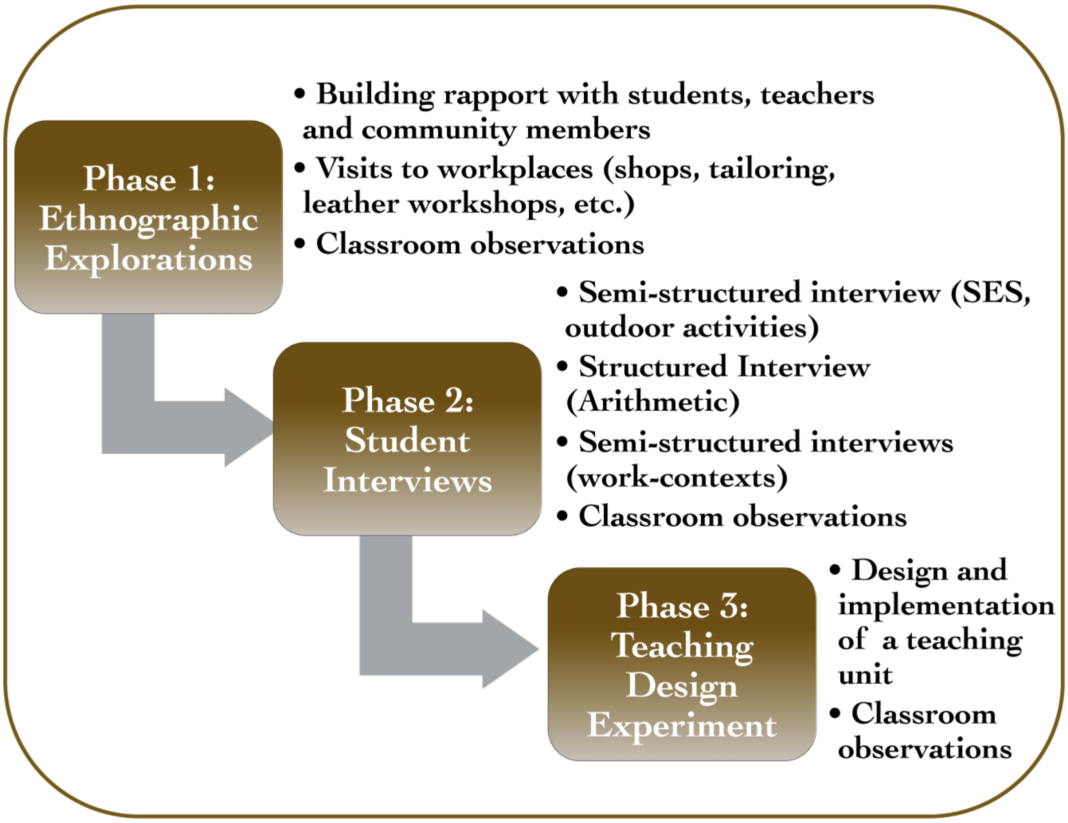


Table 3.3 Research Design & Methods

<table>
<tr><th colspan="2">Phase</th><th>Objective</th><th>Sample</th><th>Method</th><th>Data Sources</th></tr>
<tr><td colspan="2">I (Ethnographic Exploration)</td><td>Exploration of the children's life-world and opportunities to gather out-of-school mathematical knowledge</td><td>Classroom observation: Grade 5 (Urdu); Grade 7 (English)</td><td>Building a rapport with the students, teachers and community members; classroom observation; visits to field, workshops, students' home, shops.</td><td>Field notes

Photographs</td></tr>
<tr><td rowspan="3">II (Case-studies)</td><td>Part-I</td><td>To understand family background, SES, students' outdoor activities, engagement in work, parental occupation</td><td>Randomly selected representative sample of 31 students</td><td>Semi-structured interviews</td><td rowspan="3">Audio records

Transcripts

Students' worksheets

Photocopies of students' “tuition” notebooks</td></tr>
<tr><td>Part-II</td><td>To understand students' basic arithmetical knowledge</td><td>30 students from the previous sample</td><td>Interviews based on structured questionnaire</td></tr>
<tr><td>Part-III</td><td>To focus on students' knowledge about their work</td><td>Sub-sample of 10 students from the previous sample + 7 additional students who volunteered</td><td>Semi-structured interviews</td></tr>
<tr><td colspan="2">III (Teaching Design Experiment)</td><td>Connecting students' out-of-school math knowledge & school learning Exploring role of students' knowledge and identity formation</td><td>About 25 Grade 6 & 7 students of Urdu school</td><td>12-days' Teaching intervention camp</td><td>Classroom videos & logs

Transcripts (first 2 lessons)</td></tr>
</table>

3.5 Strategies and Instruments for Data Collection and analysis

3.5.1 (a) Data Collection

Classroom observation

In observing lessons of Grades 5 and 7 in the Urdu and English medium schools, the researcher typically continuously made notes of the lesson transaction and did not intervene in the classroom proceedings in any way. The researcher also built a rapport with the students as well as the teachers outside the lesson time noting whether students used their everyday mathematical knowledge during classroom interactions and problem solving, and teachers' use of their own knowledge and awareness of students' knowledge as resources during teaching.

Teaching as a participant observer

On such occasions when a teacher was on leave or not present in the classroom for any departmental work, the researcher was frequently requested by other teachers to engage some classes. These were the occasions when the researcher's role was that of an "observer as participant". Students often asked the researcher to play mathematical games and activities. The researcher complied with such requests and also discussed problems that generally came from some students. Students too came up with the topics that they wanted the researcher to teach. Such incidents gave the researcher opportunities to interact with the students in an actual classroom scenario which is rather rare to get. These short interactions provided the researcher with rich exposure of teaching, attending to students' queries, and experience to manage the classrooms and learn about the difficulties and challenges that teachers face every moment.

Visits and recording procedure

Regular engagement with the field for more than two years and a half helped the researcher build a personal rapport with the participants as well as with the community members. During this period, the researcher visited the school and the field (the community and workshops) on school days on a regular basis and sometimes on Sundays and holidays. Most visits lasted between 4-6 hours from early morning till past noon. In the last 10-12 months, the researcher visited the school and the field, three to four times a week.

During visits to a workshop for the first time, the researcher was usually accompanied by the student related to the work or known to the people in the workshop. Therefore, getting access to the workshops and interacting with the workers became easier. The researcher followed a protocol of “guiding questions” to initiate discussions about the work-contexts (see Appendix D), but no other recording protocol was used. Most discussions (other than the interviews) were one-to-one or one-to-many type without the use of any recording device or notebooks, as it was felt that any external source of data recording would cause distractions to the extent of putting off the respondents from interacting. The researcher made quick notes soon after coming out of the site. The detailed descriptive and reflective notes were taken after returning back in the afternoon. For the interviews and discussions with students (Phase II) audio recorders and recording protocols were used after taking written consent of the respondents (see Appendices F) and the teachers.

Exploratory interaction

The subsequent visits to the workshops and students' home were more like social visits and informal in nature. These interactions were exploratory in nature with an aim to unpack the mathematics embedded in the work practices. The objectives also included the exploration of the available *funds of knowledge* among the workers and the characteristics of the work-contexts. The objective was also to understand the different processes involved in a particular work and the required skills for different tasks and work practices. It was also important to understand the life-world of the people in the neighbourhood to understand the opportunities available to the students to gather out-of-school math knowledge and the available scope for them to use such knowledge.

The researcher's interaction with the community and parents was facilitated through attending the *School Management Committee* (SMC) meetings on invitation. Such meetings held annually or biannually are platforms where parents come together to raise their concerns and make plans for future welfare measures that SMC planned for the school authorities to undertake. These meetings gave opportunities to the researcher to not only interact with the parents and learn about their concerns or aspirations, but also engage with the issues. Above all, such social interactions helped the researcher build a long-term relationship with the community.

Students' interviews on SES & arithmetical tasks

The researcher's exploratory visits to the community, workshops and students' living places continued while his interaction with the students in the English and the Urdu schools increased and a rapport was built. It was soon that the researcher started getting questions from the students to attend to their mathematical queries. Students' questions, discussions with them, their problem-solving strategies helped the researcher and his mentor prepare the interview schedule and the items (Phase-II, Parts I & II). As discussed earlier, the researcher's previous visits to other low-income settlements had helped him develop a general idea about the out-of-school mathematical knowledge of students from such neighbourhoods – such pilot studies helped the researcher in narrowing down the topics for the interview on the arithmetical tasks.

Students' profiles presented in Chapter 4 draws data from the Phase-II, Part-I interviews of students' SES and family background. These data were collected through one-to-one interaction with the students following the semi-structured interview schedule. The interviews were audio recorded after taking the consent of the teachers and each student. The interview data-sheet (schedules with space) was later filled up by listening to the audio records and students' problem-solving strategies were described for analysis.

In-depth interview (work-context)

In-depth interviews of the students focused on their knowledge about their work-contexts including the skills required, stages of learning and processes involved, use of raw materials, different modes of measurement and units used, wages, decision making, and so on. These interviews helped in eliciting their meta-knowledge about the entire work process in which the student was involved or knew about. It helped the researcher get an idea of the work practice – who does which part (of a certain task/work), what is done outside, what do the adults do and what is given to the children, and so on. These interviews helped in locating the extent of mathematical knowledge that students get to use in different work-contexts and the opportunities created by such contexts to do so. The interviews helped in understanding the ways in which *funds of knowledge* become accessible to and are used by the community of workers as well as those who know about the work. The interviews lasted between 24 and 52 minutes and the transcribed interview data were coded for analysis (details of coding schemes and analysis discussed in the next section).

Teaching intervention

Phase-III of the study involved a teaching intervention following a teaching design experiment, duration of which was 12 days in the topic area of linear measurement. The broad objective of the intervention was to connect student's out-of-school mathematical knowledge with their school mathematical knowledge and math learning and also to explore the roles of their knowledge and identities in classroom learning. All the lessons were video-recorded after taking permission from the school authorities and also from the parents of the student-participants. Classroom logs were maintained separately. The classroom videos of all lessons and classroom logs were reviewed, and six lessons (first two days and the last four days) were transcribed and analysed.

3.5.2 (b) Data Analysis

The notes of classroom observations, notes taken of visits to the community or workplaces, of general visits to the school were reviewed from time to time to form a perspective that guided detailed analysis. The photocopies of the tuition class notebooks supplemented the data from classroom observation to provide a more complete picture of the formal learning that study participants were exposed to.

The data from the interviews of 30 students in Part I and II of Phase II was entered into data-sheets. Following this, a chart was prepared for each student highlighting the major points of her or his background and arithmetical knowledge. The charts were used to generate the reviews and analysis presented in Chapter 4.

All the students' work contexts interviews in Phase-II and five lessons from the teaching interaction (vacation camp) in Phase-III were fully transcribed for coding and further analysis. Written logs of the lessons in Phase-III supplemented the lesson-transcripts.

Coding scheme

The work context interview transcripts were coded at first and second levels to review what they indicated about the nature of work students are involved in, and what they know about aspects of the work. The coding scheme was developed by the researcher and his colleague and the whole interview transcripts were first divided into segments. A segment consisted of inter-connected utterances and segements ended when a new question, topic, issue began to be addressed. The unit of coding was a segment. Each segment was coded for 10 different broad categories, viz., buy, work, learning, everyday math, affect, foreground, school information, personal, nature of science/math, and play, and sub-categories thereof (discussed in Table 3.2), by the researcher and the colleague separately. The segmentation was done independently by the two coders and differences in segmentation and coding were reconciled through discussion. The descriptions of different categories and sub-categories of codes are also given in the Table 3.2 below. All the interview transcripts were coded into segments of these categories and sub-categories and these coded segments were counted at two levels: the broad categories and the sub-categories. Students have been designated with the letter “E” (for English medium school) or the letter “U” (for Urdu medium school) followed by a numerical subscript.

Lesson transcripts from Phase-III were read together with logs by three researchers. Segments of the transcripts relevant to the research questions were identified and carefully reviewed. The right most columns in Table 3.4 about the description of the categories and sub-categories indicate information contained in the participants' utterances.

Table 3.4 Codes for work context interviews, description and counts

Sr No.	Categories	Sub-Categories	No. of Segments	Description of the Categories	Description of the Sub-categories
1	Buy (26 segments)	Cost	14	Focusing on buying, selling and knowledge about them	Knowledge about the costs of the products handled
		Calc	03		Knowledge about calculations embedded in buying-selling processes
		Trans	09		Knowledge about the transaction processes involved in buying and selling in everyday or work contexts
2	Everyday Math (EV Math) (67 segments)	Estm	15	Nature and extent of everyday math and opportunities to gather them	Use of estimation techniques
		Calc	26 (3 long)		Calculation in everyday settings
		Frac	02 (both long)		Fractions used in everyday settings (e.g., binary fractions)
		PropRsng	04 (1 long)		Use of proportional reasoning
		Msmt	20 (4 long)		Use of measurement instances
3	Work (Wk) (M,F,B,S,R,N,Othr) (375 segments)	Info	147	Information about individual's work involved in everyday settings	Knowledge about work context
		Proc	41		Knowledge of work processes
		Linkg	05		Knowledge of/ awareness of the linkage in the work context
		Wage	58		Knowledge of wages and its calculation/ justification
		Obj (materials, tools, products)	26		Knowledge of the diversity of objects or tool, products, raw materials used
		Role	38		Knowledge of different roles in work

		WthWhm	09		Knowledge about co-workers
		Cost	12		Knowledge about cost of the products in work context
		MetaKn	11		Knowledge about his/her own knowledge about the work
		AgeGrp	06		Age group of the worker(s)
		HouseWk	22		Individual's house work
4	Learning (Lrng) S/T/W/B/ Othr (98 segments)	Proc	21	Child learning from his/her own work	Student's account of their learning about work related processes
		Eval	44 (8 long)		Student's evaluation of their learning from a context
		Use	12 (4 long)		Student's idea about use of learning from a context
		Obj	21 (3 long)		Student's learning from the use of different objects
5	School Info (03 seg.)		03	Student's kn about his/her school	
6	Foreground (FG) (11 Seg.)		11	Student's foreground/aspirations	
7	Personal (Pers) (95 segments)	Info	39	Personal details	Individual's personal info
		Edu	15		Information about his/her education
		Fly	18		Info on family members
		Routine	23		Day-to-day activities
8	Affect S/T/W/B/Pers/Othr (27 Seg.)		27	Affect of a given context on the student	
9	NoS/NoM (02 Seg.)		01 + 01	Reflections about the nature of science/ mathematics	
10	Play (09 Seg.)		09	Knowledge about the games students play	

3.6 Ethical considerations

The researcher ensured that respondents' and informants' identities were protected, for example, numbers have been assigned to protect the sample students' identities (described in Chapter 4). The researcher conveyed to the participants the purpose of the study in broad terms and did not engage in deception of any kind about the study. In this dissertation thesis, the researcher has presented general information and has not included such information that participants shared with him in “good faith” or to be kept “off the record”. Such information has not been made part of the analysis as well while ensuring that the analysis is not inconsistent with such information.

An anecdotal reference is necessary here to clarify the stance the researcher adopted while interacting with the community members and while visiting students' homes or workshops. It is part of the cultural practice in this part of the world to give and take things as a token of love and gratitude. However, the researcher was clear and confident not to accept or give any presents and gifts from/to different stakeholders in this research study in order to protect himself and the study from any undue pressure whatsoever. But, there were occurrences of complex circumstances when the researcher was treated with food and presents and exercise of polite refusal and expression of inability of acceptance did not work. The researcher in such cases, keeping the ethical considerations, made attempts to show solidarity with the group (for instance, by opting for water in the same glass, tumblers, bottles, etc. as used by the hosts). Such occasions created opportunities for the researcher to indicate that he is “part of the group” and not different from them or “an outsider”.

3.7 Overview findings: Children's life-world

In this section we present some findings based on visits and interaction with students and the community, from the classroom observations and from the records of students' work in the tuition classes.

3.7.1 Children's daily routine

Interactions with the students indicated that practically all of them have a packed schedule the whole day. The researcher noted that for students attending the morning-shift school, the day begins quite early. These schools work between twenty past seven in the morning and half past twelve post noon. The daily routine for most students after school hour is to attend Arabic or tuition classes (coaching classes after school hour) after lunch and then attend to the work practices that most of them were engaged in, which sometimes went on till late in the evening. The Arabic classes lasted between an hour and a half to two hours and were held at a *Maulana*'s (religious teacher) place or inside a nearby mosque for no fees. However, the “tuitions” come for a monthly fee which varies between Rs 200 and Rs 350. Because of the packed schedule, students do not have much time left for playing or for other recreational activities. There are not many open spaces for them to play and the alleys and lanes are too narrow and crowded for children to have any kind of recreation. Therefore, in the case of most students, coming to school is also an occasion to meet friends and play, chat and have fun. During the interaction, nearly every student reported that they visit shops in the neighbourhood a number of times daily to buy groceries and other provisions for the house-hold needs. Girls mostly attend to the kitchen work – cook food, wash utensils and clean the house/room, do baby sitting for their younger siblings, and so on. Some students also work for supplementing family income – either by assisting elders at home or at workshops or by working independently (details given in subsequent sections).

Interactions with students also revealed the difficult conditions of their living. Most houses do not have private bathrooms and one has to use the public toilets on payment. Water supply is time bound and many families fill water in containers once a day for daily needs.

Often children help their parents in filling water and in other house-hold chores. The researcher noticed that if students get delayed in such chores in the morning, they missed their school session for the day. Many students do not to get to bathe in the morning and do not get to eat properly before coming to school. Under the mid-day meal scheme, students get *khichdi* (cooked mix of lentil and rice) everyday during the lunch break but it is not popular among all children. Some children buy small snacks from make-shift stalls near the school entrance with the little pocket-money that they get from their parents. Some students stay hungry and eat only after returning home in the afternoon. However, as Tr S mentioned, for many students the mid-day meal was the only occasion of getting full meal in a day. Some students carried the meal in lunch boxes to their homes for their family. He further elaborated that for some students, getting mid-day meal was the prime motivation for coming to school. The cooked meals are served by a government approved organisation which delivers food to all the seven schools in the compound. The food was served to the students by a person assigned by the organisation. This person alone served the food in all the floors of the school building and therefore knocked on the classroom doors well before the recess time which was at 10:00 am. The researcher observed that at times such knocks came as early as 09:40 am which completely disturbed the teaching-learning process. Several teachers shared similar experience with the researcher.

3.7.2 The classroom setting

The researcher observed that irrespective of the language (medium) of instruction, most students in the classroom or outside speak in the local dialect (*bambaiya Hindi* – a mix of Hindi and Marathi; Marathi is the official language of the state of Maharashtra of which Mumbai is the capital). Teachers too, commonly speak in Marathi or in Hindi/Urdu among themselves. Teacher S explained that language switch (code switch) was a frequent phenomenon particularly in the English school which helped the students in better comprehension. Use of both English and Hindi/Urdu during teaching usually came during explanation of different terminologies with an objective to clarify the concepts in an effective way. Tr S further explained that “the main motto of the teachers is to clear the concept” and students often open up in their home language than in English. The following excerpts from the classroom logs of the English medium school gives an instance of language-negotiation from an actual classroom teaching scenario. “T” and “S” stand for the teacher and the student respectively while the numbers in the left column indicate the line numbers in the original transcript. These logs are taken from the researcher's classroom observation of the regular lessons in the Phase-I of the study. Third and fourth columns from the left are the teacher's utterances and their English translations respectively. The grey shaded words indicate the words actually used in English in the utterances to show the language switch.

Excerpt 3.1: Grade 6, Section A (English school); Lesson: Profit & Loss; Teacher: T_1

1	T_1	We all know about profit and loss/ what it is? Kaun batayega mujhe?	We all know about profit and loss/ what it is? Who's going to tell me?
2	S	fayda and nuksan/	profit and loss/
3	T_1	kab hota hai profit and loss?	when do profit and loss occur?
4	S	jab...	when...
5	S	jab hum kam price mein kharidkar jyada price mein bechte hain/	when we buy for lesser price and sell for more price/
6	S	agar hum bag three fifty ka lenge aur char sau mein bechenge to profit hoga/	if we buy a bag for three fifty and sell for four hundred then there'll be profit/
7	T_1	kitna profit hoga?	how much profit?
8	S	fifty/	fifty/
9	T_1	bagseller ne kitne mein kharida? Three fifty (writes 350 on BB)/ usne Wasim ko four hundred mein bech diya/ (writes 400 on BB)/	how much did the bagseller buy for? Three fifty (writes 350 on BB)/ he sold it to Wasim for four hundred/ (writes 400 on BB)/
10	T_1	to usko kitna rupees jyada mila?	then how many rupees more did he get?
11	S	humlog jitne mein bag kharidte hain, agar kam daam mein bechenge to loss hoga/	whatever price we buy a bag for, if we sell it for less then loss occurs/

3.7.3 The school setting

Due to the high demand, there are two schools of English and Urdu mediums running in the school compound in two separate shifts. The administrative set-up is different for all these schools and every school (and every shift) has a different set of teachers and supporting staff. There is no playground in the school but a concrete open space where children play. There are many permanent and make-shift stationary and eatery shops just outside the school gate which students throng during the recess and before/after the school hour. The civic body provides 27 articles related to academic needs to every enrolled student in the school at the beginning of the academic year in the month of June including two pairs of school dress, one school bag and a set of textbooks, one lunch box and a pair of shoes.

The researcher observed that the classrooms in the English school did not have desks and benches for the students, hence they sat on a long mat or rug put on the floor. They were soon to get the desks and benches. Urdu school students had benches to sit on and desks to write on. Boys and girls in the same class sat separately in groups in both the schools. One sometimes sees children dozing off in the classrooms due to over work the previous night. Teachers are aware of students' hardships and often ignore such incidents though occasional reprimands occur too. The researcher felt that it requires great amount of sensitivity and awareness on the parts of teachers about the students' conditions at home or at workplaces to tackle such situations.

Most children from the neighbourhood come to the schools walking, though some parents drop their wards and pick them up after the school hour while some others come with their older siblings who study in the same school (or otherwise). The civic body does not ply school bus/van for ferrying students, teachers and staff to these schools probably because of the lack of space for such facility and the additional cost incurred. Teachers coming to the schools from far-away places use the suburban train service which is a convenient mode of transport in the city of Mumbai.

3.7.4 One day in a teacher's life

The researcher observed that teachers in both the schools go through continued hardships that also affect the teaching process. For instance, they are deputed for non-academic work that according to most of the teachers can be done by the non-academic support staff which is not appointed. For example, the task of maintaining the “general register” (GR) that contains every student’s personal and family details, “monthly returns” about students’ attendance in the month, addressing the salary complaints themselves – are some of the non-academic work that every teacher needs to furnish at end of every month. A teacher is given charge of a particular grade who teaches all the subjects in that grade round the year. Therefore, even a day's leave by any teacher causes disruption of the teaching in that grade which also affects teaching in other grades, since students in classrooms without a teacher become noisy as they have nothing to do. Teacher S and others further clarified that in the absence of the clerical staff in these schools, even tasks like preparation of the “school leaving certificates” (referred to as “LC” by the school authorities, teachers and students), admission processes and records, preparing the hand-written question papers (absence of typing facilities), and other paper work are solely done by the teachers during school hours. Many teachers while sharing their experience explained that the new “Right to Education Act” (RTE), has promoted unnecessary paper work and that the “teaching process has become secondary” now. They further explained that frequent training programmes such as syllabus training, training for English speaking, training on “continuous and comprehensive evaluation (CCE)” amount to major loss of teaching time. Furthermore, Teacher S pointed out that there is “no encouragement or scope for experimentation” as the teachers are bound to use the prescribed books and complete the syllabus on time for revision lessons. He added that “importance is not given to student learning and comprehension but updating of paper work is largely emphasised upon”.

The researcher, to get a feel of the teachers' daily schedule, once travelled from the home of one of the teachers to the school, spent the day with him and followed the day’s proceedings at the school. Mumbai being a large and expensive agglomeration, many residents come to work from distant suburbs. The researcher noted that a day begins with jostling for space in the local trains before the day-break, to facing flak from the parents for students' fights that happened outside school premises, for not paying attention to their students, to facing rebuke from the Education Officer (EO) and so on. During SMC (school management committee) meetings, the researcher observed that teachers alone are often blamed for the lack of “students' performance” and for “poor teaching” processes. The school authorities too are non-supportive to the teachers' cause. It appeared to the researcher and was repeated by the teachers that they lacked autonomy and respect and were not empowered. A similar interpretation has been given by the eminent educationist Krishna Kumar (2005) who noticed that such situations create a “paradox of teachers’ personalities” and place them as “meek dictators” - less empowered in society and in profession and only powerful inside the classroom (p. 73).

3.7.5 Hindrances to teaching work

Teachers often spoke about the sudden and supposedly ill-timed instructions that they receive from the Education Department which cause hindrance to their teaching work. For example, in March 2014, two days after the dates for the general elections were announced, the English and Urdu medium schools received a circular from the Education Officer of MCGM on 07 March with a directive to complete the entire process of annual examination by 19 March. All the MCGM schools were asked to submit the mark-statements and other data-sheets to the Department by that date. Tr S said that in his school, the teachers were forced to commence the annual examination on Monday 10 March and complete it the same week which was around one month ahead of the regular schedule. Suddenly, the teaching work was completely stalled and the academic year came to an abrupt end. Tr S reported that the directive came as a surprise for the teachers as well as the students. He further specified that most teachers had not been able to complete the syllabi by then and did not get the crucial last three weeks of the academic year for revision of the courses, which according to many teachers are extremely beneficial for the students before the exams. Tr S and a few other teachers from both the schools mentioned that many students were not able to answer most of the questions during the examination. They said, most students submitted almost blank answer-sheets with only a few questions attempted. The teachers further shared that the students knew that would not be detained in the same class and would be promoted anyway. Tr S lamented by saying “*ek mazak ban gaya hai*” (*it's become a farce*).

Similar was the story in September 2014 after the announcement of the date for the Assembly elections of the Maharashtra state due on 15 October. Many teachers from both the schools expressed displeasure that the teaching work was going to suffer due to the training they need to undertake for the election work. Since the government school buildings are taken up by the authorities for polling, the whole process disturbs the academic calendar. The researcher had observed a similar sequence of events in the year 2010-2011 when a many teaching hours was lost due to the training that teachers had to undertake for conducting the population census work that year.

3.7.6 Different sites of learning

School

Schools are the primary sites of formal mathematics learning. The researcher observed that the classroom teaching does not involve group-work though as explained above, students discuss among themselves and/or look at others' work while doing the given exercises inside classrooms. The researcher's interaction with the teachers indicated their awareness of students' background knowledge but the lesson transaction reflected exclusive use of textbooks. No other aids including students' knowledge or resources were used. The major focus during lesson transaction was on rote memorisation of multiplication tables, facts and formulae without much promotion of conceptual understanding. Emphasis was given on working out every problem given at the end of each lesson in the textbook. Typically, the teacher solved one problem on the blackboard with explanations and occasional discussion with students but building on students' knowledge and experience rarely occurred. Rather emphasis was given on using the formulae appropriately. Thereafter, the teacher would ask the students to do other problems on their own following the “methods discussed on the black-board” or following the procedure of the “previous problem-task”. Here are two excerpts (Excerpts 3.2 and 3.3) taken from actual classroom teaching on the same topic “profit and loss” that the researcher observed in two different sections of Grade

6 by two different teachers. The emphasis in both the classes was on following the same method/procedure for solving the next problems. The transcribed conversations are taken from researcher's field notebook-I:

Excerpt 3.2: Grade 6, Section A (English school); Lesson: Profit & Loss; Teacher: T_1

76	T	yahan SP kya hai? Ek sau pandrah rupaye/ isme kya minus karunga? Sau rupaye/	what is SP [selling price] here? One hundred fifteen rupees/ what do I minus from it? Hundred rupees/
77	T	Exercise 18/ just tell us whether there is a profit or loss? CP diya hai, SP diya hai/ you have to find out profit ya loss/	Exercise 18/ just tell us whether there is a profit or loss? CP [cost price] is given, SP is given/ you have to find out profit or loss/

Excerpt 3.3: Grade 6, Section B (English school); Lesson: Profit & Loss; Teacher: T_2

36	T	yeh question aapke saamne hai, yeh try karna hai aapko abhi paanch minute mein/ ... peechhewale question ka steps laga do/	this question is in front of you, you need to give it a try in the next five minutes/ … apply the steps of the previous problem/
37	T	kya hoga isme profit ya loss?	what'll be here profit or loss?
38	C	(a girl): loss/	(a girl): loss/
39	T	come here (tr asks the student to the blackboard) and notebook mein jitna kiya hai waisa karo/	come here (tr asks the student to the blackboard) and do it as far as you've done in your notebook/

Home

Our interaction and interviews with the students revealed that most often they are asked by their parents to maintain accounts of various transactions that happen in the day, viz., buying of vegetables and provisions, procurement of raw materials as part of work requirements, delivery of “maal” [goods] made as part of the house-hold based work and so on. Students reported that they maintain accounts in “chaukri” [diary] and also do the computations. Maintaining of accounts entails diverse features of handling and juggling with numbers, refinement of estimation skills, and cross-checking of the totals, all of which students learn to do in the course of such an exercise. With such exercise comes a sense of responsibility associated with the task, which involves assisting parents or elders at home.

"Tuition" classes

"Tuition" classes or after school-hour lessons are popular in the community. School education is valorised and seen by most parents as a must for their children. However, our interaction with parents revealed that they feel extra academic guidance is necessary for supporting school learning and tuition classes can supplement for the lack of academic support that most parents are unable to give to their children. Such feelings of the parents are generated from the growing assumption that not much happens in schools in terms of teaching and learning. Thus, tuitions are seen as making up for this loss. Further, most homes lack an environment for study as most house-holds second as workshops where work continue till late night with no alternate space where one can concentrate on studies, which adds to parents' concerns. Parents therefore opt for tuition classes at the cost of money as well as time which children would have otherwise spent in assisting them in economic activities. Tuition classes mainly focus on completing every lesson in the textbooks, learning the multiplication tables, formulae, and also help children in working out after-lesson problems. These classes use the same textbooks as used in the school. Students are punished if they do not complete the tasks or are unable to reproduce them. Some tuitions classes are examples of multi-grade teaching where students from four to five classes learn together in one room. In a way tuitions or coaching classes have taken a form of a parallel system of schools. My interaction with the students and a close look at students' tuition notebooks revealed frequent use of behaviourist methods. Drilling (working out same problem a number of times) and rote learning are some of the commonly adopted strategies by the tuition teachers. Students often referred to tuition classes as the source of learning some tricks and techniques. Many of them viewed tuitions as a support to make them learn the school lessons better, for example, being able to "answer in the class" [*class mei jawab de sake*]. Tuition classes are thus significant learning sites for learning formal mathematics.

3.7.7 Thematic categories about learning

The researcher's prolonged engagement with the community and frequent interactions with them helped him in uncovering the different thematic categories about learning that were commonly used by the adults and some of which were even quoted by the students. These categories represented the perceptions about education and work in the community and were strongly connected with peoples' life-world and motivated their endeavours. In this section, some of the prominent thematic categories that came up during the discussions, interviews or interactions are discussed.

Learning hand skills (haath ka kaam)

Learning of hand skills is valorised in the community. Often people from the community express the importance of learning *haath ka kaam* (hand skill) as a gateway to securing *acchha kaam* (good job) in the longer run. Students too expressed similar feelings and offered as justification the present jobs that they were doing. U_{21} had been involved in work for more than three years and a half (till July 2013). He had started working in a bag making workshop to learn stitching. In the learning period novices do not get any wage except support for food and some pocket expenses. Wage starts when workers start contributing to the production, for example, when a novice becomes an apprentice. Getting involved in “any kind of work” (*koi bhi kaam*) is valorised in the community since it builds networks with people including *seth* (workshop owners who provide jobs), helps in learning hand skill and utilising time in a better way. It is assumed that learning hand skill early would be “useful later on” (*aage kaam aayega)* or would fetch better job (*accha kaam milegai).* U_{21} became an expert in shirt stitching work and started earning so as to be able to send money back home for his father's treatment of bronchitis and lung infection. He explained that having mastered the shirt stitching skill, he kept looking for other jobs where he could “learn something different” (*kuchh alag seekhne ko milega*) and also earn more.

Avoiding idling away time (*khaali baithna*)

It was observed among all the students that learning of a new skill is generally aspired for perhaps because such skills are seen as an opportunity to get into an income-generating practice in the future. For example, U_{22} who has learnt mobile repairing by closely watching his friend in his shop and his father and elder brother in their garment making work, takes pride in knowing about both kinds of work – mobile phone repairing as well as garment stitching work. His father runs a shirt stitching workshop where three other workers are also employed. U_{22} visits the workshop frequently and brings the material required for the work and also observes the stitching work. Learning is generally equated to the amount it can help one earn. Therefore, often children report using their "leisure" time for learning new kinds of work and developing new skills, etc. and thereby making "proper use" of the time (*samay ka sahi istemal*). It could be a reason why children from even such families also work while studying who can afford their education without a need to supplement their income.

Will learn something (*kuchh padh lega*)

During interactions and discussions, parents' and elders' concern for students' future well being frequently emerged. Many had a common concern – "*kuchh padh lega toh*", "*aage theek rahega*" or "*aage kaam aayega*" (*if they study/learn something now then, it's going to be useful later on*) or "*aage kuchh ban sakta hai*" (*can become somebody in future*). "*kuchh padh lega*" is a common category reflected in practice and works as a driving force for the parents and elder in sending their wards to school or to tuition classes. It also reflects the growing awareness for school education and the foreground that parents/elders have amidst hardships and disadvantaged conditions. Looking closely at "*kuchh padh lega*" indicates that though school education is valorised in the community, the striving for excellence is not an expectation. The very fact that mere completion of schooling or earning a matriculation certificate alone cannot take a person a long way in forming a career ("*aage kuchh ban sakta hai*") is not taken into account in such categories.

4

Participation In Work-contexts And Mathematical Knowledge: Case-studies

To learn to overcome difficulties is an integral part of education

– Mahatma Gandhi

The previous chapter presented an overview of the study setting, introducing aspects relevant to the research questions. This chapter presents detailed findings from the interview data of Phase-II of the study concerning students' immersion in work contexts and their mathematical knowledge. A representative sample of 31 students from the two classes were interviewed to obtain basic information about their family background, their participation in work and their parental occupations. We summarise this information in the next section. As discussed in Chapter 3, out of the sample of 31 students, a sub-sample of 10 students and 7 additional students were interviewed in detail to obtain further information about their knowledge of work-contexts and mathematical elements embedded therein. These in-depth interviews focused on work-contexts have been analysed through the case study approach and we discuss four case studies chosen from the reduced sample of 10 students in detail in this chapter. We use information from the remaining interviews to round-off the findings of the case studies to arrive at a reasonably comprehensive picture of the knowledge of work-contexts and knowledge of mathematics of our participants.

4.1 Work profile of participants

Tables 4.1 and 4.2 present information about the work profiles of the representative sample of 31 students and their parents. Those marked in grey signify the cohort of 10 students (5 each from English and Urdu schools) who were interviewed in detail about their knowledge of the work practices they were engaged in (Phase-II, part-III). Table 4.3 presents information about the additional 7 boys from the same class with high involvement in work-contexts, who volunteered for the interview. The alphabets “E” and “U” indicate the school, the subscripts identify the students, “M” and “F” in the bracket indicate their sex. The tables show that 29 of the 31 sample students are engaged in some work practice and such participation is frequently enabled by a family member or an acquaintance, who takes support from the student. The tables also present a picture of the diversity of work-contexts that students are involved in.

Table 4.1: Profile of the sample from English School

Work profile of sample (English School)			
Sample Students	Student’s work	Father’s work	Mother’s work
E_1 (M)	Assists father in toy selling	Runs a toy selling shop Brother: Cell (Mobile) phone shop	Does not work for wage
E_2 (M)	Helps mother in house-hold based *zari* work	Watchman	*Zari* work
E_3 (M)	Helps mother in house-hold based stitching work	Watchman	Garment repairing work (torn clothes)
E_4 (M)	Occasional singer in orchestra or bands	Masonry (civil work)	Does not work for wage
E_5 (M)	*Chindhi* collection (garment recycling)	Plastic sorting (recycling)	Works in an office (details not known)
E_6 (M)	Button-stitching Mobile-parts, mobile repairing	Owns button-stitching workshop Cell phone repairing	*Fees* cutting

		shop (brother)	
E7 (M)	Assists mother in *fees* cutting	Tailoring work	*Fees* cutting
E8 (M)	Assists brother in mobile repairing work	Deceased, did tailoring work	*Fees* cutting
E9 (M)	Tiffin delivery + *tikli* work	Tiffin delivery	Works as a helper in a school + *tikli* work at home
E10 (M)	Assists mother in house-hold based *fees* cutting	Father: deceased, Grandfather: barber	*Fees* cutting & garland (*latkan*) making
E11 (M)	Assists brother in garment selling work	Father: driver Brother: garment selling	*Tikli* stitching on garments
E12 (F)	Tailoring work at home	Left	Tailoring work
E13 (F)	*Rakhi* & *piece* making work at home	Electrician	Teacher in BMC school + *rakhi* & decorative garment pieces
E14 (F)	Assists mother in house-hold chores	Watchman	Stitching work at home
E15 (F)	Cosmetic work at parties	Mason	House-keeping, cosmetic work at parties
E16 (F)	*Stone-fixing* work	Wireman	*Stone-fixing* work

Table 4.2: Profile of the sample from Urdu School

Work profile of sample (Urdu School)			
Sample Students	Student's work	Father's work	Mother's work
U1 (M)	Assists mother in stitching work	Father: Deceased Brother: Tailoring (coat stitching)	Stitching work (vehicle gear covers)
U2 (M)	Tailoring	Current: Watchman, Previous: Carpet-weaving	*Fees* cutting

Work profile of sample (Urdu School)			
U_3 (M)	Sells tea on holidays Helps in buying provisions for stall + Accounts maintaining	Tea vendor	Ill
U_4 (M)	Works in father's workshop	Runs buckles fitting workshop	*Fees* cutting
U_5 (M)	Garment packaging	*Chappal* making	Lives in native place (village)
U_6 (M)	*Zari* work at house-hold based workshop	Grandfather: Tailoring work Maternal uncle: *zari* work, upper floor of their tenement	*Fees* cutting
U_7 (M)	Plastic recycling work	Plastic recycling work Brother: Mobile phone repairing work (employed in a shop)	Does not work for wage
U_8 (F)	*Latkan* making work	Does not know	Does not work for wage
U_9 (F)	*Piece* making work (earlier)	*Chappal* making work Uncle: Tailoring work	*Piece* making work (earlier)
U_{10} (F)	*Piece* making work (earlier) at home	Plumber Brother: Tailoring (*worlok* stitching on jeans)	*Piece* making (buckle fixing) work (earlier) at home
U_{11} (F)	*Rakhi* work (before) Garment recycling work (now)	Garment selling Brother: Stitching work (gents' *pyjama*, three-quarter trousers)	*Rakhi* work (before) Garment recycling work (now) at home
U_{12} (F)	Helps aunt in *moti* fixing work	Mutton vendor Brother: Chicken vendor	Mother: does not work Aunt: *moti* (sequin) fixing on clothes
U_{13} (F)	*Rakhi* making work	Tailoring	*Rakhi* making work
U_{14} (F)	*Chappal* making at home	*Chappal* making work at home	*Fees* cutting work (before) *Chappal* making work (now)
U_{15} (F)	Not engaged in any income-generating work practice	Teaches Arabic in a school	Does not work for wage

Table 4.3: Profile of the additional sample

Work profile of the additional sample			
	Student's work	Father's work	Mother's work
U_{21}	Tailoring	Ill, on treatment	*Bissi* work
U_{22}	Cell Phone repairing work	Tailoring	*Fees* cutting
U_{23}	Textile printing (dyeing)	Textile printing (dyeing)	*Bissi* work
U_{24}	Ready-made garment selling	Ready-made garment selling	Does not work for wage
U_{25}	*Zari* work	*Zari* work (brother)	Parents live in native village
E_{21}	Assists his father	Making of leather bag, wallet, purse	*Bissi* work
E_{22}	Previous: Bag stitching Current: Newspaper selling	Masonry (construction work) Brother: Bag stitching	*Fees* cutting

4.2 Case Studies

In the following case description, 4 representative cases are reported, one boy and one girl from each school (boys – E_5 and U_2, girls – E_{16} and U_{13}), with a focus on their exposure to work-contexts, their knowledge of arithmetic and measurement and the mathematics related to earning. The cases are taken as specific illustrations to develop an understanding of a culture-sharing group and to describe the ways in which participants acquire mathematical knowledge through their work-contexts.

Identifying the cases

The four case descriptions reported in this section were chosen since together they reflected four different kinds of work-contexts with a range of opportunities for mathematics learning, viz., independent collection of material for recycling (E_5), an employee in tailoring work who goes through different learning stages (U_2), a girl with exposure to the work of fixing stones on jewellery done at home and of running a stationary shop (E_{16}), and a girl whose exposure to work was limited to making *rakhis* at home (U_{13}). An embedded analysis of the cases has been preferred over a holistic analysis (Creswell, 2013) as the intent of conducting case study was to understand the extent of mathematical elements embedded in each work-context and students' use of them. Hence, an analysis of themes (viz., knowledge of work context, mathematical knowledge, measurement knowledge, mathematics of earning) within each case and detailed description of them is provided followed by a "cross-theme analysis" (Creswell, 2013, p. 100) across the cases focusing on the aspects of mathematics emergent from each work-context.

Mathematical knowledge

Mathematical knowledge of each case description has been analysed under the rubrics given in Table 4.4 below. These rubrics emerged prominently from the students' arithmetical knowledge interviews (Phase II, part II) and from three broad categories, viz., "Everyday Math", "Work" and "Learning", in the work-context interviews (Phase II, part III) (discussed in Chapter 3). The data is drawn from both these interviews.

Table 4.4 Rubrics for analysing mathematical knowledge

Arithmetic knowledge	Work-context related mathematical knowledge	Everyday shopping related mathematical knowledge	Measurement knowledge
• *Interview tasks (symbolic representation, arithmetic operations, currency knowledge, proportional reasoning)*	• *Decision making* • *Optimisation* • *Maintaining accounts* • *Mathematics of earning*	• *Mathematics of transaction* • *Decision making*	• *Different measurement modes, instruments, units* • *Construction of units* • *Sorting* • *Estimation*

4.2.1 Arithmetic tasks in the interviews

A brief report is presented in this section about the interview items on arithmetic and problem-tasks that were part of the structured interviews used in Phase-II (Part II) of the study. There were altogether nine items and most items had sub-parts. The interviews were between the researcher and the student, one at a time. The items were framed based on the exploratory phase (Phase-I) of the study and researcher's earlier interaction with children and impressions about their arithmetic and currency related knowledge. The items were prepared by the researcher and his colleague.

The items on "arithmetic knowledge" included reading numbers from number-cards, writing the number called out, taking out amounts of money equivalent to given numbers, counting-on using number enumeration, with or without currency notes and coins. The items on arithmetical operations were in the form of contextual word problems depicting everyday shopping involving addition, subtraction, multiplication, division and one item on proportional reasoning (See Appendix C). We discuss the arithmetic knowledge items below. The first item was based on showing altogether six number-cards to the students one at a time and they were asked to read the numbers. The numbers written on the cards were (each on one card): 279, 607, 1010, 2303, 4800 and 10010 in Roman numerals. The second item required writing down the numbers called out by the researcher. The numbers were called out in English as well as Hindi/Urdu. The numbers were "seven hundred fifty one", "one thousand one hundred", " two thousand fifty", "ten thousand sixty", "thirteen thousand two hundred six" and "one lakh twenty five thousand". The third item was about taking out currency amount equivalent to given numbers from two boxes of money-game currency notes and coins, the numbers given to them were, 165, 2725 and 13206 – the number-names were read out in English and in Hindi/Urdu.

The first "count-on" task had two numbers, "87" and "995". If the student had difficulties in enumerating numbers beyond "995" or at the transition to "1000", then count-on task with "595" was given. The next item of the "count-on" task involved the use of currency notes and coins. Students were asked to count the amounts of currency placed in the envelopes and then count-on by adding Re 1 at a time. The amounts were Rs 80 and Rs 995. The subsequent task was to count-on by adding Rs 10 each time. The amounts were

Rs 72, and Rs 970. The next task was to add Rs 100 every time and count-on. The amounts for this task were Rs 270, Rs 800 and Rs 9700. There were four contextual tasks each with different sub-parts. The first item depicted a shopping scenario and involved addition and subtraction in terms of totalling the prices of different articles and calculating the balance amounts. The first set of articles – a juice can and a football were priced at Rs 95 and Rs 265 respectively, while the second set of articles, a doll and a garment were priced as Rs 140 and Rs 199. Students were also asked to estimate if certain articles could be bought with one or two notes of Rs 500. There were two items each on multiplication and division operations and one item on proportional reasoning. The contextual problem on multiplication was about finding the solution of 35×10 and 16×7 while the division problem was about finding 315÷5 and 400÷25 respectively. The proportion problem-task was about finding the price of 25 *burfi* (a sweet) when 20 of them cost Rs 42 (see Appendix C). These contextual tasks are discussed in detail in the analysis of mathematical knowledge of each case dicussed below.

The objective of this interview was to gather the nature and extent of arithmetical knowledge from both school and out of school contexts prevalent among the middle graders. The framing of the tasks and students' responses were analysed using the distinction framework of out-of-school and school mathematics (Table 2.1, discussed in Chapter 2).

4.3 Case – I: E_5 – 13 year old boy

E_5 belongs to a Muslim family of migrants from the northern Indian state of Uttar Pradesh from where a sizeable population migrates to Mumbai in search of livelihood. He lives with his father, step mother, two siblings, cousins and a few relatives in a small single room tenement in the low-income settlement. His own mother lives in the family's native village with three other siblings. E_5 is second among six siblings. His elder sister was studying in the same school at the time of the interview. His father had had a few years of schooling while his mother is unschooled.

Like many families living in the neighbourhood, E_5's family is poor, as indicated by the fact that E_5 needed to work to supplement the family income. E_5 mentioned that he had a few relatives living in the locality and that every adult and teenager in the extended family and among relatives was engaged in various kinds of work for making a living. E_5 had done bag making work earlier and at the time of the interview he was engaged in recycling work for the past four years. He assisted his father in plastic recycling work and he himself did garment recycling work once a week, on Sundays. E_5's father had worked earlier as a watchman in a hotel in the downtown area of Mumbai which he quit and picked up plastic recycling work. His step mother did *fees* cutting work at home for sometime before taking up cosmetic (cleaning) work in different offices in the adjacent corporate office area.

Interactions with E_5 indicated that he was strongly networked with people in the neighbourhood engaged in different kinds of work. For example, he mentioned that he arranged jobs for his friends. E_5 was forthcoming in his interviews in talking about how his friends found ways of earning money in different kinds of work and also from other sources such as gambling. He mentioned that he helped his friends who sing *naath* (religious songs) at mosques seeking alms and other friends who play roulette betting games for making "easy" money. E_5 also raised issues about fair ways of earning money by working and decried the fact that teenagers like him need to earn to support family and personal needs. Interactions with E_5 reflected the extent of his access to and participation in the funds of knowledge about diverse work and other practices that happen around him.

E_5 did not go for after school coaching classes ("tuition" classes) any more. He informed the researcher that he had attended "tuitions" earlier but ever since he was scolded by the teacher he had stopped going. Following several interactions, it appeared to the researcher that E_5 was interested in getting into some kind of work and according to E_5, he got the first opportunity to work when one of his uncles asked him if he would like to learn "*haath ka kaam*" *[hand skill]*. His uncle had asked, "*bag banayega? seekha dunga*" *[will you make bags? I will teach you]*. E_5 started going to the workshop in the evenings that was located at a distance from the locality. He earned around Rs 100-150 per week but he claimed that his intention was to learn the work. He explained that he saw around him children of his age-group working which further motivated him to start working. E_5 like others in the community valued work experience and knowledge of work. When asked why did he start working, E_5 replied,

> "*aise hi sir mann kiya tha, socha bade hoke bhi kuchh, abhi thoda kar lunga to bade mein madad aa jayega*" *[just that sir I wished to, thought after growing up, if I do little now then after growing up it will be of help]*.

E_5's views also reflected gender stereotypes about the kinds of work that were prevalent in the area. For example, the household based *fees* cutting work is typically considered as a woman's job and the researcher has not come across anyone other than a woman who was engaged in this work. E_5 mentioned that his father tells him that "*ladki ka kaam ladki karegi, ladke ka kaam ladka karega*" *[a girl will do girls' work, a boy will do boys' work]*. He further classified that outdoor work are boys' (men's) work and indoor activities like cleaning work and other house-hold chores are girls' (women's) tasks. He explained that therefore when his mother goes out for work, E_5's sister does the cooking.

4.3.1 Exposure & Involvement in work-contexts

E_5's first work experience was in a bag manufacturing unit where women's bags and purses were made. E_5 had the task of putting adhesive on the leather pieces followed by sticking foam pads over them and then forwarding those pieces to the next worker for stitching. For a short period of time, E_5 was also engaged in thread removal work (*fees* cutting) at home when he helped his mother and sister. He shared with the researcher about several bad experiences while working in the bag manufacturing unit. He explained that his *seth* (boss) would yell at the workers for small mistakes, which E_5 did not like. From the conversation, it appeared to the researcher that E_5 had developed a dislike towards working for others and had quit such work. He also convinced his mother about his decision. He remarked, "*ab kaam hi nahin karta hoon, ab kaam ka naam sunne pe bhi gussa lagta hai*/" [*now I don't work, now when I hear the word "work", I feel angry*]. By work, E_5 meant working in a manufacturing unit or a workshop as an employee. He seemed to be interested in working independently and not under somebody. It appeared to the researcher that he was looking for work which could provide him flexibility of time as well as flexibility of amount of work to do. This way, E_5's present working conditions in garment recycling (*chindhi* collection work) appeared to be flexible and suited his requirement. He came to interact with the researcher on a Sunday foregoing his weekly earning that he claimed can be made up at some other time.

He discussed that he arranges for *chindhi* collection work for his friends who need financial support for various reasons. This way E_5 has helped other children of his age by creating work opportunities for them. He emphasised,

> *"toh isliye yeh bhi aa jata hai to isko bhi bhara deta hoon, chalis-ulis rupaye ka, iska bhi jeb kharch aa jata hai/" [so therefore he too comes and I help him fill up chindhi, around forty rupees or so, his pocket expenses are thus met].*

Waste collectors in the recycling work are independent workers and many children from the locality are engaged in this work. Recycling is a micro enterprise and the low-income settlement has large networks of establishments that engage in recycling various kinds of materials drawing in a sizeable population from the locality and providing them with livelihood. E_5's recycling work depends upon his direct personal contacts and rapport which he has built over three years while doing this work. He has been doing this job independently since he was in grade 4 (*sir, yeh to main kar raha hoon kabse, chauthi mein tha jab se kar raha hoon [sir, I'm doing this work since, since I was in grade four]*). He explained that personal contacts help in getting *chindhi* (explained below) and also in getting a good price for the collected material.

Garment recycling work

The garment recycling work entails collection of left over cloth pieces of varying sizes from the garment stitching workshops. These cloth pieces (called "*chindhi*") are collected for recycled garment manufacturing as well as for other purposes, such as, for use as stuffing in sofas, chair cushions, quilts or mattresses, and so on. *Chindhi* are measured not by size but by their weight. The collection is then sorted according to the colour and quality of the material and sold at different rates. E_5 is involved in this work known as *chindhi* collection or *chindhi bharna* (filling cloth pieces in gunny bags) along with many other young children from the locality.

Plastic recycling work

Similar to cloth recycling work, plastic recycling work involves collection of plastic waste. E_5 knows about his father's "*plastic ka dhandha*" or *plastic work* although he himself does not collect plastic waste. He elaborated that the plastic waste that his father collects is classified into two qualities – numbered as 1 and 2 depending upon the thickness and sold accordingly at different rates by weight. A common price is Rs 35 a kilo for the better quality. E_5 mentioned that dimensions of the plastic sheets are usually not taken into account during sorting or pricing. Rather they are sorted according to their numbers which the workers can identify using their visual and tactile senses.

4.3.2 Features of mathematical knowledge

In this section, we draw on data from the arithmetical knowledge interviews and work-context interviews (Phase-II, parts II & III) for understanding features of E_5's mathematical knowledge.

Arithmetical knowledge

We discuss here E_5's arithmetical knowledge by focusing on his strategies for completing the interview tasks and his explanation about how he tackles different kinds of mathematical problems in his work-context.

In the arithmetic tasks on number sense, E_5 could read all the numbers shown on the number cards and write the ones called out, except a six digit number. In another task, he could take out the exact amount of money equivalent to a given number using different currency denominations, which indicated his sound knowledge of currency, its different denominations and their inter-conversion. In the number counting task, he presented correct oral enumeration of numbers and often counted numbers more than the task required. He could also count on numbers with and without the support of currency.

In the arithmetic tasks on contextual word problems, E_5 relied more on mental computation while solving the tasks using convenient decompositions and groupings of numbers. For example, in the contextual problem of shopping where the price of a few articles to be purchased are given, an estimation was required whether all the articles would come against a currency note of Rs 1000 or Rs 500 and which ones would come for Rs 500 and which ones for Rs 1000. The balance amount was required to be found. E_5 came up with reasonable estimates in every problem and its sub-parts, such as he estimated "*dedh sau tak lautega*" [around hundred and a fifty would be returned], when the correct answer was Rs 140. E_5 then proceeded to calculate mentally. Price estimation of a number of articles bought, comparison of the price estimates and calculation of the balance amount are common features of the everyday shopping. The use of calculators is not common in such contexts, especially in the low-income settlement.

In other arithmetical tasks that involved multiplication, E_5 used multiplication tables, convenient groupings and repeated additions. For example, in the contextual problem of 35×10, he mentally computed 30×10 + 5×10; while in another problem involving 16×7, he used the convenient decomposition 10×7 + 6×7. His estimation of the answer in each problem task was spontaneous and close to the actual answer. In some problems, after computing mentally, he verified the results on paper as well (viz. in division problems such as 315÷15; 400÷25). In the proportional reasoning problem task ("If 20 *burfis* cost Rs 42, how much do 25 *burfis* cost?"), he chose to use the school taught *unitary method* first, but after getting stuck he used halving method and arrived at a practical answer – Rs 53, by rounding off to the next number arguing that small balances are not returned by the sellers.

Work-context related mathematical knowledge

E_5's work-context required him to use proportional reasoning while doing various types of calculations. He collects *chindhi* once a week and sells them at rates varying between Rs 4 and 20 a kilo (Kg). White *chindhi* fetches the highest value of around Rs 20 per kilo, while red coloured ones are sold for Rs 8 a kilo. *Chindhi* of smaller dimensions fetch Rs 4 per kilo. E_5 visits around 25-30 workshops on a visit often with a group of boys of similar age (his friends or known to him) and has collected up to 70 kilos of *chindhi* in a day (his highest collection was 95 kilos in a day, discussed below). Proportional reasoning is used while distributing (or sharing) the wage among themselves. This is especially so in a situation when the *chindhi* collected by the whole group is weighed as a whole and the amount is paid together and not individually. E_5 explained that he calculates an individual's earning based on estimates of the amount of *chindhi* he has collected. *Chindhi* collectors like E_5 do not have control over the kind of "*maal*" (material/goods) they would get on a given day, but in order to make more money while carrying a fixed amount of weight, E_5 uses proportional reasoning to make decisions about how much to collect of the white variety and the coloured variety. He is aware that collecting the white variety fetches him five times more than the ordinary or smaller pieces of *chindhi* (20:4), while the red variety brings only double the amount of the smaller *chindhi* (8:4).

E_5 does quick mental calculations since his job calls for cross-checking the amount quoted by the buyer for purchasing the recycled material. During the interview on the mathematical tasks and at other instances, the researcher noted that his estimations are nearly accurate and he prefers doing calculations mentally. E_5's work-context entails remembering many numbers and proportions of the material sold under each type while calculating the wage. This is reflected in the following excerpt 4.2 where E_5 spoke about an instance when he earned Rs 640 by collecting 95 kilos of *chindhi* on a single day. (In all the excerpts henceforth, "S" and "T" represent student and researcher respectively; while the numbers on the left column indicate the line numbers in the original transcript, and the English translations of the utterances are provided in the right column).

Excerpt 4.1: Interview transcript of E_5

364	S	sabse zyada to bahut pehle kamaya tha, chhe sau chalis rupaya/	maximum earning was long back, six hundred forty rupees/
365	T	kitna kilo tha usme?	how many kilos were in it?
366	S	panchanve kilo/	ninety five kilo/
367	T	panchanve kilo ka chhe sau chalis rupaya? Kaise?	six hundred forty rupees for ninety five kilo? how?
368	S	...white alag, lal alag kar liya, white becha tha bees rupaya kilo, aur lal becha tha aath rupaya kilo aur chindhi becha tha (char rupaya kilo)/	...white and red separated, sold white for twenty rupees a kilo, and sold red for eight rupees a kilo and sold *chindhi* (four rupees a kilo)/

Involvement in work also calls for management of earning. The researcher observed that E_5 was aware about his father's earnings and also calculated the profit margins. For example, he explained about his father's *plastic ka dhandha* (plastic "business") that, he buys the plastic sheets at a lesser price in Borivli (a distant neighbourhood) and sells off for more in the recycling workshops in the settlement thereby making a profit of Rs 5-10 per kilo *["Borivli mein kam mein kharid kar layenge, kabhi pandrah rupaya kilo, kabhi bees rupaya kilo, dus-paanch rupaya kilo bachta hai" "In Borivli he buys for less, sometimes fifteen rupees a kilo, sometimes twenty rupees a kilo, ten-five rupees a kilo is the saving"]*. We observed that such exposure to work practices helped in building an understanding of the mathematical concepts like "profit and loss" among the school students we interacted with. Here is an excerpt (4.3) from a conversation with E_5:

Excerpt 4.2: Interview transcript of E_5

133	T	to unko fayda kya milta hai?	so what profit does he get?
134	S	fayda milta hai roz ka teen char sau rupaya/	gets a profit of three four hundred rupees/
135	T	kaise?	how?
136	S	usi mein nikal jata hai/ wahan se kam mein kharid kar late hain na/	it comes from that/ he brings (stuff) from there having bought for less/

E_5 appeared to be more independent than many other children of his age. He managed his own contacts and network to collect and sell *chindhi* and helped his friends in the same task so that they earn some pocket money as well.

Decision making

Making quick decisions is of vital importance in this task. Negotiations happen during quotation of prices which are competitive because of the presence of many groups of waste collectors in the field who are also into *chindhi* collection. Hence, the price negotiation for selling the collection is done quickly and it is therefore imperative for E_5 to make quick decisions. For this, he claimed he uses quick computations. He also explained that his personal contacts with people on the network helps him in getting a good price.

Optimisation

Optimality of time, traverse and weight to carry is considered when E_5 does *chindhi* collection work. *Chindhi* collectors earmark workshops where they are most likely to get a handsome quantity and know how quickly they can fix a deal before other groups can drop in. One also has to keep in mind the weight that can be carried easily and which can fetch a good amount.

Maintaining accounts

E_5 like other students often reported that he maintains accounts of payments made and received, pieces of work completed and delivered, and provisions bought. He maintains accounts in a *chaukri* (diary) and assists his mother in keeping the records. E_5 further explained that while maintaining accounts, he writes the numbers in arrays (under different headings) and takes their aggregate.

Earning from roulette gaming

E_5 discussed about his friends and about his schedule on Sunday evenings when most of the establishments and workers in the settlement take time off. He mentioned visiting *dargah* or parks and friends' homes. He goes to watch movie shows in nearby theatre-halls with friends. Interaction with E_5 highlighted that children of around his age are attracted towards games like *saurat* (similar to roulette betting) that happen in the locality and take part in them. This game is seen as a gateway to make quick money. E_5 described that one of his friends plays this game and also earns from alms that he gets by singing religious songs (*naath*) in mosques. E_5 revealed that this boy puts money in *saurat* often and once he lost Rs 75 in it. E_5 mentioned that he occasionally plays *saurat*. In Excerpt 4.3 below, E_5 shared that with luck one can win a big amount.

Excerpt 4.3: Interview transcript of E_5

474	S	sir, kholo, kismat rahegi to milta hai, uss din sir do rupaya ka khele the, chhutti leke aaya tha na main, do rupaye ka khele the uske andar ek sau dus rupaye ka jeeta tha/	Sir, if luck favours then one wins, that day sir I played two rupees, I had taken leave, played two rupees and won one hundred ten rupees in return/

E_5 also asserted that "*if at all, there is a loss of five rupees, but if it comes out [i.e., if one wins] then straight away fifty*" *(kitna, paanch rupaya loss hota hai, aur nikalta hai to seedha pachas/)*.

Everyday shopping related mathematical knowledge

Children in the locality often run errands to purchase goods from nearby shops for daily household use. E_5 too visits local stores for buying daily provisions and is aware of the prices. He said that every evening he buys milk. The milk vendor puts the measuring cup once in the milk container and the quantity of milk the cup contains is half a litre for which the milk vendor charges Rs 12. He explained that measuring of milk is done by dipping the measuring can in the seller's milk container a number of times to arrive at the required amount – twice for a litre, thrice for a litre and a half and so on. Similarly, half a can for a quarter litre. E_5 was able to calculate mentally different measures of milk when the can was dipped in the milk container a given number of times.

E_5 also buys other household requirements like kerosene oil. He accompanies his mother while purchasing vegetables and regular spices. He knows that the weights for most articles are stamped on the packets. He said "*kaante pe dikhta hai aadha kilo*" *[the needle indicates half a kilo]*. He also said the prices are generally mentioned on the packets. The shopkeeper tells the final amount to be paid and E_5 checks the balance amount. The shopkeeper also sells goods on credit but to his mother and not to him.

Measurement knowledge

Different measurement modes and instruments

As part of work and everyday experience, E_5 encounters different kinds of measurement modes, instruments and processes that are different from the one that he encounters in textbooks and in the classrooms. For example, the conventional large beam scales (*kaanta*) are used for weighing large quantity of *chindhi*. This is in addition to the regular balance and weighing machines that one comes across in other everyday settings. *Kaanta* is a traditional weighing scale used for weighing large quantities. E_5 explained confidently that when the collection is large, the buyer usually brings his large beam scales (*kaanta*) for weighing,

> *(matlab ek jagah se sattar kilo nahin milta hai, kam-se-kam pachhees-tees karkhana rahta hai... ikathha karoge to bhari ho jaega to mere ko kya karne ka hai? jisko main deta hoon woh kaanta laake apna wazan karega [that is, seventy kilo is not obtained from one place, there are at least twenty five-thirty workshops... when collected becomes heavy, (but) why do I care? Whom I sell them to, brings a beam scale to weigh].*

E_5 mentioned that on an average he collects around 40 kilo in a day (*to din bhar mein main chalis kilo to bhar leta hoon, chalis-pachas kilo*). Upon being asked about the amount that he makes on a visit, E_5 confidently shared that "*kabhi sau ka der sau ka ho jata hai/ lekin jab bhi aaya hoon sir sau se zyada bechke gaya hoon*/" [*sometimes hundred or hundred-fifty/ but whenever I've come I've sold for more than hundred (rupees)*].

Sorting

Chindhi collection is done randomly by collecting whatever appears "sellable". Before selling off, the main task is to sort out the *chindhi* pieces according to i) colour, ii) size, iii) shape, and iv) texture, using only visual and tactile cues. The children engaged in the work learn to sort quickly.

Estimation

Most work-contexts entail knowledge and application of estimation skill. In both the work-contexts of plastic and garment recycling, E_5 uses estimation skill to guess the weight of the collected material. E_5 elaborated that he sometimes accompanies his father to his workplace. He described that plastic waste is sold per kilo and not per piece and that one kilo waste contains around 7-8 plastic sheets of 4'×4' dimension which fetches Rs 35 ["*ek sheet ka nahin poora ek kilo ka aur ek kilo mein chadta hai kuchh nahin to saat-aath panni chadhega/ waise hi/ tab ek kilo hoga to uska paintees*"]. Estimation as a skill is useful in deciding a "floor amount" of the material collected, i.e., a fair guess of how much the material should be at the least. This helps during weighing the collected *chindhi* in avoiding cheating and checks oneself from incurring losses.

4.4 Case – II: U_2 – 14 year old boy

U_2 is from the Urdu medium school and older than his classmates. He lives in a rented single room tenement in the low-income settlement along with his parents, 2 elder brothers, 2 sisters (one older, the other younger), a sister-in-law and her three children. The eldest brother works in a garment manufacturing workshop as a supervisor. U_2's father is old and now he works as a night watchman in a residential building in the downtown area. Earlier he was engaged in carpet and rug weaving work in their native village in the Uttar Pradesh state in north India. U_2's mother and other women in the family together do *fees* cutting (removal of extra threads from newly stitched garments) and garment packing work at home. Having been brought up in an economically active environment surrounded by people who were doing productive work, U_2 like most other children in the sample or in the settlement got engaged in income generating practices early in age. He shared that learning "*haath ka kaam*" (hand work, hand skill) is extremely useful which can provide better work options in the future (discussed in Chapters 3 & 5).

U_2's is a migrant family settled in the low-income settlement in a rented single-room dwelling. The family pays Rs 2000 a month towards room rent which is a hefty amount for people living in economically poor conditions. U_2 explained that rooms (shanties) do not have bathrooms and the public toilets in the locality are used by the residents on payment. Water supply is time bound (comes in the morning) and he helps his family members in collecting and preserving water for the entire day. Like many other households in the locality, U_2's family does not have electronic gadgets like television but his father and brother carry a mobile phone each. Interaction with U_2 revealed that he spends most of his time after school hour at the tailoring workshop and his parents do not allow him to go out with friends during Sunday breaks. Idling away time ("*khaali baithna*" [sitting idle] or *"samay barbad"* [wastage of time]) is generally discouraged in the community.

U_2 spoke about two story books that he has. He said that his class teacher gave him story books to read. These are stories with moral values or values of education. He has the popular Urdu book *paiyam-e-talim* (trans. "message of education") that presents different everyday situations and ways to deal with them through stories and he also has a copy of the Urdu monthly magazine *khatoon mashreekh* which has interesting puzzles, crosswords and other informative pieces for children.

The researcher's interaction with U_2 indicated that he does not usually participate in outdoor leisure activities. He goes out when there is some work like doing house-hold chores or buying provisions or dropping his father at the nearest suburban train station. He does not get any pocket money and does not prefer to eat out with friends, which is a common practice among children of his age group.

U_2 told the researcher that he does not attend after school tuition classes due to financial constraints but his elder sister and elder brother teach him at home. They teach subjects like Mathematics and English. There are not many such instances of students studying at home during spare time since most students either go for tuition classes and then for work or they directly join work after school hours and are left with no time to study at home. Those who get spare time like E_{11} or E_5 spend them in different ways.

4.4.1 Exposure & Involvement in work-contexts

In his work-place U_2 only attends to whatever task is assigned to him and does not have the freedom like E_5 to schedule his work according to his own terms. We discuss below his involvement in garment stitching work and the extent of his exposure to his house-hold based *fees* cutting (thread removal) work and also his father's earlier work of carpet weaving.

Garment stitching work

U_2 explained that he started working in a garment stitching workshop as a novice with an intention of learning the task of garment stitching following his father's suggestion as a way of productively using his spare time ("*samay ka sahi istemal*"). He further elaborated that novices do not get any remuneration during the initial "training" periods when they are expected to begin as "helpers" and assist other co-workers before they are given stitching work. Other students doing similar work shared similar experience (U_5, garment packaging and U_{21}, garment stitching). Tailoring is one of the common occupations in the neighbourhood. The work of tailoring as a whole is complex and involves stages of skill development from novice to apprentice to master. This process of training a novice through different stages of task learning is similar to Jean Lave and Etienne Wenger's notion of *legitimate peripheral participation* (1991). During the interview, U_2 described in detail different stages of learning involved in the tailoring work – how a novice first develops hand-stitching skills and then learns to use a sewing machine.

Stage-0

The initiation into the work begins with assisting the apprentices and observing them working. U_2 said that the beginner is asked to do "*helprei*" (assistance). In U_2's words, assistance meant, "*yaani ki dhaaga katna, aur ghadi karna, ghadi kaise hota hai, istiri maarna/*" *[that is thread cutting, folding clothes, garments, and ironing]*. He further added,

86	S	haan, helprei/ jo yeh sab karte hain na, dhaga kaichi se katna, aur ghadi karna, aur turuprei karna, colour lena, asteen lena, woh sab, aur naari dalna woh sab/	yes, *helprei* (assistance)/ who does all this, cutting threads using scissors, and folding-ironing, and hemming, putting colour, hand-cuff, all of that, and putting *naari* (round thread around waist to hold *pyjama*)/

Stage-I

70	S	pehle straight line sikhna padta hai, dhaga jaise seete hain apun woh line seedha hona mangta hai/	straight line is to be learnt first, the stitch must be straight/
74	S	uske baad jaise dhaaga chadhate hain/ pehle to sir turupei karna sikhayega/	thereafter how thread is sewn/ sir at first *turupei* (hemming) is taught/

Stage-II

92	S	uska agla darza sir machine mein baithaiyenge to aapko sirf pehle naari denge, bolenge naari banao, poore pyjame ka naari banao, jitna pyjama hai do sau, teen sau, utna poora naari banana padta hai sir/ line pe/ dekhenge line seedi ho gayi apni to apun ko denge, haan asteen denge chadhane ke liye, phin pyjama denge	in the next level, sir you're taught to use a machine, first you're asked to make a *naari* (belt strap for holding pyjama), *naari* for a whole *pyjama*, whatever the number be, two hundred, three hundred, that many *naari* are to be made sir/ on a line/ it'll be assessed, once the line becomes straight, then further work,

		banane ke liye, phin asteen denge, saada denge saada, ekdum saada chiz denge apun, uske baad fir hard kaam denge, turpei lagane ka/	hand-cuff stitching, stitching whole *pyjama*, other cuffs, only plain, simple work is given, thereafter tough work is given, like hemming (actual work)/

Stage-III

107	S	teesra darza fir wahi, hard kaam denge, woh hard kaam achhe se kar liye to fir machine par baithayenge, fir thoda pagar badhayenge, paisa denge fir/	third level is again, tough work is given, if you do the tough work nicely then again you'll be asked to handle machine, then small wage increment comes, you're given money/
108	T	achha/	okay/
109	S	haan/ aur jab aise sikhte sikhte sikh jayenge apun poora to kaam dene lagenge/ ek mahine tak aise free kaam karayenge/ ek mahine, do mahine tak/	Yes/ and when by doing these we learn then complete work starts coming/ for about a month we're made to work for free, for a month or two/

U_2 explained that although he did not earn anything from doing the work initially (as a helper) till he learnt stitching, the work he did then was taken as part of his learning a hand skill seen essential for moving ahead through the stages of skill development and valorised in the community. The researcher heard a similar narrative from U_5 who too started as a novice without earning money till he was assigned tasks after which he started getting remuneration for his work. Therefore, in the community, though earning is imperative, getting training for hand skills (even without any remuneration) is a vital part of learning, growing up and part of cultural shaping (enculturation) of the children.

Tailoring is increasingly a compartmentalised and fragmented work. For example, in garment stitching work, a group of people (mostly novices) would stitch only collars and cuffs of a shirt, while others with more experience (apprentices), do complex work like stitching all the parts together, while a third group puts buttons and yet another group removes threads from the newly stitched garments (*fees* cutting work, discussed below). This is followed by ironing of clothes and packaging work. Unlike dyeing (textile printing) and mobile phone repairing work, garment stitching work is completed by putting together smaller, compartmentalised tasks which are done at different workshops with most workers having linkages only to the next stage of the work and practically no linkage with the market network. A worker usually works at a location with materials that are provided to him. He does not deal with the customers. *Masters*, who get the orders, cut the cloth in bulk and distribute the pieces to “compartmentalised” workshops. Raw-materials like threads and needles are provided to the workers. The stitching work chain ends with the delivery of stitched materials to the dealers.

U_2 explained that workers like him get wage based on the number of pieces they stitch, for example, 50 *paise* to a *rupee* per piece of work (fragmented tasks, like stitching only collars or cuffs in case of shirt stitching work). The wage goes up to Rs 7-8 if all parts of a garment are stitched together, which workers with better expertise and experience get (for example, U_{21} reported about earning this much wage upon attaining expertise). However, there are workers who get monthly wages, which may still be quite meagre (for example, U_5 worked for a fixed monthly wage of Rs 1200 for ironing and packaging newly stitched garments). Both U_2 and U_5 worked full-time like most other students who were engaged in work, by working everyday (except Sundays) till late evenings. The researcher learnt from the students and himself observed that they missed classes when the work load was heavy. Generally, work-contexts similar to those of U_2 or U_5 did not require knowledge of linkages on the production chain or handling diverse goods. Both U_2 and U_5 were paid workers almost at the bottom of the hierarchy of workers and unlike E_5, (and others – U_{22}, U_{23}, U_{24}), who had a sense of ownership of their work. However, U_2 had other opportunities in his work setting.

U_2 explained that apart from the stitching work he was also involved in outdoor activities like accompanying the *“seth”* (employer) to bring the material (bundles of cloth and other raw materials) from a distant locality. His work demanded “*cutting sikhna rahta hai, aur maal kahan se aata hai woh bhi sikhna padta hai/*” *[cutting is to be learnt, and where is material brought from that has to be learnt too]*. U_2 indicated the processes involved in the procurement of the material – how were the negotiations done, decisions made, material loading, off loading and other tricks of the trade. When asked why learning all this is important, U_2 argued that “*hunar aa jayega na sir*” *[will learn a skill sir]*.

Thread cutting (fees cutting) work

One of the common house-hold occupations for women is thread cutting (or “*fees*” cutting) work referred to above – a part of the garment manufacture chain. This work involves removing the extra threads from the newly-stitched garments and does not require prior training, bigger space for work or fancy equipments. Generally sharp-edged cutters/knives are used. It is a routinised activity that does not demand skills or knowledge from the workers. This makes f*ees* cutting a popular house-hold based work among women and young girls. *Fees* cutting work is an example of a house-hold based income generating task that does not require awareness about other tasks beyond the immediate engagement. U_2’s mother, sisters and sister-in-law are engaged in such work at home. U_2 assists them in receiving the “*maal*” (material), keeping them aside or in their delivery of the finished product. He also assists in maintaining the accounts (discussed in the next section).

Carpet weaving

U_2’s father was involved in the carpet and rug making work while he was in their native village in Uttar Pradesh state famous for its carpet weaving work. U_2 had occasions to watch his father doing this work. Knowing about different measurement units, measurement and work processes and different measurement modes embedded in carpet weaving were part of his knowledge (discussed in the next section). U_2 acquired knowledge and procedures involved in carpet weaving work by his own observations which is a major means of enculturation in the community.

Learning from elders

U_2 discussed about things that he learnt from his father. He said that he observed his father weaving carpets and rugs and learnt about the measurement units like *guj* (Hindi word for “yard”) or use of “*naksha*” (blueprint or a layout) in such work. U_2 could make a connection between a “*naksha*” used in carpet making work and a “sample” (or a template) used in his tailoring work. When asked whether he uses a “*naksha*” in his work, U_2 claimed,

> "*mere kaam mein nakshe ka? … sample hota tha, sample yaane... usko naksha maan lo sir/*" [*blueprint in my work? There used to be sample, … sample means... you consider it a blueprint sir/*]

He discussed that he picked up different measurement units from his father,

> "*...abbu sikhate the poora..., abbu jahan lag jate hain, samajh mein aa jata hai, pauna-pav yeh sab abbu se sikha hoon/*" [*...abbu (dad) would teach me everything..., whatever abbu picked up he would make me understand, three quarter – quarter all this I learnt from him/*]

4.4.2 Features of mathematical knowledge

Arithmetical knowledge

U_2 was able to read all the numbers in Urdu that were shown on the number cards but could not write numbers bigger than 4 digits. Given a number he was able to collect an amount of money equivalent to the number. He however, had a problem in number enumeration and fumbled at the transition to the next tens and hundred, for example, while enumerating after "one thousand nine" he said "two thousand" and after "six hundred nine", "seven hundred" and so on. Interestingly, while doing a similar task of "count on" using money, while counting on beyond "995", he correctly arrived at "one thousand ten" after "one thousand nine" by adding one rupee coin at a time. After adding five one rupee coins, he was at the transition from "999" to "1000" and he figured out "1000" by mentally adding "a five rupee" to "995". Proceeding this way of adding one rupee each time, he arrived at "1009" and for the ensuing transition to "1010" he collected ten one rupee coins together and the earlier five of them to make it into 15 one rupee coins and added them to "995". He then repeated his counting by again adding "5" to "995" and arrived at "1000", and then added the balance "10" to the total number to arrive at "1010".

In the everyday shopping problem, U_2 used count on strategy to calculate the price difference between two articles priced at Rs 95 and 265 respectively and arrived at "*ek sau chausath rupaya zyada hai*" [*one hundred sixty four rupees more*]. While explaining, he said “*paanch daal diya to sau ho gaya*” [*added five to make it a hundred*] while referring to “95” and explained that he took out “five rupees” from “265” and put it in “95” to turn it into a hundred. He realised that there was some error in the answer and proceeded once again. This time he visualised the entire subtraction (column-wise subtraction as taught in schools) up in the air and concluded that “170” is the answer.

In the next shopping task, U_2 was asked to check which one of the two articles – t-shirt (priced at Rs 199) or a doll (priced at Rs 140) – was costlier and by how much. U_2 calculated the price difference mentally and quickly replied, “*pachas aur nau, unsath*” [“*fifty and nine fifty-nine*”]. The researcher noted that U_2 like other students, tended to use his own strategies at the outset and on encountering a difficulty or discomfort with the obtained results, used the school taught methods mostly for cross-checking the answers. Such tendencies were noted in other problem tasks, for example, in the subsequent contextual problem tasks, U_2 attempted to solve by using mental computation, got stuck in the middle and arrived at an incorrect solution that he himself figured out. Then, he moved to school learnt algorithms of arithmetical operations and “worked out” the computation in the air and tried to visualise it. For instance, in 35×10, he visualised the whole multiplication procedure and arrived at the answer. He however, did not use the “put a zero to the right” rule for multiplication by “10”. In the next item of finding 16×7, he orally computed, “*chhe satte bayalis aur ek satte saat, saat sau bayalis*” [*six times seven is forty two and one times seven is seven, seven hundred forty two*] and incorrectly arrived at “742” as the answer. U_2 claimed that he did not know “*takseem*” (division) and did not attempt the items on division (315÷5) and proportional reasoning. He however attempted to solve 400÷25 by grouping method but could not complete the task.

Work-context related mathematical knowledge

Maintaining accounts

U_2 maintains accounts of the fees cutting work his family members do (viz., his mother, sisters and sister-in-law). He explained that he maintains an account by noting down the amount of “maal” (material) received, the finished products after completion of task, the amount of “maal” delivered and the corresponding wage received – all these by putting the numbers in different arrays and finding the total. He uses arithmetical operations and also does cross-checking of the calculation.

Estimation in work-context

U_2's work-context required him to use estimation skills in different ways. For example, as a novice and also as an apprentice he developed estimation sense of the amount of thread required for completing certain stitching task. His work required him to choose a suitable needle and the choice was made based on his experience and practice by looking at the thread and garment quality. Estimation skill was also used while maintaining accounts of his house-hold based *fees* cutting work, when U_2 used estimation to cross-check the aggregate and the payment made.

Fairness in wage distribution

Students often make judgements about the fairness of a deal. However, students who are learning certain tasks (like U_2 as a novice in a tailoring workshop) or house-hold based "chunked" work that involved "making" (like *rakhi* making, stone-fixing work, *latkan* making work) or fragmented tasks of a larger chain of work (like *fees cutting*) often cannot question the amount of wage that comes their way. It seems that the easy availability of cheap labour in the low-income settlement is always more than the demand which forces the workers to accept the wages given howsoever meagre they could be. When U_2 was asked to discuss what he felt about the amount he gets now as an early apprentice, he replied, "*sahi hai sir*" *[it's correct sir]*. Further probing about whether it should be more or less is quoted in the following excerpt:

Excerpt 4.4: Interview transcript of U_2

394	S	jyada kaam rahega na sir to aisa lagta hai ki paisa kum de raha hai/	if there's more work sir then it seems as if the wage is less/
395	T	achha, aisa lagta hai ki paisa kum de raha hai/	okay, you feel that you are paid less/
396	S	asteen mein jaise, pyjame mein ho gaya, asteen mein ho gaya, to lagta hai paisa kum de raha hai, do rupaya-do rupaya ka dega/	like in *asteen* (cuff), or in *pyjama*, it seems as if we're getting less, two rupees – two rupees is given/

397	T	achha...	okay...
398	S	haan, aur sirf gale ka rahega to aisa lagta hai paisa sahi de raha hai, ek rupaya/	yes, for only neck it seems the wage is correct, one rupee/
399	T	aisa kyon? Matlab kum lagta hai, jyada lagta hai, aisa kyon?	why so? you feel less or more, why so?
402	S	jyada kaam ho jata hai to itna sara kaam dekh ke aur paisa sirf do rupaya milta hai to sahi nahin lagta hai/	when there's more work then by seeing so much of work and amount given is only two rupees then we feel it's not correct/

U_2 further said that the retail selling price of one shirt in the market was Rs 110 or Rs 120, "*ek piece pe usko ek sau dus, ek sau bees aisa piece pe...*" *[on one piece he makes one hundred ten, one hundred twenty, likewise per piece..."]*. He felt he should get Rs 2 per neck stitching over the current rate of Re 0.50 if it entailed complex stitching work. U_2 said that neck sizes are different and the amount of work involved in neck stitching is complex. However, in line 398, U_2 felt that for simple stitching on *gala* (neck), the wage of Re 0.50 appeared to be correct.

Everyday shopping related mathematical knowledge

Decision making

Students (children) while doing everyday shopping at times face situations when they need to take a call upon realising that they were carrying less amount of money. U_2 said he would return and take more money or he would only buy articles that require more money, choosing to buy articles of lesser price later. Here is an excerpt:

Excerpt 4.5: Interview transcript of U_2

513	S	hua hai, bahut baar hua hai/	it has happened, happened many times/ (about money falling short)
514	T	to kya karte ho uss samay?	then what do you do then?
515	S	sir, wapas aana padta hai/	sir, need to return/
516	T	nahin maan lo ki wapas aana nahin hai, jo hai wohi leke jana hai...	no assume that you need not come back, whatever is available carry that much..

517	S	haan...	yes..
518	T	to kya karte ho? kaise?	then what do you do? How?
519	S	jo saman kam daam mein hai, woh nikal dunga sir/... apne paas waise jeb mein rahega na to laga deta hoon/ khud hi lagake paisa, ammi ko bata dunga ki itna paisa lagaya karke/	the articles which are of lesser price, will drop them sir/...if I've some in the pocket will include that, will add some money, later will tell Mum that this much amount I added/

Everyday shopping calls for decision making at many instances. While we discussed about a hypothetical situation when money that children are carrying falls short or they suddenly remembered about buying some other stuff, there are other instances that crop up during everyday shopping when the buyer faces the dilemma of choosing from a variety of qualities, prices, brands, etc. Addressing such dilemma requires use of proportional reasoning and mental computations to get the best buy.

Measurement knowledge

Use of standard and non-standard units

We observed from U_2's description and interaction as well as from the visits to the workshops that work-contexts frequently involve the old standard British units for length measurement such as *inch* and *foot* wherever required. In tailoring work, use of *inch tapes* (colloquially called *inchi tape*) is very common. There are instances of the usage of non-standard units as well. *Tedha* scale (bent scale) is an example of a non-standard instrument that is used in the tailoring work particularly in the measurement of the rounded portions of garments such as neck, collar or cuffs.

Excerpt 4.6: Interview transcript of U_2

199	T	tedha scale matlab?	*tedha* (bent) scale means?
200	S	sir uska matlab yeh side ka hota hai gale ka side usse cutting hota hai/ tehra scale/	sir it means it's bent on one side, neck side is cut using it/ *tedha* (bent) scale/
201	T	tedha scale se cutting hota hai?	*tedha* (bent) scale is used for cutting?
202	T	achha/ tedha kyon hota hai lekin? Maine to hamesha seedha scale	alright/ but why is it bent? I've only seen straight scales/

		dekha hai?	
203	S	nahin tedha hi hota hai usme/ usko tedha hi, tedhe ki bhi kaam hota hai sir usme/	no there's bend in it/that's bent, bent ones are also used sir/
209	S	sir, tedha hota hai, hockey jaisa hota hai/	sir, it's bent, like hockey [stick]/

Construction of units

Tailoring work makes use of templates for cutting the cloth pieces as per the requirement. While handling bulk orders templates become essential to maintain specifications and keep the garments identical. Interaction with U_2 revealed that he was aware of the different kinds of templates that are made for the task at hand, like *farma* and *daner*. He explained that *farma* [*template*] is cut from stiff canvas called *futta* and once a *farma* is made it is then used as a unit. Templates are examples of how units are constructed in everyday work-contexts based on the requirement and convenience.

Excerpt 4.7: Interview transcript of U_2

188	S	haan, jaisa maal wahan se aayega, bolenge, sample denge ek, to sample jitne inch ka rahega utne inch ka futta katenge, wohi sample baneyenge ek futte ka/	yes, similar to the *maal* (material) that comes from there, they'll tell, they'll give a sample, so as per the sample's dimensions in inch we cut *futta* in inches, that *futta* becomes the sample then/

Notion of covering and repeated iteration

U_2's description of *farma* indicated some other underlying notions and principles associated with the measurement concept. One of them is the notion of a cover. "*Farma*" (particularly used in tailoring and leather work) is a constructed unit and applied in cutting pieces of cloth and leather as per the given specifications. As U_2 argued, *farma* is as good as a sample itself. However, when cloth or leather is cut into smaller pieces for stitching using a *farma*, the covers are marked and cut. Similarly, if the cloth or leather piece is long enough then iterations and repeated iterations of such covers are taken. This process of cloth and leather piece cutting uses a fundamental concept of measurement knowledge that novices like U_2 learn by observing. Interestingly however, such coverings are not quantified since the need does not arise. But, observing the *master* (expert) cut the *futta* into *farma* and then the cloth or leather pieces into smaller pieces provides a rich context to young children involved in such work to gather everyday measurement knowledge.

4.5 Case – III: E_{16} – 13 year old girl

E_{16} belongs to a migrant family from Uttar Pradesh state. She lived with her parents and two younger brothers in a suburb far from the low-income neighbourhood. Earlier her family had lived along with her grandparents in the low-income settlement but had moved to a separate shanty in another low-income neighbourhood recently. All the three siblings were studying in the same school in Grades 1, 3 and 6 during Phase-I of the study. They travelled by the public transport (bus) everyday and the travel time varied between an hour and an hour and a half each way. Her attendance in class depended partially on getting a bus on time. Many a times, while returning from school she visited her grandparents who still lived in the low-income settlement near the school. E_{16} discussed that due to the sudden financial trouble that befell on the family they were overburdened with loans. Her grandmother helped the family with cash and kind. E_{16}'s father worked as a freelance electrician and got hired for work on a day-to-day basis. E_{16} and her mother worked at home – they did stone-fixing work on small jewellery. E_{16} also attended to other house-hold chores, visited nearby shops to buy provisions and other articles. The researcher during the classroom observations found her to be one of the better performing students in the class who would attempt to solve the problems the teacher gave to the students during class hour and also actively took part in the classroom discussions.

During the first and second parts of the semi structured interview, E_{16} was forthcoming in discussing about the family's engagement in work past and present, routine house-hold affairs and the family's financial condition. She explained that their current financial condition was better than what it was before. She completed her initial grades (kindergarten grades) in a private school in a middle class locality. Due to financial constraints she and her siblings were shifted to the present government-run school which did not require any fees. E_{16} mentioned that her grandmother helped her open a savings account in a bank and this news was kept secret from her father. She added, "*if my father gets to know about it he will take away all the amount*". E_{16} discussed that her father did not extend any financial help to the family, rather spent his earnings on himself and took away E_{16}'s and her mother's earnings. Under such circumstances, much of the responsibility of running the family had fallen on E_{16}'s mother and herself. From the interviews, it became clear that E_{16}'s family did not possess any electrical appliances other than a mobile telephone and a fridge which was kept in the store for storing soft-drinks and other perishable goods. E_{16} shared that she watched television only when she visited her grandmother's place.

The researcher observed that managing house-hold chores is an essential part of the daily routine of women and girls in the neighbourhood. E_{16} mentioned that every evening post supper she washed utensils (*bartan ghasna*) and cleaned the cooking area (*rasoi*). She also looked after her young siblings and helped them with their studies. E_{16} went for tuition classes in the suburb where she lived. The monthly tuition fees was Rs 600. This amount was substantially more than what most students living in the low-income settlement had to pay (between Rs 150 and Rs 250). Her tuition classes ran everyday except on Sundays and all the school subjects were covered. As discussed in Chapter 3, tuition classes were seen as supplementary to the regular classroom learning. The same textbooks used in school were used during such classes and the tuition teachers ensured that students did a lot of writing work in their separate exclusive notebooks meant for such classes. It was a general practice that those supplementary classes covered textbooks topics before they were taken up during the regular teaching in school with an intention to prepare the students in advance.

4.5.1 Exposure & Involvement in work-contexts

Stone-fixing work

E_{16} had been involved in the stone-fixing work for more than three years and a half during Phase-I of the study. She referred to her work as "*maal banana*" (making *maal*/material) and "*maal bharna*" (material packaging). Stone-fixing work involves putting coloured stones (usually up to 4) on ear-rings, pendants, rings, buckles, and *mangal-sutra* (a kind of necklace). This is a fragmented task and E_{16} did not need to have knowledge about other parts of the production network such as where is the jewellery made, where are they sold, where the raw materials are produced, and so on. Although she was aware of some of these linkages and mentioned them to the researcher but such knowledge was not common among other students engaged in house-hold based fragmented tasks. The orders came from a middle-man who provided all the materials required for the task and also collected the finished-products and made the payment. E_{16} and her mother needed to only focus on the immediate task. It is a routinised work and does not call for skill or knowledge. From the interaction it appeared that E_{16} had become adept with the routine arithmetical calculations involved in the task – generally 6 pairs or 12 pairs of ear-rings were stuck in a card and a total of 1 gross (= 12 dozen) pairs were bunched together. The wage was calculated per gross of pairs. The workers did not have to deal with customers separately or sell the goods. This made such less-paying jobs preferred as it was seen as an opportunity to supplement income by working at home. The stones are colloquially referred to as *diamonds* (English word is used). She described her work in the following excerpt:

Excerpt 4.8: Interview transcript of E_{16}

63	T	to ek ear-ring mein kitna diamond baithate ho?	so, how many diamonds do you put in an ear-ring?
64	S	three/	three/
65	S	kabhi four, kabhi kabhi thirteen stones bhi lagta hai/	at times four, sometime thirteen stones are also put/
70	T	terah kyon?	why thirteen?
71	S	woh design mein aise jhumke rahte hain/	that design has this kind of *jhumke* (hanging earrings)/

In the following excerpt E_{16} explained the wage calculation with respect to ear-rings:

Excerpt 4.9: Interview transcript of E_{16}

104	S	one forty four jodi banayenge na tho one gurus hota hai tho usme eighteen rupees hi milte hain/ gurus ka full packing karenge na thaile mein daal ke...	if we make one forty four pairs then it's one *gurus* (gross) for which we get eighteen rupees/ if we pack full *gurus* in a packet..
105	T	haan, haan/	yes, yes/
106	S	to one gurus matlab one packing ka to twenty four cards honge, … matlab one card mein six jodis lagte hain, uss tarah na apun … twenty four cards ka packing karenge na, to apne ko nineteen rupees milega aur nahin	then for packing one gurus, one packing has twenty four cards, … [and] one card has six pairs, … if we pack twenty four cards, then we get nineteen rupees and if we don't do packing we get eighteen rupees only/

		packing karenge to eighteen rupees hi milega/	

Jewellery packaging work

E_{16} and her mother also packed jewellery in cards after fixing the punch-pins or holding pins (called *tachni*). They needed to pack six rings or pendants in a box and twelve such boxes are placed inside a bigger box for which they got a wage of Rs 15. She explained the process in the following excerpt:

Excerpt 4.10: Interview transcript of E_{16}

394	S	woh matlab pendant punch hai, usme ear-rings daale hain, yahan pe aisa shape rahta hai, usme apne ko pendant daal ke, chain rahta hai, chain mein pendant daal ke, usme tachni lagake dibbe mein bharna rahta hai/	That is pendant punch (speaking about *tachni*), ear-rings are put, here the shape is like this, we need to put pendant in it, there’s chain, put pendant on a chain, then after putting *tachni* in it, they are placed in the box/
399	S	aisa ek bada box hota hai, usme six ring aate hain, aisa six cheez bharna rahta hai, aisa twelve, twelve bade bade box bharenge na twelve, one box mein six pairs six aate hain, aisa twelve box bharenge na bade bade to fifteen rupees milte hain/	There’s a big box like this, there’re six rings in it, likewise six articles are placed in it, if twelve, twelve big boxes are filled, one box has six pairs, likewise when twelve big big boxes are filled, we get fifteen rupees/

Stone-fixing work carries an extra rupee for packaging. For example, for fixing three-stone one gross ear-rings E_{16} gets Rs 18 and packing all the twelve or six cards together (totalling one gross), she earns an extra rupee, i.e., Rs 19. Though a meagre amount, E_{16} makes the effort to earn the extra rupee. The adhesive used in the stone-fixing work is purchased by the workers. E_{16} explained that two different kinds of adhesives are bought, yellow and white and while pasting they are mixed together. E_{16} could not explain how much adhesive one bottle contained. But, she estimated that one bottle can be used to make up to 200 “gurus” of ear rings.

Stages of learning

E_{16} discussed the procedures through which workers like her start getting work beginning with *chalu maal* (regular material) and gradually moving up to do *fancy maal* (fancy, expensive material). She explained that novice workers or beginners in the stone-fixing work are first asked to undergo a drilling practice – *haath saaf karna [hand cleansing]* which involves working on “simple cards” (a typology used by E_{16}) using non-expensive, simple stones. Here is an excerpt from E_{16}’s interview where she explained about the different stages through which one learns the task:

Excerpt 4.11: Interview transcript of E_{16}

229	T	ok/ achha, tum yeh batao yeh jo tumlog kaam karte ho na yeh wala ear rings banane ka, to isko kaise sikha? Isko sikhne ke liye koi training leni padti hai?	ok/ alright, tell me the work that you do of earrings, so how did you learn this? do you need to undergo some training to learn this?
230	S	woh log first of all, matlab wohlog ke paas jaise yeh ear-rings aate hai, wohlog aise simple cards dete hain/	they first of all, like when they get the earrings, they give simple cards/ (to work on simple cards)
231	T	sample?	sample?
232	S	sirf card/ to usme apne ko ear-rings daal-daal ke sikhna, page lagane padte hain/ pehle wohlog aise hi dete hain, baad mein bolna padta hai apne ko ki stone lagane ka kaam humlogon ko milega? humlogon ko aata hai; wohlog pehle jo unlogon ke paas chalu maal rahta hai, wohlog sirf fevicol dete hain, fevicol sirf card mein, ear-ring mein phir fevicol lagake chalu stone lagane ka rahta hai, woh matlab hi-fi stone nahin rahta hai, yeh wala stone nahin rahta hai, woh chalu stone rahta hai, pehle log chalu kaam se shuruwat karte hain phir wohlog apne ko fancy maal dete hain/	only card/ then we learn to mount earrings on them, we learn to put page [holder]/ first they give like this, later we ask them if we can get stone-fixing work? that we’ve learnt the task; first they ask us to work on the *chalu maal* (regular material) that they’ve, they only give fevicol (a popular brand of adhesive used in house-hold chores), we learn to use fevicol on the cards, on earrings then learn to fix stones using fevicol. those are not *hi-fi* stones, first people ask to begin the task using the *chalu kaam* (regular work), then we are given the *fancy maal* (fancy material)/

The wage given during this stage (training phase) is less. For filling cards with earrings bearing *chalu maal* they are given Rs 4 instead of Rs 6 (regular wage in case of 3-stone, 1 gross of earrings). In the following excerpt 4.12, E_{16} described the wage:

Excerpt 4.12: Interview transcript of E_{16}

233	T	accha/ to chalu kaam se jab shuruwat karte hain to koi paise milta hai?	okay/ so when you begin with *chalu kaam* do you get some money?
234	S	haan/	yes/
235	T	kitna paisa milta hai?	how much money is given?
236	S	jaise woh stone ka kaisa, sirf card bharenge na chalu to four rupees/	like for that stone, for filling only cards *chalu*, then four rupees/

Running stationary store

Apart from the stone-fixing work, E_{16} and her mother also ran a small store of stationary goods in the front portion of their one-room tenement. One corner of the store had a telephone booth from where customers could make telephone calls on payment. The family dealt in stationary goods like chips and snacks, biscuits, chocolates, toffees, soft-drinks, provisions like cereals, eggs, noodles, and so on. They also sold a variety of chewable tobacco of different brands. E_{16} said that she managed the counter in her parents' absence. She kept accounts of the store in a diary and reported to her mother about the transaction that occurred in her charge. E_{16} explained that most articles and goods came in small sachets or packets which had their weights and prices marked on them. She simply checked the price and quoted the same to the customers. She also said that the articles came in different weight measures and she pulled out the one that customers asked for. She read out price to the customers. At the time of the third interview (on work-contexts) E_{16} informed the researcher that the store was closed down. E_{16} and her mother managed running the store and their jewellery work simultaneously.

E_{16} described that on holidays like Sundays and sometime on Saturdays she got time to work more and did stone-fixing work the whole day. Hence, she did not go out on holidays unlike some other students (like E_5) who took time off on Sundays to be with friends. E_{16} mentioned that it was customary that girls did not venture out much and without purpose. In the following excerpt E_{16} described the amount of work she does on holidays:

Excerpt 4.13: Interview transcript of E_{16}

367	T	...saturday, sunday, jab chhutti rahta hai na, chhutti ke din kitna banta hai?	on saturdays, sundays, when there's an off day, how many [card] is made on holidays?
368	S	mera kum-se-kum forty, forty-five rupees ka kaam karti hoon/	I do at least make forty, forty-five rupees worth of work/
369	T	forty, forty-five rupees ka kaam?	work worth forty, forty-five rupees?
370	S	haan/	yes/
371	T	matlab kitna card banta hai?	so how many cards are made?
372	S	main to kabhi-kabhi subah se baithte hain na humlog, recess dopahar mein hoti hai, phir wapas se shaam se raat mein to kum-se-kum seventy-four rupees ka kaam kar hee lete hain ek din mein/	I make sometimes by working since morning, with a recess in the afternoon, again from evening till night, I work at least seventy-four rupees worth of work in a day/
373	T	Seventy-four rupees ka/	of seventy-four rupees?
374	S	ek din mein/	in a day/
375	T	kitna card ban jata hai?	how many cards are made?
376	S	fifty, sixty/	fifty, sixty/

4.5.2 Features of mathematical knowledge

Arithmetical knowledge

E_{16} could read all the numbers shown on the number cards including the 5 digit numbers and wrote correctly all the numbers called out. In the currency task, she was able to take out amounts of money equivalent to a given number. E_{16}'s oral enumeration of numbers was correct and so was her count-on beyond a number using currency.

While solving the contextual problems posed during the interview, E_{16} used convenient strategies and not the school learnt algorithms, unlike U_2 who preferred his own out-of-school strategies at the outset and then fell back upon school taught algorithms. For example, for finding 35×10 in a contextual problem, E_{16} used "doubling" strategy and doubled 35 three times to arrive at "280" and then she added "20" and "50" respectively to conclude that "350" was the answer. One can notice here that having computed "280", E_{16} realised that she had calculated 8 times 35 and two more 35s remained to be added and that she needed to add 70 more. She conveniently decomposed 70 into 20 and 50 to do the remaining addition. Interestingly, she did not use the "rule of 10" for multiplication i.e., "to put a zero to the right" the common school-taught strategy. In other problem tasks of multiplication and division, she first estimated the answer and her estimations were either accurate or close to the actual answers. In the proportional reasoning task, she did not use the unitary method (school taught) but used halving strategy.

In the contextual shopping task, E_{16} used oral computation for finding the price differences between two sets of articles. She estimated which all articles could come against a note of Rs 500 and Rs 1000. E_{16} neither used any school taught algorithm for solving any of the problem tasks, nor did she use paper and pen for writing while calculating. All her computations were oral. As mentioned before, it was interesting to see that E_{16} relied more on her convenient strategies and everyday mathematics for computations, though she was a better performing student in the class and knew to use the formal, school learnt algorithms well.

Work-context related mathematical knowledge

Mental Calculation

Store managing work provided E_{16} with opportunities to carry out calculations. She did calculations either on paper or mentally. During interactions it appeared that she knew the prices of the popularly sold goods like chewable tobacco (Re 1 or 2 per small sachet), sugar (Rs 34 for a kilo and Rs 17 for half a kilo) and so on.

In stone-fixing work, E_{16} reported that she calculated the wage that is due to them and she did oral computations. Explaining the wage of 3-stone earrings she said, "*agar three stones ka twenty-four ear-rings banayenge na to one rupee fifty paisa aata hai*/ " [*if twenty-four ear-rings are made of three stones each then we get one rupee fifty paisa*/] The wage for fixing stones for one gross of earring pairs is tabulated below:

Table 4.5 E_{16}'s wage for stone-fixing work on jewellery

Ear-rings (in pairs)	Single-stone	2-stone	3-stone	4-stone
Wage per dozen	Re 0.50	Re 1.00	Re 1.50	Rs 2.00
Wage per gross	Rs 6.00	Rs 12.00	Rs 18.00	Rs 24.00

The wage for fixing stones on other artefacts like buckles, rings, pendants and necklace varied roughly proportionately. E_{16} knew about the wages for each of the various artefacts and did quick calculation of the wage for a given quantity of work. The researcher noted that the entire process of stone-fixing, packaging and wage calculation involved mental computations. The researcher observed that E_{16}'s involvement with stone-fixing work for more than three years had helped her develop number sense that involved multiplication of numbers that make one gross (144 pieces). For example, 6 × 24 and 12 × 12 and their multiples. She was conversant handling such numbers which were multiples of 12 (a *dozen*) and the wages which were multiples of 6 (half a *dozen*). She was swift in switching between these numbers and came up with quick mental computations.

Arrays of numbers

As discussed above and also from the interview, it appeared that in her work, E_{16} uses array of numbers while mounting the jewellery (e.g., earrings) on cards. The arrays are usually of the 6 × 2 or 6 × 4 or 12 × 2 format per card. In other words, each card consists of either one or two dozens of earrings mounted on them and 12 or 6 such cards are packed respectively so as to make one gross. However, from the interaction with E_{16}, it was not clear whether she was comfortable with other arrays that added to 144 or to other numbers.

Maintaining accounts

The stone-fixing and jewellery work involves maintaining accounts of the finished products and the left overs. E_{16} reported that she maintained an account in a *chaukri* (diary). Similarly, she kept an account of the store when she ran the counter. On the whole, E_{16}'s all the three work-contexts involved use of numbers in some ways.

Decision making

Interactions with E_{16} revealed that she had to take a call for the kind of jewellery she would like to work on. She made a decision based on her own calculation of the better deal coming from the comparative wage calculation and the relative profit made. Here is an excerpt from the interview where E_{16} compared the wages:

Excerpt 4.14: Interview transcript of E_{16}

452	T	accha/ kisme jyada fayda hai? Ear rings mein, pendants mein?	okay/ which [work] gives more profit? earrings or pendants?
453	S	ear ring mein/	earring [work] gives/
454	T	ear ring mein fayda hai? Kyon?	earring gives profit/ why?
455	S	kyunki, woh fatafat banta hai, yeh sab jaldi nahin banta hai, kantal aata hai/	because, it is quickly done, rest of these can't be done fast, we get tired/
456	T	jaldi banta hai to fayda kyon hai?	why is quickly made profitable?
457	S	kyunki woh banta hai phir delivery ho jata hai phir dusra maal aata hai, aisa karke, jyada maal banta hai to jyada paisa aata hai/	since it's made and delivered and then next *maal* (material) comes, like this, more work is done and we get more money/

Fairness in wage distribution

E_{16} informed the researcher that the usual rate of selling one pair of a three-stone earring in the market was Rs 5[1] or more per pair. She showed the researcher the three-stone earring she was wearing and said that it was bought for the same amount. At this point, the researcher asked her to calculate the wage that she got for making one card that contained twenty-four pairs of the such earrings. E_{16} mentally computed the selling price as Rs 120 (= 5 × 24). The researcher pointed out that for making six such cards (1 gross) she got Rs 18. E_{16} calculated that the wage for making one card with 24 three-stone earrings on it was Rs 3 compared to its selling price of Rs 120. The researcher then asked E_{16} what she thought about the wage given to her. She responded that others in the production chain too did not make much profit. She added that the seller at the market hardly saved only Rs 0.50 per

1 The researcher however felt that it was a conservative price estimation and the actual market retail price was much higher.

pair of earring by selling it for Rs 5. The researcher indicated to her that at this rate the seller made a profit of at least Rs 12 per card of 24 earrings whereas workers like her got only Rs 3 for making one such card.

The discussion on low wage distribution failed to evoke any response possibly because of the underlying power structure. E_{16} did not seem to realise the injustice and the low wage given to her and to other workers like her. Rather her response indicated justification for the wage structure. She claimed that other costs were incurred before the material went to the retail shopkeepers for selling. According to her, there were underlying invisible costs like, those incurred during dispatch, designing and polishing charges, and others' wages. She added that she was "happy" with the wage she was getting. Here is an excerpt of the conversation:

Excerpt 4.15: Interview transcript of E_{16}

346	T	one twenty rupees mein becha/ to tumko sirf three rupees mila aur usne kitne mein becha?	sold for one twenty rupees/ so you got only three rupees and how much was it sold for?
347	S	one twenty/	one twenty/
348	T	one twenty mein/	for one twenty/
349	T	to yeh tumko kya lagta hai?	so what do you think about it?
350	S	unlogon ko profit nahin hota hai sir, kyunki humlog yahan pe banate hain, humlog company mein dete hain, company se tempo mein jata hai, wahan se display matlab dusre aadmi ke paas jata hai, wahan se bhav badh-badh ke wohlog ke paas aata hai, wohlog ke paas bhi...	they don't have profit sir because we work here, we hand over to the company, from company all this goes in a tempo, from there it goes for display, i.e., goes to some other person, from there the rate rises higher and higher and comes to the sellers, sellers too...
351	T	lekin phir bhi, tumhe nahin lagta ki ek sau...	even then, don't you feel one hundred...
352	S	unlogon ko jyada profit nahin rahta hai, unlog four rupees fifty paisa mein kharidte hain to five rupees mein bechte hain, unlogon ko bhi aise hee hota hai/	they don't have much profit, they buy [one pair] for four rupees fifty paisa and sell for five rupees, they too need to face this/
…	…	…	…

356	S	sir, wohlog jo company se humlog laate hain, woh bhi design banata hai, to design ko polish karta hai, polish karke design ko card mein lagane ka hota hai, ..., to unlogon ko bhi paisa dena rahta hai, to isliye bhav karke hee market mein jata hai/	sir, the company from where we bring [work], they make designs too, then they polish [refine] the design, after polishing the designs are put in the cards, …, so they're required to be paid too, so, the rate rises and rises before it [material] goes to the market/
357	T	achha, to tumko aisa lagta hai ki yeh bhav jo tumko mil raha hai woh,	okay, so you feel that the rate that you're getting is,
358	S	Sahi hai/	it is fair/
359	T	sahi hai?	is it fair?
360	S	(nodding in affirmation)	(nodding in affirmation)
361	T	achha/ kum nahin lagta tumko?	okay/ don't you feel it's less?
362	S	(nodding in 'no')	(nodding in negation)
363	T	Sahi lagta hai/ tum khush ho iss bhav se?	feel it's fair? are you happy with this rate?
364	S	(nodding in 'yes')	(nodding in affirmation)

Everyday shopping related mathematical knowledge

E_{16} reported that she buys provisions like milk, rice, *daal* (lentil), sugar everyday. She explained that she bought rice and *daal* in the evening. She carried eighteen rupees for buying rice and *daal*. Here is an excerpt:

Excerpt 4.16: Interview transcript of E_{16}

507	S	kyunki, nine rupees ke daal laate hain aur eight rupees ke chawal/	since, I bring *daal* of nine rupees and rice of eight rupees/
508	T	nine rupees ka kitna daal aata hai?	how much *daal* comes for nine rupees?
509	S	one fifty grams/	one fifty grams/
520	T	chawal eight rupees ka kitna aata hai?	for eight rupees how much rice comes?
521	S	aadha kilo/	half a kilo/

Measurement knowledge

E_{16}'s work-context entailed use of different measurement modes and units and she used such knowledge. We discuss in this section features of measurement knowledge that E_{16} encountered and used in her work.

Sorting

E_{16}'s work involved sorting of jewellery based on four categories: *ear-rings, buckles, pendants* or *rings*. Each of these jewellery was further grouped as having single-stone, two, three or four stones. These were sorted, collected together, worked on and packed separately. Sorting was also done based on the number of jewellery mounted on the cards, viz., the 6 × 2, 6 × 4 and 12 × 2 arrays of cards were collected separately.

Use of multiple units

Stone-fixing work entailed use of different units of counting. Some of the old British units are still in practice like *dozen* and *gross* (twelve dozen). E_{16} seemed to be conversant not only with these units but also with their inter-conversions.

Construction of units

For convenience, people involved with the stone-fixing work construct a different unit – instead of 144 pieces (12 dozen; 1 dozen = 12 pieces), E_{16} reported that sometimes 1 gross was taken to be equal to 140 pieces. The unit name remained the same (gross) but its representation changed to 140 from the standard 144. E_{16} described that handling 140 and multiples or parts thereof was easier to work with than those of 144.

4.6 Case – IV: U_{13} – 13 year old girl

U_{13} is from the Urdu medium school. She lives in the low-income settlement with her parents, two sisters and a younger brother. U_{13} is the eldest among her siblings. One of her sisters was 3 months old at the time of the work-context interview. The family lived in a rented room on a rent of Rs 1000 per month. Her father did stitching work and made shirts and trousers. He was employed as a *karigar* (worker) in a garment making workshop in the locality. U_{13}'s mother was involved in *rakhi* making work (decorative wrist bands which sisters tie on brothers' wrists during an annual festival of the Hindus) but now she had taken a break from work to attend to her infant child. U_{13} remained engaged in different kinds of work at home doing them at different times of the year. She mentioned that she was involved in making *rakhis*, *fees* cutting work, garment repairing work at home apart from other house-hold chores.

U_{13} explained that while her mother prepared lunch, she cooked supper everyday for the family and cleaned the cooking area. She said, "*subah school chali aati hoon to subah ammi pakati hai aur shaam ko main rahti hoon to main pakati hoon/ shaam ko meri chhoti behen hai na* [ammi] *usko sambhalte hain*/ [*mornings I come to school so mum cooks and in the evenings I'm around so I cook/ mum spends evenings attending my younger sister*]. U_{13} accompanies her mother sometimes for buying vegetables. She also goes to buy provisions when sudden need arises, like need of spices, salt, sugar, tea powder and so on. She does baby sitting when her mother goes out. U_{13} also reported that in the locality, one has to look out for work and often the workshop owners shift places and workers who cannot relocate look for fresh work. For example, U_{13} reported that the garment workshop which provided her *fees* cutting work had moved to some other place, "*fees jahan se humlog laate the wahan se karkhana khali karke kahin aur chale gaye humlog ko nahin maloom*" [*place from where we would bring fees from, they have vacated the place and moved to some other place, where we do not know*]. She added that once her *rakhi* work was over, she would have to look for some other work afresh.

During interaction with the researcher, U_{13} mentioned that her family had financial constraints and therefore they did not keep expensive objects, electronic gadgets or a mobile phone. She seldom brought “tiffin” (lunch box) to school and mostly took the mid-day meal provided by the school authorities during the school recess.

U_{13} shared that she spent her leisure time doing *rakhi* work and did not go out for play or to spend time with friends who she said also worked. The family did not have savings accounts in any bank. U_{13} mentioned that at times, her mother borrowed money from a relative who did not charge any interest. U_{13} went for tuition classes till she was in Grade 4. She cited financial crisis for not going to tuition classes anymore. None of her siblings attended tuition classes either. She claimed that she taught her siblings and added, “*main unhe lessons yaad karwati hoon*” [*I make them learn (memorize) the lessons*].

U_{13} had a limited outdoor activity and she did not go out of home often unless there was some work (like buying some stuff or fetching or giving something to the neighbour). She said her father did not allow her to go out. When asked if she liked visiting some place, she replied, “*kahin gayi nahin to kaise bataun kya achcha lagta hai*” [*never visited any place so how do I say what do I like*].

We came across some prejudices while interacting with the students. Celebrating birthdays for instance, was considered a taboo among some families. U_{13} quoted her father saying, “*janamdin manana gunah hai*” [*celebrating birthdays is a sin*]. Similarly, watching movies is prohibited for many children in the locality since watching movies are seen as bad by some elders. Financial constraint is not the only reason for not allowing children to watch movies.

4.6.1 Exposure & Involvement in work-contexts

U_{13} has exposure to different work-contexts because of frequent change of jobs. She and her mother make *rakhi* for half a year. This work begins in February-March every year and lasts till the *rakhi* festival which falls in the month of August. August onwards the family shifts to other house-hold based work like *fees* cutting or garment “repairing” (darning work). In this section, we discuss U_{13}’s exposure to some of the work-contexts.

Rakhi making work

U_{13} was involved in the rakhi making work for more than 3 years at the time of the interview. *Rakhi* making is a common household based seasonal work in which mostly women are involved. The raw material for making *rakhi* is delivered at the worker's home by middle-men who collect the finished product and send them for packaging. U_{13} explained that her mother has contacts with people who give work but sometimes she also goes to fetch work. Similar to stone-fixing work, *rakhi* making work does not require awareness about other aspects or linkages on the production network. The workers do not have to deal with retail sellers or retail customers. They only focus on their fragmented task at hand. U_{13} told the researcher that her 9 year old sister, studying in Grade 3 then, also knew *rakhi* making. She observed and also assisted U_{13} and her mother in their work.

Rakhi making involves putting together on the base paper, a decorative typically round shaped, symmetrical, flower-like shining disc or a design sequinned on top with "*zari*" design or a decorative stone. The symmetrical, decorative discs are supplied by the middleman who brings work. Workers like U_{13} only need to prepare the bottom base which is usually made of paper and glue all the materials together before passing a thread using a needle from the middle of the base so that all the parts of *rakhi* are tied and glued together and another thread at the bottom is stitched that is tied on the wrist. A sample *rakhi* is given to the workers who cut a template as per the specifications of the base paper and using this template other base papers are cut. The decorative flower-like discs come ready made and *rakhi* makers use them as they are.

The wage for *rakhi* making is given per gurus (gross). The amount varies between Rs 10 and 15 per gross of *rakhi*. U_{13} reported that usually she gets orders for making around 20-25 gross of *rakhi*. U_{13} said that she and her mother together made 1 to 2 gross of *rakhi* per day. According to her, making of one *rakhi* took around 2 minutes of time and going by this rate, making of 2 gross of *rakhi* took around 4-5 hours of continuous work in a day. Like stone-fixing work, *rakhi* making too is a minute task needing deeper concentration that causes strain on the eyes. Once *rakhi* making is over, they are counted and bunched together in gross and then placed in *thaili* (small bags). U_{13} mentioned that she did not require to do the packaging work which was done by another group of workers.

Stages of learning

U_{13} discussed that *rakhi* making required the doer to learn *banana* (making) and *urana* (thread work using a needle while mounting sequins on a *rakhi*). Like stone-fixing work, when the prospective worker feels she is ready to make *rakhis*, she can approach a middle-man for getting work. U_{13} clarified that anyone can learn *rakhi* making or *fees* cutting and only a few days' of practice is sufficient to start the work.

"Fees" cutting work

U_{13} and her mother were involved in *fees* cutting work in the past. They got the orders from a nearby garment stitching workshop. U_{13} informed the researcher that the wage was Re 1 or 2 per garment piece for fees cutting and folding. She along with her mother removed threads from newly stitched shirts which came in the bundles of 50 or 100. She explained, "*fees mein sirf dhaage hi katna rahta hai na, fatafat katke rakh deti jitna bhi rahta hai, jyada dhaaga nahin laga rahta hai/*" [fees requires only thread removal, I cut them quickly and keep them, there aren't much threads]. She used a cutter for removing the extra threads from the shirts. However, at the time of the interview, the garment making workshop had relocated to a place that was not known to U_{13}. Therefore, she did not know what she would do post-August when *rakhi* work would get over.

Garment "repairing" and darning work

U_{13} spoke about repairing work that she did on garments. This is also a popular house-hold based work that women and girls do at their homes. Small tears or holes on clothes are mended by darning the torn parts or stitching patches. The wage depended upon the nature of work and varied between Rs 0.50 and Rs 2 per piece of work. U_{13} said she did this work more often before getting into *rakhi* making. U_{13} knew about her father's stitching work of shirts and trousers that he did as a *karigar* (worker) in a garment-making workshop. From the interview it appeared that she was also aware of the work processes and the stages of learning involved in her father's work.

4.6.2 Features of mathematical knowledge

Arithmetical knowledge

U_{13} faced difficulty in reading and writing numbers bigger than 3 digits and made place value errors. But, in the currency task, she could take out requisite amount of money equivalent to the given numbers – 165, 2725 and 13206. In the number enumeration task, she got stuck at the transition to the next hundred or thousand (viz., enumeration beyond 88 and 995). She however correctly enumerated beyond 595. In the “count on” activity using currency notes and coins, she could complete most tasks though she had some confusion at the transition from 3 to 4 digits.

In the contextual shopping task, U_{13} could estimate that each of the two sets of article would come against a 500-rupee note, but could not calculate the balance amount. She did not use paper for the calculation involved in this task. However, in the subsequent problem tasks, she used formal algorithms for finding 35×10 and 16×7 and computed them on the given paper. But, in the division problem of 315 ÷ 5 and 400 ÷ 25 (where she had to find the price that one family pays if 5 of them together purchased one large can of oil for Rs 315; and the number of *bindi* packets made out of a collection of 400 *bindis* when each packet contained 25 *bindis*), she struggled to figure out the appropriate arithmetical operation to be used and tried multiplication and addition but did not proceed further upon realising that the answers she got were absurd and unrealistic (see fig. 4.1). In the proportional reasoning problem, she thought for a while but did not proceed.

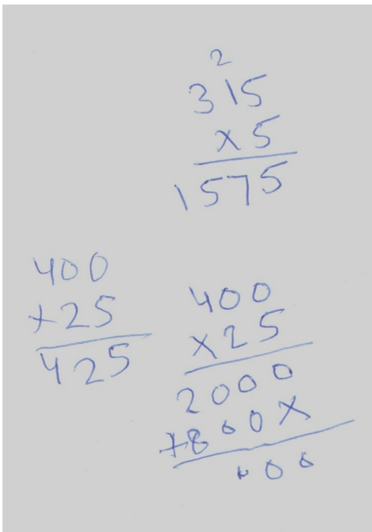


Fig. 4.1 Glimpse from U_{13}’s work sheet

Work-context problems related mathematical knowledge

Proportional reasoning

U_{13} used proportional reasoning to ascertain which work gave her relatively more money. She compared the wage she got in *rakhi* and *fees* cutting work and concluded that *fees* cutting work was more profitable since the payment was more and the work took less time.

Excerpt 4.17: Interview transcript of U_{13}

327	T	to tumko kya lagta hai kisme jyada fayda tha woh feeswale kaam mein ya rakhi wale kaam mein?	so what do you feel which work gave you more profit, that *fees* work or *rakhi* making work?
328	S	fees/	*fees/*
329	T	fees wale kaam mein fayda hai?	*fees* making work gives more profit?
330	S	(yes)	(yes)
331	T	kyun?	why?
332	S	kyunki isme ek fees mein ek ya do rupaya milta hai aur usme barah darjan banane mein pandrah milta hai/	because cutting one *fees* fetches one or two rupees and in this [*rakhi* work] making of twelve dozen fetches fifteen/

Use of symmetry and congruence

From the description of U_{13}, it appeared that *rakhi* making entailed use of symmetry and congruence. U_{13} described how she used a *rakhi*-template to cut the base paper and also used thread and needle to stitch *rakhi* over the thread. The notion of symmetry was used in *rakhi* work while passing the needle from the middle and equally dividing the *rakhi* into two parts, and also while fixing sequins (*zari*) symmetrically. Use of the notion of congruence in the task appeared to be implicit, for example, while using the template to cut the base paper. Visual cue was used for maintaining symmetry and congruence. U_{13} explained the process in the following excerpt which underlines the use of symmetry in her work:

Excerpt 4.18: Interview transcript of E_{16}

364	S	... yeh dekho, yeh sui hai, woh sui hai, abhi pehle yeh daalne ka, pehle isko daale, fansa diye, uske baad ek zari daalne ka, fansa diye, uske baad ek sa kagaj rahta hai usko daal ke isko fansa dene ka aur iske baad neeche se dusra dhaga jismein baandhte hain haath mein woh dhaaga daalke isme khinch ke nikal lene ka/	..look here, here's a needle, there's a needle, now first put this [decorative flower-like design], fix it [to the thread], thereafter put a *zari* [sequin], fixed, now put a paper [base paper] and fix it [glued], beneath which another thread is put the one which is tied on the wrist, put the thread and pull it across.

Maintaining accounts

U_{13} described that she maintained accounts in a *chaukri* (diary). The researcher however, has not looked at any *chaukri* of any student as he did not find it ethically sound to see someone's accounts. The description of accounts is presented here based on students' explanation. U_{13} kept accounts of the *rakhi* work – amount of *rakhi* raw materials received and the number of *rakhis* delivered – all in different columns. She also kept an account of the daily provisions that comes. She said that her mother asks her to keep this account. She explained that she computes the aggregates on a paper.

Fairness issue

Fairness is seldom taken into consideration in the world of work, which is governed far more by possibilities and bargains. For poor children in the metropolis, fairness is not easy to grasp. Wages are often not correlated with hours of work, entitlements are not equally or fairly distributed in society, rewards and punishments are socially manipulated to favour a few over the majority (Bose & Kantha, 2014). To cite an example, when the researcher discussed with U_{13} whether she was satisfied with the wage for making *rakhi* she answered in the affirmative. On asking she could compute and tell the retail price of one dozen *Rakhi* – at least Rs 60 (one *rakhi* is sold for Rs 5; 1 USD = Rs 60 approx.), whereas for making one gross (12 dozen) *rakhi*, she got Rs 15 or less. The researcher helped her calculate the retail price of one gross *rakhi* – Rs 720 and compared it with her wage (Rs 15 or less). Subsequently, U_{13} estimated that she perhaps did not even get half a rupee for making one *rakhi* [*humko, ek rakhi to aath aana bhi nahin milta hoga*]. But in broader terms, the discussion did not trigger any concern about fairness of wages in the student. The payment for making the *rakhis* is held up till all *rakhis* are sold out. As they are sold only near the festival time that falls in August, workers like U_{13} wait for the payment till then. The following is an excerpt from the conversation:

Excerpt 4.19: Interview transcript of U_{13}:

185	T	To ab, paisa kaise milta hai?	so now, how much wage is given?
186	S	paisa gurus ke hisaab se milta hai/	wage is given according to gurus/
187	T	matlab, ek gurus ka kitna milta hai?	so, how much for a gurus?
188	S	ek gurus ka pandrah rupaya, pandrah, barah, aise, dus/	for one gurus, fifteen, fifteen, twelve, may be, ten/
189	T	poore ek gurus ka?	for a whole gurus?
190	S	Haan/	yes/
191	T	matlab ek sau chhauuwalis rakhi ka pandrah rupaya milta hai?	so, for one hundred forty-four rakhis fifteen rupees you get?
192	S	Haan/	yes/
193	T	achha, yeh, tum iss rate se khush ho? Ya aur kam milna chahiye, jyada milna chahiye, kya lagta hai?	ok, so are you happy with this rate? or, you should get less, more, what do you feel?
194	S	pandrah, pandrah bees rupaya milna chahiye/	fifteen, fifteen twenty rupees we should get/
195	T	pandrah bees rupaya milna chahiye? Matlab kam milta hai?	should get fifteen twenty rupees? so, getting less/
196	S	Haan/	yes/
197	T	accha, to wohlog ek rakhi kitne rupaye mein bechte hain?	so, how many rupees do they sell one rakhi for?
198	S	ek rakhi wohlog paanch,dus rupaye mein bechte honge/	they must be selling one rakhi for five, ten rupees/
199	T	Paanch-dus rupaye mein bechte hain? Aur tumko, tumko kitna milta hoga ek rakhi ke liye?	sell for five-ten rupees? and you, you get how much for one rakhi?
200	S	humko, ek rakhi to aath aana bhi nahin milta hoga/	perhaps don’t even get *aath aana* (eight aana; equal to 50 paise or half a rupee) for one rakhi/

Everyday shopping related mathematical knowledge

As mentioned before, U_{13} had limited opportunity to venture out and therefore unlike other students she went for everyday shopping less frequently. She mentioned occasionally going for buying daily requirements for cooking like chilly, coriander, turmeric, etc. She said that she buys on cash and never on credit. She counts the balance amount returned and also checks the measurements.

Measurement knowledge

U_{13}'s work-contexts entailed comparison, putting marks and cutting. For instance, following a sample *rakhi* provided by the person who brings work, other *rakhi*-templates (paper base) are cut following different designs prescribed implicitly using the notion of congruence. U_{13} explained that her task did not require frequent use of measuring scales or tapes. She does visual comparison and cuts one template or "paper base" which is then used to make other templates. She used SI units like *centimetre* while describing the procedure for cutting templates. She explained the varying lengths of *murabba* (square) in *centimetres* and did not use old standard units like *inch* or *foot* that are popular in the locality and frequently used by other students. She also described the use of other geometric figures like *circle* for cutting the templates. The researcher noted that in case of U_{13}'s work-context though there was a limited requirement of using diverse measurement instruments, but it involved abstract notions of measurement knowledge such as the notions of covering, iteration, concepts of similarity and congruence. Such notions however remained implicit and embedded in the work-context.

4.7 Drawing the cases together

Similar to the "stories" of the four cases discussed above, the other six cases too reflect engagement with different work practices that bring varied experiences that involve mathematical aspects. For example, E_6 (boy) has experience of button-stitching through assisting his father in running the workshop and of mobile phone repairing through helping at his brothers' shop; E_8's (boy) experience is similar and involves handling different mobile-parts and optimisation of costs and repairing charges. U_8 (girl) has experience of *latkan* (door hanging) making work that involves handling different kinds of sequins and decorative articles. All these work practices involve different arithmetical skills such as computation, maintaining accounts, optimisation, sorting, estimation and decision making to varying extent. In addition, there are tasks that require specialised skills, viz., block printing work (dyeing) or *zari* stitching or tailoring work that requires training through a series of stages. These are in contrast to other fragmented tasks that do not have any prerequisites at all and one can quickly get into the earning mode. We observed in not only the 10 interviews of the sub-sample of students, but also in the interviews of the additional 7 students, that all the 17 students had in-depth understanding of their respective work-context, about the procedures involved, the raw-materials and their costs and so on. This was evident from the minute details that each one of them was able to present while explaining his or her work. In addition, some students knew about other work-contexts happening around them. We noted that some kinds of work, mostly done by men, involve diverse interactions with people and material, leading to greater opportunities to acquire knowledge and skill. We thus find economic activities that are varied and call for a range of knowledge and skills as well as activities that are routinized, making little demand on skills and knowledge.

4.7.1 Gender aspects of work-context

In some work-contexts, especially those which are typically done by women and girls at their home as in the case of U_{13}, U_{11}, E_{13} (engaged in *rakhi* making), E_{16} (engaged in stone-fixing work on jewellery), E_{15} (cleaning/cosmetic work in parties), *piece* making work (hand-stitching work, especially designs or sequins on garments) of U_9, U_{10}, and E_{13} – the opportunities to use diverse goods or raw materials or awareness about the linkages that their work has with other tasks on the production network are very constrained. Women in the community and school going girls like those mentioned above are mostly involved in those kinds of work which require working at home and are perceived as less technical. For instance, in the cases of U_{11} (*rakhi* work earlier, garment recycling work during interview), U_{13} (*rakhi* making) and E_{13} (*rakhi* and “piece” making work) they did only a small chunk of the entire *rakhi* work or *garment manufacturing* work. Although their work was large in terms of quantity of output and the economic production, there was little diversity in the work and such tasks were mostly "isolated" and "fragmented" (Bose & Subramaniam, 2013). Managing house-hold chores remains as essential part of the daily routine of women and girls, and only spare time is used to generate some income on the side, except in some cases where we found women worked for longer duration (E_8's mother following his father's death). We observed the practice of gender-labeling of certain work practices which are deemed as "women's work". Such stereotypes are often heard from the children at different locations – classrooms, school ground, out-of-school contexts, workplaces, and so on. Typically, house-hold based fragmented tasks are deemed as women's work such as *fees* cutting, *piece* making, *rakhi* making, etc.

We observed that the opportunities that the work-contexts provide for most of the girls in terms of learning are severely constrained. An indication of this is the sparse and limited responses that we got, from students with limited exposure and no sense of ownership towards the work, to questions involving the application of mathematics to questions about income and fairness of earnings (discussed in chapter 5). The case of E_{16} is an exception but even for her the awareness of the linkages of her work-context was limited in comparison to E_5, E_{11}, $U_{22,}$, U_{23}, and U_{24} (some of these are discussed below) though there was diversity in the kinds of jewellry that she handled. It should also be noted that E_{16} also had experience of running the small shop along with her mother.

4.7.2 Connections with family work

We observe from Tables 4.1, 4.2 and 4.3 that in most cases (all except 4) students were engaged in the work practices of their parents. There were two students (both girls) in the sample who did not work at all. There were only 5 students among the sample of 31, viz., E_4 (singer in orchestra bands), E_5 (*chindhi* collection), U_2 (tailoring work to learn hand skills), U_8 (*latkan* making) and U_5 (Garment-packaging as an employee) whose work differed from their parents' possibly because of their circumstances, interests or special skills. While E_4 was good at singing, we observed that E_5 was independent in nature and had leadership traits[2]. U_2 on the other hand opted for the current work to learn hand skills

2 Based on researcher's prolonged interaction with the students, we noted that E_5 was popular among his friends as a "solver" of their problems. In many cases, the researcher observed that E_5 would arrange small jobs for his friends or solve their problems. From his body-language, use of words, tone, external appearance and independent nature, he was deemed as a "leader" in

but U_5 seemed to be working for income. U_8 joined other children in a neighbourhood house-hold to make *latkans* as she found the work interesting although it was not a work practice her parents were involved in and she claimed that she worked only occasionally.

The above observation suggests that the opportunities that students get to learn about work practices come primarily from their parents' or close relative's work or work done at home. Such opportunities also draw the contours of their mathematical learning. Thus, *funds of knowledge* available at home are the first step towards development of skills and out of school learning. Thus participation of children in work practices cannot in all cases be equated with exploitative child labour as discused in Chapter 2.

4.7.3 Connections between out-of-school and school mathematics

From a viewpoint of looking at the overlaps between out-of-school and school mathematics, there emerged contrasting features in students' propensities for using out-of-school or school learnt maths knowledge. While, there was a prevalence of out-of-school techniques in most students' work-contexts as well as interview problem-tasks, it was also observed that U_2 and U_{13} used concepts and artefacts that they learnt at school in their respective work-contexts. For instance,

- In U_{13}'s work, she used the metre scale that is used in schools. She also used SI units (taught in schools) and not the old British units that the researcher observed being used in most of the work-contexts in the low-income settlement and also popular among other students of her grade.

- In the arithmetic tasks, both U_2 and U_{13} showed a preference for school learnt techniques. They relied on formal algorithms and according to the interview description, they also used school learnt strategies in maintaining accounts or doing calculations.

- On the other hand, E_{16} preferred to use her own convenient and situation-specific strategies despite having learnt formal algorithms well. E_5 and U_2 similarly relied on their own strategies for computations. Such characteristic features of students'

his group of friends.

> work-contexts and everyday experience indicate the hybrid nature of mathematical knowledge prevalent among children and that they draw from both school and everyday mathematical experience. Most mathematical procedures that the middle graders used show inter-penetration of both school and everyday mathematics and the separation is rather blurred and not distinct.

It appeared to the researcher that the mathematical computations done by the students in their work-contexts were driven by the fact that wrong or incorrect answers would lead to loss.

4.7.4 Other sample students

The researcher observed that students in the sample and other students across different grades have similar life world as those mentioned in the case studies. Some students had a different profile like U_{21} (part of additional sample) who immigrated to Mumbai to earn a livelihood to support his family back home in Bihar and took up the work of stitching shirts. He showed a keenness to spend time on school studies everyday since he thought and mentioned many times to the researcher that learning arithmetical operations would help him in his work. He mentioned an instance where he needed to use the long division method (which he claimed he did not know), which presumably motivated him to pay attention to school learning. He therefore managed both attending school in the morning and working in the afternoon through late in the evening. There was only one student in our sample, U_{15}, who was not engaged in any work. Belonging to a *maulana's* (a religious scholar) family, interaction with her revealed that her family did not like doing the kind of work that happens around them. "*yeh sab kaam humlog nahin karte*" - *we don't do such work* – was her reply. U_{15}'s family live in a 4-storied redeveloped building that houses 64 families near the edge of the low-income settlement, and are not engaged in any house-hold based work. Community leaders express concern that moving to such dormitory-style dwellings in buildings snatch away not only the work opportunities for such families but also detach them from the rich social network present in the settlement.

Among the sample were students with comparatively more independent access to their work practices as E_5 and U_{21} had and also students with limited and fragmented chunks of a larger work requirement at hand. The latter kind of work was more visible among our participants who mostly assisted their parents or relatives while some were engaged in work for wages (viz. U_2 and U_5 who did tailoring and ironing/packaging garments respectively). Examples of *fees* cutting, *rakhi* making or similar other micro enterprise done at home are instances of such fragmented tasks. On the other hand are the examples of U_{22} (mobile repairing work), U_{23} (textile printing work) and U_{24} (ready-made garment selling) whose work-contexts required them to interact with different people starting from negotiations with different customers to shopkeepers selling raw materials and spare parts. These are the instances where our participants had diverse opportunities to deal with and gather out-of-school mathematics. It was observed that most engagement in economic activities entailed varying degree of decision making related to income generation. We discuss in brief the engagement of some participants, other than the four cases discussed, in some of these work-contexts here:

Mobile phone repairing (U_{22}, 14 year old boy)

Several young men in the neighbourhood are involved in this work. U_{22} got into this work by spending time in his friend's shop observing him repair phones. His work-context requires him to interact with different people starting from negotiations with a variety of customers who bring the defective mobiles. U_{22} is led to diverse sites in connection to his work, for example, to places where he buys mobile phone spares, parts, repairing tools, electrical appliances like soldering machine, etc. He has knowledge of different mobile-parts and their functions; he knows the costs of both original spare parts and low-cost substitute parts that are made locally. He travels to distant markets in the city which sell spares at lower prices than the neighbourhood shops. Knowledge of a range of products and brands and their prices is required for his work. He has to quote a price for a job by guessing the customer's paying capacity. This helps in deciding whether to use an original part or a low-cost substitute, keeping in mind the expected profit. Quoting a price may often call for mental computation of quantity and price of required parts and the time required to carry out repairs. U_{22} also has to keep in mind what other shopkeepers in the vicinity are charging for the repair work. Thus, U_{22}'s decisions are similar to the ones made by his friend, the repair shop owner. The researcher observed that U_{22} has an awareness of different aspects of the entire job and has had exposure to linkages in which the work of mobile phone repairing is situated. At present he works with his shop-owner friend, but he plans to run a similar business all by himself.

Textile printing (U_{23}, 13 year old boy)

U_{23} helps his elder brother and father in their textile printing ("dyeing") workshop where three more employees work. The workshop is located in a tiny first floor room of a rented tenement while the ground floor room is used as the living room and kitchen for the family of seven. The work involves block printing of patterns or "logos" on textiles, school bags and gunny bags or sacks and is referred to as "dyeing work". U_{23} showed awareness of linkages in which this work is situated, which include the place from where the printing orders come, place where the design is drawn using computer graphics based on the logo template, the place from where raw materials are procured, and the place where delivery of the printed material is made.

U_{23} knows about the different raw materials used in the work, viz., stoppers (blocks used in printing the design), dyeing frames of varying sizes, different dye colours, thinner, adhesive, etc. He explained that different sizes of the dyeing frames and stoppers are made by carpenters on order. U_{23} explained that the typical frame-sizes are 16"×12", 28"×12" making of which cost between Rs 2000-3000. Two types of colouring material producing either shining or mat-finished effect are used. U_{23} knows the prices of the colours, thinner, coating material and so on. He knows which colours are to be mixed for a particular colour or shade to emerge and the proportion in which they are mixed.

Bulk orders for printing logos on the school bags for municipal corporation run schools in Mumbai are usually placed in the months of March, April and May. For U_{23}, this heavy work pressure coincides with the preparation for the year-end examinations. In the peak season of work, the daily turnover varies between Rs 2000 to 3000. The workers get a monthly salary of Rs 4500 with an option of withdrawing up to Rs 500 per week which is then adjusted against the salary. U_{23} gets around Rs 100 per week for pocket expenses.

U_{23} informed the researcher that he makes a choice of the suitable dye-frame and stopper of a particular size by looking at the logo design to be printed in a given task. The unit used for measuring is inch. When the interviewer (researcher) asked U_{23} to make a "guess" of the dimensions of few objects lying around, viz. voice recorder, desk, note-books, he gave nearly accurate answers – all in inches. It is interesting to note that inch is a measuring unit that is not taught in schools.

Ready-made garment selling (U_{24}, 13 year old boy)

Ready-made garment selling is a popular business that is seen in most localities. Interview with U_{24} revealed that he visits his father's ready-made garment shop in downtown Mumbai on Sundays and on holidays. U_{24} knew the connection between garment size and age: that frock-size 22 is for 4-year olds while, size no. 30 for 10-12 year old girls and 32 for girls who are 13-14 years old. Knowledge of profit margins of different sizes helps in making decisions about quoted price and discount at the time of bargaining with customers. The frocks that he deals in sell for about Rs 70-160 with a profit margin of Rs 5 to Rs 40 depending upon the sizes. He claims that sizes like 30, 32 incur loss because their making charges are more in comparison to other sizes and hence the profit margin is less. U_{24}'s job hence entails quoting price and discount to customers while ensuring a reasonable profit. He also takes up other responsibilities in the job: keeping track of stocks and sales, and maintaining accounts. U_{24} is familiar with some of the backward linkages: his uncle runs a *zari* workshop where designs are made and stitched on to clothes. The newly-stitched dresses are then sent for "thread-cutting" ("*fees*" cutting described earlier) and subsequently come to U_{24}'s shop for packaging and selling.

Most of the students that the researcher came across spoke about the work-contexts happening around them with confidence. The middle graders as a group had access to *funds of knowledge* which included not only the kinds of work that they themselves participated in, but also about other work-contexts that occur in the settlement. This phenomenon was interesting since within a single class, students had peers who were engaged in diverse work practices and practically covering all of them, and therefore created opportunities for learning about them. This indicates the depth of immersion and access of the students and children/teenagers to the work-contexts around them.

4.7.5 Performance in arithmetic tasks

We draw from the interview data from both the Exploratory Phase (Phase-I) and Case-studies Phase (Phase-II, part-II) to discuss in this section students' performance in the arithmetical tasks. In Phase-I, an eclectic exploratory approach was adopted to understand students' everyday mathematical knowledge and the discussion protocol involved items from "currency knowledge", "arithmetical operations on numbers" and "use of units" in number knowledge. No structured questionnaire or protocol was adopted in this phase but open-ended discussion was held with the students. The interview protocol in Phase-II had structured questions focused on two broad categories, i) arithmetic knowledge, and ii) problem solving (see Table 4.4 and Appendix C). These tasks have been discussed briefly in the earlier section 4.2.1 and in detail later in this section while discussing "data from Phase-II".

Data from Phase-I (Exploratory Phase)

Knowledge about currency

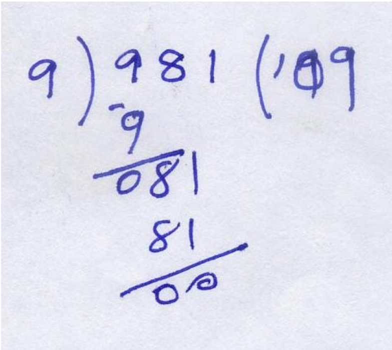


Fig. 4.2 Glimpse from U_{21}'s work sheet (a)

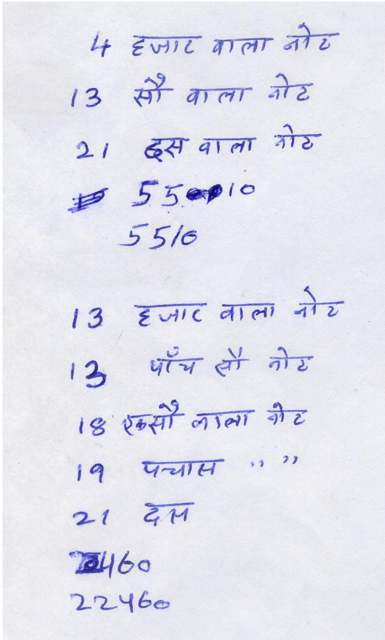


Fig. 4.3 Glimpse from U_{21}'s work sheet (b)

The exploratory interaction with students in Phase-I of the study helped the researcher in knowing the broad features of students' mathematical knowledge. Most visible was their sound knowledge about different denominations of the Indian currency and their inter-conversions. Many students had developed number sense building on their currency knowledge. Numbers for them were amounts of money and arithmetic operations signified "summing up", "getting more", "giving away" or "distribution" and so on (Bose & Subramaniam, 2011). For example, when asked to divide 981 by 9, one student (U_{21}) of grade 5 of the Urdu school (at the time of interview in 2010) looked at the problem as "equally distributing" Rs 981 among 9 children and arrived at 109 as the answer. His explanation was to divide Rs 900 among 9 children thereby arriving at Rs 100 for each of them and then to divide the remaining Rs 81 among 9 children for each to get Rs 9. Hence, each child gets Rs 100 plus Rs 9, i.e. Rs 109. Interestingly, when U_{21} was first presented with the problem, he did the calculation on the worksheet and he arrived at "19" as the answer, making the common error of omitting the zero (*shown in Fig. 4.2 above*). He himself noticed the discrepancy in the answer and hesitatingly put a “0” between “1” and “9” probably because he had “more faith” in the oral procedure than school taught algorithms. U_{21} like other students in the same grade had number sense built on their currency knowledge. During Phase-I of the study he worked as a learner (novice) in a garment making workshop after the school hours. Interactions with him indicated that his interest in school studies brought him back to studies after a two-year gap when his financial condition of his family forced him to work than attending school. Discussions with him earlier had shown that he could add currency-values sometimes involving 5 digit numbers purely mentally. For example, when asked how much money would be represented by, 4 thousand rupee notes, 13 hundred rupee notes, and 21 ten rupee notes, U_{21} correctly replied, “*five thousand five hundred ten rupees*” but initially wrote the sum as 550010 and subsequently corrected it to write 5510. When asked to add 13 thousand rupee notes with 13 five-hundred rupee notes, 18 one-hundred rupees notes, 19 fifty rupees notes and 21 ten rupees notes, U_{21} had the accurate answer as, “*twenty two thousand four hundred sixty*” (see *Fig. 4.3, previous page*).

Figures 4.4 and 4.5 are two students’ calculation of total when certain number of currency-denominations were given. For example, while adding the total amount obtained from 2 notes of Rs 1000, 7 notes of Rs 100, 1 note of Rs 20 and 5 notes/coins of Re 1 (Fig. 4.4), a student orally calculated Rs 2725 but writing on the paper, he wrote it as “207025”. The same student made place value error in adding the numbers and arrived at “16000” but immediately realised his error that the amount could not be that bigger. Similar were other calculations shown in the above photographs.

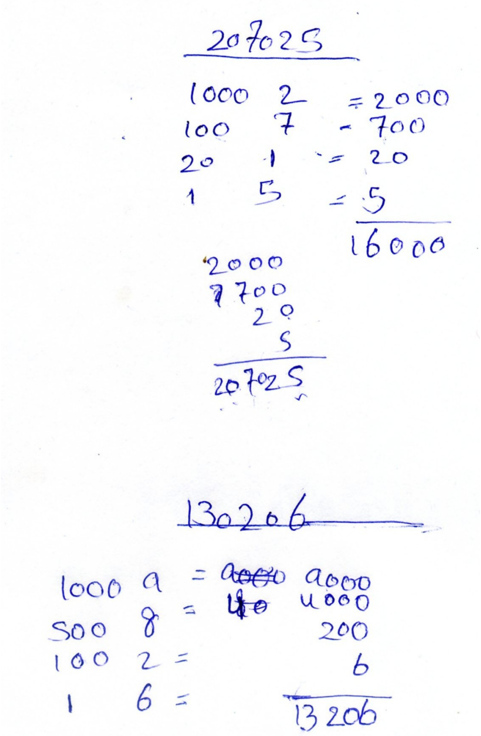


Fig. 4.4 Currency calcuation (a)

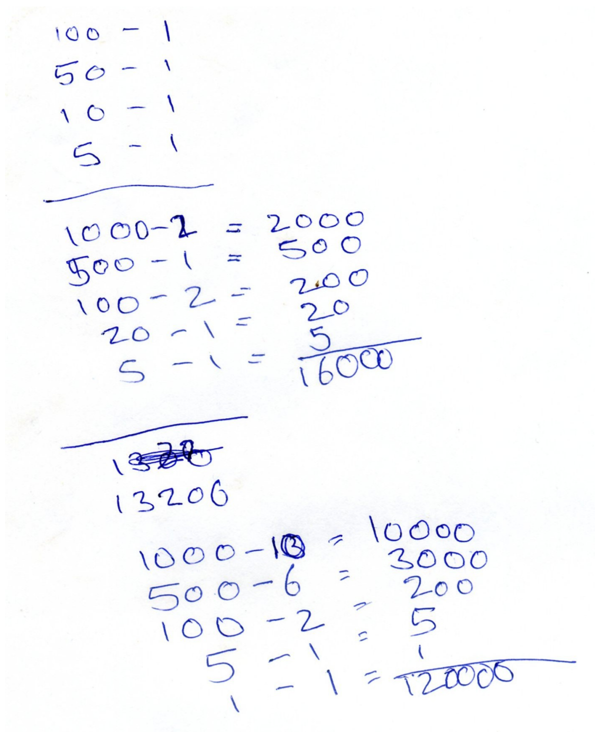


Fig. 4.5 Currency calcuation (b)

Arithmetic Operations on Numbers

It was interesting to find students using their out-of-school mathematical knowledge to solve problem-tasks given to them by the researcher, despite the fact they were regular students in the school and therefore had exposure to school mathematics. For instance, one girl from Grade 7 of the English Medium School (during Phase-I in 2010) who belonged to a low socio-economic family of five, had a natural propensity to attend to math problem-tasks mentally despite having had the option to use paper and pen. She had an exposure of buying everyday articles such as kerosene oil for cooking (sold in bottles), milk and other groceries. Her father did scavenging work of removing debris from the road sides while her mother worked as a domestic help. She informed the researcher that milk is sold for Rs 12 per packet. On asking how much milk a packet contains, she quickly replied *"aadha litre"* ("half a litre"). When asked for the price of 2 packets, she immediately replied, "24". She claimed that she knew this as she often hears the milk-seller telling this to the customers. When she was asked to find the price of 5 packets, she paused and started thinking. She then added 24 and 12 and arrived at 36 and then added 36 and 24 and arrived at 60. Her strategy was to use to the known values, viz. 12 and 24, adding them to first arrive at the price of 3 packets, and then to add 24 to find the price of 5 packets. She did not use paper and pen despite being reminded that she could them if required.

She further told the researcher that a bottle of kerosene oil comes for Rs 28. On being asked to find the price of 5 bottles, she calculated mentally and came up with *"one forty rupees"* as the answer. Her argument was, *"bees ke hisaab se paanch bottle ka hundred aur aath ke hisaab se paanch ka forty"* ["price of five bottles at the rate of twenty is hundred and at rate of eight is forty"]. Then for 15 bottles, she added 140 twice and again added 140 to the sum to get 420. To find the price of 7 bottles, she added 28 twice and then added the sum (i.e. 56) to 140 thereby getting 196 as the answer. Similarly for 22 bottles she added 280 twice and got 560 and then added 56 to it to get 616 as the answer. Following the distinction framework (Table 2.1), we analysed that the strategy was to use addition that included "continuous monitoring" about "where she was" in the midst of a calculation and that gave her confidence in the procedures and meaningfulness in the results obtained.

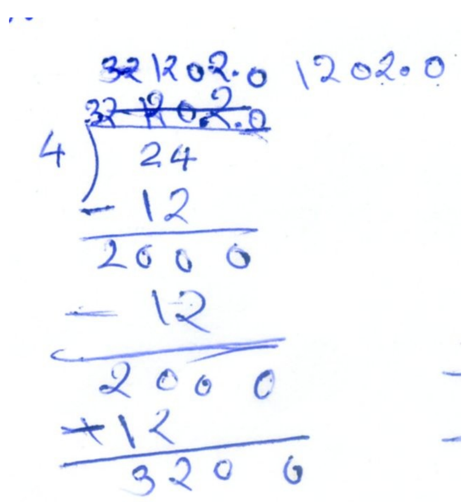


Fig. 4.6 Difficulty with long division method (a)

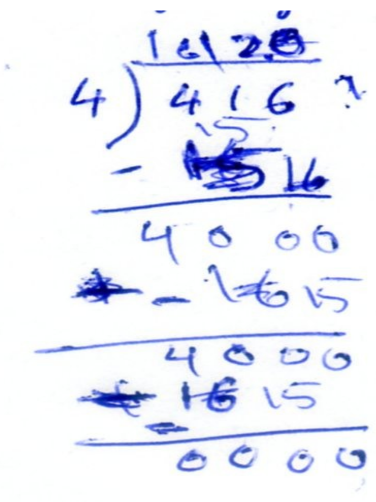


Fig. 4.7 Difficulty with long division method (b)

Interestingly, all the students that the researcher interacted with claimed difficulty in the division algorithm though many of them could orally divide two numbers considering them as referents of some familiar contexts. For example, one student (another girl from Grade 7, Eng. school) repeatedly obtained absurd results like getting quotients bigger than dividends (for all positive dividend, divisor and quotient). She however, did the seemingly easy division orally in a contextual problem situation instantaneously (as shown in Figs. 4.6 & 4.7).

Different Representations

Discussions with the students showed that children make use of a variety of units mostly based on the convenience and syntactic support from prevalent practices. For example, one Grade-7 student from the English medium school wrote "six hundred sixty" as 6005010 and read it as "*chhe sau pachaas aur upar se dus"* ("six hundred fifty and ten more") but for "one hundred seventy four" she wrote 10074. It was interesting to note that she used "fifty and ten more" for the representation of "sixty" while in case of "seventy four" it was the formal decimal representation "74". This probably happens because the numbers "50" (*pachas*) and "10" (*dus*) are commonly used numbers in daily parlance in terms of weight measures in everyday shopping or as commonly used currency notes and therefore these two numbers are often used as stand-alone units. Several such examples of the use of different representations emerged during the exploratory interactions.

Data from Phase-II (Arithmetic knowledge)

Data presented in this section is drawn from the students' interviews about their arithmetical knowledge and problem solving from Phase-II (part II). Wherever required, we have repeated the description of the task below for readers' convenience.

Number-sense and currency-knowledge

We noted that some students had difficulty in reading and writing numbers bigger than 3 digits (sometimes bigger than "500") and they made place value errors or wrote numbers the way they are called out (for example, writing "1000100" for "one thousand one hundred"), however it seemed that most students had a sense of "bigness" of a number derived from their currency knowledge.

As discussed before, we interacted with many students during our study and noted that they all had sound knowledge of different currency-denominations and their conversions. In the problem-task of taking out currency amount equivalent to given numbers (refer to Section 4.2.1) which were 165, 2725 and 13206, every student could take out Rs 165 from the box correctly. Only 3 students – 2 from English School and 1 from Urdu School could not take out the money equivalent to the last two numbers. All the students also had awareness during currency calculation as to "where they were" in the midst of currency counting – how much has been counted, how much left; or, how many notes/coins of which denomination to choose; or, how to make do in lieu of the notes that were not available, and so on. In the process of counting money, most students used Re 1 coins (called "*dollar*" colloquially) and not Re 1 notes kept together, possibly because Re 1 coins are common in everyday practice while Re 1 paper notes have slowly faded away from use. Three students who had difficulty writing or handling large numbers could actually take out small amounts correctly presumably due to exposure to handling smaller amount but not bigger amounts in their everyday practices.

Similarly, in the next item, though some students faced difficulties in the number enumeration task at the transition to next "hundred", viz. from "five hundred ninety nine" to "six hundred", they could actually complete the enumeration task using currency notes and coins. However, currency as a cue is not adequately exploited during classroom teaching.

Use of Arithmetical Operations & Multiplication Tables

We noted that in solving problems the students used addition as a build-up strategy and made convenient groupings to arrive at the required results. In case of subtraction, many students used "count on" technique orally. for example while calculating the price difference (costlier by what amount?) of two sets of two articles in each – costing Rs 95 and 265 & Rs 140 and 199, some students "counted on" from 140 to 199. A few of them "counted on" from 65 to 95 and then adjusted the count from 300, though often arriving at 270 in the process. Some did find the correct answer 170 as well.

The multiplication problem (35 × 10) was attempted by doing "repeated addition" and taking convenient groupings and smart build-up strategies (suitable decompositions). Only a few students used multiplication-tables. In case of another contextual problem that involved multiplying 16 with 7, only one student used the multiplication table of 16 while some students used the tables of 10 and 6 and added the partial products. It was observed that many students were not comfortable with the use of multiplication tables though some did use them. The rest used decomposition and repeated addition strategies. We further observed that many students were not familiar with the common school taught rule for "multiplication by ten" as "put a zero to the right". This became evident when a few students actually added 35 ten times to find the price of ten balls each costing 35 rupees. Some also did this problem-task by decomposing 35 into 30 and 5 and adding their ten counts using the respective multiplication tables.

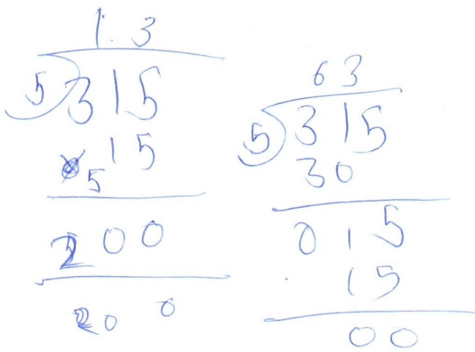


Fig. 4.8 Excerpt from E_{11}'s work sheet

Many students faced difficulties in using the formal algorithm for division. However, they could use currency knowledge as a tool to cross-check the validity of answers to math problems both in school context or outside which gave them a sense of the "range" or possible "variations" in the arrived answers. For example, E_{11} actually divided 315 by 5 on the paper and obtained 13 as the answer. He soon realised the error as the obtained answer could not be that small a number and did the computation afresh and arrived at 63 (See Fig. 4.8).

We observed that among the common strategies that many students used was the one to begin with “potential” or “possible” answer and then narrowing down to the correct answer. The procedure adopted by U_{21} in solving 981 ÷ 9 (discussed above) was similar in which after dividing 900 by 9, he approximated 81 by choosing 10 and adding it nine times (arrived at 90) and then subtracted 9 from 90 to arrive at the balance 81, thereby “refining” 10 to 9 to arrive at the 100 + 9 = 109 as the answer.

Approximation, Estimation and other strategies

We noted that exposure to everyday contexts helps students to develop good approximation and estimation skill. Students often came up with approximations that were close to the correct answers. Price estimation was strong too as was evident while estimating what all articles could be bought with a given amount of money. The estimation for the balance amount was correct for many students who mostly used oral techniques. This was evident from the problem-task in which almost all the students, except a few, could estimate whether the articles shown (priced at Rs 95 and 265 in one set and Rs 140 and 199 in another set, discussed in the previous section) would come against a note of Rs 500 or two notes of the same. We started off with the first set of articles then considered both the sets. The estimation for the balance amount was quite close to the actual answer for 21 students out of 30 who took the interviews on arithmetic knowledge. Barring eight students, the remaining could also tell which articles would come for Rs 500. They could calculate how much more they needed to pay for buying all the four articles. While doing this task all students except four used oral techniques. Of these, two students visualised the computation using school learnt algorithms mentally and some others used paper and pen for doing the subtraction – for which they used formal method. The remaining four students could not even begin to solve the problem. In other problem-tasks, many students were able to give approximate answers before doing actual calculations. Such approximations in some cases were very close to the actual answers. The reality orientation for some students was strong. For example, one student said 7 kerosene cans would last for “three to four months” (one can lasts for 16 days) which is quite close to the actual answer – 112 days.

Students also used strategies like halving and made use of convenient numbers. In a proportion problem task, while finding the price of 25 *burfi* when 20 *burfi* cost 42 rupees, 14 out of 30 students found the prices of 10 and 5 *burfi* by halving 42 and 21. However, some of those students who could do it this way opted for the school learnt “unitary” method of finding the price of one *burfi* first and got stuck. Only 1 out of 30 students could correctly complete the task using unitary method. Students also used strategies like build-on or “workable guesses”. Many students used “closed” numbers, “convenient” numbers and different “units” from daily usage. Interestingly, some students arrived at 53 as the answer and justified that sellers and shopkeepers often do not return changes as balance amount rather round-off to next rupee. Routine experience from the everyday settings reflected in the calculations of some students who used the reality perspective as in the previous example.

Knowledge of fractions

From the interviews about work contexts, it could be seen that many students were comfortable in using binary fractions that are part of everyday discourse like half *(aadha*), quarter (*paav*) and half-quarter *(aadha-paav, i.e.,* one-eighth). However, fractions other than these were difficult to comprehend for most of them and poorly developed despite these being present in the school curriculum. Their everyday experience does not include such fractions, or any kind of visual support for arbitrary equal partitions. In their experiential world there is not much insistence on precision, or fair division.

The researcher also noted that there were a handful of students who would come to him with problems from the math and science textbook or to learn topics like long division method, fractions and operations on them. Such interactions indicated that some of the students (fifth graders then) knew exactly where they lacked in arithmetical proficiency and wanted the researcher to address them.

The above examples underline our claims that the whole gamut of everyday experiences including diversity of cultural and work practices shape students’ everyday mathematical knowledge and has structural difference with school mathematics. However, the inter-penetration between everyday and school mathematics indicates that learning in one domain has relevance for the other which remains to be unpacked. From the standpoint of socio-economic influence of math learning, analysis of such hybridised embeddings of one domain knowledge onto the other has remained an area that calls for systematic exploration.

Use of "convenient bases" as units

On different occasions, we observed that some students made use of units involving "closed" numbers or "convenient base". For example, in one of the "count-on" problem-tasks, while calculating the money kept in one of the envelopes (actual sum was Rs 995) one student in the midst of calculation arrived at "970" and referred to it as "*saare nau sau bees*" [*nine and a half hundred, twenty*]. Here, "nine hundred and a half" stands for 950 and is taken as a stand-alone "unit". Similarly, while calculating Rs 275 kept in an envelope, another student in an intermediate counting step called "264" as "*do sau saathh aur char*" [*two hundred sixty and four*]. The same student while taking out Rs 165 from the box had referred to "65" as "*sattar mein paanch kam*" [*five less than seventy*]. Most of these usages conform with the local parlance.

While solving the contextual problems, most students used strategies that involved use of convenient bases and decomposition. This was seen in the problem involving price difference in two sets of articles (described in the above section). Some students tried to convert "295" into a "closed number" or a "convenient base" by adding 5 to it and arrived at "300". One of them decomposed 265 and 95 and removed 65 from both. The new set of numbers that remained were 200 and 30. He then subtracted 30 from 200 to get 170 as the answer. In another problem (finding 400 ÷ 25), most students grouped four "25" together and made it into "one hundred" and then built it up to "four hundred" and subsequently arriving at 16 as the answer. Here, "100" was a "convenient base" to handle the calculation. Interestingly, none of the students attempted to do this problem using the formal division technique.

4.8 Discussion

Our observations indicate that most of the school going children living in the low-income settlement have an exposure to currency handling and in ensuring its optimal use. In some cases, they also handle operations with multi-digit numbers that represent currency denominations, adopting the oral mode of computation. Children use different forms of currencies as tools for mental (oral) activities. The resultant cognitive activity of (viz., in the case of U_{21}, discussed above) involving operations on multi-digit numbers were shaped, dependent and governed by the use of "currency" as a cue. In lieu of this, when students attempted to write the resultant amount obtained after addition, they expressed the numbers according to the number-names and not using the school taught multi-digit representations which carry the positional values of the respective digits. This indicates the disconnect between the syntactic as well as semantic differences in the language used in everyday contexts and the language used during classroom-teaching. Multi-digit representation of numbers and algorithms used in the number-operations have remained hard-spots for students in the middle grades. But interestingly, when using arithmetic operations on the currency denominations including multi-digit numbers, the resultant answers obtained by the students were correct when computed orally and not when the multi-digit number representations were expressed in the written form.

Language has an important role in gaining everyday mathematical knowledge in out-of-school contexts and also in facilitating mathematics learning in the classrooms while drawing upon from familiar contexts. Out-of-school mathematics bears the functional aspect of mathematical knowledge that is available to all and not hidden (Subramaniam, 2010). This calls for bringing together everyday mathematical knowledge and school mathematics which can possibly pave way for developing skills and interests in learning mathematics.

5

Learning, mathematical knowledge and identity in out-of-school contexts

The defect of equality is that we only desire it with our superiors

- Henry Becque

In this chapter, we analyse the data from the case-studies to show how opportunities for learning in general and learning mathematics in particular arise in work contexts. We discuss aspects of the mathematical knowledge gained in out-of-school contexts and its relation to school mathematics. We also discuss how work-contexts shape the identities of participants in our study as learners. Finally we draw some implications from these analyses for the teaching and learning of school mathematics aimed at making connections with out-of-school knowledge.

5.1 How do work contexts create opportunities for learning?

Features of work contexts and the degree of students' engagement in them shape the learning experience of students who participate in the work-contexts and the richness of the knowledge that they acquire. From an analysis of the data, we discuss the aspects and ways in which work-contexts create opportunities and affordances for learning in general and learning mathematics in particular. (A preliminary version of the analysis was presented in Bose & Subramaniam, 2013).

5.1.1 Diversity

In several work contexts discussed in the previous chapter, we find students dealing with a diversity of goods or artifacts in the course of their work. Some kinds of work involve diverse interactions with people and material, leading to greater opportunities to acquire knowledge and skill, while some tasks are fragmented and create limited or no scope for interaction with people. Thus we find economic activity that is varied and calls on a range of knowledge and skills as well as activity that is routinized, making little demand on skills and knowledge. Hence, one of the factors in creating opportunities for varied learning experience is the nature of work itself, which may be difficult and demanding, or repetitive and mechanical, or characterized by diversity (Bose & Subramaniam, 2013). The importance of such diversity for mathematical learning has been noted by other researchers like Khan in her study of *paan* (betel leaf)-cigarette sellers' and newspaper vendors' out-of-school mathematical knowledge (2004). She argued that diversity in the type of goods sold by *paan*-cigarette sellers created opportunities for them to acquire greater proficiency in arithmetic skills (mentioned in Chapter 2).

Handling diverse goods: examples from some work-contexts

In our study, we found several examples of how diversity of the goods dealt with gave rise to learning opportunities. For example, U_{22}'s mobile repairing work calls for interactions with different people starting from negotiations with a variety of customers who bring the defective mobiles to those from whom he buys spare parts. U_{22} has knowledge of the spares and tools market in connection to his work – with shops selling mobile phone spares, parts, repairing tools, electrical appliances like soldering machine, and other instruments. He has knowledge of different mobile parts and their functions; he knows the costs of both original spare parts and low-cost substitute parts that are made locally and about different products of different brands and their prices. He travels to far-away markets which sell spares at lower prices than the neighbourhood shops. Knowledge of a range of products and brands and their prices is required for his work.

U_{23} knows about the different raw materials used in his textile printing (called “dyeing”) work, viz., *stoppers* (blocks used in printing the design), dyeing frames of varying sizes, dyes producing glossy or matt finish of different colours, thinner, adhesive, coating material, their prices and the units in which they are sold. He knows which colours are to be mixed to obtain a particular colour or shade and the proportion in which they are mixed.

The work of fixing stones on jewellery (ear-rings, buckles, pendants), which is usually done at home also reflects diversity, although limited in comparison with other kinds of work. Those doing this work like E_{16} have to deal with pieces of jewellry of different kinds on which a certain number of stones need to be fixed. The variation in such pieces also is an occasion for learning. The finished pieces are mounted on a card in different array arrangements that E_{16} is intimately familiar with and she associates multiplication facts with these arrangements (details given in Section 4.2.3). E_{16} knows about the wages for fixing different number of stones, different types of adhesives, the proportion in which they are mixed and their prices.

U_{24}'s involvement in his father's ready-made garment business gives him an opportunity to handle different kinds of garments and of different sizes with varying price-tags. He has learned the connection between garment size and age: that frock-size 22 is for 4-year olds while, size no. 30 for 10-12 year old girls and 32 for girls who are 13-14 years old. Further, he knows costs incurred in making of different sizes of garments and therefore the profit margins. He reported that he calculates (“*hisaab karta hoon*”) and prepares the bills (“*fatafat jodkar usko bill banana rahta hai*”). We noted that U_{24}’s involvement in his father’s work practices creates opportunities for him to handle numbers in different ways. In the following excerpts he explains the connection between frock-sizes and the age, and where did he learn about it:

Excerpt 5.1: Work-context interview of U_{24}

87	T	… isme yeh number kya hai? yeh baaees kya hai?	…what these numbers here are? What is this twenty-two?
88	S	size hai, baaees size/ jaisa baaees size kitne saal ko? char saal ke ladki ko aayega/	it's the size, size twenty-two/ like size twenty-two is for what age? it'll fit a four year old girl/
89	T	achha, char saal ki ladki ko aayega?	so, it'll fit a four year old girl?
90	S	haan, waise hi, do number bole to ek saal do saal ka fark hota hai/	yes, likewise, there is a difference of two years between two numbers [sizes]/

115	T	to yeh char saal ka baaees hua to maan lo tees wala size kitne saal wale bachche?	so for a four year old it's twenty-two, then suppose for size thirty which age-group of children?
116	S	Dus-barah saal wala/	for ten-twelve year olds/
117	T	Dus barah saal wale? Aur battees?	for ten-twelve year olds? and thirty-two?
118	S	Battees terah-chaudah ko aayega/	thirty-two will fit thirteen-fourteen year olds/
119	T	to yeh sab tumhe kahan se pata chala? Kaun kisne sikhaya?	so where did you get to learn all this? who taught this?
120	S	abbu ne bataya/ abbu, bahrhal na mere ghar mein na kuchh bhi aata hai na to woh number sab pata kar lete hain kitne number hai, woh shirt ko dekh lete hain to bata dete hain/	Dad told me/ Dad meanwhile when anything comes in my house then he finds out the number, he finds out everything what all numbers are there, he looks at shirt and tells us [the numbers]/

We observed that E_6's involvement in the mobile repairing shop that his elder brothers run as well as his father's button stitching workshop creates occasions for him to learn about diverse sets of articles handled in different work practices. For example, E_6 spoke about different mobile parts and repairing materials such as IC, iron machine (soldering rod), blower, mike and the cleaning liquid (discussed in a later section on "*identity as a learner*") and also about different kinds of buttons and needles, sewing machines.

We see similar features in other kinds of work such as recycling, where the worker handles diversity of goods, their varying features, different rates at which they are sold, and different variables that need to be checked.

Fragmented tasks: limited access to diversity

Tailoring work

As described before, the work of tailoring as a whole is complex and involves stages of skill development from novice to apprentice to master. However, tailoring work, as commonly done in the settlement is increasingly compartmentalised and fragmented into smaller tasks. For example, in garment stitching work, a group of people (mostly novices) would stitch only collars and cuffs of a shirt, while others with more experience (apprentices), do complex work like stitching all the parts together, while a third group puts buttons and yet another group removes threads ("*fees*" cutting work; discussed below) from the newly stitched shirts. This is followed by ironing of clothes and packaging work. Unlike dyeing and mobile phone repairing work, tailoring work (garment stitching work) is completed by putting together smaller, compartmentalised tasks which are done at different workshops. Thus, people involved in tailoring work like U_2 or U_5 get limited access to handling diverse goods or diverse tasks. However, as a worker moves up through different stages of a work practice (in this case, tailoring work), he gets to handle diversity of goods, learns about the connections between the different stages, and linkages on the production chain. By the time one becomes an expert in the work practice, as in the case of U_{21}, one gradually learns about the broader connections, prices, wages and other related information. Students in our sample, other than U_{21} did not display the "expert" knowledge of tailoring work that U_{21} did.

A worker engaged in tailoring usually works at a location with materials that are provided to him (usually male workers are engaged in the workshops) and does not deal with the customer. *Masters*, who get the orders, cut the cloth in bulk and distribute the pieces to "compartmentalised" workshops. Raw-materials like threads and needles are provided to the workers. They only need to focus on the task at hand. The amount of work is large but repetitive and mechanical in nature. Hence, not only are such tasks fragmented, but the scope of handling diversity is also minimal.

Thread cutting (or "*fees*" cutting) from newly stitched garments is one of the most common house-hold occupations for women. This work is a part of the garment manufacture chain. It does not require training, bigger space or any special equipment. Generally sharp-edged cutters/knives are used in removing the extra threads. Workers involved in this task need to only focus on garments at hand and try and complete as quickly as many of them as possible. The routinised and fragmented nature of the work, with no variation or innovation involved, neither demands any skills or knowledge from the workers nor any pre-experience or learning. Opportunity for handling diversity in this task is minimal. Similarly, *rakhi* making work in which E_{13}, U_{11} and U_{13} are involved or U_8's *latkan* making work get the corresponding raw-materials delivered at their homes by the middle-man who subsequently collects the finished product, sends them for packaging and makes the payment. Both these kinds of work are mechanical based on a given sample *rakhi* or *latkan* whose specifications are to be followed in making the new ones. Workers do not need to make innovative designs or use different colour combinations or calculate the proportion of adhesive to be mixed. Such low-paying jobs are preferred as it is seen as an opportunity to supplement income by working at home. Thus, the opportunity to handle diversity in these work practices is limited.

5.1.2 Making decisions in relation to work; optimising resources and earnings

Many work contexts require participants to optimize resources and earnings and to make decisions, which create the opportunity and the need to learn and use computation and knowledge of proportionality. In U_{22}'s mobile repairing work that we discussed earlier, optimisation is required for quoting a price by guessing the customer's paying capacity. This helps in deciding whether to use an original part or a low-cost substitute, keeping in the mind the expected profit. Such optimisation requires quick mental computation and making estimations such as how much of a particular part to buy and stock and at what price, time required to carry out repairs and repair charges. U_{22} also has to keep in mind what other shopkeepers in the vicinity are charging for the repair work.

Textile printing (dyeing) work requires U_{23} to frequently make a choice of the suitable dye-frame and "stopper" of a particular size by looking at the logo design to be printed. The unit used for measuring is inch and U_{23} makes the estimation of the design in inches and accordingly takes a call about the "stopper" to be used. Similarly in *chindhi* (garment recycling) work, E_5 is required to make quick decisions about the rate at which he can close a deal. He mentioned in the course of the interview proudly that once he earned Rs 640 on a single day's collection of 95 kg of *chindhi*. This was like a benchmark against which he could make a decision of when to stop collection for the day and to assess the success of a particular day's collection and earning.

In ready-made garment selling business, U_{24}'s knowledge of the profit margins of different garment sizes helps him in making decisions about the quoted price and discount to offer at the time of bargaining with customers. The frocks that he deals in sell for about Rs 70-160 with a profit margin of Rs 5 to Rs 40 depending upon the sizes. He claimed that bigger sizes like 30, 32 incur loss because more amount of cloth is wasted in comparison to other smaller sizes and their making charges are more. The selling price of frocks of the size-range 22-32 is the same, while that of the size-range 16-20 is same. According to U_{24}, the profit margin for frocks of sizes viz., 30 and 32 is less, and sometimes they incur loss. U_{24}'s job hence entails quoting price and taking decisions on discount to customers while ensuring a reasonable profit. Here is an excerpt from the interview with U_{24}:

Excerpt 5.2: Work-context interview of U_{24}

124	S	aur yeh solah se bees, iski bhi kimat same aayegi/	And these sixteen to twenty, their price are going to be the same/
125	T	accha, solah se bees ki alag kimat hai aur baaees se battees ki alag kimat hai?	Ok, sixteen to twenty have the same price while twenty-two to thirty-two have the same price?
126	S	yes sir/ yes sir/	yes sir/ yes sir/
127	T	aisa kyon?	Why is so?
128	S	kyunki isme yeh chhota ho gaya, yeh bada ho gaya, chhota mein maal thoda kam lagta hai, bade mein maal thoda jyada waste hota hai/	Because these are smaller, these are bigger, smaller ones require less cloth, more cloth is wasted in [making] the bigger ones/
129	T	lekin battees to sabse bada hai, baaees bhi chhota hai, to battees se baaees ka daam ek hi rahega kya?	But thirty-two is the biggest [among the lot], twenty-two is smaller too, then why is the price same for twenty-two to thirty-two?

130	S	haan/ kyunki yeh pattern ek hi rahta hai, kapda ek hi, kapda jahan se uthaya jata hai na woh ek hi rahta hai/ grahak ko dene ke liye kisime fayda, kisime nuksan hota hi hai/	Yes/ because the pattern is the same, cloth is same, the place from where the cloth-piece is taken up is the same/ while selling to customers some of these give profit, some give loss/
131	T	achha, to kis kis mein nuksan hota hai?	Alright, so which ones give loss?
132	S	jyadatar na humlog ko battees number aur tees mein/	Mostly for us thirty-two and thirty/

Many students spoke of the kinds of work that they had given up because the earning was too meager. Such decisions are continuously made as workers choose different kinds of work. For example, U_{21} switched to being an apprentice to three-quarter trouser stitching work after becoming an expert in the shirt stitching work aiming at better wage, less work pressure and learning a new skill. He compared the wage, number of pieces he could stitch in a day and his earnings in both the jobs before taking a decision in favour of switching jobs. He explained that shirt stitching work has different patterns and designs, has more work whereas three-quarter pant stitching work is relatively easier and offers more wage,

> *"shirt mein zyada kaam hai, rate zyada nahin mil pata hai" (shirt has more work, rate [wage] is not as good)*

U_{21} elaborated that he earned Rs 8.50, 9.00 or 10.00 per piece of a three-quarter pant stitching 35-40 pants a day and while working for more than twelve hours a day during normal work pressure, compared to the wage of Rs 10-14 per shirt that he got. The normal working hours varied between twelve to fourteen hours usually between nine in the morning till eleven in the night with a break in between for lunch and evening snacks. He explained that shirt stitching takes more time compared to three-quarter pant stitching and in a day he could stitch twenty shirts and with lots of efforts up to twenty five shirts. So, in a week he could earn a maximum of Rs 1500-2000 in the shirt stitching work, while he earns more money in the current job of three-quarter trouser stitching. U_{21} explained that stitching work varies depending upon the demand and therefore it was required for him to keep an account of the number of trouser he stitched each day and the expected wage per week. At the time of the work-context interview, U_{21} had almost quit his studies and was engaged in work full time. Though his name still appeared in the Grade 7 attendance register, he could seldom attend school lessons.

Engagement in economic activity thus involves making decisions relating to income: what work to engage in, how much time to spend, what the possibility of income is, how steady or reliable it is and so on. Calculation is an important input in the decision making process: calculating costs, incomes, opportunity costs, optimising earning, and so on. Calculation is also involved in making a judgement about the fairness of a deal. These aspects are important to take account of in mathematics education since judgements about personal and public finances often need one to apply mathematical knowledge.

5.1.3 Involvement in the work; awareness of linkages & ownership

Although all work-contexts call for some level of decision making, the nature of the workers' involvement determines the kind of decisions and the extent of control over decision making. We found for example that some of our participants identified themselves with owners or proprietors, and took decisions on their behalf. The nature of involvement also influenced the extent of knowledge that participants gained in terms of the forward and backward linkages of the work-context. We found that some work-contexts facilitated participants in gaining such knowledge, which was visible, for example, in the mobile repairing work of U_{22}, in the tailoring work of U_{21}, in U_{23}'s textile printing work, in the garment selling work of U_{24}, and in the recycling work of E_5. U_{23}'s awareness of linkages in which this work is situated became visible from the interactions with him that the researcher has had on several occasions. He knew the place from where the printing orders come, place where the design is drawn using computer graphics based on the logo template, the place from where raw materials are procured, and the place where delivery of the printed material is made. Similarly, interactions with U_{22} revealed his extensive knowledge about the linkages associated with his mobile phone repairing work. As discussed before, U_{22} knew about the prices of different mobile parts of several brands and who is offering them at what price. His knowledge about the linkages helped him manage his work.

The control over and extent of decision making, need for optimisation, knowledge of backward and forward linkages are strongly related to the sense of ownership that participants had about their work. Study participants whose close relatives, friends or families own businesses have a stronger sense of ownership of the work, in comparison to those who work merely for wages. With the sense of ownership comes greater involvement in the processes of decision making and optimisation with regard to the work. Such involvement creates greater opportunities to gather and use mathematical knowledge. For example, in addition to awareness of linkages, we noticed that E_5, U_{22}, U_{23} and U_{24}, also had a sense of ownership and were aware of diverse aspects of their work as well as the forward and backward linkages that the work had. Except for E_5, these students did not participate in the work primarily for the income, but rather also to learn something and to pick up useful skills which are valued as they are perceived as securing opportunities to get future employment. For example, U_{22} takes pride in knowing about both kinds of work – mobile phone repairing as well as garment stitching work. His father runs a shirt stitching workshop where three other workers are employed and U_{22} does not particularly need to earn to support the family as is the case with other children like U_{21} who need to earn to support their families.

Knowledge of forward and backward linkages are required in other tasks too. For example, in garment selling work, U_{24} takes up other responsibilities such as keeping track of the available stock, sales done and maintaining accounts. U_{24} is familiar with some of the backward linkages – his uncle runs a *zari* workshop where designs are made and stitched on to clothes. The newly-stitched dresses are then sent for “thread-cutting” (also called “*fees*” cutting) and subsequently come to U_{24}'s shop for packaging and selling. Knowledge of forward linkage comes through the requirement of knowing about the small retailers who purchase garments from his shops in bulk.

In the case of E_5 (recycling), U_{22} (mobile repairing), U_{23} (textile printing), and U_{24} (garment selling), where the sense of ownership and control over decisions was strong, frequent references were made to decisions over deals. Most students in the original sample (barring E_5) did not have encounter with decision making processes as much as U_{22}, U_{23} or U_{24} had. In the case of other students, we noticed hesitation to carry out mathematical calculation to engage with questions of fairness of income, and in some cases inappropriate use of calculation. For instance, U_8 (*latkan* work) and U_{13} (*rakhi* making) both were hesitant to do mathematical calculations based on their work-contexts and earnings which was otherwise prominently visible in the discussion with others, especially with U_{21}, U_{24}, E_5, E_{16} and so on. U_8 described seemingly incorrect size of the *latkans* made, mixing up between different units of length measurement: inches, foot, metres and centimetres. However, she remembers the sequence in which the sequins are placed in the *latkan* and such knowledge of work is sufficient for her task. She does not really require to deal with the lengths of *latkans* and it does not matter if she is not clear about the inter-conversions of measurement units from different systems (British and SI). A few other students' interviews (viz., E_6, E_8, E_{15}, U_2, U_{13}) also indicated that they were not conversant with such conversions though most of them had a comparative sense of different units in practice. During conversation with the researcher it appeared that these students did not know the relation between units from different systems.

Excerpt 5.3: Work-context interview of U_8

48	S	pehle teen moti lagega...	There are three *motis* (pearls) at the start..
49	R	moti kitna badi-badi hoti hai?	How big are the *motis*?
50	S	itni chhoti-chhoti (gestures)/ itne badewale ka bataungi main/	This small (gestures)/ I'll tell about the bigger ones/
51	S	teen moti rahega, chhoti chakli, badi chakli, fir ball, fir badi chakli, chhoti chakli, fir ek moti, fir ek nahin do moti, fir chhoti chakli, badi chakli, ball, badi chakli, chhoti chakli, ek moti, ghanti/	Three *motis*, small *chakli*, bigger *chakli*, next is ball, then bigger *chakli*, small *chakli*, next one *moti*, then not one two *motis*, then small *chakli*, bigger *chakli*, ball, bigger *chakli*, small *chakli*, one *moti*, and a bell/

71	T	to ek latkan kitna lamba, kitni badi hoti hai?	So, how big, how long is one *latkan*?
72	S	itni badi hoti hai/	This big/
77	T	fir se lambai batao andaz se/	Say the length again, approximately?

78	S	ek foot hoga/	It'll be one foot/
79	T	ek foot hoga? Ya usse lamba hoga? Aisa? Ek foot kitna hota hai?	Will it be one foot? Or is it longer? This much? How much is one foot?
80	S	aadha foot lo/	Take half a foot/
81	T	aadha foot?	Half a foot?
82	S	ek latkan mein teen moti rahega/ teen, char, paanch, chhe, saat, aath, nau, dus, gyarah, barah, terah, chaudah, pandrah, solah, satrah. Ek latkan mein satrah chizen/	One *latkan* has three *moti* (pearls)/ three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen/ one *latkan* has seventeen items/

We further noted that the student's sense of ownership determined the nature of involvement and engagement in the work. A sense of ownership was often accompanied by decision making about the expenditure on raw-materials and outsourcing. For example, U_{23}'s textile printing work required outsourcing of the design outlays using computer graphics from a neighbourhood friend on payment. It also was evident from U_{23}'s response that he felt a sense of ownership over the work. When asked if he helps the workers, he said, "*apna kaam hai to madad to karna padega na*" [*it's our work so need to help*]. His statements also indicated a concern about optimal use of resources and time,

> *"colour-valor jo hai, cutter-wutter saman laker de deta hoon, karigar jayenge to time laga dete hain na... isiliye humlog jate hain saman kharidte hain...pagar chalu hai na..." [I go to buy colour, cutter, whatever, if the workers go they take long and they are being paid (for their time)... so we go and buy stuff].*

We noticed similar expression of concern from E_6 during the interview where he discussed how he and his father attended to work when all the workers went on leave and his brothers were not around. He described how his father was tensed and he joined in to help his father and worked for two months. E_6's description reflected his sense of ownership over the workshop that his "father ran" ["*papa chalate hain*"] and that he owned responsibility when it was required. He also referred to the workers as "my workers" ["*mere karigarlog*"] (See Excerpt 5.4 below). Though such utterances might sound precocious for a young boy like E_6, they signify E_6's sense of ownership. Such utterances are shaded with grey in the excerpt below.

Excerpt 5.4: Work-context interview of E_6

185	S	… main do mahine, jab mere karigarlog sab chhorkar bhag gaye the na, tab main akela hee tha, bhai bhi gaon chale gaye the, main hee akela tha, to papa bahut tension mein the, … fir main gaya tha karkhane, main bhi marking kiya, aur ek the sajju bhai kar ke, woh the, papa the, papa button laga rahe the aur sajju bhai card maar rahe the, main marking maarta tha, marking maarta maarta tha, khelta bhi tha/	… for two months I, when all my workers left everything and ran away, then I was alone, brother too had been to village [native place], I was the only one [around], then Dad was in tension, then I went to the workshop, and there was one sajju brother, he was present, Dad was present, and sajju brother was working on the cards, I was putting the markings, would put markings and play too/

Limited awareness of linkages and ownership

As discussed before, workers in the tailoring tasks get the garment pieces cut to the required specification and they are only required to stitch small, specific parts in large numbers. They do not directly deal with the middle-man who brings the work orders and therefore have no connection with the forward or backward linkages associated with their tasks. Even the workshop owners (where large cloth-pieces are cut) have limited linkages only to the next stage of the work and practically no linkage with the market network.

E_{16}'s stone fixing work involves putting coloured stones (usually up to 4) on ear-rings, pendants, rings, buckles, and *mangal-sutra* (a kind of necklace). As discussed before, workers involved in this work do not need to have knowledge about other parts of the production network, such as where or how the pendants, rings or buckles are made, and what the costs involved are. The worker needs to only focus on her immediate task. It is also routinised work and does not call for skill or other peripheral knowledge. The order comes from a middle-man who provides all the materials required for the task and also collects the finished-products and makes the payment. The workers do not have to deal with customers separately or sell the goods. Similar is the work requirement of *rakhi* or *latkan* making in which the workers only need to focus on the task at hand and do not require the awareness about the aspects or other linkages on the production network/chain. In these tasks too, the workers do not have to deal with retail sellers or retail customers.

Employees in the tailoring workshops like U_2 and U_5 have limited awareness about the aspects of linkages of their work on the production chain as compared to U_{21} who is engaged in the same work practice but has gained expertise in shirt making. Workers at the early stages of learning like U_2 and U_5 only need to complete the task in hand and their wage either depends on the pieces stitched or is fixed as monthly/weekly wages. For example, U_5 who is employed in a garment packing workshop along with four other children of his age, explained that their monthly salary ranged between Rs 1200 – 1500 depending upon how fast they could pack the garments. Similarly, U_2's wage was between Rs 0.50 to *a rupee* per garment-piece that he stitched. For neither of them did their work require knowledge of linkages on the production chain or have opportunities to handle diverse goods unlike others that we have described above. U_2 and U_5 are paid workers almost at the bottom of the hierarchy of workers and unlike U_{22} (mobile repairing), U_{23} (textile printing) and U_{24} (ready-made garment selling), did not have a sense of ownership of the work.

From the conversation with E_6, it appeared that he sensed competition among the apprentices and novice workers and sharing of skills and knowledge between does not happen easily. E_6 shared his belief that not every worker is fortunate to quickly move up through the stages of learning. When the researcher asked him how he could get into stitching work, E_6 described the hardships a novice faces in the beginning. Here is an excerpt:

Excerpt 5.5: Work-context interview of E_6

225	T	… to maan lo agar main silai ka kaam karna chahun to mujhe kya seekhna padega?	Then suppose if I were to learn the stitching work then what would I need to learn?
226	S	aapko pehle, pehle sir maloom bahut satate hain, pehle helper banayenge, yeh saman udhar se idhar karo, uske baad thoda pareshan karenge, daurayenge...	First of all you, sir you know they will treat you badly, first of all you'll be made a helper, put this stuff here to there, then they'll bother you, make you run around...
227	T	dauranyenge matlab?	Running around? Means?
228	S	matlab ki, yeh le aao, woh le aao, yeh ghero, jaldi sir seekhne nahin dete hain innlog karigar log/	Means, bring this here, bring that here, put a mark here, these apprentices don't let you learn easily/

229	T	kyon nahin seekhne dete?	Why don’t they let learn?
230	S	helper jo rahte hain na woh sabko help karte hain/	Those who are helpers help everyone/
231	T	lekin karigar log seekhne kyon nahin dete hain?	But why do apprentices don’t let [others] learn?
232	S	agar yeh seekh jayega to phir wohlog kya karenge?	If these [people] learn then what would they do?
233	T	ohh, aisa hota hai?	Ohh, does that happen?
234	S	aisa hai sir/	It is sir/

It appeared to the researcher that other respondents including the students and some community members with whom the researcher had conversations, did not describe such instances of non-sharing of knowledge and skills.

5.2 Features of participants' mathematical knowledge in relation to out-of-school context

Although many research studies have distinguished out-of-school and school mathematical knowledge, it may be misleading to think of them as completely unconnected kinds of knowledge existing in the same individual. We identify knowledge that students demonstrate as out-of-school knowledge on the basis of features identified in the literature, summarised in the “Distinction Framework” given in Chapter 2 (Table 2.1). These features include the accompaniment of an “out-of-school” context in which a task or question is presented to the student, or in which the student makes a remark, the presence of oral computation strategies and the reference to mathematical entities that do not appear in the school curriculum such as words for binary fractions. In the mathematical tasks presented to students during the interviews, some tasks were presented in both context-rich and purely symbolic forms. As expected, the performance in context-rich forms was slightly better (see Section 4.3.4). Tasks which were formulated with contextual detail (viz., *burfi* task, price comparison and balance calculating task) usually elicited oral computation strategies and factoring in of reality perspective. On several occasions participants used school math knowledge in the form of formal algorithms like unitary method as well as oral computation strategies. Although, the tasks included prompts located in out-of-school contexts, some participants' responses began with using school learnt method, subsequently falling back on their out-of-school math knowledge. Thus on multiple occasions, we found students using methods that they had learnt at school together with those that were likely not taught explicitly at school.

Several features reported in the literature were also found in our study participants. For example, situation specific strategies in problem solving drawing on their everyday mathematical knowledge, use of common fractions, proportionality strategies, use of convenient numbers and decomposition.

Besides students' response to mathematical tasks posed to them, in the course of the interview they discussed mathematical aspects present in out of school contexts and responded to the researcher’s mathematical questions posed in the immediate context being discussed. (For example, the interviewer would ask about cost or payment for a quantity different from what was mentioned.)

This allowed us to get a sense of the kinds of mathematical elements embedded in out of school contexts, and also the nature of such knowledge. We describe aspects of such out-of-school mathematical knowledge of the study participants.

5.2.1 Limited combinations and fragmented knowledge

A feature that we noticed about the mathematical aspects embedded in work contexts was that variation was limited to what the context itself included. Thus the mathematical experience of students was constrained and limited in terms of variations and possibilities, the exploration of which is an essential part of mathematical abstraction. For example, as mentioned earlier, E_{16}'s stone-fixing work required her to make arrays of the finished jewellery pieces in only limited arrangements: 6 × 4 or 12 × 2 arrays on a card so that six such cards put together can make one gross (144 units). In the following excerpt, E_{16} spoke fluently and at length about such combinations reflecting familiarity and ease of calculation.

Excerpt 5.6: Work-context interview of E_{16}

106	S	To one gurus matlab one packing ka to twenty four cards honge, dekha jaye to, matlab one card mein six jodis lagte hain, uss tarah na apun twenty four cards banayenge na packing ke baad, to kya hoga, twenty four cards ka packing karenge na .. cards ka, tho apne ko nineteen rupees milega aur nahin packing karenge to eighteen rupees hi milega/	then for packing one gurus, one packing has twenty four cards, by looking at it, means one card has six pairs, that way if we make twenty four cards then after packing, if we pack twenty four cards, then we get nineteen rupees and without packing we get eighteen rupees only/
107	T	achha, achha, ok/ to yeh jo tumne chhe banaya hai, chhe pair, aisa tumko chaubees pair banana hoga...	ok, ok/ so here you've made six, six pairs, like that you need to make twenty four pairs...
108	S	haan/	Yes/
109	T	to usme one forty four pair aa jata hai/	then you get one forty four pairs in them/
110	S	haan/	Yes/
111	T	matlab kitna gurus?	so, how many gurus?
112	S	one gurus/	One gurus/
113	T	achha, ek gurus kitna hota hai?	ok, one gurus makes how much?
114	S	twenty four cards/	twenty four cards/
122	S	one dozen hi hua hai, aisa matlab.. ab sir aisa card aate hain na to usme twelve, twelve jodis rahte hain, twelve pairs hote hain, aise card aate hain na to usme twelve pairs hote hain/	It is one dozen, this means... now sir in such cards there are twelve, twelve pairs, twelve pairs are there, such cards have twelve pairs/

It was not however clear from E_{16}'s responses whether she considered other combinations or total quantities other than 144. It appeared to the researcher that her work did not require her to explore other possibilities of numbers. She just needed to focus on the work instructions and the suggested patterns.

As mentioned before, in most work-contexts, inch and metre are both used for length measurement, but we noticed that students were unaware of the connection. Similarly, in small neighbourhood outlets, milk in small quantities is sold by weight as well as by volume. Our participants did not show awareness of the relation between these two measures. Indeed, sometimes volume and weight units were interchanged ("kilo" instead of "litre"). For example, E_8 purchased milk measured by weight for small quantities, but for larger quantity, milk is measured in volume – in litres and multiples or fractions. Interestingly, small quantities of milk are available in fractions of a litre as well, viz., half or quarter litre. Here is a small excerpt (5.7) from a conversation with E_8:

Excerpt 5.7: Work-context interview of E_8

202	T	to doodh kis bhav se milta hai?	So at what rate is milk available?
203	S	aap kaisa loge waisa hee milega/	As much you take based on that/
204	T	tumne kitna liya tha?	How much did you take?
205	S	main paanch rupaya ka liya tha/	I bought for five rupees/
206	T	kitna diya?	How much was given?
207	S	der sau gram/	One hundred fifty gram/

235	T	der sau gram ka kitna paisa liya?	How much was charged for one hundred fifty gram?
236	S	paanch rupaya/	Five rupees/
237	T	achha, phir ek kilo ka kitna?	Ok, so how much for a kilo?
238	S	ek kilo ka nahin, litre milta hai, ek litre/	Not a kilo, it comes in litre, one litre/
248	T	ek kilo ka nahin aata hai, ek litre milega/	(repeats) It doesn't come in kilos, one litre is available/

Thus everyday contexts give rise to pieces of mathematical knowledge that may be intimately familiar to students but may be unconnected to other mathematically related pieces of knowledge. The familiarity and confidence that students display about what they know suggests however that even such fragmented knowledge can be a potential resource for classroom learning.

5.2.2 Knowledge for use rather than conceptual knowledge

In the work-contexts, non-transparent mathematical artefacts are used which are familiar to the users in practice but the conceptual underpinnings are blurred. For example, inch tape is used for quantification of length, but the principles underlying its construction remain unclear to the users. Students may be aware that length can be measured (quantified) by iteratively covering with a unit, but may not be aware that this principle is the basis for constructing the measuring tapes (as was revealed in the teaching intervention discussed in Chapter 7). Similarly, they may be aware of how shopkeepers construct their own weight measures for small weights, but may not know about the sub units marked on a scale. For example, during vegetable selling vendors commonly use small stones or brick pieces in lieu of bar weights for measuring small quantities such as 250, 200, 100, and 50 grams. The researcher observed that many fruit and vegetable vendors in the neighbourhood had adopted such measurement modes.

Some students are familiar with the use of length dimensions for designating “size” (area), such as in frame sizes, without being aware that the product of the “length” gives the area measure. The connection between length and area remains opaque. The templates used in leather work such as purse, wallet making or bag or file making are essentially area measures but doubled as discrete length measures in work practice (discussed in Chapter 6). Students involved in these work practices are familiar with such measurement modes. In readymade garment selling, a unique number assigned to each garment and referred to as “size” (like, shirt size, frock size) is widely used as a label used to mark garment sizes. However, we observed that not many students, or even older people including some associated either with tailoring work or with garment selling know what the ”size” numbers signify, or how these numbers are arrived at. In the teaching intervention phase (Phase-III) of our study, we attempted to address the issue of the connection between fragmented everyday knowledge of mathematics and their conceptual underpinnings. For example, the participants (middle graders) measured different parts of the garments given to them as part of the activity, and tried to connect the measures with the “size number”. We therefore argue that in most everyday or work contexts, knowledge and mathematical artefacts though frequently used, remain opaque and non-transparent when it comes to the conceptual knowledge associated with those artefacts.

These have implications for school teaching and learning which aims at making connections with out-of-school knowledge of mathematics. These are explored in the context of a particular topic area, that of measurement, in Chapter 7.

5.2.3 Demathematisation of knowledge

In contemporary society, industrialisation is frequently accompanied by deskilling and loss of craft-based knowledge due to mechanisation and fragmentation of the labour process. House-hold based small scale production, like factory based production, is also subject to the processes of fragmentation, routinisation, mechanisation, and deskilling. One of the aspects of deskilling is the demathematisation of knowledge within the community. The need for and the opportunities to acquire mathematical knowledge diminish. For example, standardisation is one of the means by which demathematisation takes place. One can argue that the adoption of the standardised decimal system of units and measures at the national and the international level, has made the skills of computing with a variety of compound units redundant. Thus the mathematical skills needed to compute with a variety of units, to deal with non-decimal subunits, to inter-convert between these units are now unnecessary. In older school math textbooks, we find a topic referred to as the "arithmetic of compound operations", which involves column based arithmetic algorithms dealing with non-decimal subunits (Subramaniam, 2012). A surviving example is addition or subtraction of time durations expressed in hours, minutes and seconds. With the adoption of the decimal system the arithmetic of compound operations are no longer required or used. This can be seen as an example of demathematisation of knowledge.

Embeddings of arithmetic in modern artefacts and devices are other ways in which demathematisation takes place. For instance, calculators make paper-pencil calculation redundant, while comparative EMI tables make it unnecessary to calculate interests. Demathematisation as a notion has been discussed by several mathematics education researchers. "[It] refers to the trivialisation and devaluation which accompany the development of materialized mathematics; mathematical skills and knowledge acquired in schools and which in former time served as a prerequisite of vocation and daily life lose their importance" (Keitel, Kotzmann & Skovsmose, 1993, quoted in Jablonka & Gellert, 2007, p. 8). Demathematization with respect to explicit knowledge and skill accompanies the process of the mathematization of society, i.e., the incorporation of implicit mathematical knowledge in artefacts, instruments and practices. "The greatest achievement of mathematics... can paradoxically be seen in the never-ending, two-fold process of (explicit) *demathematizing* of social *practices* and (implicit) *mathematizing* of socially produced objects and techniques" (Chevellard, 2007, p. 60, emphasis original).

Counter-trends to demathematisation

It is well argued now that children who participate in the informal economy possess mathematical abilities and skills, which are different from those that they learn in school. Some of the previous studies have explored the nature of such knowledge and their distinction with school mathematics (viz., Nunes, Carraher & Schliemann, 1993; Resnick, 1987) while others have looked into the implications for teaching and learning of school mathematics (e.g., Nasir, Hand & Taylor, 2008). We argue, following Subramaniam and Bose (2012), that the circulation of the mathematical knowledge in the informal economy – in micro, small and medium scale enterprises (MSME) and those embedded in the artefacts and objects offers a counter-trend to the process of demathematisation that is rapidly taking place in our contemporary society. We have argued that the counter-trend to demathematisation comes concurrently through the enterprise of individuals and groups seeking out avenues for making a living.

Mukhopadhyay while adapting from one of the Paulo Freire's quotes mentions that "mathematical practices of those without power are too often characterised as non-mathematical" (2013, p. 1). Such devaluation that Mukhopadhyay has pointed to can be both a consequence and a cause of demathematisation. One form of devaluation occurs when mathematics embedded in the cultural and social practices are not taken on board or acknowledged or identified but replaced by the dominant "academic" practices. For instance, the interesting modes of transmission of mathematical knowledge and skills through riddles, activities during festivals, or mnemonic tricks for tables or calculations are fading away in many cultural or subaltern groups such as *Mushars* (Bose & Kantha, 2014). The multiplication tables of fractional numbers (such as quarter, half, three-quarters, one and a quarter, one and a half, two and a half and so on) that were once part of the village community's *funds of knowledge* in many parts of India and passed on through oral practices and traditions from the community elders are rapidly disappearing as schools no longer use or acknowledge them. In this way, demathematisation is devaluation which impacts learning opportunities that are largely shaped by what the dominant culture values and perceives as useful. As it turns out, demathematisation amounts to carrying forward the hegemony of academic mathematics, or in other words, the hegemony of devalorising non-academic mathematics.

One of the counter-trends to demathematisation that we observed in the low-income settlement was the persistence of older forms of practice. For example, the use of non-decimal system of units, such as the old British units of *inch, foot, ounce, dozen*, are still common. Other similar examples are the prevalence of older, traditional Indian units and other non-standard units in commerce and trade, and in the informal economy. There was frequent use of oral computation and less frequent use of calculators or use of mobile phones for calculation, which may be interpreted as a form of resistance to demathematisation.

The general trend of a shift from craft based industry to large scale factory based manufacture leads to deskilling and the expertise situated in the craft based knowledge thus becomes redundant. Such a shift leads to demathematisation. The emergence of the informal sector may be seen as a counter-trend to industrial factory-based production, and hence generates its own set of mathematical practices. The micro and the small-scale enterprise of the economically disadvantaged people from the low-income settlement can be seen as a form of resistance to such a shift towards large-scale factory-based production and the subsequent deskilling (Subramaniam, 2012). The counter-trend against such deskilling is embedded in the valorisation of the funds of knowledge that the community in the low-income settlement possesses.

In the informal sector therefore, we see not only the circulation of a form of arithmetic knowledge and standardised skill, but also knowledge related to measurement that may not be “standardised”. One aspect of such knowledge is the familiarity with and use of informal measurement units. Another aspect is the experience of “non-standard” modes of quantification (discussed in Chapter 6).

The framework of demathematisation helps explain why informal practices and contexts have disappeared partly from social practices and wholly from the curriculum and how their importance and value is diminished. However, in household based occupations, measurement in a diversity of modes and with a variety of units always plays a role. The emergence and survival of such informal mathematics can be seen as a counter-trend to the broad process of demathematisation.

5.2.4 Transfer and hybridity of knowledge

We have argued before that students in our sample have diverse opportunities to gather out-of-school mathematical knowledge through the requirement of their work practices, optimal use of limited resources, decision making, management of house-hold chores, routine purchase of provisions and so on. All the students in our sample also had access to school mathematical knowledge. Therefore, it becomes interesting to explore the interplay between both the forms of mathematical knowledge (school and out-of-school) available to the students. In this section we analyse this interplay by investigating if the students were able to transfer mathematical knowledge learnt or experienced at two domains – classroom/school context and out-of-school context. That is, whether students were able or unable to interchangeably draw on mathematical knowledge learnt in one context and use it in the other (school and out-of-school), or if there was an overlap of both the forms of mathematical knowledge in their problem solving strategies. As mentioned in Chapter 2, some previous studies, viz., Saxe’s study with candy sellers in Brazil (1988) has indicated instances of knowledge transfer. Saxe contended that some sellers who had had exposure of a few years’ of schooling could interchangeably use strategies learnt at work-context as those learnt at schools and transfer learning of one domain into the other. We have considered transfer of knowledge as a process of using knowledge learnt in one domain in the other.

As discussed in Chapter 2, researchers have identified and distinguished between mathematical knowledge acquired in the above two contexts as school mathematics and out-of-school mathematics respectively (see Table 2.1) and we use the "distinction framework" for locating transfer or hybridity of knowledge. We have analysed our arithmetic interviews and interactions with the students using the "distinction framework" to observe the ways in which they attempted to solve the mathematical tasks or problems and how they arrived at the solutions. Analysis of the students' computation strategies determined whether there was transfer between school and out-of-school mathematical knowledge or whether the strategy reflected a hybridised form of both the kinds of mathematical knowledge. We have observed that students often showed competence and flexibility in their choice of drawing on school mathematical knowledge or out-of-school mathematical knowledge or both.

We illustrate an instance of a student's problem-solving strategy to highlight the interplay between his school and out-of-school mathematical knowledge. U_{21} was forthcoming in sharing with the researcher about what he perceived as a gap in his knowledge of the school-learnt long division algorithm. He shared that he arrived at incorrect answers and that he needed to fix the error since he required such skills in his work-context. The researcher then gave U_{21} a series of division tasks beginning with the division of small two-digit numbers by single digit numbers. U_{21} could use the algorithm and complete the division tasks correctly. However, soon after carrying out "981 ÷ 9" using the long-division method (discussed in Chapter 4), he started pondering over the obtained answer of "19". He explained to the researcher that the actual answer cannot be that low, and emphatically mentioned "*kam-se-kam sau-sau to milega na sir*" ["*at least hundred-hundred would be obtained sir*"]. The researcher observed that U_{21} went on and used his out-of-school mathematical knowledge of "convenient decomposition" and arrived at "109" as the answer. However, it appeared to the researcher that U_{21} still believed that he had "correctly" used the long division algorithm that his teacher had taught during lessons in the school but seemed confused about why the algorithm gave "19" as the answer. Yet he seemed to be convinced that "19" cannot be the correct answer and explained that if Rs 981 was distributed among 9 children then each one would get Rs 100 and more and so the answer could not have been 19.

We observe that U_{21} possessed the school mathematical knowledge of the long-division algorithm for solving simple division tasks (involving smaller numbers) but faltered when the need to focus on the place values occurred. However, it is also true that having arrived at incorrect answer, U_{21} could realise the error spontaneously without any external prompt and quickly used his out-of-school mathematical knowledge for obtaining the correct result. We argue that this was a case where a student was able to transfer his out-of-school mathematical knowledge successfully to the solving a school mathematical problem. Other similar instances of students' use of different mathematical strategies drawing from different domain of knowledge encouraged the researcher to look at other problem-solving tasks to explore further instances of transfer or hybridity of mathematical knowledge and what implications such instances might suggest for the classroom pedagogy.

We have drawn data from the arithmetical tasks and work-context interviews to look at the notion of transfer. Our sample children's work-contexts are diverse and consequently, the extent and type of mathematical knowledge that students acquire outside school can be expected to show diversity. Such diverse engagement with contexts help children develop effective context specific problem solving ability that could be used for effective mathematics learning in the classrooms (Bose & Subramaniam, 2013). Occurrence of (partial) transfer of learning emerged among the sample middle graders while they solve mathematical problems reflecting everyday contextual situations in the school set up as well as in the situations that emerge in the work contexts. It is claimed in this study that the occurrence of partial transfer can work as scaffolds for better learning of different components of the algorithms and principles, unlike Bransford and Schwartz's relatively indefinite notion of "preparation for future learning" (p. 69).

Framework to locate transfer and hybridity of knowledge

The arguments and counter-arguments about the occurrence and non-occurrence of transfer of knowledge or hybridity of both the forms of knowledge depend on what is considered as transfer and how we define it. As discussed in Chapter 2, one of the major paradigms in "transfer of learning" has been "Direct Application (DA)" of knowledge gathered in one domain to another, for instance, to new problem situations. The DA paradigm evaluates cases of transfer by following learner's problem solving in isolation, individually, and without help from others (referred to as "Sequestered Problem Solving" method by

Bransford & Schwartz, 2001, p. 67). We have argued that the DA paradigm carries a narrow view of transfer which could be a possible reason for the vast transfer failure reported in the literature (Bransford & Schwartz, 2001; Bose, 2014). Using the distinction framework of Table 2.1 we have looked at transfer or hybridity of knowledge through a two-stage filter – “direct application” as the first filter and partial application of knowledge as the second filter.

The first filter [DA filter] considers definition of transfer on the following criterion:

- Are the students able to use elements of mathematical knowledge learnt in one domain to the other, for instance, use of algorithms?

If the first filter suggests inconclusive answer, by using the second filter [PFL filter], it is checked whether the students are able to transfer at least elements from their school or out-of-school (everyday) knowledge in the form of some components of the underlying principles or the algorithms. Here is the criterion for the second filter:

- If the direct application (first filter) of knowledge from one domain to the other did not occur in, for e.g., use of algorithms, we check whether there was application of components of the underlying principles implicitly or explicitly.

In the present analysis, we have expanded Bransford and Schwartz' notion of “*preparation for future learning*” to include the possibility that elements of everyday knowledge contribute to and scaffold mathematical learning.

Transfer from Out-of-school to school setting: contextual problem-solving

We now discuss an example of a problem-task that required application of proportional reasoning. The problem-task was to solve a proportion problem of finding the price of 25 *burfi* when 20 *burfi* cost 42 rupees. Students’ strategies varied from using school-learnt “unitary method” of finding the price of one *burfi* first and then that of 25, to out-of-school mathematical knowledge involving “halving methods” and “convenient groupings”. For example, 14 students out of 30 who took the arithmetic interviews used the halving method to find the prices of 10 and 5 *burfi* by halving 42 and 21. One student explained, "*bees ka bayalees, dus ka ikkis aur paanch ka gyarah*" [*forty two for twenty, twenty one for ten and eleven for five*]. However, there were 12 students who could not proceed in this problem-task. The remaining 3 students, who initially used the school taught "unitary" method, eventually got stuck in the middle while handling the decimal numbers and could not proceed further. Out of these 4, three students then switched over to the halving and decomposition method. Hence, altogether 17 students (14 + 3) used out-of-school mathematical knowledge and strategies. Some students arrived at 53 as the answer and justified that sellers often do not return a small change of Re 0.50 [*aath anna nahin chhorta sir*]. Some of the students who could not proceed, actually struggled to figure out an arithmetical operation for solving the problem.

Under the first filter of DA, we observed that 4 students could not use their knowledge of school mathematics to complete the task. Thus, those 4 students could not transfer their school math knowledge in solving the task and under the first filter it was not a clear case of knowledge transfer. But, using the second filter, we noticed that three out of those 4 students could accomplish the task following their everyday contexts and reality perspective in using the halving technique that allowed them to find the price of the "difference" in the number of *burfis.* Though not all the students could successfully arrive at the correct answer, but they used "convenient strategies" from their out-of-school math knowledge and hence such instances qualify the second filter and we characterise them as cases of partial transfer of knowledge.

In another problem-task (*finding the number of days 16 kerosene oil cans can last if one can lasts for 7 days*), 10 students used their everyday mathematical knowledge. For example, one student while computing orally, grouped 15 days for 2 cans and arrived at 4 months for 16 cans (considering 30 days per month) and then compensated the extra counts of 1 day per 2 cans, by subtracting 8 days and arrived at 112 days. Although the student did not use the multiplication algorithm for arriving at the answer, she could successfully use her out-of-school mathematical knowledge for solving a multiplication problem-task. Her solution strategy exhibited transfer of knowledge from everyday problem-solving context to school context since the student could build on her everyday mathematical knowledge to use a "convenient strategy" to solve the problem. Interestingly, only one student used the multiplication table of 16 while some students used the tables of 10 and 6 and added the partial products.

E_{16} computed mentally by carrying out the multiplication task in the air and visualised it. U_1 used both oral computation and multiplication table for solving and cross-checking. These instances show mixed use of both forms of knowledge which indicates students possess hybrid knowledge. In some other problem-tasks too, one finds use of students' hybrid knowledge, where instead of characterising knowledge as drawn from a particular setting, one gets to see an overlap of both school math knowledge as well as elements of out-of-school math knowledge.

Many students found the formal algorithm for division difficult to use and they often used situation specific convenient strategies. For example, E_{11} actually divided 315 by 5 and obtained 13 as the answer. He immediately realised that the actual result cannot be that low. Subsequently, he aborted the formal division and did mental computation. We argue that in this case the student could not directly apply school-learnt long division method but he could transfer his out-of-school math knowledge from everyday setting to school-type problem-solving setting. Student's use of everyday experience emerges in realising the error and his use of "repeated distribution" as the underlying principle.

Use of fractions

Binary fractions like *aadha* (half), *paav* (quarter), *aadha-paav* (half-quarter, i.e., one-eighth) and *sava* (five quarters) are part of the everyday discourse that most students were exposed to and comfortable in using. The common contexts where binary fractions are used and which students regularly come across, are while buying provisions, vegetables, milk, and so on. Non-routine fractions remain difficult for most students to comprehend and are poorly developed, whereas binary fractions are easy for them to visualise. The researcher noticed during his classroom observation that students used such mathematical terminologies from their out-of-school experiences while conversing between themselves during the lessons. Often the contexts were the short discussion while solving problems from the textbooks. We claim that these are instances where students drew on their mathematical knowledge from one context to another and hence they transferred knowledge from everyday setting to the classroom setting.

Classroom activities[1]

In the vacation course, during a classroom activity of shirt measurement which aimed at drawing on students' everyday mathematical knowledge for informing classroom teaching, almost all the students preferred using *inch* tape (a popular measurement instrument in the everyday settings) for taking different measurements of the shirts, although textbooks deal only with the international standard units like centimetre, metre and so on. Most students, as mentioned earlier, did not know the relation between old British units (inch, foot) and standard international units. Students also came up with other indigenous measuring units like *bitta* (palm length) and use of templates in the form of *rassi* (rope) during classroom discussions in the teaching intervention (discussed in Chapter 7). One student used her palm-length to measure the desk while another student explained, "*jo log ghar banate hain na wohlog rassi se na dekhe hain naapte hain wohlog*" ["*those who make houses they measure and compare using ropes*"]. Although the students could not inter-convert between the two systems of units, but they could draw on their out-of-school knowledge and use them in the classroom learning context.

Transfer from school to out-of-school setting: Problem-solving

In the work-contexts

Work-contexts of some students require doing quick calculations and use of approximation and estimation skills (for example, in the work-context of E_5's garment recycling work or E_8's mobile repairing work). As discussed before, many children and adults are involved in garment recycling work (as the researcher gathered from E_5, E_6 and some adults in the community), which involves weight measurement of the collected cloth pieces of varying size, colour and texture. The collected pieces are then sold off and the price is negotiated which requires children to make quick decisions and calculations. Children use convenient strategies and develop situation specific competencies, some of them reported use of

1 The classroom activities are drawn from the vacation camp classes for the grades 6 and 7 students of the Urdu school that the researcher and his colleagues conducted. The activities do not reflect actual classroom teaching.

multiplication tables that they learned at school. Like E_{16}, E_5 who does recycling work, explained that he does the multiplication “up in the air” by visualising the whole operation. He claimed that he does multiplication to cross-check the money he received. This is an example where students used their school mathematics in their work-contexts and therefore we argue that these instances present examples of “Direct Application” of school learnt algorithms and methods in the work practices and are instances of transfer of learning.

Other students’ accounts such as U_{24}’s (ready-made garment selling) revealed that they used mathematical knowledge acquired in school as well as in out-of-school contexts. As mentioned before, such instances of hybrid form of mathematical knowledge emerged in students’ solution strategies.

Everyday shopping

School taught formal algorithms are often part of the daily routine, for example, at general stationary stores, sellers apart from using oral computations also use paper and pen to arrive at the total cost when the list of provisions is longer. Oral computations are preferred while dealing with small amount of goods. Some students claimed that they cross-check the calculations using the school learnt algorithms on paper. Such examples indicate direct application of school taught methods as well as out-of-school math knowledge and show transfer of learning in both the directions. Number estimations used in routine computations, glimpses of which came through in the arithmetic interviews with the sample students indicated use of both forms of mathematical knowledge – hybridised form of mathematical knowledge.

Non-occurrence of knowledge-transfer

There are occasions where transfer of learning did not seem to occur. The reason could be poor mathematical learning and lack of preparedness to handle complex calculations. For example, when the researcher discussed fairness of wage with a student (U_{13}) and whether she was satisfied with the wage she received for making *Rakhi* (discussed in Section 4.2.4), the researcher realised that she was not able to calculate the amount she earns for making a single *rakhi*. She could neither use the school taught multiplication algorithm nor any other computation strategy. There was no reflection of the use of any form of mathematical knowledge – school or out-of-school. It occurred to the researcher that poor learning of school mathematics may impede workers like U_{13} from checking the fairness of a deal or the wage and entitlements that are distributed among the workers.

There were other similar cases where students were not able to use mathematical knowledge learnt in one domain in another. Discussion with U_8 about her *latkan* work indicated that she was reluctant to do computations in either way – using school taught methods or out-of-school math strategies. When she tried using the latter (out-of-school strategies), she arrived at absurd answers. These are some instances where knowledge-transfer was not visible.

Pedagogic Implications?

The examples discussed above presented some clear instances of transfer of knowledge from one domain to the other while there were instances where the knowledge source of the target transfer could not be determined or characterised as definitely belonging to either of the two contexts – school or out-of-school. Occurrence of such instances indicate that in everyday contexts students often gather and develop their mathematical understanding from different sources. Such instances question the validity of the distinction between different forms of mathematical knowledge. We noted that although the sample students were able to inter-changeably use their mathematical knowledge drawn from both school and out-of-school contexts but in many instances (transactions during everyday shopping, maintaining accounts) categories of such knowledge were blurred showing an overlap of knowledge from different sources.

Some of the examples indicated that students were not able to directly transfer their knowledge from one context to another but their strategies carried some components of the underlying principles either from school math or out-of-school math knowledge. Difficulty in using formal algorithms could be a possible reason for transfer failing to occur, for instance, where students were unable to use the long division method. Thus, instances where transfer occurred partially emerging from the use of components of the underlying principles has strong pedagogic relevance for better learning. From a pedagogic viewpoint these are also the pointers that can help the educators connect everyday mathematics with school mathematics. The arguments and claims made here are however evolving and calls for deeper exploration of their systemic and pedagogic underpinnings.

5.3 How does out-of-school knowledge shape learners' identities?

Researchers have argued that “issues of identity are critical to understanding the development of mathematical knowledge for individuals and communities …” (Nasir, Hand & Taylor, 2008, p. 188). It is important to focus on the issue of identities since “how students view themselves as learners can influence how they participate in educational activities and settings, [which] is shaped by how teachers and institutions create or limit opportunities for participation” (Nasir & de Royston, 2013, p. 266). Identity is viewed as “becoming a member of a community or social network and is related to an individual's acquisition and alignment with particular bodies of knowledge, goals, and practices valued by that community” (pp. 269-270). Nasir (2002) has illustrated that an individual's identity is recreated through her “agency” during meaning making processes and when she engages in “social practices” and aligns with the community (p. 220).

We note that this notion of identity is aligned with identity in a community of practice as analysed by Wenger (1998). For Wenger, identity and learning were mutually implicated – identity was constituted through participation in the practices of the community, and through sharing the knowledge resources or funds of knowledge of the community. Thus participation and acquiring knowledge are central to the process of becoming a member of a community of practice. Learning is interpreted as this process of “enculturation”. Thus three central notions implicated in characterizations of identity are learning, acquiring knowledge and participation in a community of practice. Accordingly, in our analysis of markers of identity, we adopt the following three lenses:

- Identity as learners (in school and workplaces)
- Identity as knowers (everyday shopping, work-context shopping, discussions about work)

- Identity as doers (in the role of student and worker)

5.3.1 Value as precondition for identity formation

As indicated in the quote from Nasir and de Royston in the previous section, identity in terms of belonging to a community of practice requires that the practice and the associated knowledge and skills be valued in the community. Our interaction with the community members indicated that learning of work skills as well as learning at school are both valued in the community. Though we came across many children during the study who dropped out of school for different reasons, most parents and elders whom we met expressed concern about their children's school learning while at the same time they wished that their children learnt hand skills during their spare time. Participation in the work-contexts is thus a valued feature in the life-world of the children living in the neighbourhood.

Value of learning hand skills (haath ka kaam sikhna)

Learning of hand skills is valorised in the community where people often express the importance of learning *hunar* (skill) and *haath ka kaam* (hand skill) which is seen as laying a foundation and making it easier to get *acchha kaam* (good job) in the future. This view was frequently echoed by students. For instance, 12-year old U_{21} had a work experience of more than three and a half years. He first worked in a bag making workshop to learn the stitching skill where as a novice he was not paid except for food and some pocket expenses. Thereafter, he switched to garment (shirt) stitching work and gained expertise and moved on to three-quarter trouser making work. He explained that workers start getting wage when they start contributing to the production, i.e., when a novice becomes an apprentice. Getting involved in “any kind of work” (*koi bhi kaam*) is valorised in the community since it builds networks with people including *seth* (workshop owners who provide jobs), helps in learning hand skill and utilising time in a better way. It is believed in the community that learning hand skill early would be “useful later on” (*aage kaam aayega).* U_{21} got into work to learn a skill and to start earning so as to be able to send money back home for his father's treatment of bronchitis and lung infection. As mentioned before, U_{21} explained that having mastered the shirt stitching skill he kept looking for other jobs where he could “learn something different” (*kuchh alag seekhne ko milega*) and also earn more. Interaction with U_2 (engaged in tailoring work) raised this issue when he discussed why learning and doing some work is important:

Excerpt 5.8: Work-context interview of U_2

155	T	to woh sab sikhna zaruri kyon hai?	so why is it important to learn these? (pointing at work practices)
156	S	sir, ek haath mein hunar ho jayega na sir/	sir, it's like getting a skill on hand, sir/
157	T	hunar?	skill?
158	S	haan/	yes/

Just as learning a hand skill is valued, idling time away ("*khaali baithna")* without doing any productive work is considered bad. We observed among all the students that learning of a new skill is sought after and seen as better than idling away time (*time barbad karne se achha hai*). For example, U_{22} who has learnt mobile repairing and garment making by closely watching his friend in his mobile phone shop and his father and elder brother in their garment making workshop, takes pride in having learnt about different kinds of work and having not been idle *(khali nahin baithta*). His father runs a shirt stitching workshop where three other workers are also employed. U_{22} visits the workshop frequently and brings the material required for the work and also observes the stitching work. Learning is generally evaluated by the amount it can help one earn. Therefore, often children report about using their "leisure" time for learning new work and developing new skills, etc. and thereby making "proper use" of the time (*samay ka sahi istemal*). It could be a reason why children belonging to such families, who can afford their education without a need to supplement their income, also work while studying.

Prolonged interactions with U_2 and other students and community members indicated that learning of "hand skills" (*haath ka kaam* and *hunar*) are valorised for three main reasons: i) helping family members and acquaintances, ii) self reliance, iii) getting work in future. Most students that we interacted with appeared to be self dependent and managed their affairs themselves. For example, U_2 manages without any help when small stitching work on his garments is needed – buttons are to be replaced or repairing cuts or tears. He claimed that attends to all such needs himself - "*khud se karta hoon*" (*I do it myself*).

Similar was our observation with other students who are self-reliant when it comes to repairing of electrical appliances (for example, E_8) or mobile phones (E_{22}, E_8) and so on.

Anybody can learn

There is also a belief in the community that anybody can learn any task. People believe that one can learn by observing others carrying out the tasks. With such a belief young children are sent to workshops and manufacturing units early in age to learn "*haath ka kaam*" and "*hunar*". U_2 started going to the tailoring workshop with similar intention. The following conversation with U_2 highlights this belief:

Excerpt 5.9: Work-context interview of U_2

219	T	jaise maan lo..., aaj agar main tumhare wahan jakar bolu ki mujhe sikhaoge, to mujhe sikhayega kya? Ya woh poochega na, kuchh sawalat karega na ki aapko yeh aata hai, woh aata hai?	so assume, if I go to your place today and ask would you teach me, so would they teach me? Or will they ask me, some questions, whether I know this, know that?
220	S	nahin, nahin woh sab nahin/ aapko sikhna hai, jaise aap kabhi machine nahin chalaye to koi baat nahin, aapko batayenge woh/	no, no nothing of that sort/ you want to learn, for example if you have never used a [sewing] machine, no problem they'll teach you/

During our interactions with U_{23} (involved in dyeing work), he emphasised that "*nobody comes having learnt since childhood*" [*bachpan se thode na koi sikhkar aata hai*] and that they all learnt the skills by watching others. He argued that therefore anybody can learn even if slowly. Here are two excerpts from the interview with U_{23}:

Excerpt 5.10: Work-context interview of U_{23}

132	T	to uske liye kya kya janna hota hai? Maan lo mujhe bhi sikhna hai painting, to mujhe kya kya sikhna hoga?	So what all do you need to know? Suppose I want to learn painting, then what all would I need to learn?
133	S	abhi jo karigar karte hain, chhape kaisa, abhi isko kaise karne ka, dyeing kaise karne ka, banate log to	Now what the apprentices do, how to print, how to do this, how to do dyeing, all this is done by apprentice,

		karigar hai, woh log to sikh hi lete hain na? Woh dekh-dekh ke humlog sikh hi lete hain, isme kya hai?	don’t they learn anyway? We learn anyway by watching, what’s the big deal in it?

405	S	dheere dheere aap sikh jayenge kaam ko/	Slowly you'll learn the work/
406	T	tab to koi bhi kaam mein aa sakta hai/	Then anyone can get into a work/
407	S	haan, koi bhi/ dheere dheere time lagega to sikh lenge bilkul/ bachpan se thode na koi sikh kar aata hai/	Yes, anyone/ it takes time but one can learn totally/ no one comes fully learnt since the childhood days/
408	T	lekin paisa kaisa milta hai?	But how much money is given?
409	S	mahine ka to paanch hazar hai, saare char bhi hai, jo jitna fast karega, jitna fatafat piece maar le, jisko experience hai bahut saal se kaam kar raha hai, usi hisaab se usko pagar milta hai; agar accha kaam karke dega to usko achha pagar dena padega na/	Monthly it's five thousand, four thousand even, who can work faster, how much fast can one complete a piece, one who's experienced working for many years, payment is made accordingly; if one does good work then better wage is given to him/

The quotes above indicate an openness to learning, as well as indicate the integral connection between learning and participation in a practice. As noted earlier, both learning and participation are central to the construct of identity in socio-cultural analyses.

Value of school learning

As described before, school learning is valorised in the community and seen as a gateway to future opportunities. Graduating from school is taken as an important milestone and parents often urge their children to complete schooling. The excerpt presented below highlights how E_5's parents and relatives suggest that he continue his studies so that he can be placed in a job, though E_5 himself does not want to study further. The interview took place when E_5 was finishing his seventh grade.

Excerpt 5.11: Work-context interview of E_5

201	S	Sir, padhai to pehle karna chahiye, kaam baad mein/ kyunki padhai karenge to aage chalke kuchh ban bhi	Sir, study should come first, work later/ because if we study then later on we can become somebody/

		sakte hain/	

447	T	aage, aage kya soch rahe ho kya karna hai iske baad?	Next, next what are you planning to do after this?
448	S	sir, iske baad soch raha tha ki iske baad padhoon nahin kaam karoon, lekin abbu bol rahe hain aur padh le phir aage chal ke tere ko hee kaam aayega, main bola theek hai/	Sir, after this I'm thinking not to study but to work, but *abbu* (Dad) is asking me to study more so it will be useful later on, I said alright/

456	S	meri ammi boli barawi padh le, mera babora bola chaudha padh lega to tere ko police ki naukri mein daal dunga/	My *ammi* (mother) is asking me to study till twelfth, my uncle said if you study till fourteenth then I will put you in the police's job/

In the excerpt below, U_2 reiterates his interest in learning mathematics. On earlier occasions too, he came to the researcher and expressed his concern that he is not good at math and that he wants to learn it well. U_2's utterance below also reflects the value that some participants gave to learning of mathematics (another example, U_{21}). In other words, U_2 wished to identify himself as a learner of mathematics.

Excerpt 5.12: Work-context interview of U_2

573	S	Main bus sir math padhna chahta hoon/ math achhi se koi padha de na...	Sir, I just want to study math/ if only someone can teach me math well...

Some participants explained how school maths is different from out-of-school maths, but it may be useful in keeping accounts, etc. As described in previous chapter, maintaining accounts is a common house-hold practice that parents usually ask their children to do.

5.4 Identities in work-contexts

In the interviews with students about their work-contexts, they spoke at length often describing many details that indicated the strong sense of belonging that they had in relation to their work practice. Researchers have suggested that the sense of belonging to or membership in a group (of learners or practitioners), especially when such participation includes a sense of competence acquired in and through practice, is constitutive for the notion of identity (Nasir, 2002; Nasir & de Royston, 2013; Gee, 2001). Nasir has further argued that the "relation between learning and identity is bidirectional" and access to learning supports formation of stronger identities which then supports further learning (Nasir & de Royston, 2013, p. 264). In the earlier sections, we have illustrated how some students had a sense of ownership about their task or work-context, which creates affordances for math learning. Such "identification" with the work practice conveys a particularly strong sense of identity derived from the work context. However, even for other respondents, work-contexts built strong sense of identity through the extent of their participation and learning in the contexts.

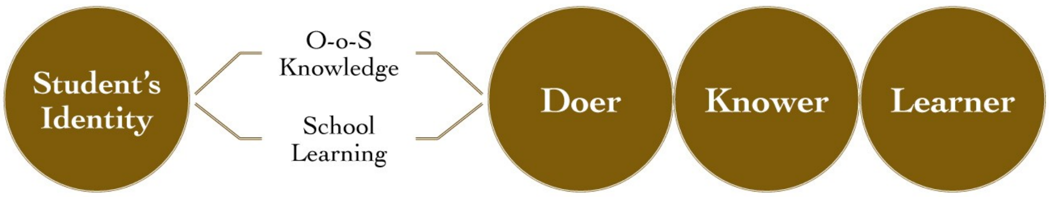
Students with a strong sense of ownership often made explicit remarks identifying themselves with the owners of a business (such as U_{23}, who worked in the block printing workshop, or U_{22} who worked in the mobile repairing shop). Other students, on occasion, used forms of self-reference that explicitly indicated belongingness to a group. For example, in the interviews, E_{16} used expressions such as "*humlog*" (we) and "*unlog*" or "*wohlog*" (those people) as explicit markers of belonging to a group, while E_6 and U_{23} used the term "*karigarlog*" (workers). In informal talk, students often used the word "*taporilog*" (signifying street toughs or rowdies) as a judgement about persons who did not pursue any useful activity. We reproduce below an excerpt from the interview with E_{16}(involved in stone-fixing work), which indicates the context in which she used explicit markers of belonging to a group. Words which function as such markers are shaded in grey.

Excerpt 5.13: Work-context interview of E_{16}

350	S	unlogon ko profit nahin hota hai sir, kyunki humlog yahan pe banate hain, humlog company mein dete hain, company se tempo mein jata hai, wahan se display matlab dusre aadmi ke paas jata hai, wahan se bhav badh-badh ke wohlog ke paas aata hai, wohlog ke paas bhi...	Those people don't make profit sir, because we these here, we send them to the company, from company these go in a tempo, from there to display that is to other people, from there these go to those people following increase in the price-tags, those people too have...

To explicate students' sense of identity, we need to go beyond such explicit markers. More generally, for each of the students with experience of work, a sense of identity derived from participation in work practices, learning work processes and gaining competence in work (hand) skills, that is their participation in the practice as doers, as learners and as knowers. In this section, we discuss students' sense of identity by tracing their assertions during the interviews reflecting their identities as doers, knowers and learners.

Fig. 5.1 Students' identities and learning



Participation in work practices: identities as doers

We have argued earlier that the *funds of knowledge*, and availability of strong network within the community help the children gather and master social skills, skills of cooperation and skills of the work practices. Participation in work practices is the most important way to acquire such skills. Children participate in work primarily as doers – they have productive capacities that are actualized in a work setting through the products that they make. In our interviews with the children, they repeatedly spoke about what they do, what they make and how much they make. Performance criteria are important to the extent that one has taken on the identity of a doer. Similarly assessment of how much one produces and earns, what kind of work yields a better income, are judgements that children make as doers or producers. U_{21} for example, had decided to move from stitching shirts to stitching three-quarter length trousers since it yields a better income. The following excerpt indicates the considerations that underlie the choices made by U_{21} as a doer. In the Excerpt, U_{21}'s identity as that of a doer is reflected where he talks about the number of garments stitched in a day by other workers that he as a doer can aim to achieve. The account is also a reflection of the competence that doers like U_{21} aim at.

Excerpt 5.14: Work-context interview of U_{21}

95	T	Achha...to abhi ye shirt silne ka kaam chhod kar tum three fourth...?	Okk... so now after shirt stitching work you've moved to three fourth...?
96	S	Three four/	Three-four/
97	T	Three fourth pant silne mein aaye ho na.... to ye kyun chhoda aur ye kaam kyun pakda?	You've joined three-fourth pant stitching work right... so why did you leave this and joined this?
98	S	Is kaam me kya hai jaise ki....bhari-vaari maal banta hai to koi pattern matlab banta hai usme se ek nahi mil pata hai, kam milta hai, aur isme thoda kaam bhi jyada dikhta hai, shirt mein kaam bhi thoda jyada hai, achha three-four me kya hai jyada kaam hai nahi, do pocket lagana padta hai aur seedhi seedhi silai marna padta hai bas/	This work has... heavy (complex) materials are made some patterns, don’t get single ones, less of similar (patterns), and this has more work, shirt (stitching) has more work, ok, and three-four has not of much work, two pockets are put and straight stitches are required to be put/ that's it/
99	T	Hmmm... to dono me muqabla karein to kisme jyada paisa milta hai?	Hmmm... so we compare both then which has more money?
100	S	Three-four mein/	Three-four has/
101	T	Three four me jyada, okk/ kitna jyada milta hai?	Three four has more, okk/ how much more do you get?
102	S	Dus rupaya/ aur isme kaam bhi jyada rehta hai../	Ten rupees/ and this (shirt stitching) has more work too/
103	T	Three-fourth me kaam jyada hota hai ki shirt mein?	Three-fourth has more work or shirt stitching has?
104	S	Shirt me jyada hota hai jaise ki three-four banane ko chahein to jyada uttarega, banega jyada, shirt kam banega/	Shirt has more work, for example, when we make three four we make more (in numbers), more can be made, shirts – fewer/

105	T	Oh achha...tum bol rahe ho ki ek hi samay me three-fourth jyada utar jaega matlab jyada nikaal paoge. Achha...kitna nikalte ho ek din me?	Oh ok... so you're saying at a given time you can make more of three-fourths, more of them can be readied/ ok.. how many do you make in a day?
106	S	Jaise ki mein to naya naya gaya hun mein chahun to paintees chalis tak nikal dun/	So as I've newly joined, if I try I can make upto thirty five to forty/
107	T	Achha/	Ok/
108	S	Aur baad mein...aur bhi karigar hain mere se pehle se kaam karte hain woh sattar assi panchhattar nikal lete hain/	And later... there are some other workers who're working before I joined they can make seventy eighty seventy five/
109	T	Ek din me?	In a day?
110	S	Ek din me aise nikal dete hain/	They make that many in a day/

Students identify themselves not only as knowledgeable, but also as having skills and capabilities in work contexts. They often described their work with pride. For example, E_5 mentioned his record collection, when he once collected 95 kilos of *chindhi* and earned Rs 640 (see Excerpt 4.1, p. 109 and discussion on it). This functioned as a benchmark and helped him to decide how much time to spend on a given visit. Most students described in great detail how much they could produce as workers and the different kinds of articles that they make, collect or sell. For instance, E_{16}'s description of the small details about her stone-fixing work and the array arrangements that she makes or E_5's detailed illustration about his *chindhi* collection work or U_{21}'s and U_{23}'s detailed discussion about their tailoring and textile printing work reflect their identities as doers.

Competence at work: identities as knowers

As mentioned earlier, in the interview with students, they were eager to share many details about their work and to explain its intricacies to the researcher. We have spoken earlier of how U_{24} knew about the various garment sizes, the ages of the children they would fit, their prices and profit margins. We have spoken of the details that U_{23} knew about block printing (dyeing) work, or the details that E_{16} knew about stone-fixing work. In the following chapter, we discuss in detail the knowledge that children gain from work-contexts specifically about measurement. The identity as knower is expressed when children not only know about their work-contexts, but know that they know, share this knowledge with others and value what they know. In the case of E_5, who collected chindi, he also helped other children with what he knew.

The work-context interviews contained instances where most students were forthcoming about discussing their work at length and sharing the nitty-gritty of the tasks. Such dispositions reflected their taking pride about presenting details of work which shaped their identities as knowers. Students often invited the researcher to their workplaces/workshops and explained their work-context and the related work processes. The researcher visited U_{21}'s garment stitching workshop and U_{25}'s *zari* workshop several times on their invitations. During these visits, the researcher was invited to learn a few skills like holding the needle used in *zari* work and picking the sequins. In the process, the researcher's action was evaluated and often corrected. The researcher came across many students outside the sample who were keen to talk about their work-contexts. This included the seven students who volunteered to take part in the in-depth interviews as well as other students. We argue that such instances reflected students' identities as knowers who were ready to share their knowledge about the work-contexts. E_{16}'s illustration of different array structures for forming a set of 144 jewellery pieces or E_5's discussion about different measurement modes used in weighing *chindhi* or U_{24}'s description of relation between age and size numbers of the garments – all indicate that students behaved as knowers with skills and possessed *funds of knowledge*. The following conversation (Excerpt 5.15) shows that E_{16} needed to arrange different arrays of jewellery on the cards totalling 144 pairs, in the arrays of 24 × 6 or 12 × 12.

Excerpt 5.15: Work-context interview of E_{16}

104	S	one forty four jodi banayenge na tho one gurus hota hai tho usme eighteen rupees hi milte hain/ …	if we make one forty four pairs then it’s one gurus [gross = 12 dozen], then we get eighteen rupees for that/ ...
105	T	haan, haan/	yes, yes/

106	S	to one gurus matlab one packing ka to twenty four cards honge, dekha jaye to, matlab one card mein six jodis lagte hain/ …	Then one gurus means in one packing there'll be twenty four cards, so if we see, it amounts to six pairs on one card/ …

132	S	sir, apun aur jaise twelve cards banayenge na three stones ke to apne ko isme ka twelve cards hi banane padenge, agar aise card rakhenge na to twenty four card banane padenge/ agar aise rakhenge to twelve cards hi banane padenge/	Sir, if we make twelve cards of three stoned [earring], then twelve cards are needed to be made, if we put this way [*she indicated on worksheet, 6 pairs on a card*] then twenty four cards are made/ if we put this way [*12 pairs on a card*] then twelve cards are made/

U_{24} shared with the researcher his knowledge about the garment sizes and the age-group of children those fit to. U_{24} used his knowledge of the connection between garment-size and age-group in quoting price and offering discounts and also for quickly choosing a garment of proper size for the waiting customer. Similarly, U_{22} had exposure of handling and repairing not just different parts of a cell (mobile) phone but he was also aware of where he could get different products (branded or locally made) and at what rates. Here are excerpts from the interview:

Excerpt 5.16: Work-context interview of U_{22}

116	S	mike baithana, speaker, chhoti-moti IC, mobile band rahta hai to thodi-mori chalu deta hoon/	Putting mike, speaker, big or small IC, if mobile if switched off, I can repair to turn it on/

142	S	jaise sir mike liye, terah ru ka ek mike aata hai nokia ka, china ka lenge to saat-aath ru ka aayega ek mike, jaise barah-terah lenge to pachas-saath sattar aisa kuchh aayega/	For instance sir, for mikes, one costs thirteen rupees for nokia [brand], local products cost seven-eight rupees per mike, if we purchase twelve-thirteen [pieces] of them, they come for fifty-sixty or seventy/
143	S	matlab kam aayega/	So comes for less/
144	T	matlab kam paisa lagega/	So requires less money/

145	S	aur battery lenge to woh bhi kam mein aayega, jaise yahan deta hai dhai sau, teen sau ru mein, to wahan sau rupaya/	And if we take battery, it costs less too, for instance here it's sold for two hundred and for fifty or three hundred rupees, but over there for hundred rupees/

Moving within and between work contexts: Identities as learners

As discussed before, students often shared what they have learnt or are learning in their work-contexts. U_2, E_5, U_5, U_{21} explained that they started working "to learn" hand skills often with no remuneration till they started contributing to the production. Since the community valorises "learning" gathered at school or workplaces, it was pertinent that students had a disposition to associate themselves with learning something. Such disposition shaped their identities as learners. The excerpts 5.17 and 5.18 given below illustrate two different contexts – while in both the cases the students initially went to learn about work and to gather "hand skills", U_{21} subsequently had to support his family and required to work to earn an income. U_2 on the other hand continued working since his parents asked him to learn some hand skills and not waste time.

Excerpt 5.17: Work-context interview of U_2

38	T	achha, to usme tumko kya karna hota tha?	Ok, so what did you have to do there?
39	S	sir, main sikhne jata tha/	Sir, I was going there to learn/

Excerpt 5.18: Work-context interview of U_{21}

681	S	Nahi uss time jarurat nahi thi kaam karne ki magar phir bhi mein jaata tha kaam sikhne ke liye/	No that time working wasn't required but still I used to go to learn work/
682	T	Achha lekin aaj kal tum ek dum school aana kam ker diye ho aate hi nahi ho balki/	Alright but now you come to school less frequently, rather you've stopped coming altogether/
683	T	To aaisa kyun hai, abhi kaam karne ki jarurat hai?	So why is so, is it required to work now?

684	S	Abhi thoda kaam karne ki jarurat hai, ghar me bhi abbu abhi kaam karne jaate nahi hain. abbu ka elaaz bhi chal raha hai unka/ to unke liye thoda paisa-vaisa chahiye aajkal/	Now it's required to work, at home Dad doesn't go for work these days, Dad's treatment is going on/ therefore for him some money is needed/

We observed from the interviews that most students viewed participation in work-contexts as an opportunity for learning and gaining competence. The commonly used categories like "*kuchh sikhne ko milega*" (*will get to learn something*), "*abhi seekh raha hoon*" (*learning now*), "*yeh abbu se seekha*" (*learnt from father*) indicate that learning in work-contexts is not just valued by the community and the students but they looked forward to gaining competence and such dispositions are markers of students' identities as learners.

There are similar accounts of other students who learnt about their work by being part of the family that owned workshops and it was considered as obligatory to support family business. Some of these families did not require supplementary income and their children often worked not for earning but to support their parents or elders and to learn about the work. E_6 for example helped his father in his button stitching workshop when it was required, at other times he sat in his brother's mobile repairing shop. He explained that he learned about different mobile-parts and repairing tools such as, IC, mike, blower, iron machine, liquid for cleaning. We observed that E_6's identity as a learner grew through these kinds of work exposure. Here is an excerpt:

Excerpt 5.19: Work-context interview of E_6

237	S	Main, main sir jyada der mobile ki dukan mein hee baithta hoon, mere ko maloom hai ki mobile kahan, kaun saman kahan lagti hai/	I sir spend more time sitting in the mobile [reparing] shop, which parts are put where inside a mobile is known to me/

We obtained similar account of development of the identity as a learner in the cases of U_{24} (garment selling) or U_{23} (textile printing) who were engaged in their respective work practices, since those were their family work and they needed to take part in them and learn about them. Skill learning was considered important in such cases as well. Such work exposure established in these students an identity of a learner. As mentioned before, U_{24} had once clarified that "*apna kaam hai to karna padega na*" ["*it's own work so needed to work*"]. The families of these children did not require them to work to supplement the family income. In some other cases, students got into work by being in the company of friends (E_8) or as independent workers (E_5). All the students that we interacted with reflected identities as learners and in some cases, they looked forward to getting better jobs in future. Similar account of learning and competence requirement was noted when U_{21} discussed why he switched to a new job, since his objective was to learn different types of stitching skills that can help him open a tailoring workshop in future.

As discussed before, U_2 and U_{21} (and other students) had approached the researcher to with an intention to learn the method of carrying out long division. They had explained that such knowledge was required in their work-contexts. It was interesting as well to note that working students like them actually knew where they were falling behind and what shortcoming was needed to be fixed. We claim that learning in work-contexts and importance of gaining competence framed their identities as learners that encouraged them to learn different such traits in order to make progress.

Some of the participants responded to questions about how they had learnt a particular kind of work responded by saying that they had learnt by "watching others".

Excerpt 5.20: Work-context interview of U_{22}

125	S:	jaise woh kuchh kaam karta rahega, jaise IC baithana hua, to dhyan se dekhunga/	For instance, if he's continuing with his work [U_{22}'s friend], say, fixing of IC, then I watch carefully/

Such accounts reflected students' initiative in creating learning opportunities for themselves, which are constitutive of identities as learners. Learning by observation should not be interpreted literally as a silent interaction, but as a way of learning from others while on the job. Such expressions were used especially for people with whom the participant had close relations. This contrasts with learning in more formal apprentice-like contexts such as in tailoring where learning happens through stages (for example, U_2 went through similar stages of learning garment stitching, see Section 4.4.1). It is important to note that children also actively created learning opportunities for themselves in the social networks generated by the work context.

When children's learning occurs in community links that are especially close, it becomes embedded in life histories reflecting their being a part of shared funds of knowledge. For example, U_2 mentioned that he learnt about carpet making work done in mosques in his native village in Uttar Pradesh when he watched his father working. In the course of the interview, U_2 remembered that he had learnt about a traditional unit of length measure from his father and tried to recall what it meant – carpet making work used old Indian equivalent of the British unit of yard (in Hindi: "*guj*"). U_2's connection to such community practice was also a site where out-of-school learning and knowledge sharing occurs through community networks by making use of funds of knowledge. Such life-histories and connection to work-contexts help children form their identities as learners. Here is an excerpt of U_2's old connection with carpet making:

Excerpt 5.21: Work-context interview of U_2

323	S	barah inch ka... gaon mein abbu kya sikhate the? Ek guj/ gaon mein guj...	twelve inch is.. what did dad teach in village? One yard/ yard in village.../
324	T	achha, guj bhi hota hai kuchh?	alright, is there something called yard?
325	S	haan sir, guj... jo woh bolte hain na galicha...	yes sir, yard... there's something called galicha (carpet)..
326	T	haan, galicha, haan...	yes, carpet, yes...
327	S	galicha, masjid waigarah mein kalin...	carpet, inside mosques, etc., carpet...
328	T	kalin haan/	carpet, yes/
329	S	Haan to mere abbu jab kalin karte the to usme ek guj rahta tha/	yes, so when my dad did carpet making work yard was used there/

5.4.1 Students' identities and classroom pedagogy

It is commonly seen that classroom pedagogy remains limited in scope by not moving beyond what is prescribed in the syllabus[2] and consequently, students' identities as learners in the classroom remains isolated from their identity as knowers, doers and learners in out-of-school contexts. In others words, merging of such identities can facilitate more instances of learning transfer which in turn can facilitate better understanding of the concepts. We claim that the processes of transfer can be facilitated if there is a back-and-forth movement between the three identities in a student – that of a learner, knower and a doer. Such movement can promote drawing of knowledge from one domain to another which amounts to transfer of learning. Students' identities when kept distinct and isolated do not promote transfer of knowledge from out-of-school contexts to the classroom context and building connections between school and out-of-school mathematical knowledge remains a non-starter. We discuss these aspects in Chapter 7 in the context of the teaching design experiment that we conducted. We also argue that students' participation, learning, competence and belonging to groups (some researchers like Nasir, Hand & Taylor [2008] term it as “affinity groups”) are the markers of their identity formation which in turn shape transfer.

5.5 Implications

Participation in work-contexts and in the classroom lessons, competence in work practices and learners in work-contexts and in schools, co-construct students’ identities of doers, knowers and learners respectively as described above. Students’ engagement in the work-contexts and their acquisition and assimilation of the hand skills are some of the processes that shape one’s identity as a doer. The acquisition of competence comes through stages of learning which shapes one’s identity as a knower. This entire process of learning the task shapes one’s identity of a learner. We argue that children's identities are shaped at the interplay of their viewing themselves as “doers”, “knowers” and “learners” which come through their encounter with the diverse out-of-school contexts and school contexts.

The above discussion indicates that work-contexts have rich resources and opportunities for mathematical learning though such contexts often extend only limited possibilities of using mathematics, which is largely focused on use and devoid of the requirement of

2 This observation matches with the researcher's account of the classroom pedagogy as described in his classroom observation narratives discussed in Chapter 3.

conceptual understanding. However, the affordances created by the work-contexts can scaffold school learning towards conceptual development. The case-studies also indicated the formation of identities among the participants through their out-of-school knowledge. These implications can also address the school experience which as of now tends to reinforce the disconnect between out-of-school and school learning.

The above examples also underline our claims that the whole gamut of everyday experiences including diversity of cultural and work practices shape students' everyday mathematical knowledge and has structural difference with school mathematics. However, the inter-penetration between everyday and school mathematics indicates that learning in one domain has relevance for the other which remains to be unpacked. From the standpoint of socio-economic influence of math learning, analysis of such hybridised embeddings of one domain knowledge onto the other has remained an area that calls for systematic exploration.

The separation of school and everyday learning is also internalised by students, who do not expect that what they learn in school will be related to the knowledge that they acquire from out-of-school contexts. Learning skills and acquiring knowledge through participation in work is valorised in the community that we have studied, although some families discourage their children from participating in work because they think it would affect their studies. School learning too is valued, although for different reasons and as a different kind of learning. It has aspirational value, and the community believes that education is the route to social and economic mobility. However, it is self-defeating for an education system to merely aim to produce the trappings of social class, while depriving learners of knowledge that has power because it illuminates aspects of life. Students from deprived backgrounds enter the classroom with their own rich complement of experiences. In the case of measurement, we see that such experience is diverse and incorporates familiarity and intimate knowledge of measures, of measuring instruments and embodied skills. Our perspective is that education that shuts this rich resource out of the classroom is a recipe for failure.

6

Opportunities and Affordances for Measurement Knowledge

Reason has always existed, but not always in a reasonable form

– Karl Marx

The discussion in previous chapters on students' out-of-school mathematical knowledge and experience has indicated the prevalence of measurement related knowledge in the diverse work settings that participants in the study have access to. Children acquire and make use of measurement related knowledge in the workplaces, in economic exchange, in homes and in other everyday life settings. In this chapter we explore and characterise the out-of-school measurement knowledge possessed by school children embedded in the micro enterprises. We then inquire into the implications of such knowledge for school mathematical learning.

We have located our discussion in the existing research literature on knowledge of measurement in out-of-school contexts, and in the research studies on the school learning of measurement by highlighting the affordances that out-of-school settings create for measurement learning. The diversity of measurement knowledge that is prevalent in out-of-school micro-enterprises have deeper implications for school measurement learning that has not emerged in the literature explicitly. The chapter presents a characterisation of out-of-school measurement related experience that may allow integration of out-of-school knowledge with school learning. We note that the topic of measurement is a common core area in both science and mathematics education. Hence it has potential to not only make connections with out-of-school experience, but also between curricular subjects.

Inclusive math education has remained one of the underlying goals of the school education policies adopted in India and strongly advocated in the policy document currently in practice – *National Curriculum Framework-2005*. One of the goals of inclusive mathematics education is to make strong connections between school mathematics and the life of children outside the school. When school mathematics is seen as relevant and useful and is perceived by the culture as valuable, then the motivation for and participation in learning school mathematics is enhanced. This is one of the assumptions underlying the currently adopted school curriculum framework in India (NCERT, 2005). From this perspective, measurement is an important topic area because it makes strong connections with real life and with other school subjects, and is also a compulsory section of the school math curriculum in the middle grades. The notion of measurement occurs frequently in the diverse everyday settings and remains ubiquitous in our everyday life contexts, whether in relation to activities in the home or the workplace. As many studies have pointed out, in India, like many developing countries, children from low-income urban homes who participate in work related activity acquire familiarity with different measurement units, measurement processes, ability to estimate quantities, or knowledge of the costs of different kinds of materials or goods (Subramaniam & Bose, 2012). Hence, for educators, it is important to explore the opportunities and affordances that such knowledge has for furthering and deepening measurement learning in schools.

6.1 Previous studies of measurement in the everyday context

Previous research on measurement within work-contexts or in other everyday settings was carried out alongside or within the research on everyday mathematics, with a particular focus on the alternative ways of thinking in different everyday contexts. Such research provided evidence of how mathematical ideas were developed and framed within work-contexts. Lave (1985) described the use of different units by Liberian tailors and dairy workers in their work-contexts. Her work showed that adults evolved techniques for mental estimation that were markedly different from the school learnt techniques (Lave, 1988). Millroy's ethnographic study (1992) with South African carpenters in their everyday woodworking activities noted extensive use of conventional mathematical concepts like congruence, symmetry, proportional reasoning and optimisation. Her study discussed the use of spatial visualisation and ways in which visual and tactile cues were incorporated. As discussed in Chapter 2, Nunes, Schliemann and Carraher (1993) studied how construction foremen applied multiplicative thinking in everyday work-contexts using proportions and inversion techniques that school students, despite having learnt to solve proportional problems, could not use as these tasks were not among the routine school-type problems. Scale-drawings used by the foremen are examples of how measurement knowledge and proportional reasoning come together in work-contexts. The recent work of Mukhopadhyay (2013) on “vernacular boat making” in the Indian state of Bengal, showed that builders of fishing boats made use of a blue-print that they saw once and subsequently drew on the knowledge of design and construction that they all possessed together as a community. The boat-builders were unschooled and used spatial visualisation and estimation skills and locally made indigenous tools. Mukhopadhyay's work as also Pande and Ramdas' study (2013) with middle graders and Joram, Gabriele, Bertheau, Gelman and Subrahmanyam's work (2005) showed that quantitative measuring skills often develop through qualitative experiences of handling different physical attributes of objects like comparison, seriation or by using reference-point or benchmark strategies.

Saraswathi (1989) studied the measurement practices of agriculture labourers in the Indian state of Tamil Nadu. She reported the use of a variety of measurement modes and units to describe the linear dimensions of routine objects used in everyday contexts. The units were standard (old British, metric) and non-standard (body parts, indigenous units). Linear dimensions often served as an object's identity and description. Estimation skills depended more on experience and mental measurement. The implications that Saraswathi drew for an adult education curriculum involved systematisation of experience in linear measurement beginning with understanding the evolution of processes of linear measurement and overlap of measurement stages (recognition of linear dimension, its comparison with other similar object, choice of suitable units of continuous measure and need for standard units and tools), followed by introduction of standard tools.

Most of the above studies that focused on participants' measurement knowledge involved adults in their singular work-contexts. We have not come across studies that looked at the varied contexts in the everyday settings that students from low socio-economic backgrounds are exposed to and the affordances of these settings for school learning about measurement. The literature mentioned above has led to a cumulative understanding of the skills, procedures and strategies based on mathematical principles that are acquired in out of school work contexts. The focus has been on oral computation strategies, proportional reasoning strategies, visuo-spatial and geometric reasoning and estimation skills and strategies. In this article, we restrict focus to the topic of measurement, but take a broader view derived from the funds of knowledge framework. Thus we are interested in not only what our participants know or can do, but also what they have observed and are familiar with even if the mathematical knowledge associated with these aspects is partial and fragmented. Our perspective is to explore what aspects can serve as starting points or building blocks for mathematical exploration in the classroom. We are also interested in how mathematical learning can strengthen the understanding of measurement practices in the real world.

6.1.1 Research on the learning of measurement as a school curriculum topic

Research on the teaching and learning of measurement as a topic in the school curriculum has been influenced greatly by the work of Piaget. This research conducted over the last several decades has focused on the conceptual aspects of early measurement learning, with a large focus on length measurement. Measurement refers to the quantification of an attribute of interest for purposes of comparison and for use in a calculation. Research studies have highlighted the key concepts that underlie the understanding of measurement. Piaget himself stressed the key notions of conservation, transitivity, equi-partitioning, displacement and iterative covering (Stephan, 2003) as underlying length measurement. Subsequent research has added the notion of accumulation of distance and additivity and the role of the origin on scales (Sarama & Clements, 2009). These ideas have also been extended to the learning of area and volume measurement.

Such research insights about conceptual building blocks of measurement understanding are important for guiding the redesign of the curriculum and teaching. It must be mentioned that current mathematics textbooks do not adequately cover many of these key ideas. A look at textbooks prescribed by the central and state governments (followed by the vast majority of students in India) reveals that the dominant emphasis is on acquiring measurement skills and on knowledge of the international system of units (for example, Maharashtra Textbook Bureau, 2006; NCERT, 2006b, c). Conceptual issues are dealt with briefly under the rubrics of “use of non-standard units” and “need for standard units”, before the treatment moves over wholly to the development of skills. These include familiarity with common measurement instruments, use of standard measurement procedures, interconverting between smaller and larger international units and computing with units. Classroom teaching in the schools that formed part of the study revealed that there is even greater emphasis on paper and pencil computation skills with very little treatment of either conceptual matters or even of practical measurement.

We note that the group of students with whom we carried out the study were 12-13 year olds in Grades 6 and 7. For them, some elementary notions such as transitivity, conservation and unit iteration are likely to be known, although they may be unclear about how these notions form the basis for common measurement procedures, tools and conventions. However, the curriculum for these older children needs to go beyond the aspects emphasised so far in the research on the learning of measurement or the aspects that have been included in school textbooks. The curriculum and research agenda needs to include concepts that connect with and illuminate the diversity of measurement related practice encountered in work and everyday contexts. It needs to focus on the idea that quantification is at the heart of measurement and that quantification is achieved in different ways for different attributes and for different purposes. It needs to develop an appreciation of the difference between scientific measurement and measurement in the everyday world. In this chapter, we have argued for the inclusion of conceptual aspects that have so far not been included either in the curriculum or in the research on measurement learning. We argue that the diversity of measurement experiences in work contexts and everyday settings justify inclusion of these aspects in a curriculum that connects school learning with out-of-school experience. The knowledge that children bring into the classroom from out-of-school contexts supports learning of these ideas.

6.2 Characterising out-of-school measurement related experience

The diversity of out-of-school settings discussed in the case studies in Chapter 4 give rise, as one may expect, to diverse experiences of measurement. In this section, an analysis of the diversity of measurement related experience is presented from the point of view of portraying the inherent richness of concepts implicated in such experiences, which can help connecting out-of-school knowledge with classroom learning. The diversity of measurement related experience is discussed under two rubrics: (i) comparison, estimation, quantification and construction in relation to measurement and (ii) diversity of objects, measurement units and tools.

6.2.1 Comparison, estimation, quantification and construction in relation to measurement

Measurement in everyday contexts including work and domestic settings is different from measurement in the scientific world. Precision and accuracy are not as important as convenience. In many situations approximate measurements suffice. However, many of the processes and concepts that underlie measurement in the everyday world are centrally relevant to a conceptual understanding of measurement. The everyday measurement contexts present diverse and extensive use of comparison and estimation, and varied processes of quantification. Such diverse aspects of measurement knowledge are described below together with excerpts from students' interviews about their work-contexts.

Comparison

In the everyday contexts of measurement, comparison is far more frequently used than full-fledged measurement and quantification is used for the purposes of comparison as well as for use in calculation. For example, as described earlier, in the plastic recycling work, the plastic waste were classified by numbers (1 or 2) to distinguish them from one another in terms of thickness and quality. Here, quantification was used as a tool for comparison. However, such quantification is not based on a unit, and has the function of qualitative ordering an attribute. Comparison at times draws on embodied kinesthetic knowledge, as for example, while judging the suitability of a decorative piece within a defined space in *zari* work or in *latkan* making. The lengths of the *latkan* strings are fixed by comparing them with the sample *latkan* and often in such cases visual comparison is done. Comparison with a template occurs frequently in work and everyday contexts. The use of templates is based on the notion of congruence involving attributes such as shape, size (length), area, volume and weight. In bag, wallet and purse making work, the notions of congruence and similarity are commonly used in comparing the leather/rexin pieces, their shapes and designs. Another example is the use of a filled packet of snacks (*nalli*) as a template to check against, while filling other snack packets. Some students described how they had observed this being done by visual comparison in roadside snack making joints. Estimations are done based on the visual cues while doing physical comparison of the sample/template and the article under production, allowing for rounding off the slight differences in measurements, and not looking for full accuracy.

Templates for length measurement are often used in tailoring or making of leather bags, wallets and files. In tailoring, the work begins with cutting a "*futta*" which is a stiff fabric or a canvas that is cut as per the dimension specifications of the garment to be stitched and made into a template called "*farma*". Both tailoring and leather work use "*farma*" (templates) which are cut to specific dimensions. *Farma* of shirt-collars and shirt-pockets, and "*farma*" of wallets and purses are commonly used in garment manufacturing units and leather workshops respectively. Figures 6.1 and 6.2 below are of the pieces of rexin cut according to the *farma* of a wallet and a ladies' purse respectively in a purse-wallet-bag making workshop. *Farma* is always based on the design and specifications of a product to be made. Hence, in work-contexts *farma* presents instances where comparison is used.

Fig. 6.1 Rexin pieces cut according to *farma* of a wallet

Fig. 6.2 Rexin pieces cut according to *farma* of a ladies' purse

In leather work, a "*farma*" is cut corresponding to every product being made. For example, the "*farma*" (template) for making wallets is often cut from a rexin piece of size of 33" × 39" (1 metre ~ 39"), while "*farma*" of different parts of shirts and other garments is cut from stiff canvas (called "*futta*"). In Figures 6.3 and 6.4, the rexin can be seen rolled up and corresponding to the *farma* of the bag under making, other rexin pieces cut to size (see Figures) are kept for further processing – stitching, hemming, pasting and so on.

Fig. 6.3 Nearly finished purses

Fig. 6.4 Rexin pieces cut according to *farma* of a bag

Estimation

In everyday contexts, estimation is a common measurement mode used with continuous as well as discrete attributes. Children like adult workers learn different kinds of estimation skills based on their work requirement. Work-contexts like *zari* work entail frequent use of estimation by the workers in choosing the quantity of sequins to be stitched in a marked area or a specified design laid out on a part of a garment. Similarly, in leather and tailoring work too, estimation skill is used while deciding the amount of adhesive to be used or while choosing the needle of a certain grade (called number) and amount and types of threads for stitching. "*Chindhi*" work makes use of both estimation and visual comparison skills while sorting. Cloth pieces of similar size are sorted and collected together and the weight of the collection is estimated. Other work like textile printing requires estimating the lengths of the cloth pieces on which block printing is to be done and choosing a suitable "stopper" (i.e., printing block) whose dimensions are known to the workers. During the interview with U_{23} (engaged in textile printing work), he gave detailed explanation about the estimation of the quantity of the colour that is required in printing designs on cloth-pieces of different dimensions. For example, he said in simple designs, one kg colour is sufficient to print the design on 2000 small cloth-pieces. Here is an excerpt from the interview with U_{23} in which he discusses his estimates of the amount of colour:

Excerpt 6.1: Work-context interview of U_{23}

659	T	accha ab ye batao maanlo ke koi design karna hai, manlo koi bhi design to us me kitna color lagega ye kaise andaz hota hai ki kitna colour chahiye, kitne saare chahiye?	Ok now tell suppose a design is to be made, suppose any design, then how much colour is required how is that estimated, how much colour and how many of them?
660	S	… yeh ikkees by dus ka design hoyega (s*howing the logo on a school bag kept aside*) bada design matlab itna bada design, isme colour kam se kam ek, isme pachas gram jayega/	… this is a design of twenty-one by ten (s*howing the logo on a school bag kept aside*), a bigger design, so this bigger design will take at least one, will take fifty grams/
661	T	pachas gram jayega?	will take fifty grams?
662	S	ha hmm.. hoga/	Yes hmm.. it will/
663	T	ye kaise pata chala?	How do you know this?
664	S	hum naapte hai na piece to maloom padta hai na, abhi ek bar me hi/	I take measures of the pieces then I get to know at once/
665	T	to tum pachas gram wazan kar ke rakh lete ho kya?	Do you weigh fifty gram and keep aside?

666	S	nahi andaz se utna colour lagsakta aur lagta hi hai/	No by estimation that much colour might be used and so it happens/
667	T	lekin ye andaz aya kaise matlab?	But this estimation, how did it come?
668	S	andaz abhi hum log kaam karte karte experience se itna colour jata hai...	Estimation, now by doing work, by experience this much colour is used up...
669	T	wahi poochh raha hoon ki kaise pata chala kitna colour gaya, kaise … ?	That's what I'm asking how did you realise how much colour is used up, how … ?
670	S	… abhi dekh rahe hain woh log kaam kar rahe hai to pata chalega kitna ho gaya ya jo maar rahe hai isme to bahut colour jayega isme kam se kam pachas gram colour ja raha hai abhi.../	… now when we watch them working then we get to know how much is made. The one being made [now] will take a lot of colour, at least fifty grams of colour is being used up now.../

The researcher also observed that U_{23} had a strong estimation sense and he was skilled in estimating the dimensions of different objects lying around. While talking about the length and breadth of the *stoppers* used in printing work, he indicated that a commonly used *stopper* was about the same length as the audio-recorder that the researcher was holding in one hand at that time. The researcher asked U_{23} to tell the sizes of some of the objects such as the pen the researcher was carrying, the audio-recorder and the length of the desk. After returning, the researcher and his colleague found that the length of the pen and the length and breadth of the audio-recorder that U_{23} had estimated were surprisingly accurate. Here is an excerpt from the conversation:

Excerpt 6.2: Work-context interview of U_{23}

269	S	yeh der inch hoga shayad chaurai iska. Lambai kam-se-kam paanch inch/ (speaking about the dimension of the voice-recorder that the researcher was using at the time)	this would be one inch and a half its breadth. length would be at least five inches/ (speaking about the dimension of the voice-recorder that the researcher was using at the time)

283	S	haan lambai utna hi rahta hai, kyunki haath mein utna hi aata hai na, jyada bada rahega to maar nahin sakta hai na apun/ (discussion on	yes the length is kept that much because the hand [palm] can hold only that long, if it’s longer then I can’t put the mark/ (discussion on

		stopper-size)	stopper-size)
284	T	achha/	ok/
285	S	ab, itna hai ki haath mein aa jaye	now, it's as much as can fit in hand [palm]/
286	T	to apne haath ka lambai kitna hota hai?	so how much is the length of our hand [palm]?
287	S	apne haath ka to yahan tak chhe inch hoga/ (shows his palm length from the top of middle finger to the end of the palm)	our hand [palm] would be six inches from here till here/ (shows his palm length from the top of middle finger to the end of the palm)
288	T	mera bhi chhe inch hai?	mine is six inches too?
289	S	nahin aapka thoda bada hai/ saare chhe inch hoga/	no yours is little bigger/ would be six inches and a half/

339	S	frame to … bees inch lambai aur barah inch chaurai/ iss hisab se aata hai frame, isse bada bhi chahiye to isse bada bhi mil sakta hai/	frame is … twenty inches long and twelve inches wide/ frame comes at the rate, if you want bigger than this then you can get bigger than this/
340	T	achha, to bees inch lambai kitna hota hai?	alright, then how long is twenty-inch?
341	S	bees inch kitna hoga, yahan tak aayega bees inch; dus inch, dus inch/ itna ho gaya bees inch, aur barah inch chaura, itna ho gaya/ (shows by hand gesture)	how long would twenty-inch be, twenty-inch come till here; ten inch, ten inch/ this much would be twenty-inch, and twelve inches breadth, so this much/ (shows by hand gesture)

Another similar instance of estimation skill came up when the researcher and his colleague were visiting a *zari* workshop. One student who was completing Grade 7 in the English school at that time and assisted the workers in the *zari* workshop that his elder brother ran, could estimate the weight of different objects, viz., the weight of the garment with *zari* sequins stitched on them and weights of various quantities of a variety of sequins. While U_{23} had estimation skill for length measures, this student was conversant in estimating weight measures – skills learnt based on the requirement of their respective work.

The practice of drinking tea in roadside stalls is very common throughout Mumbai city. Tea is often served as "cutting", which refers to a half filled glass tumbler, which may actually be a little more than half a glass. It is sold at half the price of a full cup, sometimes a little more. The “cutting” is a unit of convenience using estimation. It functions as a separate stand alone unit, and not as a part of a fraction of a cup. This practice is completely based on visual estimation skill in deciding how much amounts to “half a tumbler”. Similarly, as reported by students, grocers frequently sell small quantities of articles like tea powder or poppy seeds that are generally expensive, which requires estimation skills. Often small amounts are estimated in measures of “fistful” (called “*mutthi*”). Instances of the use of *mutthi* also emerged during our visit to the *zari* workshop when the workers referred to small quantities of tiny sequins in terms of *mutthi*.

In *rakhi* or *latkan* making, students may need to make estimations about whether a decorative element fits well in a certain space, which they may do visually or by handling. Here the length measures together with geometrical considerations are used in an implicit manner. Estimates of quality are also a part of the work contexts, although these are rarely quantified. An exception is the practice of “grading” found in recycling work described above in which visual estimation and tactile sense are used to designate numbers to plastic waste based on their quality.

Quantification & Construction of measurement units

All measurement depends on the use of measurement units. In school learning, children largely encounter standard units that are pre-given in the form of measuring instruments (tapes, weights, etc.). The choice of a unit and the construction of a convenient unit are the first steps towards quantification of an attribute, and are important aspects of the concept of measurement. In the classroom, these steps are rarely emphasised. In many classrooms, they may be at best be explained verbally. However, there are several out-of-school contexts where children encounter the construction of a unit.

Use of body parts in measurement

The use of the body for purposes of measurement in everyday contexts is ubiquitous. This may be in the form of directly forming units from parts of the body, as in the case of measuring length using hand-spans, palm-lengths, finger bands or finger widths.

Alternatively, kinesthetic knowledge encoded in the body may be used to arrive at estimates of length or weight in terms of standard units. As described in the previous sub-section, U_{23}, who has experience in textile block printing, was confident and accurate in his estimation of the length in inches of small articles merely by sight.

In tailoring, the "finger band" (phalanx) and "finger width" are commonly used to estimate length and length intervals. E_6 who regularly visits his father's button-stitching workshop and also manages its running at times, mentioned the use of finger bands to quantify and measure the distance between every two buttons – about four to seven fingers width distance is maintained between them. E_6 was aware that this may vary between persons and knew that the measurement does not need to be highly accurate. While placing buttons both visual estimation and finger width measurement is used. E_6 also knew that one "inch" is roughly equal to the length of one "finger band". He knew that the lengths of finger bands differ among people while one inch is a fixed measure. Here is an excerpt from the interview:

Excerpt 6.3: Work-context interview of E_6

102	S	isme dur se pakad ke karna chahiye/ (distance to be maintained from the sewing machine needle)	one should hold it far while doing/ (distance to be maintained from the sewing machine needle)
103	T	kitne dur se?	how far?
104	S	char inch ya paanch inch/	four inch[es] or five inch[es]/
105	T	char inch kitna hota hai?	how much is four inch[es]?
106	S	char inch itna hota hai/ yeh ek inch, do inch, teen inch, char inch/ (showed finger-bands)	four-inch is this much/ this is one inch, two inch[es], three inch[es], four inch[es]/ (showed finger-bands)

E_6 was familiar with measuring tapes and knew that they are marked in both centimetres as well as inches on either side. He explained that an "inch" is smaller than a "metre" but he did not know their relation or inter-conversion. Here is an excerpt from the interview with E_6 where he explains the button stitching work, how the markings on the cloth done, how he has to do the work when workers proceed on leave and so on:

Excerpt 6.4: Work-context interview of E_6

184	T	accha, main yeh poochhna chah raha tha, ki tum jo woh kaanch button ke baare mein baat kar rahe the na, ki woh haath kat jata hai, aur kuchh batao na, kaise lagaya jata hai woh?	ok, I wanted to ask this, that you were telling about the button-stitching work, that results in cuts in hands, tell some more, how are the buttons put?
185	S	marking kiya jata hai sir/ main do mahine, jab mere karigarlog sab chhorkar bhag gaye the na, tab main akela hee tha, bhai bhi gaon chale gaye the, main hee akela tha, ... aur ek the sajju bhai kar ke, woh the, papa the, papa button laga rahe the aur sajju bhai card maar rahe the, main marking maarta tha, marking maarta tha, khelta bhi tha/	markings are made sir/ I for two months, when our workers ran away leaving behind everything, then I was all alone, brother was in village, I was alone, … there was one sajju bhai, there was papa (father), papa was stitching buttons and sajju bhai was making the cards, I was putting the markings, put markings, played as well/
186	T	to donolog kaam karte the?	then both of you worked?
187	S	(nodding in 'yes')	(nodding in 'yes')
188	T	arrey wah/	ohh nice/
189	S	dukan bhi chalata tha, mobile ka/	was running the shop too, mobile's/
190	T	acccha, marking kaise maara jata hai?	alright, how are markings put?
191	S	marking shirt pakadke na sir, idhar pakadke idhar pakadke aisa nibat pen se bhi aisa marte hain/	markings are put by holding the shirt, sir, by holding on this side and this side, then by putting pen marks/
198	T	usse kya hota hai?	so what happens with that?
199	S	usse hota hai sir, button jo rahta hai na woh tedha-medha nahin lagta hai, ek hee jagah pe lagta hai/ kanch ke daag pe../	by that sir, the buttons are not put in curvy manner, it's put at the same place/ on the button's marks../
200	T	achha aur do button ke beech mein kitna, kitne ka gap, kitne ka fark hai, kitni duri rahti hai?	ok and how much gap is kept between two button, how much space, how much distance?
201	S	char ungli ka, paanch ungli ka/	of four fingers, five fingers/
202	T	paanch ungli/	five fingers/
203	T	kiska paanch ungli?	whose five fingers?
204	S	mera sir/	mine sir/

Equi-partitioning of units

The use of parts of the body as convenient units is one example of the construction of units. Another example is the practice commonly found in work-contexts of the creation of convenient smaller units by partitioning bigger units. Children often see convenient weight “templates” being used for small weights (50g, 100g or 250g). These weight templates, like the templates used in tailoring or leather work, are created in the work settings and such constructions of sub-units from bigger units are a common process in the work practices. The construction of convenient units or templates derived from standard units is a conceptually rich activity, since it may involve partitioning, combining or otherwise manipulating a given standard measure. It is a step beyond using ready made measuring instruments that are pre-encoded with standard units, in the direction of understanding measurement conceptually rather than learning it merely as a skill.

The practice of “cutting tea” discussed above is an example of quantification through the construction of a sub-unit. In this case, there is no visible process of equal partitioning; a “cutting” as a quantity is offered and accepted through a shared understanding of approximately how much it is. Thus the quantification is convenient, imprecise and limited. Vendors usually do not partition the cutting further, nor sell tea in other fractional quantities. However, they may do so for a group of favoured customers, creating fractions like ¾ or even 5/7. Such fractions of a cup are indeed made while serving tea in other cities in India. In such cases, there may be a more visible and complex process of equal partitioning.

Some units like the “*mutthi*” or “fistful” are imprecise, but may be partitioned: shops may be willing to sell half a mutthi of tea powder or poppy seeds. Indeed, in the context of cooking, a fistful is a unit that may be partitioned or reduced proportionately when the number of servings is changed. “*Mutthi*” is also a unit that can be iterated, typically a small number of times. Like the hand span, this is a commonly used and “handy” body based unit.

Iteration

In the world of work, we also find instances of precise quantification using a non-standard template length though such instances were not familiar to the students in our study. An example taken from another study is the use of a measuring rope in the building of traditional fishing boats (Mukhopadhyay, 2013). The master builder fixes a basic length which is specified by a length of rope between two knots. All measurement is done with this rope, which may be folded several times to measure small lengths in a precise fashion. The folding of rope to make smaller lengths is an example of equi-partitioning. The rope is iterated to measure longer lengths, and the smaller folded lengths are iterated to form fractional measures. Thus these smaller lengths in turn function as units that can be iterated. Such partitioning is a common measurement feature in this task. Although the measuring rope is itself unmarked and bare, the quantification is more transparent in the use of the rope and its folded parts than in the use of a marked tape. In our study we have not come across explicit use of ropes for length measurement in the work-contexts but in the vacation course we saw students mention use of rope and folding of rope for length measurement. When the teacher (researcher's colleague) asked how would they go about measuring different objects if all the measuring scales vanished or they did not exist, the students came up with different modes of measurement such as use of ropes or use of cubit and their iterative covering for measuring lengths.

An artefact like the measuring tape may become so familiar that children may estimate lengths with surprising accuracy without the use of the tape. As described before, we came across a few students from different grades in the Urdu school (not part of the sample) who had strong embodied estimation skill that were nearly accurate almost every time we interacted with them. U_{23} (textile printing work) had an estimation skill using which he could mentally partition an inch into half and quarter and iterated them to arrive at the length estimation of an object. Similarly there were students who could accurately estimate the weight of an object that is within the range of a few kilograms or fractions of a kilogram. In such cases, students used the construct of equi-partitioning mentally and also used the sub-units maximum up to two levels – half and quarter, and their iteration and combination. Here we see an example of how an artefact in the form of a measuring tool is integrated with bodily proprioception to create a form of embodied knowledge or skill.

However this does not necessarily imply that children are aware of the principles underlying the quantification of weight or length.

Children of the owner of the purse-wallet-bag making workshop shown in the figures were very young, the eldest was studying in Grade 5 of the English school and the yougest among the 5 siblings was a few months old when the researcher began the ethnographic visits. This student (the eldest boy) was forthcoming in discussing about his father’s work and invited the researcher to visit his home-cum-workshop. The researcher visited similar bag making workshop in the neighbourhood invited by another fifth-grader from the Urdu school. Both the students, the researcher observed, had knowledge of different processes involved in the task and they took part in them occasionally. Both the students knew about the use of templates (*farma*) and how the templates were used in the iterative covering of the an area, for example for carving out smaller pieces of rexin from a bigger piece as shown in the Figures 6.1 and 6.2 which also provided the quantification of the larger rexin piece.

Discrete quantification

Children exposed to tailoring or attending to customers in a garment shop are familiar with shirt sizes. Some shirt sizes are marked with a letter (for example, “S” for small), but more commonly, a number like “38” or “40” is indicated in the label. Although most adults and many children are familiar with these shirt sizes, whether and how these numbers are obtained through measurement is not clear to most people. Students in our study interpreted these numbers as unrelated to any units like inch or centimetre, and as merely indicating increasing sizes. Only some tailors were aware that this indicates the person's chest measurement (not chest measurement of the garment, which is larger) in inches. Here we have an instance of a measure familiar from experience, but whose origin in quantification is obscure.

6.2.2 Diversity of objects, measurement instruments and units

As discussed above, the elementary school mathematics curriculum includes the measurement of attributes commonly encountered in the everyday settings such as length, area, volume, weight, time and money. Measurement in everyday and work-contexts is typically restricted to these attributes. However, the attributes are associated with diverse objects in a range of contexts and are carried out with a variety of measurement units and tools. In this section, the variety of objects that are measured and the variety of measurement units, tools and modes that students encounter in their everyday settings are discussed.

Objects and attributes measured

We observed that length as a salient attribute of an object is measured in unary as well as in multiple dimensions. As a unary dimension, length may refer to length or distance, for example, length of a strap sewn on a bag, distance between two buttons, or the depth of a pouch or a bag. Sometimes area measures are indicated by specifying two length dimensions, as for example, when a rectangular textile printing frame is indicated by specifying the length of its sides (16”×12”) or when different sizes of rectangular plastic packets as given by their length dimensions “*satrah paanch*” (seventeen by five), “pandrah dus” (fifteen by ten). Here the underlying connection between length of sides and area of a rectangle is implicit. Areas is also “measured” using area units. For example, in leather work, a template in the form of a square leather piece called *desi* (4” × 4” leather piece used in purse making) is commonly used. Although “*desi*” is used as a unit of area, the language and vocabulary used may not make clear distinctions between length and area. For example, “*nau desi se ek foot banta hai*” [nine *desis* make a foot]” means that 9 *desis* cover and are equal to a square foot. What may seem improper or ambiguous use of measurement units is however commonly used and understood in the community, possibly from the context. The desi is also used as a unit to iteratively mark and cut out pieces from a large piece of leather, an action that is frequently seen in leather work. “*Desi*” is used as a discrete unit and not in fractions like half a *desi* or quarter *desi*, etc.

Volume is commonly measured using both standard and informal units. Volume measures are often interchangeably used with weight measures. For example, although it is more common to measure quantities of tea powder or other grocery items by weight, shops sell these in *mutthis* or fistfuls. Mutthi is also a unit in measuring sequins used in *zari* work, although weight measures are also used. The word “kilo” commonly means “kilogram” and is a unit of weight. However “kilo” is often used as a synonym for “litre”, a unit of volume. For example, E_8 and U_{23} referred to kilos of milk and colour used in everyday shopping and textile printing work respectively, although they actually meant “litres”. Similarly while measuring quantities of milk or *ghee* (molten/clarified butter), their volumes may be specified in “kilo”, which means “litres”. Measures of weight as such, figure centrally in many work-contexts, such as recycling work, or in grocery selling where standard weight measures like *kilo* and its various sub-units as multiples of *gram* are used. Convenient templates of weight measures such as pieces of stone are commonly used for measuring fractions of a kilo – 250g, 200g, 100g, 50g or 25g, and so on. Here is an excerpt from E_8’s interview:

Excerpt 6.5: Work-context interview of E_8

218	T	to der sau gram kaise taula?	so how was one hundred fifty grams measured?
219	S	ek sau gram ka patthar rakha hai aur ek pachas gram ka/	one stone of [weight] one hundred grams was kept and another of fifty gram/

Measuring instruments & tools

It was observed in the study that students encounter a diversity of measuring instruments in work-contexts. Weight measurement, for example, is done with the help of spring balances, two-pan balances of various designs, beam balances, electronic single pan balances, and platform weighing machines for large weights. Besides the use of tapes marked in both inches and centimeters for length measurement, shops and workplaces use steel rulers, which may contain other kinds of markings. Shops selling cloth use steel meter scales usually with markings for every 5 or 10 cm. Steel rulers often contain binary divisions of the inch up to 1/32 of an inch. Volume measures used to measure grain, oil or milk come in a variety of shapes and sizes. Such volume measuring instruments are not always properly calibrated or marked. Students reported that sellers often use other containers such as different plastic or tin bottles by making them close to giving agreeable measures. The researcher thus noted that students were familiar with different measurement instruments. Two students complained about the *kerosene* measuring containers which according to them were squeezed inside from the bottom thereby holding lesser quantity than its containing capacity which is usually a litre. Students thus alleged that such containers weighed less than a litre of *kerosene* but they are charged for a full litre.

In micro and small manufacturing units, diverse measuring instruments and tools are used. As discussed before in different contexts, length is often measured using a convenient template length unit that is usually non-standard. In tailoring or leather work, measuring tapes and scales (commonly 24-inch steel scale and 60-inch plastic tape) are used to cut “farma” (templates) which are then used to indicate measurement and design of different parts of the object being manufactured. As discussed in the previous section, Mukhopadhyay’s study (2011) showed that the traditional construction of wooden fishing boats on the Eastern coast of India, uses rope-templates for length measurement where this measuring tool (rope) has a knot tied at both ends, and it is folded one or more times based on the requirement of measuring smaller objects or parts. Though it is not part of our study, the rope-template presents an example of a traditional measurement tool based on non-standard units. Such traditional instruments and other cultural practices associated with measurement have largely been overlooked in the literature in Indian context till recently.

Diverse measurement units

As discussed earlier, children in the study participated in and were exposed to a variety of activities that characterize the house-hold based informal economy such as tailoring, sequin-stitching, leather work, catering, recycling, mobile repairing, and so on. A variety of formal and informal units are used in such work. By formal units, we indicate units belonging to the International system and also those that have remained in practice from an older system of units. Informal units are units of convenience, and may not be defined precisely quantitatively. We describe some of the informal and formal units in use below.

In the above examples of work contexts, we have elaborated that old British measurement units like *inch*, *foot*, *yard* are still being used. One also finds the use of older indigenous units like "cubit", together with non-standard units like "finger bands", "fistful", and so on. For example, an elder in a garment workshop mentioned that the common unit to measure the length of a *sari* is in terms of a "cubit": 12 cubits is a standard length of a traditional *sari*. Bundles of cloth are sold in "yards" in the wholesale markets from where supply of cloth is made to the manufacturing units, while in retail market/shops cloth pieces are sold in metres. Therefore, the workers in a garment manufacturing workshop usually become aware of such old British units like "yard" and depending upon their exposure to the local retail shops they also learn about the International Standard units like "metre", etc.

Units used in Zari work

Zari work essentially involves thread work and stitching a variety of decorative sequins on to pieces of cloth (see Figure 6.5 below), and is a common occupation in the low-income area where the study was conducted. Workers and many students (we came across many school students who do part-time *zari* work) are familiar with different units in which sequins and other raw-materials are sold in the market and required for certain specific

Fig. 6.5 Sequin stitching in *zari* work

tasks. Some decorative elements are sold by weight in pound units and some in gram and kilogram, some are sold in "*laris*" (a bunch of tiny spherical sequins strung on a thread), while some items are sold by metre. "Pound" is a unit of weight from the older British system that is no longer a part of the school curriculum. Some decorative elements are tiny and light in weight. These are frequently measured using the informal unit of a "*mutthi*" or "fistful". Apprentices make an estimation of the amount of some of the raw materials required in fistfuls.

Units used in leather work

Leather work includes the making of bags, wallets, purses, files, shoes, etc. Both length and area units are used here. Standard units from the British system like inch, foot, and international standard units like metre are both used. Some indigenous units are also used, viz., "*waar*", "*desi*", etc. Interestingly, zips usually come in "*waar*" (also called "*guj*", the Hindi word for yard). The sizes of zips are indicated using the counting numbers (No. 1 to 8) that show varying width and size.

We have discussed above the use of templates for comparison. In certain contexts, templates also serve as units of measurement. For example, wallet making work often involves cutting square shaped leather pieces of dimension 4" × 4" referred to as "*desi*". A "*desi*" is a template and, as described earlier, also used as a measuring unit. For example, typically a square leather piece that is one foot long and one foot wide, used for making portfolio files, is measured and referred to as 9 *desi*. Interaction with a bag-wallet-purse maker revealed that leather for the inner-tiling in bags usually come in terms of 15 *desi* lengthwise (15 *desi* refers to one side of a square leather-piece of dimension 5' × 5'). Thus, although "*desi*" is an area measure, "*desi*" is also used as a discrete length unit to specify the length of the side of a large leather piece.

The researcher learnt about different measurement units used in the purchase of raw materials in the leather work. For example, leather-piece (or rexin pieces) were sold in terms of *desi*, a different kind of rexin called "foam" (foam leather) was sold in metre as well as in foot, sponge (foam) in kilo while threads came in metre as well as in ream. Zips were sold in *waar*, adhesives (called *doodh* or milk, presumably because of their white colour and texture) were sold both in litres and in kilograms, while magnets (put in ladies' purse) were sold per piece.

Discrete units

In stone fixing work, the jewelry pieces are counted in "*gurus*" ("*Gurus*" *or* "*Gross*" is an British unit for 12 dozen). E_{16} counts 144 pairs of earrings (or other ornaments) as one *gurus*. Her work requires the task of putting one *gurus* of earring pairs on one card in different rectangular arrangements. However, for the sake of convenience, in counting they sometime consider one *gurus* as one hundred forty units instead of one hundred forty four units (12 dozen). Everyday work-contexts frequently call for such flexibility.

Other units

Children are also familiar with the traditional measurement units for area of land, viz., "*kattha*" and "*bigha*" (1 "*bigha*" = 20 "*kattha*", 1 kattha is about 1300 square feet, but varies from place to place). The British system of "acres" and "square feet" and metric units like square metres are also used. "Square feet" is the common unit for measuring the floor-size of apartments in cities and prices are quoted using this unit. Flower garlands are commonly used in India for ornament and for religious occasions. These are typically sold in the market by cubits measured from the elbow to the tip of the finger. As described before, "cutting" is an informal unit often encountered in the tea-joints in Mumbai that refers to approximately half a cup of tea.

Table 6.1 below presents a summary of the diversity of measurement modes, units, processes and attributes in a few illustrative contexts related to work, school, shopping and running a house-hold.

Table 6.1 Diversity of measurement related experience

Contexts	***Objects & Attributes measured***	***Measuring instruments***	***Measurement units***	***Measurement modes: Quantification, Estimation, Construction***
Tailoring, leather work	Length of cloth, Area of leather pieces	24” steel scale, 1m steel bar, 60” plastic tape *tedha* scale (bent scale used in tailoring)	Old British units (inch, foot, *guj* or yard, ream) International units (metre, centimetre) Non-standard units (cubit, finger-band) Indigenous units (*desi*, *waar*, *kattha*, *bigha*)	Construction of Templates (*farma*), iterating to measure length and area Estimation Comparison
Recycling	Grading of plastic sheets (recycling work) Weight of *chindhi*	Weighing hook/beam scale (*kaanta*)	Standard units (kg)	Ordinal numbers (grades of plastic sheets) Estimation Comparison
Shopping	Weight of provisions, goods length volume counts of discrete objects sizes of garments, shoes (denoted by number)	Balances of different kinds, meter scales, volume measures	Old British units (dozen, gross, ream) International units (metre, centimetre) Mutthi (fistful) cutting (tea)	Construction of standard units by partitioning, construction of convenient units
School	Length	Standard ruler (6” or 12”)	International units (metre, cm,	Measurement by reading from a

	Weight	None	mm) (Kg, g)	scale

6.3 Connecting measurement learning in school and out-of-school

We have highlighted in Section 6.1.1 above that the requirements and use of measurement learning in school curriculum and in the out-of-school contexts emerge in different ways. While in school textbooks, measurement learning is envisioned in terms of skill development of carrying out measurement tasks through activities and exercises given at the end of the lessons in the textbooks, the out-of-school settings present diverse measurement contexts which often make use of seemingly subtle underlying concepts of measurement. However, such underlying mathematical concepts remain hidden and doers/workers in the out-of-school contexts do not necessarily realise the conceptual underpinnings or connections that are present in their measurement related experience.

The topic of measurement as covered in the textbooks reflects a disconnection with the out-of-school contexts and the separation is even more than that with arithmetic, which has some overlaps and connections with everyday calculation. The mathematics textbooks of Grades 5, 6 and 7 followed in the government-run schools in the state of Maharashtra, where the field of this study was located, indicate that the textbook treatment of measurement makes no references to the everyday world, and everyday experiences of measurement do not enter classroom discussion (Maharashtra Textbook Bureau, 2006). A reflection of this is the fact that the textbooks faithfully implement the government directive of including only metric units and banning inches from textbooks, while in out-of-school contexts such directives are ignored. A consequence is that the school mathematics topic of measurement produces disconnects between itself and measurement in the real world.

The manner in which measurement occurs in everyday commerce or in work contexts, especially in the informal economy may be significantly different from the picture of measurement in scientific or engineering contexts characterised by precision and exact quantification. The features of out-of-school measurement contexts described above lead to the use of a diversity of measurement modes in everyday contexts. In contrast, the school curriculum emphasises scientific measurement which is based on full quantification achieved within a system of units, with well-defined relationships between sub-units and between fundamental and derived units. It has been noted that the notions of abstraction vary between work-contexts and differ from abstractions handled at schools. For example, diverse measurement work-contexts implicitly use abstract notions like construction of units and sub-units, chunking of measures, partitioning, unit iteration, covering, use of convenient units and modes (like templates) which are available to students as part of the everyday mathematical knowledge. The school curriculum, in contrast, treats learning of measurement as a skill development and then moves towards abstraction without building on the knowledge resource already available to the children from the work-contexts. The abstractions available to students in implicit form through their exposure and experience in work-contexts are potentially rich resources for building on measurement knowledge in the classrooms. Similarly, conservation of attributes, transitivity and seriation that are foundation of comparison and hence of measurement are not sufficiently emphasised while handling abstractions in the school context. Thus, although experience at work-contexts or in the cultural practices helps in broadening children's learning potential, they are not leveraged in the formal learning situation.

In this section we summarize the distinct features of the measurement learning that occurs in both the domains – school and out-of-school contexts and explore possible inter-connections that are pedagogically significant.

Construction of templates and units

We have discussed in the earlier sections that templates are constructed and used locally for purposes of comparison. Units are different from templates in being used for measurement, i.e., for the quantification of an attribute. As discussed earlier, a template may occasionally be used to iteratively cover an area. Although, iterative covering is the basic operation to quantify area in terms of a unit, the context in which children observe the operation may not have measurement as an overt aim. For example, a *farma* in leather work may be used to cut out pieces of the required size, or may be iterated to optimize the use of a large piece of leather. Further differences between templates and units are that units are chunked or partitioned in systematic ways to obtain larger or smaller units. Units are also typically generalized beyond the immediate context of application. As discussed above, children in our study were familiar with the construction of templates to measure length, area and weight. They constructed units out of parts of the body to measure and estimate length. Familiarity with the idea of such construction is valuable in the learning of measurement. It also gives rise to questions and problems that can lead to fruitful mathematical work in the classroom: why is the construction of units or templates needed? How do we construct new templates or units from given templates? What attributes of templates allow them to be used as units (for instance, in measuring area)? In what contexts are units partitioned to yield smaller units? What quantities can be measured with a given combination of templates?

Non-transparent knowledge of measurement instruments

In the context of tailoring, length is measured using an inexpensive plastic tape that has both inches and centimetres marked on it. The researcher observed that almost every student was familiar with such tapes but lacked understanding of the different units present in the tape, the partitions of the units and the smaller sub-units indicated by the markings on the tape. Further, students often confused between the units from two different systems (inch and cm) and were not clear about the distinction. Knowledge about the underlying construction of measuring tapes and the relation to units remained unclear to most students, as evidenced in the teaching intervention. To cite an example, E_6 during the interview clarified that the difference between an "inch" and a "metre" (Excerpt 6.5, line 132) is that "*inch chhota rahta hai, metre bada rahta hai*" ["*inch is smaller, metre is bigger*"] but that he was unable to say anything further about the relationship between the two units. Similar were the responses from a few other respondents like E_8 to whom the difference between an inch and a centimetre/metre was not clear.

Despite not knowing the construction underlying a measuring tape, the students may be able to carry out measurements of acceptable accuracy by reading off the length from the tape. This instance was noticed among the sixth-seventh graders during the garment measuring activity in the vacation camp. However, the measurement itself remains critically dependent on the integrity of the artefact. For example, as we noticed, some plastic scales bore different calibrations that led to differences in the measures they showed, that students did not think was a problem. During the classroom observation the researcher noted that some students found broken scales difficult to work with and some students made errors if the scale was used in a non-standard way (for e.g., measuring from a point other than zero). Some students counted the markings on the scale starting from the zero point. E_6, for example, counted the finger-bands starting from the first mark on the finger and arrived at "four inches" instead of three inches corresponding to four "markings" on each finger. He described "*char inch itna hota hai/ yeh ek inch, do inch, teen inch, char inch/* (showed finger-bands)" [*four-inch is this much/ this is one inch, two inch[es], three inch[es], four inch[es]*/] (see Excerpt 6.3, line 106).

Excerpt 6.6 : Work-context interview of E_6

107	T	to yeh jo finger band yeh jo ungli ka nishan yeh ek inch hai?	so is this finger-band these marks on finger, is this one inch?
108	S	ek inch/	one inch/

However, in a formal classroom setup, unlike out-of-school practices, there is a single mode of quantifying an object – by its length/area/volume or its weight by noting the measurement readings from the calibrated, scientific scales. Our classroom observation indicated that no other modes of quantification was put to use, such as, estimations or construction of newer and convenient units other than the formal ones mentioned in the textbooks. Measurement learning in the classrooms amounts to learning of the measuring skill which most of the students are anyway familiar with. What they are not familiar with is conceptual underpinnings of measurement. Classroom learning also focuses on unit conversion and calculations, which do not connect to situations familiar to the students.

Some quantifications were familiar to students but their origin was obscure, as in the example of shirt and garment sizes. The students were familiar with the garment sizes and U_{24} was skilled in connecting garment-sizes with age, but they were not clear as to what those numbers signified or how those numbers were arrived at. Similarly as referred to before, the making of a measuring scale (inch-centimetre relation) and the quantification generated from it remain unclear to the students.

The measures of *desi* were described as the "size of a Rs 2 note" by adults who were involved in bags-wallet-purse making work. The researcher came across identical description from different other people engaged in leather work on different occasions. On one such occasion when the researcher asked one person (a fifth-grader's elder brother) who ran one such workshop about the breadth of a *desi*, he replied "*shayad utna hi hoga*" [*probably it's that much*] referring to its length. We argue here that the quantification was opaque to not just novice workers but even to the expert workers.

Archaeology of measurement tools

We argue that measuring scales and templates are examples of objects that have embedded in them mathematical ideas and elements which remain hidden even for those who frequently use such objects. As noted before, even the mathematics textbooks or curricula do not require one to explicitly uncover the hidden or embedded mathematics from such objects, viz., the frequently used measuring scales or tapes. Such uncovering or unpacking task of the underlying principles and concepts is what we have referred to as "archaeology"[1]. To begin with, unpacking of the embedded mathematics in measuring scales can be the starting point of an "archaeological" exploration, that can lead to learning about length measurement and its uses. Such archeaology can have an important role in supporting the mathematical learning of students who gather, as evidenced from our study, fragmented and embedded mathematical knowledge from their work-contexts.

The following excerpt from the work-context interview with E_6 (involvement in button stitching work and mobile repairing work) shows that although he is familiar with both

1 The idea of "archaeology" of embodied mathematics or mathematical ideas was developed through personal communication with K. Subramaniam, the researcher's mentor. The researcher gratefully acknowledges the major contribution of Subramaniam in the formulation of this notion.

inch and metre as measuring units and that the former is smaller than the latter, the quantification remains opaque (see Excerpt 6.7, line 124). Hence the requirement of unpacking the underlying mathematical concepts which we call “archaeology”.

Excerpt 6.7: Work-context interview of E_6

120	S	naapne ke liye jo naapne wala tape aata hai na...	For measuring those measuring tape are available...
121	T	Hmm../	Hmm../
122	S	woh/	That/
123	T	to woh tape kitna bada hota hai?	Then how long is that tape?
124	S	woh rahta hai sau metre, der metre, hazar metre, aisa hai, jitna lamba chahiye kaapda rahta hai napte hai/	That comes in a humdred meter, one metre and a half, a thousand metre, likewise, howsoever long a cloth-piece may be, it can measure/
125	T	hmm/ yeh kaun sa tape rahta hai?	Hmm/ so what kind of tape is this?
126	S	yeh plastic ka rahta hai/	It's of plastic/
127	T	plastic ka?	Of plastic?
128	S	rubber jaisa rahta hai/	Like that of a rubber/
129	T	accha/ to tum abhi inch bol rahe the abhi metre bol rahe ho? Dono same hai kya?	Alright/ so you were saying inch, now you're saying metre? Are they the same?
130	S	nahin/ inch alag hai metre alag hai/	No/ inch is different and metre is diff./
131	T	achcha, dono mein kya fark hai?	Ok, what's the difference between the two?
132	S	inch chhota rahta hai metre bada rahta hai/	Inch is smaller, metre is bigger/
133	T	kitna bada rahta hai/	How much bigger?
134	S	metre jaise yahan se wahan tak, inch khali wahin jo red middle line hai na uske baaju mein utna hee/	Metre is from here till there, inch is only upto there till that red line, near that upto there/
135	T	achha, to kitna inch milke metre banega?	Ok, so how many inch will make a metre?
136	S	yeh nahin maloom/	I don't know this/
137	T	nahin maloom, achha aur kya maloom hai jaise inch hua, metre hua, aur iske alawe?	Don't know, ok then so what else do you know like inch or metre or anything other than these?
138	T	naapne mein tumhe aur kya istemal	What else do you use for measuring?

		hota hai?	
139	S	kabhi doodh lene jaate hain, ghaslet laane jate hain na to nikalte hain/	Sometime when I go to fetch milk or to buy kerosene then they use it/
140	T	kya nikalte hain?	What do they use?
141	S	ek litre ka, aadha litre ka, pav litre ka nikalte hain na	They use [instrument] for a litre, half a litre, quarter a litre/
142	T	woh kya hota hai?	What's that?
143	S	usse na doodh barobar se nikalke daalte hain/	They use that to take out milk in a correct quantity/

Children in our study were familiar with common measuring tools such as the inch tape, but were unclear about the meaning and construction of the markings on the tape. The above excerpt further shows the demathematisation of the measurement tasks. It is not required to understand the construction behind a measuring scale or the meanings and inter-connections between the different markings on it. What has become important now in the school curriculum is to be able to use the scale and be able to measure the length of a given object. We can observe in the above excerpt that E_6 admitted that he did not know the connection between an inch and a centimeter but nevertheless he had a fair estimation of how much distance both signify. In line 134, E_6 gives a rough estimation of an inch and a meter to the researcher which was not accurate but nonetheless a close estimation. We argue that such instances indicate that students from the neighbourhood favoured less use of tools and instruments and relied more on visual, tactile aids and on their own mental computations. The ability to estimate as a skill or reliance over mental computations devalue use of measuring scales and calculators and such propensity comes as a resistance to the prevalence of demathematisation processes. We argue that archaeological exploration resists the processes of demathematisation as well and stresses on the comprehension of the hidden underlying concepts. Such explorations therefore have strong potential to become effective pedagogic modes. A teaching intervention was made in the vacation camp (discussed in Chapter 7). Discussion on archaeological exploration as a possible starting point for building a connection between school and out-of-school mathematical knowledge is presented in Chapter 8.

U_2, who had started working as a helper (novice) in a tailoring unit discussed the use of *futta* in making *farma* by giving the specifications "inches". He was also aware of the use of different types of scales (regular and bent) depending on the context and was aware of the techniques of taking and recording measurements. U_2 explained that inch tapes are used in tailoring work while *guj* (Hindi word for "yard") is used as a unit in "qaaleen" (carpet) stitching work in which U_2's father used to work before. However, U_2's knowledge about the connections between inch and *guj* in the same system of units or inter-connections of these units with units from different systems was not clear and U_2 himself claimed that he did not know the relation between these units. His knowledge of the units was confined to the use of a particular unit in a particular work-context and hence fragmented. The diversity of units prevalent in the world of work, has underlying it historical and cultural reasons, which could also be explored in an archaeological approach.

Limited opportunity to explore variations

As noted before, work practices do not call for understanding of the conceptual constructions underlying measurement activities or tools. Familiarity with the artefacts or tools, and knowledge and skill related to their use is sufficient. In work contexts, the opportunities to explore variations different from those already contained in the traditional work processes is limited. Often the work-contexts which involve "making" call for following the instructions and patterns that come with the work orders (discussed in Chapter 5) which limits the opportunities for further exploration or archaeology. For instance, in E_{16}'s stone-fixing work, the jewellery cards followed the array structure of 6 × 24 or 12 × 12 but there was no effort or requirement of exploring other arrays that could also amount to 144 – viz., 9 × 16 or 8 × 18 or 3 × 48 and so on. The work context did not give rise to questions like why the jewellery pieces were arranged in particular arrays and not in others. Engagement with such explorative ideas was not required in the work-contexts. Mathematical learning in the classroom is different in that it contains the possibility of exploring variations outside those that are needed in practical contexts. This is important for understanding the general or invariant structures or concepts that underlie variations, and the affordances or limitations of such structures. In this important sense, school mathematics learning goes beyond the mathematical knowledge embedded in practical contexts. Though such instances of exploring variations on what students encountered in out-of-school contexts were not taken up during the teaching intervention, we claim that such opportunities in the work-contexts can enable a shift towards understanding which can illuminate both out-of-school experience and mathematics learning in school.

Prevalence of different units and systems

Students in our study used different kinds of units: international units, old Indian units, old British units and non-standard units. Besides suggesting the idea that units are purely conventional creations and are embedded in cultural and political histories, such knowledge is useful in exploring the relation and differences between different systems. Questions that can be fruitfully explored for example are, why do we need unit systems rather than just units? What are the different principles of subdivision and the advantages and disadvantages of the binary based and decimal based subdivisions? We observed that students were more familiar and conversant with the words for binary fractional measures (half, quarter, half-quarter, etc.), than with the decimal fractions. Conversion between binary system to decimal system was a challenge. For example[2], in response to a question about how to express *pauna* (three-quarters) in decimal representation, students came up with alternative representations in the binary system itself and not in decimal, viz., *teen paav* (three quarters), *aadha aur paav* (half and a quarter), *ek se paav kam* (quarter less than a whole) and also *pachhattar* (seventy five). However, students had difficulty in arriving at the decimal representations in the class. Sudents were able to connect the multiplicative relations of the above binary fractions with their geometric meanings and arrived at their inter-connections. But, it is important to explore if similar inter-connections can be made between multiplicative and geometric relationships of the decimal units and between binary and decimal units. Such connections can facilitate building a pedagogic mode to connect school mathematics learning with out-of-school knowledge.

2 We revisit this classroom instance in Chapter 7 in the context of making conceptual connections (Section 7.3).

Measurement of area

Area as seen in the examples above was frequently specified using a rectangular template. Further, the template could serve as a unit that could be iterated. This can give rise to a discussion about the interesting variations in the measurement of area and the use of different shaped units to measure area, their relationships and equivalences. Many students are familiar with the iterative use of templates for covering a space, which essentially quantifies the area of the space. However, this way of measuring and quantifying area – that the quantification of iteration is another representation of the area concept – is perhaps not clear to them. There is an apparent disconnect between the numerical representation of area as taught in schools and the quantification of covering of a space by pre-determined area-units (or templates). Recent studies by the researcher's colleagues, Rahaman, Subramaniam and Chandrasekharan (2012) have highlighted such disconnects in students' understanding of area measurement and have argued for connecting the numerical (multiplication relationship) and geometrical relationship in the learning of area measurement. Hence, such out-of-school experiences can work as starting points for building on students' knowledge of iteration and covering notion embedded in the use of templates for connecting with the formal numerical methods of teaching and learning area measurement. Deepening of such conceptual aspects of measurement can lead to its effective learning.

Quantification of various attributes

We have noted that the out-of-school work-contexts possessed a diversity of attributes that were quantified. The ways of quantification were diverse too. By drawing on students' familiarity with the range of objects and attributes that are quantified, students can explore questions such as what is common and what is different in how we quantify different attributes such as length, area, volume and weight? In what situations can one attribute substitute for the measurement of another attribute? What conditions or properties allow for such substitution? One can also raise questions about the quantification of other salient attributes. How is an abstract attribute like monetary (exchange) value quantified? How do we quantify different aspects of labour such as time, effort and expertise? Such questions are important to build an holistic understanding of the measurement concept that students get to handle in different domains of their lives. Students can also explore the need not only for units, but also for systems of units. In the first instance, each measurement system has its own units and sub-units. Within a particular measurement system, the same attribute can be measured in different objects using different units. For example, the attribute of weight is measured using different units for different objects – the units used to measure grain by weight are different from the units used to measure precious metals like silver and gold and the units used to measure quantities of salt and sand. However, quantification of such different attributes for different objects are not dealt with in the mathematics textbooks that we have analysed (for e.g., Maharashtra state mathematics textbooks). We understand that measurement learning cannot be complete unless the curriculum engages with the complexity of different attributes (in the above case, weights) and measures prevalent in the world of commerce. Interestingly, the old textbooks reveal that different forms of tables with information about measurement units of various kinds followed in different systems and their inter-conversions. For example, four different systems of weight measures for weighing metals such as gold and silver were presented in the math textbook written by Gopal Krishna Gokhale that was in practice in the last two decades of the 19th century: the system in the state of Maharashtra, the “old” system, the system prevalent in the city of Bombay and the system in England (Subramaniam & Bose, 2012). Ironically, other similar textbooks from a hundred years ago show a strong connection with life outside school, which have faded away from the contemporary textbooks, while educators worry about the lack of such connections in the modern textbooks.

6.4 Implications for classroom learning

We highlight below some of the specific ways in which measurement knowledge gathered from out-of-school contexts discussed in this chapter may be used as learning resources in the mathematics classroom. It was observed in this study that measurement experience in the everyday context is richer and more sophisticated than measurement experience that arises in the classroom context. This is due to the diversity of measurement modes and aspects of construction of units and tools that are often encountered in everyday contexts.

Treatment given to the topic of measurement in school mathematics is structurally different from the measurement experiences embedded in the out-of-school contexts which are mostly characterised by diversity. However, at the same time it would not be correct to assume that there does not exist any connection between the content prescribed in school mathematics and out-of-school measurement experience or to assume that both the forms of measurement knowledge have no relevance for drawing from each other. The distinctness as well as the inter-penetration of out-of-school and school knowledge has been long recognized in the Vygotskian approach to education and psychology. On the similar lines, Moll (1992) has argued that "everyday and scientific concepts are interconnected and inter-dependent", and draw on each other in their mutual development. Science is not something that stands over as distinct and apart from the everyday, but must illuminate, question and re-invent the everyday (Subramaniam, 2012).

We have noted in our discussion over the distinction between school and out-of-school mathematical knowledge that both these forms of knowledge have different and distinct outcomes and goals (Resnick, 1987). Interestingly, the mathematical knowledge that one acquires in the out-of-school contexts is distinct from what school mathematics recognises as its outcome. School mathematics aims for producing knowledge that is generalisable and not bound to specificities of particular contexts. In contrast to generalised knowledge, knowledge which are specialised are necessarily bound to contexts and embodied in individuals. Specialised knowledge can be effective in action and expertise driven only in limited domains and contexts. Knowledge that is generalisable on the other hand has wide applicability and generality but may not lead to expertise and efficiency in specific task contexts (Sfard & Cole, 2003). Generalised forms of knowledge however, also contain elements of situational and contextual knowledge and it has the ability to re-invent and shape the everyday knowledge. Generalised forms of knowledge are neither about abstraction without the concrete content, not are they about mere induction from a number of instances. Rather, generalisation is all about arriving at or holding an idea or a construct that can illuminate and be applicable in diverse instances. Valuing generalisability as an outcome of school learning in fact places greater importance to the diversity of out-of-school experiences, for such diversity actually creates contexts for school learning. From this standpoint, we understand that mathematical aspects are present in the work-contexts as hybridized and opaque embeddings and it would not be correct to look at such practices as reflecting mathematical thinking and understanding. At the same time, we argue that it would be fallicious to look for elements of school learning in a particular work-context or to expect school mathematics to increase proficiency in specific practices. We claim that the formal mathematical learning can illuminate the diversity of practices as a whole and strengthen understanding, but not practice.

The funds of knowledge perspective illuminates how the connectedness of social networks gives rise to diverse and rich knowledge and experience that can be drawn on for the purposes of school learning. In our study, which is set in an urban, developing world context, we found that students often directly participate in work, or are closely aware of work contexts and practices. Experiences and knowledge of measurement drawn from such contexts are intimately familiar, with aspects of it even embodied in students and present in the classroom. All the students in our study were from two Grade 6 classes that were co-located in one school building. Such diversity of experience, within a school community hence presents a great opportunity for learning that has been largely ignored in formal school education.

Educational thinkers in the developing world, and particularly in India, have recognized the value of work experience for education conceived in a broad sense. Policy documents on education have taken on board this insight. The quotation from Gandhi on the role of work in education (cited in chapter 2) emphasises that modern education centred around work is different from the traditional education in the crafts. Its aim is a well rounded education and not just training in a particular craft (Gandhi, 1927). The learning of measurement in school must therefore be framed in broad terms. It is aimed at acquiring understanding and insight and not at practical training.

Existing curricula and teaching practices, in contrast to policy documents, serve to reinforce the separation of the everyday from formal school learning. Underpinning of this may be an implicit awareness of the structural differences between these forms of knowledge and learning. This may combine with an anxiety about the potential distractions caused by the contextual details of the everyday. Thus teaching practice typically keeps the everyday out of the classroom and creates school mathematics as a culture and practice that is not only distinct, but also disconnected.

One of the challenges before the teacher or the instructional designer is to imagine connections between school and out-of-school knowledge that can produce powerful learning. We point to two aspects of out-of-school knowledge that can lead to powerful connections. The first involves conceptual construction. Mathematics education researchers have pointed to the fact that in existing literature, the treatment of the topic of measurement largely ignores foundational concepts and emphasises the "physical act of measuring" (Sarama & Clements, 2009, p. 275). The notions of conservation of an attribute, transitivity and seriation form the foundation for comparison, which in turn, forms the foundation for ideas of measurement. Conceptual aspects of measurement such as equi-partitioning, conservation, transitivity, unit iteration and covering, structuring of unit coverings, accumulation of measure and additivity have been highlighted by researchers as critical to the understanding of measurement (Sarama & Clements, 2009). These aspects are not adequately treated in existing curricula in the Indian context.

From the point of view of the diversity of out-of-school experience, we need to go beyond critical concepts listed above to include construction of units and templates, equi-partitioning and chunking of measures and unit, construction of measuring scales, design of convenient measuring instruments and units. Further aspects critical to the understanding of measurement that have not been adequately addressed in the curriculum include the extensive use of comparison and estimation in real life contexts, the use of the body as a measuring instrument, the trade-offs between convenience and accuracy, the variety of purposes of measurement, the variety of modes of quantification and the limits of informal quantification, and the cultural-historical origins of units and systems of units. These aspects, with the exception of estimation, have also not received adequate attention from mathematics education researchers. The diversity of measurement experiences in out-of-school work contexts can be drawn upon to illustrate each of these concepts and ideas, and also for understanding the difference between comparison, estimation and measurement and their purposes.

6.5 Summary

A study of measurement in informal work contexts reveals a situation characterised by a rich diversity of measurement units and modes of quantification. Besides informal units and units of convenience, older units may still be in use in the culture together with standard international units. Mathematics textbooks from a century ago reflect something of the diversity and richness of measurement units in the everyday world, but stress the arithmetic of conversion and computation rather than the concepts of measurement. In modern Indian school mathematics textbooks this diversity is not found and only standard units are taught using standard measuring instruments like a ruler or weighing balance. School curriculum designers do not consider it worthwhile to deal with a variety of units even though they may still be used. The framework of demathematisation helps explain why informal practices and contexts have disappeared partly from social practices and wholly from the curriculum and how their importance and value is diminished. However, in household based occupations, measurement in a diversity of modes and with a variety of units always plays a role. The emergence and survival of such informal mathematics can be seen as a counter-trend to the broad process of demathematisation.

When school mathematics textbooks adopt a restricted view of measurement, children may fail to see any connection between their classroom experience and the rich world of measurement outside school. Further, how an attribute is quantified may not be clear from classroom learning. Children also need to appreciate the fact that measurement as it occurs in the world of commerce or in work contexts in the informal sector may show characteristics quite different from precise, scientific measurement. The extent of precise quantification may be limited, and may be just sufficient for the purposes at hand. The quantification may be incomplete or if embedded in cultural artefacts, may be opaque. Even so, it may be embodied in the form of a skill at estimating quantities.

The prevalence of diversity in measurement units and modes in the culture suggests that more than teaching measurement as a skill, it is the conceptual aspects of measurement that are important to learn. Understanding how quantification is achieved in various modes may allow children to understand and make connections among the diverse ways of measuring that they encounter. It may lead them to appreciate the possibilities and limits of different kinds of informal measurement, and the distinctiveness of these from scientific measurement. Further, an inquiry into the history of older units still in use may provide interesting avenues of exploration and possibilities of connection with other curricular subjects. An inquiry into familiar measurement tools which embody measurement ideas in a “materialised” form, but where the process of quantification is obscure (like the inch tape or shirt sizes) can potentially become an important part of school learning. These ideas need to be explored further. It is likely therefore that more research about how measurement plays a role in the everyday world in diverse ways will have an impact on the school mathematics curriculum.

7

The Teaching Intervention

"… Without a sense of identity, there can be no real struggle…"

– Paulo Freire, *Pedagogy of the Oppressed*

The first two phases of the research study have shown active immersion of the middle graders in out-of-school work practices which create varied opportunities and affordances for them to acquire and use mathematical knowledge embedded in such practices. Chapters 4, 5 and 6 have collectively indicated the hybrid nature of students' mathematical knowledge formed by the overlap of school mathematics and mathematics embedded in the work-contexts, and how work contexts impact student identities. In addition, our study has also shown that students' knowledge of mathematics in both the domains may be fragmented and non-transparent. As educational researchers, it is important to think about ways to bring together different forms of mathematical knowledge that students possess and see if and how the connection between knowledge drawn from one domain can illuminate the knowledge used in the other. In particular, it is challenging to chart a pedagogical approach that can build such a connection. As an endeavour towards drawing such a pedagogical approach for enhancing students' conceptual understanding of mathematics, this chapter analyses a particular intervention in a mathematics classroom that aimed at integrating mathematical knowledge gained from everyday and work-contexts with school learning. Although the analysis is of a specific intervention, it is intended to throw light on general aspects that facilitate such integration (Research question 5).

The classroom intervention mentioned above was planned in Phase III of the study. The intervention consisted of a teaching design experiment (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003) in the form of a two-week long (12 days) vacation course aimed at drawing curricular and pedagogic implications of connecting everyday and school math knowledge. The course was attended by sixth and seventh graders from the Urdu school and only a few sixth graders of the English school who were available and volunteered to participate. Some of the seventh grade students were from the sample while others were non-sample students from the same grade. The classes were conducted by the researcher's senior colleague for one hour and a half every day for 12 days. All the lessons were recorded on video with prior permission from the school authority as well as the parents of the participating students. The course was held after the annual exams in April 2012 and before the students went on vacation. It was therefore a convenient period of holding classes without affecting the regular teaching schedule.

The researcher was on several occasions invited by the Urdu school Grade 6 and 7 teachers to hold a series of classes for the students. There was a similar request from the students as well. On a number of occasions even during the regular classroom teaching-learning process, the researcher was asked by the Grade 7 (the class of the sample students then) class teacher to teach a certain mathematical topic to the students, or to clarify their queries and doubts which the researcher gladly obliged. It was the academic year of 2011-2012 when the researcher would visit Grade 7 every morning and attend the lessons. Such requests were timely as the research design anyway had a plan for a teaching intervention. Moreover, since students and teachers were keen on a vacation course themselves, getting permission for the same from the school authorities became easier.

7.1 Teaching Design Experiment

The design experiment is an “extended (iterative), interventionist (innovative and design based) and theory oriented enterprise whose *theories* do real work in practical educational contexts” (Banerjee, 2008, p. 131; Cobb *et al.*, 2003, p. 13). Design experiments based on prior research and thinking, and carried out in educational settings, seek to trace the evolution of learning in complex classroom and school situations, test and build theories of teaching and learning, and produce instructional tools that survive the challenge of everyday experience (Shavelson *et al.*, 2003, p. 25). In design experiments, the theory intends to “identify and account for successive patterns of student thinking by relating these patterns to the means by which their development was supported and organised” (Banerjee, 2008, p. 131; Cobb *et al.*, 2003, p. 11). The purpose of adopting design experiments is to allow formative evaluation of research and to study learning processes in a context with an objective of supporting them (Banerjee, 2008; Cobb *et al.*, 2003; Collins, Joseph & Bielaczyc, 2004). The design is then put into practice, tested and revised based on experience to

1. lead to the development of some local domain-specific theory, or
2. address the theoretical questions and issues delineating why it works or understand the relationships between theory, artefact and practice.

There are researchers who consider the term “design study” as more appropriate than “design experiment” since the latter “does not conform to the requirements of an experiment” (Cohen, Manion & Morrison, 2013, p. 330). Rather, such efforts are aimed at engineering innovative, iterative, process-focused, development and reflection guided intervention which occurs in cycle of refinement, testing and feedback. Thus, design experiment (study) focuses on the changing practice “instead of the static, frozen input-output model” kinds of interventions (p. 331).

This chapter reports the analysis of the teaching design experiment that we adopted in the vacation course, the focus of which was on integrating students' out-of-school measurement knowledge with formal teaching aimed at building conceptual understanding among the participants. The design experiment was limited to a single instance and hence not iterative in nature. However, the intervention threw light on the possible features of an approach to teaching mathematics that integrates out-of-school and school knowledge, that we will explore in this chapter.

7.1.1 Design experiment in the vacation course

Based on the earlier phases of this research, by the time the teaching intervention was designed, we had learnt about the nature and extent of students' out-of-school mathematical knowledge, and had gathered a fair idea about the students' varied measurement knowledge and use of measurement modes and units (Research Questions 1 through 4). In addition, we also had learnt by then, about the opportunities and affordances that the work-contexts create for students to gather out-of-school mathematical knowledge (discussed in Chapter 5). The teaching design experiment therefore was intended to explore the possibilities and limits of connecting everyday mathematical knowledge with school learning. The emphasis was on *exploration and establishing feasibility*, rather than effectiveness of instruction in terms of learning outcomes. We wanted to focus on the conceptual connections that could be made in the classroom and the take up by the students in terms of their participation. Classroom activities during the course were designed based on the outcomes of the earlier phases of the study and focused on the familiar activities from the common work-contexts that were known to the children. The instructional tools drew on the everyday experience and invited students' alternative thinking while building connections with school mathematics. At the surface level, two kinds of devices were brought into the classroom from out-of-school contexts:

i) physical artefacts that were known to the students from out-of-school contexts such as measuring tapes, calculators, garments;

ii) practices known to students from out-of-school contexts such as measurement practices, construction and use of templates and units, partitioning of units, everyday language and vocabulary for mathematical objects (such as the words for binary fractions).

The analysis revealed deeper levels of connections between the out-of-school and school learning contexts that we analyse in the sections to follow.

7.1.2 Goals and objectives of the vacation course

One of the primary goals of this chapter is to reconstruct the goals and objectives of the teaching intervention in an explicit manner. The reconstructed goals and objectives were discussed and arrived at keeping in mind the planned objectives as well as the enactment aspects based on lesson records. The analysis presented in the chapter describes the goals together with the enacted episodes from the classroom that illustrate the implementation of these goals. The enactment episodes are taken from the lesson logs and the transcripts. The analysis is focused broadly on two aspects:

i. conceptual connections between everyday mathematical knowledge and school mathematical knowledge with a focus on the topic of measurement, and

ii. agency and negotiation of identity in the classroom in relation to the connection between out of school and school learning.

The broad goals of the vacation course, as planned, and as supported by the reconstruction from enactment episodes, were to

1. Make connections between out-of-school mathematical knowledge of school children and learning of school mathematics

   1.1. by using their out-of-school knowledge to organise and build conceptual learning of school mathematical topics,

   1.2. by using school mathematical learning to illuminate aspects of out-of-school knowledge.

2. Foster identities that allow connections to be made between out-of-school and school math knowledge and to align students' identities as learners of mathematics, as doers and as being experienced and knowers in everyday contexts

   2.1. by legitimising the sharing of everyday knowledge in the classroom,

   2.2. by encouraging explanations that connect everyday and school knowledge,

   2.3. by building a culture of shared learning in the classroom.

The specific instructional objectives of the vacation course were formulated in the light of the broad goals, and evolved over the lessons based on students' response. The enacted objectives focused on two mathematical topics –

(i) length measurement, and

(ii) fractions and decimals.

On the topic of measurement, the objectives included drawing on students' out-of-school knowledge of length measurement to deepen conceptual understanding of units and sub-units in measurement by connecting them with out-of-school contexts known to children such as tailoring. Connection between the topics of measurement and fractions was sought to be established through the construction of units and sub-units, and using them in measuring length and in creating measurement situations to learn fractions and decimal numbers. For the topic of fractions and decimals, the instructional objectives included connecting students' out-of-school knowledge of fractions and proportional reasoning to the school topic of fractions and decimals by

(i) strengthening and extending students' understanding of binary fractions gained from everyday contexts,

(ii) making connections between binary and decimal fractions, and

(iii) building students' understanding of decimal fractions.

7.2 Analysis of the design experiment

As mentioned above, a major purpose of the analysis of the vacation course was to elaborate the enacted goals of the teaching design experiment. The goals as elaborated above propose an answer to the question "*what should be the goals of an approach to teaching that attempts to connect out of school knowledge with school mathematical learning?*" The analysis to follow supports the articulation of these goals. The analysis is based on lesson records, which included detailed logs of all twelve days of teaching by the main instructor prepared from the video recordings and notes of classroom observers. Six of the twelve lessons were transcribed in full. Reviews of the logs, the video recordings and the transcripts provided information about the enactment aspects of the course. These reviews were carried out independently by the researcher and his colleague, and a third researcher R_3 with several years of experience of teaching design experiments.

7.2.1 Choice of transcript segments and video excerpts

To begin with, three transcripts of the lessons were chosen and carefully read by the researcher, the vacation course instructor (researcher's supervisor) and researcher R_3. The three chosen transcripts were from the Days 1, 9 and 12. These were among the lessons, where the activities marked a departure from the usual classroom practice or an important shift took place in terms of topic of the study. For example, the classroom activity on Day 1 employed measurement of garments using *inchi* tape (common measuring tape used in tailoring work) which was followed by a similar activity but using a non-standard yet “fixed” measure (viz., a paper-strip cut breadth-wise from A4 size paper sheet, whose length was hence equal to the breadth of A4 size paper). The focus on Day 9 was on making connections between different oral representations of the binary fractions, as used in daily parlance, and their decimal representations, and other decimal fractions up to two digits from the decimal point. Day 12 was the last day of the course and focused on strengthening decimal fraction knowledge by introducing new contexts and using the Standard International and old British units. These three transcripts were analysed for the enacted goals (discussed below) and corresponding video excerpts were looked into.

In the second stage of the analysis, we transcribed three additional lesson-videos, those of Days 2, 10 and 11. These lessons were also on length and volume measurement and the corresponding classroom discussion was on their decimal representation. Based on the goals of the teaching design experiment, we now looked for instances where agency and students’ identities emerged during the lessons. Our analysis is based on closely looking at the particular chunks in the transcripts that showed such emergence. We also looked at the videos for non-verbal gestures, activities and noted a series of “shifts” during the enactment of the goals by the teacher. We discuss in the following section the instances where such “shifts” were enabled by the teacher and also the instances which depict negotiation of students’ identities, based on the analysis of the transcripts of the six lesson-videos.

7.3 Drawing from the analysis

From the transcript of the lessons, we reviewed episodes that revealed the teacher's and learners' engagement in (i) requesting and sharing out-of-school knowledge and (ii) asking for and providing explanations, clarifications and justifications. Students readily participated in such interaction suggesting acceptance of the norm of sharing knowledge about work and other out of school contexts. In this section, we begin with the examples of co-construction of the normative identities in the classroom followed by a detailed discussion on enabling of shifts as a pedagogical approach.

7.3.1 Student participation: negotiation of identities

A practice in a community, be it inside the mathematics classroom or in the work contexts, entails "negotiation of ways of being a person in that context" (Wenger, 1998, p. 149). Negotiation in these practices comes through engagement in action, interaction with colleagues (co-learners, co-doers, co-knowers) including conversation and discussion with them. It involves how clarifications are sought and questions get addressed, how assistance and support are extended, and contentions are challenged or explained. Such negotiations involve negotiation of the identities of the practitioners (students or doers) (Wenger, 1998). One way in which the identities are formed is by the negotiated experience in terms of participation in the practice and its reification.

One of the major purposes of the analysis was to arrive at an understanding of students' receptivity to the instructional goals from the nature of participation in the classroom. Students' participation during an instruction is linked with how their identities are shaped in the classroom practice and connect with their identities shaped in outside contexts. The classroom practice is constructed on the negotiated participation structure by the students and the teacher, and interaction between them. The analysis draws on the notion of "normative identity" as a construct illuminating the participation structure in the classroom (Cobb, Gresalfi & Hodge 2009, see Chapter 2). This refers to the set of norms co-constructed by teacher and students in the classroom that determine expectations about how students should ideally participate in the classroom. With regard to mathematics, the normative identity refers to what is considered appropriate mathematical engagement on the part of a student. Individual students may accept the normative identity, merely co-operate without accepting the identity, or actively resist the identity.

The efforts of the teacher from the very beginning to invite students to share what they knew about work contexts was a striking feature of the initial teaching episodes, where setting up of norms is a primary goal. The co-constructed normative identities in the classroom were shaped by means of students' and teacher's engagement during the lesson. An example was the teacher's invitation to the students to share what they knew from outside and treating their knowledge from out-of-school contexts as valid and important in the pedagogical processes. On Day 1, there was a long initial interaction sequence with participation from several students focused on the processes that lead up to the manufacture of a garment. Students were invited to share the processes in detail and almost every student in the classroom took part in the discussion beginning with who cuts the cloth, using which tools, where do the large cloth-pieces come from, how cloth is made, and so on. The agency of students as knowledgeable agents during the classroom discussions was reinforced and their identities as knowers and doers were acknowledged and valued.

The other ways in which normative identities were shaped emerged, for instance, by establishing the black-board as a space that belonged to everyone in the class and not just for the teacher to use. Use of the black-board was promoted for making an individual's expressions public (discussed as a "shift" below). The classroom norms were co-constructed through different ways of engaging in the classroom proceedings by building tools for justification and sharing and justifying the answers. The teacher encouraged students to share their justification with others in the classroom and it was alright to use their own solution techniques and to make mistakes. Arriving at proper justification discussed in the following section can also be seen as a way of legitimising the co-constructed norms in the classroom. Communicating to the teacher if one did not understand also was a classroom norm.

Another device implemented by the teacher was to bring artefacts from work contexts into the classroom, setting up a difference from a typical school classroom. For example, shirts and kurtas of different sizes as well as the measuring tape were introduced in the classroom followed by a non-standard but fixed unit (a paper-strip made from the smaller side of the standard A4 paper sheet, discussed in the next section). These moves by the teacher elicited enthusiastic participation from the students. The students worked in groups and quickly taught each other the correct ways of taking the measurements of a shirt. Several students could be seen wearing the tape around the shoulders in the manner of tailors (discussed below). One student, who was mystified at the way this "mathematics" lesson was going, asked the researchers if they were all being trained because the researchers wanted to open a garment making business employing the students! At a later point in the design experiment the simple calculator commonly used by shopkeepers was introduced eliciting similar enthusiastic participation from the students.

7.3.2 Enabling shifts

The instructional goals described above spell out how classroom teaching might draw on and connect with out-of-school knowledge that students bring to the classroom. However, school learning is not the same as out-of-school learning and the goals need to acknowledge the complementary dimension of the differences between out-of-school and school learning. This complementary dimension of the instructional objectives could be viewed through the lens of enabling a series of shifts. They are:

- Shift from oral to written mathematics
- Shift from knowledge about use of a tool or artefact to understanding the tool (e.g., measuring tape, numbered sizes of garments)
- Shift from co-operation to a mathematically focused discourse community (e.g., shift from making assertions to providing clarifications, justifications, explanations; moving from "helpful" interactions to a discursive culture)
- Shift from individual expression in private to shared, public expression
- Shift from identities that are disconnected (or reinforce disconnection) to identities that are connected

From our analysis, we try to show that the normative identities indicated above can be characterised through these shifts. We discuss the shifts below.

Shift from oral to written mathematics

The first kind of shift is from *oral* mathematics to *written* mathematics. An aspect of written mathematics that was focused, was the development of consistent symbolic representations for binary and decimal fractions. For example, while the students were aware of the words for fractions such as quarter (*paav*), three-fourths (*pauna*) and even one-eighth (*adha paav*), one-and-a-quarter (*sawa*), one-and-a-half (*dedh*), and two-and-a-half (*adhai*) they were unsure about how to represent these fractions symbolically. The teacher initiated a discussion by asking “how will you show this using language of maths”. Developing the fraction notation to represent binary fractions was an important focus of the lessons.

Another aspect of written mathematics involved use of the phrase “language of maths” as a prompting device to elicit precise quantification. For example, when students said that a particular length was smaller than a unit, the teacher asked by how much is it smaller. When the students replied saying “*thoda kam hai*” [it is a little less], the teacher’s prompt was to ask “*math me bataa sakte hain kya*?” [can you say that in math?]. Another prompt, “*kaise likhenge asariya mein*” [how to write in decimals”] aimed at encouraging a shift from oral to written/formal math (excerpts from the transcripts are given below). Another device used in the instruction was to create a need for symbolic mathematics in the context of using a particular tool – the calculator, which was familiar to students from everyday contexts. Specifically, in order to use the calculator, students needed to know decimal fractions to carry out arithmetic operations with fractions such as half, quarter, etc., since the calculator did not have keys to enter fractions directly in the standard notation. The following excerpts present instances of the teacher’s attempt to encourage a shift from oral to written mathematics.

Excerpt 7.1 Day 9, Topic: Decimal and Binary Fractions

139	T	… kitana chhota hai exactly bata sakte hai kya? Koi maths mein bata sakte hain ki kitna chhota hai? yaani kitna aur usme dalenge ki wo pura ek ekai ban jayega?	… can you tell exactly how much smaller? Can anyone say that in math how much smaller? That is, how much to add in it so as to make it one unit?

Excerpt 7.2 Day 9, Topic: Decimal and Binary Fractions

177	T	… dedh ko kaise likhate hai hum?	How do we write *dedh* (one and a half)?
178	S	ek ek bate do/	One one by two/
179	T	ek aur ek bate do, haina dedh yani isko asariya me kaise likhenge/	One and one by two, right *dedh* so how to write it in decimals?
180	S	One point five/	One point five/

Excerpt 7.3 Day 9, Topic: Decimal and Binary Fractions

153	T	… ye hamne kaise likhe hain? bate me likhe hain/ hai ki nahi? usko agar asariya mein likhenge to ?	… how have I written it? Written in fractions/ isn't it? If I were to write it in decimals, then?
154	S	Zero point fifty, zero point fifty/	Zero point fifty, zero point fifty/
155	T	Zero point fifty kaha aata hai isme? Zero point five zero (writes 0.50) ka matlab ?	Where does zero point fifty come on this? Zero point five zero (writes 0.50) means ?
156	S	Zero point five/	Zero point five/
157	T	Zero point five (shows 0.5 on blackboard) yani aadha, to bate me kitna hota hai?	Zero point five (shows 0.5 on blackboard) means half, so how much in fraction (notation)?
158	S	Pachas batte sau/	Fifty by hundred/
159	T	Pachas batte sau kasar me pachas batte sau ya paanch batte dus, lekin mujhe kya chahiye paanch bate sau, usko kaise likhenge Aasariya mein? pachas bate, [pause] zero point five likhne se chalega? haan?	Fifty by hundred in fractions, fifty by hundred or fifty by ten, but what I want is five by hundred, how to write that in decimals? Fifty by, [pause] is it okay to write zero point five? Yes?
160	S	No sir/	No sir/

Excerpt 7.4 Day 10, Topic: Volume measure and decimal representation

452	T	*paav*, *paav* kaise likhenge?	*paav,* how to write *paav*?
453	S	mathematics mein ek batte char/	in mathematics one by four/

The following excerpt shows the teacher's use of the prompt of "expressing in mathematics" which students responded to by thinking about how to "express in mathematics" and coming up with "mathematical forms of expression".

An aspect of the shift from oral to written mathematics is the move from informal expressions from out-of-school contexts and to mathematical expressions. The phrase "how to say in mathematics" marks this shift. In the following excerpt, the teacher again explicitly invokes this phrase in the context of asking for greater "accuracy" (line 138):

Excerpt 7.5 Day 9, Topic: Decimal and binary fractions

133	T	sahi lag raha hai 0.99? bate me kaise likhenge?	is 0.99 correctly placed? how would you write it as a fraction?
134	S	ninety nine bate sau/	ninety nine by hundred/
135	T	achha, thank you, woh kya ek ekai se chhota hai ki bada?	alright, thank you, is that smaller or bigger than one unit?
136	S	chhota hai, thoda sa/	smaller, by a little/
137	T	thoda sa chhota hai? kitna? kitna chhota hai?	little less? how much? how much less?
138	S	ek/	one/
139	T	ek? ek tukda na? kitana chhota hai exactly bata sakte hai kya? koi maths mein bata sakte hain ki kitna chhota hai? yaani kitna aur usme dalenge ki wo pura ek ekai ban jayega?	one? One piece right? can you tell exactly how much less? can anyone say in maths how much less it is? that is how much more does one put in it so as to make it into a unit?

In the above excerpt, the teacher asked the students to locate "zero point nine nine" on the number line and encouraged them to frame their arguments "in the language of maths" ("*maths mein batana*"), i.e., by using mathematical expressions. Shift to "mathematical expression" was adopted by asking students to express in decimals (*ashariya mein kaise likhenge? - how do we write in decimals?*).

Shift from knowledge for use to understanding

A second shift was from *knowing how to use* a tool or artefact to *understanding* the tool or artefact. An example is the analysis of the measuring tape – students were familiar with this tool, but did not know the principles underlying its construction or the relation between the different units found on the measuring tape. A major focus of the teaching intervention was the construction of measuring strips and tapes based on non-standard and standard units by the students. A shift was exercised from just knowing and using an artefact or a tool to understanding the underlying conceptual construction embedded in the artefact or the tool.

In another instance, the teacher asked students to find the meaning and origin of the "numbers" or "labels" found on the garments that indicated their sizes. These numbers ("28", "38", "40", etc.) or alphabets ("S", "M", "L", "XL" or "XXL") are used as labels on the inside of garments. As mentioned before, every student was familiar with the garment sizes but no one could explain how exactly those "size numbers" were arrived at or what was the connection of those "numbers" or "labels" with the garments or its parts. The familiarity of the students with garment sizes is indicated in the excerpt below by the fact that many of them guessed the size of a shirt just by looking at the shirt.

Excerpt 7.6 Day 1, Topic: Length measurement

365	T	mera size ka hai? [referring to a shirt]	is it of my size? [referring to a shirt]
366	S	haan, sir/	yes, sir/
367	T	mera size kya hoga? kya hoga mera size?	my size is what? what can be my size?
368	S	sir, naapna padega/	sir, need to measure/
369	T	iska size kitna hoga?	what can be its size?
370	S	sir, naapna padega/	sir, need to measure/
371	T	aap dukan par jate hain to napkar dekhte hain kya?	when you visit a shop do you measure to check?
372	S	size hota hai uska/	it has a size/
373	T	to size likha hota hai kya?	so is the size written?

374	S	battees number sir, battees number/	number thirty two sir, thirty two number/
375	T	battees number? yeh kaun sa hai?	Thirty two number? what is this one?
376	S	chalis/	forty/
377	T	chalis?	forty?
378	T	haan? yeh kahte hain chalis, to kuchh andaz laga sakte hain yeh kaun se size ka hai?	yes? they say forty, so can you make a guess which size could it be of?
379	S	(chorus) sir, chalis hai chalis/	(chorus) sir, it's forty, forty/

In the above excerpt, the respondents showed a visual estimation skill and claimed that the shirt would fit the teacher. A few of them guessed that the size of the shirt was “40” (lines 376, 379). The teacher checked the label on the shirt and confirmed that this was correct.

The following excerpt from the conversation between the teacher and the students shows that the latter did not know exactly what the garment “sizes” indicated. Guesses like “length” or “breadth” came up as answers. Teacher’s question (line 402, 404) in the Excerpt 7.7 below called for a shift from simply knowing or having familiarity with an artefact or a “mathematical sign” (number indicating garment-size) towards understanding the underlying concepts behind such practices or signs.

Excerpt 7.7 Day 1, Topic: Length measurement (using standard measuring tape)

397	T	to chalis kya hota hai? chalis kahan se aaya?	so what is forty? where has forty come from? (“40” was the size indicated on the garment.)
398	C	sir, iska size hai na, naap hai, who chalis hai/	sir, it’s its size, it’s a measure, that's forty/
399	T	size? kaun sa?	size? which one?
400	C	lambai, lambai, chaurai/	length, length, breadth/
401	T	lambai, chaurai/	length, breadth/
402	T	theekhai, abhi mujhe pata nahin hai ki chalis kya hai/ theek hai? yeh chalis kya hai yeh aapko pata karna	alright, now I need to find out what is forty/ right? what this forty is you need to find this out/ the number

		hai/ chalis yeh number jo aaya hai, adad jo aaya hai, kaise aaya, kaun sa naap hai, yeh pata karna hai/ pata kar sakte hain?	forty that has come, the number that has come, which measure is this, you need to find this/ can you find it?
403	C	haan/	yes/
404	T	haan? kya chahiye pata karne ke liye?	yes? what do you need for finding out?
405	C	inchi tape/	inch tape/

The shirt activity

In the above context, the teacher suggested an activity during the class which students liked and participated in. The activity was to measure different parts of the garments which the students thought would be necessary to make a connection with the respective "garment size". Students discussed and collectively decided to take the measures of length ("L"), breadth ("B"), neck ("G" for *gala* in Urdu), shoulder ("S"), waist ("K" for *kamar*) and cuff ("A" for *asti*) of the garments. Two, three or four students in a group took these measurements of a garment as part of the activity. Every group recorded their measurements on the chalk board for everyone to see and arrive at a general pattern. Together the whole class then deliberated as to which measure was associated with the garment size and in what way, for instance, could there be a generic relation applicable in all instances.

The students were also familiar with different parts of garments that are measured by the tailors. Their collective consensus on which garment-parts were to be measured was a reflection of such familiarity. Based on the data that each group collected, the numbers under each heading was filled up on the board by all the seven groups. Figures 7.1 and 7.2 depict the compiled data on the board. The data compiled on the board generated a discussion among the students to come up with a "relation" to relate "size" with one of the measures they had taken.

PHOTOS FROM THE VACATION COURSE CLASS (MEASUREMENT ACTIVITY)

Fig. 7.1 A group records their data on the chalk board

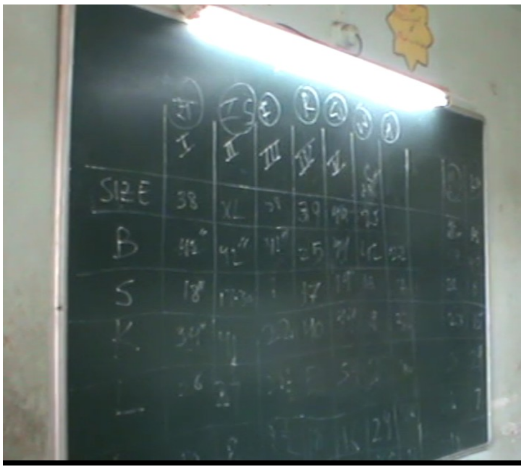


Fig. 7.2 Data recorded by different groups on the chalk board

Fig. 7.3 Taking measurement of different garment-parts using a paper strip (a)

Fig. 7.4 Taking measurement of different garment-parts using a paper strip (b)

Fig. 7.5 The shirt activity in progress

Towards construction

The shirt measuring continued the next day (Day 2) but with a different focus. In Chapter 6, we have noted that many students were familiar with the notion of construction, namely, construction of new and sub-units, different measuring scales and equi-partitioning. The activity of Day 2 drew on this resource that students had from their exposure to the out-of-school contexts. The objective was the construction of a unit and scale to measure length. Students were asked to imagine that all the measuring tapes and scales that existed had vanished and to think of a way to measure the shirts (or other objects) if the regular measuring tools were not available.

During the activity while finding ways to measure lengths without the use of common measuring instruments, students came up with the idea of body-part measurement and measurement using ropes/strips in the absence of the regular measuring tapes/scales. On the teacher's suggestion, the students made a new unit in the form of a paper strip to measure lengths, which they used to measure the length of objects by iterative covering. This was followed by making a scale based on the A4 width unit, that contained subdivisions of the unit, obtained through equal partitioning by means of paper-folding. Though, it was not clear if the students understood well the conceptual underpinnings embedded in the measuring scales that they made in the classroom, they took a step in this direction by exploring and extending their familiarity with the *inchi* tape.

Templates are important measurement tools used in leather and tailoring work-contexts and as we have noted before, students were familiar with such objects and used them in their work. Templates are often used in the context of leather or tailoring work as units to estimate or measure a length or area by iterative covering. We noted further that in everyday contexts, students come across units that have been constructed through equal partitioning, such as the 250g or 100 g weight units used by grocery or vegetable vendors. These experiences are related to the activity described above of constructing and using measurement units or scales, although no explicit connection was made in the classroom with specific work or everyday contexts. In making use of the non-standard measuring tools, and constructing new units, the classroom teaching was directed towards enabling a shift towards understanding the artefact, viz., the measuring scale and measuring tape. Subsequent to the measurements that students made with the A4-width scale, they also made their own one-metre measuring tape on a paper strip, and made marks for every centimetre.

The above instances call for a shift towards understanding the underlying mathematical constructions than just knowing about the tools or routine procedures and working with them. Hence the need to employ the archaeological exploration or unpacking (discussed in Chapters 6 and 8) of the embedded mathematics and illuminate further learning.

Shift towards a mathematical discourse community

A third shift was in terms of students' *identities* as participants in a mathematically focused discourse community. This included moving from making assertions about what they know or what is correct, to seeking and providing clarifications, explanations and justifications. Students' identities as learners and doers in out of school contexts include elements of co-operation, of learning from each other and sharing what they know, interactions which mediate membership in a community. However, as we discussed in Chapter 5, the cooperative engagement with others is in the mode of "helping" each other, which is also the way children often interact with each other in the classroom. The shift that the teaching intervention aimed at was moving from such "helpful" interaction *towards a more discursive culture* in which reasoning is central, where statements are listened to with attention, are challenged, elaborated and justified.

The teacher encouraged the students to come up with proper reasoning and justification and not to accept or believe any result without a justification. For example, the teacher used phrases like "*hum kyun maane*" [*why should I accept*] as a trigger to move towards a mathematically discursive practice. Here is an excerpt from the classroom conversation:

Excerpt 7.8 Day 10, Topic: Volume measurement and decimal representation

152	T	kyun teen batte chaar ka matlab pauna hai? hum kyun maane? aap keh rahe hain to sahi hai to aap prove kijieye ki teen batte chaar pauna hai/	why is three-quarter three by four? why do I accept? if you're saying it is correct then you prove that three by four is three-quarter/
155	S_1 (G)	sir pauna hota hai to teen batte chaar hota hai/	sir if it's *pauna [three-quarter]*, it's three by four/
156	T	kyun?	why?
158	S_2 (G)	teen batte chaar/	three by four/
160	T	yahan aake koi explain ker sakt hai ki kyun … pauna ko teen batte chaar kehte hain?	can anyone come here and explain why … *pauna* is called three by four?
162	T	kyun maane?	why should [we] accept?
163	S_1 (G)	kyunki wo teen paav hai/	because that's three quarters/
164	T	ok. board pe explain kerna/ jao jao jao/	ok/ explain on the board/ go, go, go/
165	S_1 (G)	mujhe nahi pata kaise kerna hai.	I don't know how to do it/
166	T	jo aap bol rahe the wahi batana/ koshish ker sakti hai na/ aap ye bata rahe hain ki kya soch rahe hain/ pata nahi hai to chalega/ riyazi me sab	You can show what you just said/ you can try right/ you can say what you're thinking/ if you don't know it it's alright/ it is not necessary to

		cheez pata hone ki jarurat nahi/	know everything in maths/
167	S_1 (G)	kyunki teen paav... do paav se aadha banta hai aur ek paav me pauna hai na isilieye teen...	Because three quarters … two quarters make one half and one quarter [more] gives *pauna*, isn't it therefore three...
168	T	teen paav/ woh to bilkul sahi hai lekin teen batte chaar kaise aaya teen paav se?	three quarters/ that's perfectly correct but how is three by four obtained from three *paav* [quarter]?
169	S_2 (G)	ek paav/ ek paav ho gaya aur ek paav/	one *paav*/ one *paav* and one more *paav*/
170	S_3 (G)	ek batte chaar jama ek batte chaar jama ek batte/	one by four plus one by four plus one by four/
171	T	haan?	yes?
172	S_1 (G)	(writes on board) 1/4 + 1/4 + 1/4 = 3/4	(writes on board) 1/4 + 1/4 + 1/4 = 3/4

The shift towards a mathematical discursive classroom community was exercised by emphasising the need for providing reasons behind any claims made with proper justification. The students subsequently figured out that *pauna* is “zero point seven five” by adding “zero point two five” (*paav*) three times. While doing so, they discussed, helped and challenged each other.

The following excerpt from the transcript shows a mathematical discussion where a girl (S_3) sought clarification from a boy (S_1) about the measure of waist of the corresponding garment that S_1's group measured using the paper strip. The subscripts given to the alphabet are meant to distinguish between the five students who took part in this conversation (mainly between two students and the teacher). The mathematical discourse between the students revolved around explanation and justification in response to S_3's query that S_1's garment measurements indicated that the “breadth has become wider than the length”. S_3 sought an explanation of how S_1 arrived at the measures. Some of the confusion arose from the fact that the “breadth” needed to be doubled since the chest measurement included the front as well as the back.

Excerpt 7.9 Day 2, Topic: Length measurement (using non-standard measuring strips)

244	S_1	sir yeh breadth/ chaurai/ isko bhi aise hi napenge/	sir this is breadth/ *breadth*/ will measure this the same way/
245	T	thik hai/ aise naapne ke baad kya aaya woh batana/	alright/ tell us what you got after measuring this way/
246	S_1 (B)	(a boy): woh aaya char ikai ek paav aaya/	(a boy): it came [as] four units [and] one quarter/
247	T	char ikai aur...	four units and...
248	S_2 (B)	(another boy) iska bhi double karna hai/ iska aage aur peeche ka/	(another boy) need to double this/ its front and back side/
249	T	yaani aage aur peeche ka double kiya unhone/ double kiya na?	meaning he doubled front and back side/ doubled right?
250	S_3 (G)	(a girl): yaani breadth lambai se chaura ho gaya/	(a girl): it means then breadth has become wider than the length/
251	T	Kya kuchh bolna hai aapko? (pointing to S_3 who was asking Q)	do you want to say something? (pointing to S_3 who was asking Q)
252	S_3 (G)	sir, main bol rahi hoon na ki breadth length se bada ho gaya, char sahi hai aur woh utna chaura bhi nahin hai/	sir, I'm saying that breadth has become larger than the length, four is right and it's not that wide too/
253	S_4 (G)	nahin to aage peeche ka ho jayega na?	no but front and back will come right?
254	S_3 (G)	nahin to char double kiya to aath ho jayega na/	but then doubling four will give eight, right?
255	S_5 (G)	(another girl): arrey jod na usse badhaya/	(another girl): ohh add it neatly/
256	T	aapne naapa kitna aur uska double kaise kiya yeh woh poochh rahi hai/	she's asking how much did you measure and how did you double it that's what she's asking/
257	S_3 (G)	haan/	yes/
258	S_1 (B)	accha, yeh iska, iska, iska maloom kitna aaya? Iska do ikai aadha paav aaya/ to peeche ka bhi do ikai aadha paav ek hi hoga na sir/	right, you know its, its, how much is it? it's two units [and] half-quarter/ then the back portion will also be two units [and] half-quarter, right sir/
259	S_3 (G)	haan/	yes/

260	S_1 (B)	do ikai do ikai char pakdo paav ho gaya...	take two units two units as four [and] a quarter...
261	S	(other girls in chorus): char ikai ho gaya/	(other girls in chorus): it's four units/
262	S_1 (B)	char ikai ho gaya aur aadha aadha ek paav ho gaya/	it's four units and half [and] half [of a quarter] make one quarter/
263	S_3 (G)	haan to?	yes so?
264	S_1 (B)	haan, to char ikai ek paav ho gaya na?	yes,, so it becomes four units [and] a quarter, right?
265	S_3 (G)	(nods in affirmation)	(nods in affirmation)
266	T	sahi hai?	is it correct?
267	S_3 (G)	haan/	yes/

Although S_3 knew that as a measure of full waist line, measure of the front side needed to be doubled, but she did not notice that S_1 had already doubled the waist measure (front side) before recording the measure on the board. S_3 thought S_1 had incorrectly measured the waist as "*char ikai ek paav*" [*four units and a quarter*]. According to S_3, the waist did not look as big (Excerpt 7.9, line 252). S_1 explained that one side of the waist (say, front portion) was "*do ikai aadha paav*" [*two units and a half-quarter*] and after doubling it he arrived at "*char ikai ek paav*". Hence, further doubling of "*char ikai ek paav*" was not required. Following this discussion S_3 was convinced and satisfied. It is important to note here that enabling a shift towards such discursive practices in the classroom reinforced normative identity of the students that engaging in such mathematical discourses was expected and accepted. Furthermore, S_3 applied her visual estimation skill in verifying the measure. In the transcript above we observe the students adopting the norm of a discursive community, and note the teacher's intervention in supporting questioning and responding by the students (Excerpt 7.9, lines 251, 256 and 266).

Students often used proportional reasoning and convenient decompositions in computing answers when they were encouraged to do so in the classroom. Such instances also facilitated discursive practices among the students. An example was the task of finding out ½, ¼, 1½, 2½ and 3½ times a given number. Students explained the various strategies that they used to compute these multiples to their peers, which indicated that they had a robust and confident awareness about decompositions of fractions (twice of 1½ [*dedh*] is the same as three; half of 1½ [*dedh*] is equal to ¾ [*pauna*], half of 2½ [*dhai* or *adhai*] is equal to 1¼ [*sawa*], etc.). Here is an excerpt from the transcript.

Excerpt 7.10 Day 12, Topic: Measurement units

298	T	… dedh aur pauna me koi rishta hai kya?	Is there a relation between one-and-a-half [*dedh*]and three-quarters [*pauna*]?
299	S_1	sir paav aur paav aadha hota hai/	sir quarter and a quarter makes one half/
300	S_2	sir pauna aur pauna dedh hota hai na/	sir *pauna* [three-quarters] and *pauna* make one-and-a-half/

Shift from individual to public expression

In relation to the culture of the typical classroom that the students had experience of, the shift towards a mathematical discourse community entailed a fourth shift from individual expression in private to shared public expression (e.g., use of blackboard by students to record the results they had obtained or to record their thinking or reasoning, which others could interpret and respond to). The normative identity co-constructed in the classroom is reflected in this shift where students were encouraged to discuss the results that they obtained for everyone else in the classroom to think over and respond. This was done by inviting a student to share her answer/finding or justification with the whole class either verbally or by writing on the board. This was followed by discussion over the obtained answer which also led to verification and cross-checking of answers. Such a shift encouraged the students to disprove the answers if they disagreed or were not convinced or to come up with counter-arguments or to ask questions to clarify their doubts. For example, as described in the previous Excerpt 7.9, one student asked several questions about the garment measures that another group obtained. She sought clarifications from the other group members to understand how they arrived at the measurements recorded on the blackboard. She correctly pointed out that the measure taken from the front of the garment needed to be doubled so as to add the measure at the back. However, she misunderstood what the other group had reported and instead of doubling, quadrupled the answer. We claim that such questioning-counter questioning happened since the attempt was made to share one's results and arguments with the entire classroom. In this way, the discursive practice was to make ideas and expressions public and discuss them in front of the classroom. Using the chalk board not only as a space for the teacher to write, but also as a space for students to record their thinking was part of enabling a shift from individual to public expressions. A similar classroom scenario is reflected in the following Excerpt 7.11.

Excerpt 7.11 Day 12, Topic: Measurement units

31	T	… aap boliye/ sun rahe hain kya? peechhe mud ke dekho sab log sun rahe hain kya/ class ko bolna hai/	… you tell/ are you listening? (to the class) look back and see if all others are listening/ need to tell the class/
32	S	do batte dus ke pandrahwe hisse ko pandrah batte dus kahte hain/	one-fifteenth part of two by ten is called fifteen by ten/
33	S_1	arrey pandrah batte sau kahte hain/	ohh it's called fifteen by hundred/
34	S_2	nahin samajh mein aaya/	haven't understood/
35	T	samajh mein nahin aaya/ aap logon ko sunai diya kya?	haven't understood/ could you all hear?
36	S	nahin sir sunai bhi nahin diya/	could not even hear/
37	S	do batte dus ke pandrahwe hisse ko pandrah batte sau kahte hain/	one-fifteenth part of two by ten is called fifteen by hundred/
40	T	samajh mein nahin aaya/ samajh mein aaya kya?	could not understand/ have you understood?
41	S	(many voices) nahin sir/	(many voices) no sir/
42	T	mujhe to samajh mein nahin aaya/ aap ja ke board pe likh rahe hain kya?	I haven't understood/ are you going to write on the board?

We describe another example of this shift in the context of a discussion of decimal addition. Students were adding 0.1 to itself a number of times and arrived at a pattern. They noticed that by adding two "0.1"s, they obtained "zero point two", by adding three

"0.1"s, they obtained "zero point three" and so on. The teacher asked the student to use simple calculators[1] (distributed among the students in groups of two) for doing the addition as a means of cross-check. By following the pattern, students noticed adding nine "0.1"s led to "zero point nine" and the teacher asked the students to first guess what would be the sum of ten "0.1"s. Students' immediate response was "zero point ten". The students were then asked to add ten 0.1 on the calculator and see what was the answer. The answers that students had anticipated ("zero point ten") did not match with the one shown on the calculator.

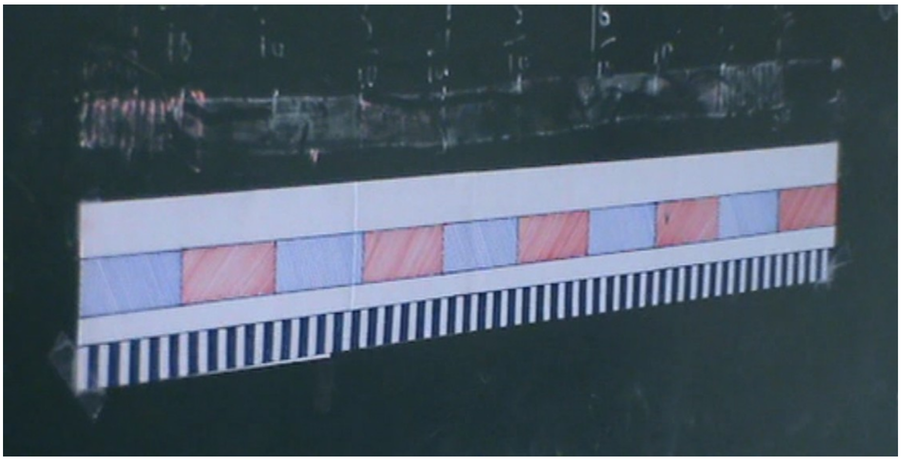
We noticed that such classroom experiences that were "counter-intuitive" to the students and came as a "surprise" to them were enabled by a shift towards public expressions, by sharing of answers and also by listening to others' arguments and explanations. The discrepancies in the answers were addressed by the teacher later, a part of which is discussed in Section 7.4.

The teacher and students co-constructed a classroom norm of explaining on the black-board for everyone to see and respond. Telling the answers individually was not the norm. Rather than tackling individual's answers, answers were collectively sought and discussed often using the chalk board in the process. We present below a couple of instances of how use of the black-board elicited students' responses that sought to achieve conceptual understanding.

The teacher employed an activity in the classroom in which students made one-metre long measuring tapes for using them to measure various objects. As a device to project students' understanding of the tape, and the structure of sub-divisions on it, a blackboard version was developed and used (Fig. 7.6). The shifts towards providing reasoning and justification encouraged students to use these constructed artefacts to verify for themselves the location of different points on the number line.

1 Use of calculators in Indian classrooms is rare unlike in some Western and African countries. School students are discouraged from using calculators during formal learning, though they have exposure of calculator-use in the everyday contexts.

Fig. 7.6 One metre-long measuring tape used during lesson



On Day 10, the understanding of length measurement and its units, and the understanding of fractions and decimals was extended to a new context – that of volume measurement. The teacher drew four glasses of equal shapes and sizes on the blackboard (assumed to hold one litre of water). The glasses were filled to different levels, and the task was to identify the fraction filled, or to fill water to the required level when the fraction was given. The students' response to this new context is discussed in a later subsection.

Shift from disconnected to connected identities

There was an increased focus in the vacation course on merging of students' identities – as doers, knowers and learners (discussed in Chapter 5). Based on the experience gathered from engagement in the work-contexts, students came to the classroom lessons with out-of-school mathematical knowledge, even if they were fragmented and non-transparent in nature. Snippets from the intervention classroom showed that when students' identities were acknowledged and their responses linked with the out-of-school knowledge were valued, they came up with justifications that were "out-of-school" in nature (distinction made using the framework, Table 2.1). For example, the shirt measuring activity triggered an inflow of students' out-of-school knowledge and identities where most students connected the activity with their knowledge of tailoring work in different forms and in varying capacities. As discussed earlier (section 7.3.1), the teacher invited the students to share what they knew about the process of making a garment and its various stages. The shirt measuring activity allowed them to participate as "knowers" of the task in contrast to the usual practice of "learners" in the classrooms. There were a few students in the classroom who started enacting the way a tailor would take shirt measurements. For instance, one student (U_{22}) who regularly visits his father's tailoring workshop and therefore knew about the task well, put around his neck the long measuring tape copying a tailor's typical gesture. We argue that at this instance, U_{22}'s identity as a "knower" of the activity as well as a "doer" of the task came together in the classroom activity and the identities as learner, doer and knower were merged.

Similarly, the instance where students came up with three different representations of "three quarter" (discussed in Chapter 6 and in Section 7.4 below) drawing from their out-of-school knowledge while attempting to find its decimal representation, was a reflection of a shift where students' identities as "knowers", "doers" and "learners" were merged and connected. Such merger of identities facilitated students to make conceptual connections. This was a departure from the regular classroom practice where students' identities as knowers and doers remain disconnected with their identity as learners.

Table 7.1 Summary of the "shifts" in classroom norms

A pedagogic approach to connect out-of-school and school mathematics learning by entailing a series of shifts in classroom norms				
Oral to Written	Knowledge about use to understanding tools	Co-operation to Mathematically focused discourse community	Private, individual expression to shared, public expression	Disconnected to connected identities

The first previous three shifts reflected movements from out-of-school contexts to school contexts, more specifically, from culturally embedded everyday contexts or practices towards a mathematics classroom culture in schools. In contrast, the last two shifts described movements within the existing mathematics classroom culture.

7.4 Significance and necessity of the “shifts”

This section discusses the significance of the shifts and why they are required in the pedagogical process. It also discusses how enabling of shifts can be effective in connecting out-of-school mathematical knowledge with school mathematics and for the development of students’ identities.

The instructional goals described above spell out how classroom teaching might draw on and connect with out of school knowledge that students bring to the classroom. However, school learning is not the same as out-of-school learning and the goals need to acknowledge the complementary dimension of the differences between the two forms of learning. It is with this need that the above five shifts are viewed as enabling building connections between out-of-school and school knowledge. Furthermore, enabling of the shifts as a pedagogical approach provides opportunities for merging of students’ identities as knowers, doers and learners which, as described earlier, mediates the connection of prior knowledge with the classroom topic and facilitates knowledge transfer. The significance of the shifts further accords power and legitimacy to the agency of the students as knowledgeable persons who can contribute to the pedagogical processes during classroom learning. That the agency of the students is valued, respected and acknowledged in the classroom is concretised by adopting the shifts. Some previous research studies have also called for valuing students’ agency and have indicated that the mathematics classroom practices of “negotiations and interpretations” help students to make better connections between mathematics they learnt in the class and the out-of-school situations or practices where they are used, in comparison to those students who predominantly used only textbooks (Boaler & Greeno, 2000, p. 172).

One of the foci of the analysis of the teaching design experiment was on the conceptual connections between out of school and school mathematical knowledge with specific reference to the topics covered in the lessons of measurement and fractions. Classroom interaction in the course of the design experiment supported some of the findings concerning mathematical knowledge of students from earlier phases of the study and elaborated on others. The fragmented nature of knowledge from out of school contexts was reinforced on multiple occasions. For example, while students had an idea that the common plastic measuring tape was marked in inches, most did not have any idea of what the other units marked on the tape were (cm). Although they all knew the word "centimetre" from classroom contexts, they also did not have an idea of the relation between the inch and the centimetre. Similarly, several students seemed familiar with shirt sizes while they did not have any idea of how the number was obtained. Most students knew that "0.5" meant the same as half, but did not have any further understanding of decimal numbers.

Nearly all students were comfortable in using binary fractions which are part of the daily parlance, namely, *aadha* (half), *paav* (quarter) and *pauna* (three quarters). Some students appeared to be aware of the use of *aadha paav* (half-quarter) and *sawa* (one and a quarter) *dedh* (one and a half) and *dhai* (two and a half). In a teaching episode, students were presented with the challenge of finding names for smaller binary fractions. Students readily extended their repertoire of binary fractions by not only including "*aadha paav*" and also by inventing the term "*aadha ka aadha paav*" (half-quarter of half) or "*paav paav*" (quarter of a quarter) for one-sixteenth. There were also occasions where students combined their understanding of proportionality and binary fractions (see Excerpt 7.10 and the preceding discussion above). We have noted before that although students were familiar with the words for binary fractions, the written notation was not known to many students, barring a few. They could make the connection between *pauna* (three-quarter) and *teen paav* [three quarter]. On one such occasion, a student could make the connection, but repeatedly gave an incorrect fractional representation of *teen paav* as *ek batta teen* [one upon three]. He was not convinced that his answer was incorrect even when the teacher indicated this. But he offered *pachhattar* [seventy five] as an alternative representation for three-quarters, possibly indicating the equivalence of ¾ and 75%. Such instances indicated that students possessed fragments of knowledge about the fraction notation for binary fractions or the equivalent decimal fractions, but the connections between these fragments were fragile.

We claim that eliciting such discussion reinforced the fact that when classroom practices acknowledged and valued students' identities as knowers, that facilitated enabling of the shifts described above. Such shifts were necessary in the classroom as they encouraged moving towards mathematically discursive practices where students were eager to share their knowledge publicly in the classroom, and also extended help to their peers who had difficulty in understanding the concepts. Such practices went beyond mere helping but established a classroom community practice of arriving at conceptual clarity and valuing of it. Exercising of shifts by the teacher also helped in bringing together student's identities as learners, doers and knowers which in turn facilitated drawing on students' out-of-school mathematical knowledge. Building connections with out-of-school mathematical knowledge with school mathematics and exploring the implications of such processes was the underlying objective of the teaching design experiment.

7.5 Negotiating the shifts to integrate out-of-school and school mathematics

In this section, we discuss two kinds of activities in the classroom that depict the manner in which the five shifts were negotiated in an inter-connected manner in facilitating integration of out-of-school and school mathematics. The first of these is seeking and providing justifications for assertions made in the mathematics classroom, the second is developing consistent mathematical representations, which is centrally connected with the movement from oral to written mathematics.

7.5.1 Seeking and providing justifications

Several of the excerpts presented above indicate that among the norms that the teacher sought to establish was that of providing justifications of assertions. The teachers' questioning frequently focused on providing explanations, clarifications and justifications. Students were asked to verbalise or show on the board a strategy that they had used. Alternative solutions were invited and students were asked to choose the correct or the better solution. The teacher then typically asked students "how do we know/ decide which is correct". Students offered justifications which were accepted or contested by their peers or by the teacher. Three kinds of sources of justification were accepted in resolving "how do we know" questions. They were *invoking of authority*, *use of prior knowledge of mathematics*, and *use of experiential knowledge*. Some of these have surfaced in the excerpts discussed above, but here we clarify the difference in the sources of justification with brief discussions about each one of them. The following table (Table 7.2) summarises the different sources of justification:

Table 7.2: Sources of justification

Sources of Justification	Invoking of authority
	Prior knowledge of mathematics
	Experiential knowledge

The first source of justification was *invoking of authority,* which was done in cases where information was to be shared, or conventions about symbols needed to be cited. Examples where the authority of the teacher was exercised included decisions about conventions of written representations for fractions or decimals, or relation between well known units such as the meter and centimetre. Information relevant to the activity at hand was also supplied by the teacher, such as the standard size of paper sheets used for photocopying (A4). An interesting source of authority that was often invoked in justifying a response or assertion was an artefact – the calculator. Computations done on the calculator were invoked to judge the correct decimal representation of a known fraction, or the outcome of an operation on decimals. We illustrate how calculators were used as a source of justification below.

A second source of justification was *prior knowledge of mathematics*. For example, a relation between fractions may be justified using a computation procedure. A specific example (discussed above) is to explain that 1/10 is the same as 0.1 because adding 0.1 ten times using the vertical addition algorithm gives 1.0. A justification of this kind is lengthy and not mathematically elegant. Many justifications of the mathematical kind were of this nature. In an earlier excerpt,we noted how students offered a justification for why ¾ was the correct representation for pauna (three-quarters), by showing that adding ¼ three times gives ¾ (see Transcript 7.8, Day 10 in the section on "Shift towards mathematical discourse community" above.) For a justification such as this to be accepted, such procedures needed to be part of the shared knowledge of several students in the classroom. Such prior knowledge was typically restricted to mathematical procedures learnt at school.

A third important source of justification was *experiential knowledge*, which typically was from out-of-school contexts. Students' knowledge of binary units and fractions was frequently invoked, as was proportional reasoning and convenient decompositions. Knowledge about units and relations between units were sometimes cited. One student frequently justified his oral computation strategies using money as a convenient representation for quantity. That such justifications were accepted by other students indicated a level of shared knowledge drawn from out-of-school contexts.

We have mentioned how an artifact drawn from an out-of-school context, viz., the calculator played a role in securing justifications for assertions. According legitimacy to the use of artefacts that are typically viewed as used in out-of-school work-contexts in the classroom is part of the significance of the shift from disconnected to connected identities. We have observed in this study that there remains a disconnect between the artefacts that are used during formal lessons, viz., standard instruments and units, and those which are used in the out-of-school work practices and other settings. Bringing the resources from the community based *funds of knowledge*, such as the use of artefacts, was one way of building connections with out-of-school knowledge. As discussed above, one of the artefacts from out-of-school contexts introduced in the classroom was the measuring tape. Another was the simple calculator, which is commonly used in shops in the neighbourhood, and is familiar to most students. While the use of calculators was practically forbidden during lessons in the schools where the study was conducted, measuring scales were commonly used in the classrooms but not the measuring tapes. In the intervention lessons, the calculator was used as a resource for exploring variations on calculations and for cross-checking answers.

Excerpt 7.12 Day 9, Topic: Decimal and binary fractions

195	T	... thik hai to aap calculator mein ye dekh sakte hain ki paanch bate sau jama paanch bate sau kitna aana chahiye?	… alright so you can look at the calculator to see how much would five upon hundred plus five upon hundred comes out to be?
196	S	dus batte hundred/	Ten upon hundred/
197	T	paanch bate sau jama paanch bate sau kitna aana chahiye? dus batte sau? dus batte sau yani kiske	How much should five upon hundred plus five upon hundred be? ten upon hundred? ten upon

		barabar?	hundred is equal to what?
198	S	Zero point one ke barabar/	Equal to zero point one/
199	T	0.1 ke barabar, to ye aata hai kya calculator me aap dekh sakte hain. janchenge?	Equal to zero point one, so does it come you can see in the calculator/ will you check this?

In the ensuing discussion, the teacher elaborated,

213	T	Yani ki hame yeh jachna hai ki paanch batte sau ko calculator ki zuban mein ya ashariya mein kaise likhenge?	That is, we need to check how to write five upon hundred in the language of the calculator or in decimals?

There was an emphasis by the teacher on "*karke dekhna*" (observe by working out). Expressions like "*karke dekhna woh aata hai kya/ aata hai? Sahi hai?*" [*observe by working out whether it comes (number under discussion)/ does it come? Is it correct?*]. The following excerpt exemplifies such an instance.

Excerpt 7.13 Day 9, Topic: Decimal and binary fractions

246	T	bilkul sahi... to yani ki yeh barah batte sau hai aur yeh zero point one two bhi yahi hai/ iska matlab barah batte sau/ chhe batte dus ya agar batte sau mein likhenge to?	perfectly correct... so it means it's twelve by hundred and this also is zero point one two/ it means twelve by hundred/ if we were to write six by ten in hundredths then?
	S	(parallel voice): sixty batte hundred/	(parallel voice): sixty by hundred/
	T	ab aap dekh sakte hain ki kaun sa bada hai/ (students: upar wala sixty batte hundred)/ kyunki yeh twelve batte hundred hai aur yeh sixty batte hundred hai yani bahut bada hai, bahut farak hai, kitna guna hai? barah batte sau ki kitna guna hai zero point six? double hai? teen guna hai? char guna hai?	now you can see which one is bigger/ (students: the above one sixty by hundred: indicating the figure drawn on board)/ since this is twelve by hundred and this is sixty by hundred, so it's much bigger, much of a difference, how many times? how many times of twelve by hundred is zero point six? is it double? is it three times?

			is it four times?
247	S	paanch guna hai sir, paanch guna hai sir/	it's five times sir, it's five times sir/
248	T	zero point one two ko paanch se guna karenge to kya aana chahiye? zero point six aana chahiye/ yeh aata hai kya dekhiye aap calculator se/	what do we get by multiplying zero point one two by five? zero point six should come/ do we actually get this you check using a calculator/
249	S	sir, zero point six aaya sir/	sir, zero point six came sir/

As we observe from the excerpts quoted, a socio-mathematical norm that the teacher sought to establish was to justify i) different representations of answers, ii) different ways to arrive at answers, and iii) verification of the equivalence of all the answers arrived at. Such an approach adopted by the teacher firmly established acceptance of multiple answers as part of a discursive practice among the students and encouraged them to come up with different strategies following different lines of thought which could vary from those suggested in the textbooks. This approach catalysed a shift towards building a mathematically focused discourse community which moved away from making assertions towards giving explanation, justification and verification. *Jachna* was accomplished in different ways by using – an artefact (calculator), visual cues (in the form of artefacts like measuring strips) and blackboard space (use of diagrams and a representative number line). These methods were employed to arrive at justification. For example, students were encouraged to locate different decimal fractions on the metre-long measuring strips that they had made and by using a standard metre-strip stuck on the blackboard as a visible analog.

Consistent mathematical representation

A feature of school mathematics, as different from out-of-school mathematics, is the emphasis on consistent mathematical representation. This is part of the culture of written mathematics. Thus the shift from oral to written mathematics entailed developing consistent mathematical representations. This was continuously negotiated in the lessons of the vacation course, particularly in the later sessions that involved using symbolic representations for fractions and for decimals. In these negotiations, there were repeated indications of the familiarity and comfort that students had with binary fractions and their verbal representation, a strong characteristic of out-of-school fraction knowledge. While the teacher made efforts to draw on such knowledge to move towards the complex formal notation for fractions and decimals, students used their understanding of binary fractions as a scaffold. Their efforts to connect binary fractions with other fractions or decimals were not always successful, which points to the need to pay greater attention in the curriculum to how binary fractions relate to other fractions.

In a task related to the metre scale, students when asked what was “zero point five” on the number line, came up with different words that expressed the same meaning – for example, “*aadha*” (half), “*ek batte do*” (one by two) and “*paanch batte dus*” (five by ten). When asked to locate the point on the number line drawn on the board, one student indicated the distance between the point marked as “0.5” and “0” which was at the left end of the number line that began at “0”. However, when asked to tell where would “zero point three” be on the number line, some students referred to it as *paav* (quarter). As mentioned in Chapter 6, in common everyday practices, approximation overrides accuracy, and often small quantities are referred to by smaller units such as *paav* (quarter) or *aadha paav* (half-quarter) and not in the accurate sense of the terms. This could be possibly the reason behind the students’ response, since they realised that “zero point three” is close to a quarter – “*yeh sir paav hai*” (sir this is a quarter), justified one student who also mentioned that it is “*teen batte dus*” (three upon ten). It is important to note here that the same student gave two answers – one of them (“*teen batte dus*”) was mathematically accurate while the other (“*yeh sir paav hai*”) was socially acceptable. The teacher’s exercise of shifts towards public expressions and his assurance that it was alright to make mistakes and give incorrect answers (normative identity in place), encouraged the students to put forth different answers and the above instance is an example of this. There seemed to be variations in students' replies when some student referred to “zero point three” as *pauna* (three quarters), probably because *pauna* refers to three parts out of four equal parts. Some students incorrectly mentioned that “zero point three” is less than a *paav* (quarter) and equal to *pauna* (three quarters). Such instances of variations in answers also came up during other classroom discussions.

On Day 10, when students had developed some familarity with the use of binary and decimal fractions for length measurement, the teacher introduced a new context, of identifying the quantity of liquid in a container. Four equal sized jars were drawn on the board. The first jar was marked as fully filled with water (assuming the volume capacity of one litre), next was half filled, the third one was quarter filled, while the fourth jar was empty. The researchers were aware from earlier students' responses that some students knew the decimal representations of half (aadha) and quarter (paav) as "zero point five" and "zero point two five" respectively and they also knew that those can also be represented as "ek batte do" [one by two] and "ek batte char" [one by four] respectively. The discussion began with where would "zero point four" be marked. The next excerpt is about this discussion.

Excerpt 7.14 Day 10, Topic: Volume measurement and decimal representation

228	T	litre bharna hai/ sifar ashaariya chaar/ zero point four/ itna bharna hai to kitna tak bharega paani? kaun bhar sakta hai isko?	litre to be filled/ zero point four/ zero point four/ if this much is filled then till where will water be? who can fill this here?
231	S_1	sir pura bharega/	sir it'll be full/
232	T	haan? poora? full?	yes? full? full?
233	S_1	haan/	yes/
238	T	one litre aapko maloom hai/ zero point five litre aapko maloom hai/ zero point two five maloom hai/ zero point seven-five maloom hai/ ab aapko zero point four...	you know one litre/ you know zero point five/ you know zero point two/ zero point two five is known/ zero point seven five is known/ now you need zero point four...
240	T	zero point four litre bharna hai, to kitna bhareinge? aur kaise bhareinge?	need to fill zero point four, then how much to fill? and how to fill?
245	S_1	sir, ek sawal puchein?	sir may I ask a question?
246	T	haan/	yes/
247	S_1	zero point seven-five wahan tak bharein, to zero point four yani ek litre.	if zero point seven-five is till there, then zero point four means one litre/
248	T	zero point four yani ek litre? Woh kaise?	zero point four means one litre? how's that?

249	S_1	paav...	quarter...
250	T	kaise prove kar sakte hain?	how do you prove this?
251	S_1	paav paav matlab aadha...	quarter quarter means a half/
252	T	hmm...	hmm...
253	S_1	phir paav paav milke aadha (a girl seated beside her repeated this sentence)	then *paav paav* make a half (a girl seated beside her repeated this sentence)
254	S_2	ek/	one/
255	S_1	to Ek litre bana na sir.	so doesn't that make one litre sir/
256	T	to paav ko kya bolenge hum?	so what's *paav* [quarter] then?
257	S_1	paav ko zero point five	*paav* is zero point five/
258	S_2	nahin/ zero point two five/	no/ zero point two five/
259	S_1	two five/	two five/
260	T	zero point two five/ to aapka kya kehna hai ki zero point two five ko chaar baar agar milaenge to zero point four aaega?	zero point two five/ so are you saying if zero point two five is added four times will make zero point four?

The decimal representations and their connections to the commonly used binary fractions was not clear to some students. Based on the normative identity that was developed in the classroom, students asked questions while in some cases, some other student asked the question or offered justifications on behalf of her friend. There was a detailed discussion about what would "*ek batte dus*" [*one by ten*] be. As we noted above, students knew that "one by four" is also represented as a *paav* [a quarter], but a few of them also referred to "one by ten" as a *paav* and interpreted "zero point four" as *paav*. It was possibly because in the routine everyday setting, a small quantity is often seen as *paav* or *aadha paav*. Here is an excerpt from one such conversantion:

Excerpt 7.15 Day 10, Topic: Volume measurement and decimal representation

465	T	usme se ek hissa yaani chaar hissa humne banaya aur sirf ek hissa humne bhara hai/ theek hai to ek batte chaar/ ab ek batte dus me kya hoga?	in that one part, that is we made four parts and filled only one part/ alright so one upon four/ now what'll be one upon ten?

466	S_1	dus hissa banaeyenge to ek hissa.	one part out of ten parts made/
467	T	aaisa hai kya? … sahi hai kya dekhna?	is it so? … check if that's correct?
468	S_2	sir wo bhi paav hoga sir.	sir that'll be *paav* too sir/

The decimal representations of the commonly used binary fractions appeared to be clear to most students in the class. For example, the same student (S_2) who said *paav* for "one upon ten" in the previous excerpt, could actually justify why a half is taken as "one upon two". Here is an excerpt:

Excerpt 7.16 Day 10, Topic: Volume measurement and decimal representation

475	T	aadha ko ek batte do kyun likhte hain?	why is a half written as one upon two?
476	S_1	kyunki whan uske do hisse kerte hain/	because there we make two parts of it/
477	S_2	sir, ek litre ke do hisse karein aur sirf ek hissa humne le liya/	sir, we made two parts of a litre and took only one part/

The conceptual connection between "one upon ten" and one of "ten equal parts" emerged gradually when the teacher pointed out that by calling "one upon ten" as a *paav*, "one upon four" and "one upon ten" would refer to the same parts. When this discrepancy was pointed out in the following excerpt (7.17), students realised that ten parts were needed to be made. Initial part of the discussion in the following excerpt was to figure out why is half written as "one by two"? [*aadha ko ek batte do kyon likhte hai?*]. As an answer to this question put up by the teacher, one student replied that "since there are two parts in it" [*kyunki wahan iske do hisse karte hain*]. The teacher then extended the discussion to the figure on the board (a glass was drawn on the board with different parts filled with water) and asked the number of total parts made on it and the number of parts chosen. The students replied four and one part respectively.

Excerpt 7.17 Day 10, Topic: Volume measurement and decimal representation

482	T	...ussi liye ek batte chaar likh diya/ ab ek batte dus kaise...kitna bharenge?	...[accepting students' explanation] that's why wrote one upon four/ now how... how much to fill for one upon ten?

483	S_1	woh bhi sir paav bharenge/	will fill *paav* too/
484	S_2	paav ke barabar hai/	equal to *paav*?
485	T	woh bhi paav ke barabar hai? to kitna hissa banaun iski?	that's equal to *paav* too? so how many parts to make of it?
486	S_2	chaar/	four/
487	T	chaar hissa/ theek hai, chaar hissa banaya mein ne. Phir?.....ye aaisa hi rahega?.... iska (pointing at the figure – glass with water) aur ek batte dus ka kuch rishta hai kya? chaar hissa banane ka rishta?	four parts/ alright, I've made four parts/ next? … will it remain like this? … this (pointing at the figure – glass with water) and one upon ten do they have some relation? relation behind making four parts?
488	S_3	sir dus hisse to banaiye/	sir make ten parts/
489	T	kitna hissa?	how many parts?
490	S	(chorus: a few students): dus hissa/	(chorus: a few students): ten parts/
491	T	kyun? kyun dus hissa banana hai?...yahan pe dus likha hai na?	why? Why to make ten parts? … ten is written here (on the board) right?
492	S	sir, isko...	sir, this one... (pointing to the one shaded part of the glass out of ten parts)
493	T	isko kya kahte hain?	what is it called?
494	S	sir, ek ka na dus hissa bananaenge...	sir, each one of ten parts to be made...

The last part of the above excerpt shows that the students were beginning to grasp the meaning of choosing "one part out of a number of equal parts" and could figure out that "one by ten" was not same as "one by four" rather refers to a much smaller quantity.

7.6 Summary

The broad goals of the vacation course were to explore how to make connections between out-of-school and school mathematical knowledge by fostering students' identities that allowed building such connections. The teaching design experiment that was adopted during the vacation course, attempted to operationalize classroom pedagogy, that allowed for the negotiation of students' identities and made use of students' out-of-school measurement knowledge through the use and construction of artefacts, viz., measuring tapes, non-standard measuring strips (similar to templates used in work-contexts), calculators, metre scales and volume measuring jars.

The teaching approach adopted in the course was based on prior inputs from the earlier phases of the research study discussed in Chapters 4, 5 and 6. These inputs suggested a pedagogical approach by tracing how students' learning and conceptual constructions were shaped in their work-contexts and other out-of-school settings. The larger objective was to support students' learning and understanding of basic mathematical concepts. We have tried to illustrate how findings about students' knowledge from out-of-school context played a role in designing instruction with an example. We have mentioned earlier that tailoring work is one of the prominent occupations in the neighbourhood and tailoring workshops or garment whole-selling shops are commonly visible in the alleys. This was possibly a reason why almost every student in the classroom was familiar with the tailoring work. Such exposure made them familiar with the notion of "template" commonly used in tailoring and leather work, as noted in Chapter 4 and 6. Most students were also familiar with the notion of the construction of templates and its application in the iterative covering of a larger cloth or leather piece and comparison between pieces. Hence the choice of a measurement activity using garments as artefacts. The objective was to use an artefact that was commonly available and familiar to the students.

The analysis of the vacation course presented in this chapter shows the enactment of the goals of the teaching design experiment based on the earlier phases of this study and the ideas generated from reflecting on them. As speculated from the outcomes and observations of the earlier phases of the study, merging of students' identities shaped in out-of-school and school contexts allowed building of connections between both the forms of mathematical knowledge. Although the teaching design experiment was limited to 12 days of teaching in one-and-a-half hour sessions, it indicated important features that can guide further pedagogical exploration.

8

Conclusions, implications and future directions

We know what we are, but not what we may be

– William Shakespeare

This chapter attempts to bring together the results and findings of the study in terms of out-of-school knowledge of children immersed in work-contexts and its curricular and pedagogic implications under a unifying perspective. Future directions and road-maps are also discussed.

The funds of knowledge perspective illuminates how the connectedness of social networks gives rise to diverse and rich knowledge and experience that can be drawn on for the purposes of school learning. In our study, which is set in an urban, developing world context, we found that students often directly participate in work, or are closely aware of work contexts and practices. Experiences and knowledge drawn from such contexts are intimately familiar and include elements of mathematical knowledge, with aspects of it even embodied in students and present in the classroom. All the students in our study were from two Grade 6 classes that were co-located in one school building. Such diversity of experience, within a school community hence presents potentially rich opportunity for learning that has been largely ignored in formal school education. Although education policy documents currently followed in India call for integration of children's out-of-school knowledge with school learning, there is a lack of concerted effort to implement education practice that draws on knowledge related to manual vocation and work, similar to Gandhi’s vision about education centred around work (Gandhi, 1927).

Participation in work is not always beneficial to the child. Indeed it can be harmful, exploitative and violative of children’s rights. Debates concerning child labour, and child rights activism have pointed out the danger of exploitation of children under the umbrella of weak legislation and regulation. Nevertheless, it is important to make a distinction between child labour that is exploitative and children’s participation in work, that is often a means of socializing children into the adult world. The plea to include work as a central feature of education, even in Gandhi’s time, had to contend with opposition from those who thought that his ideas amounted to advocating child labour (Sykes, 1988). As we have discussed in Chapter 2, as educators, we should not project an image of childhood derived largely from middle-class sensibilities as a universal notion. There are different forms of socialization of children into adulthood, and some of these forms, especially in low-income communities include participation in adult activity from a young age. Education policy documents in India, in fact, point to participation in work as a potential corrective to the bookish, and disconnected learning that happens in most schools. We have adopted the perspective in this study, that the participation in work practices of children from low-income communities equips them with knowledge and with identities as capable individuals, which must play a part in classroom learning. Out-of-school experience and knowledge includes elements of mathematics, which must be taken into account in designing mathematical learning in school.

Mathematics in workplace activities has distinct features as compared to the mathematics used in the formal school contexts. School mathematics is largely textbook driven which, as Freudenthal says, comes as a packaged “ready made mathematics” (1971, p. 431). Contrastingly however, one may draw a parallel between what Freudenthal calls “mathematics as an activity” which is about creating and applying mathematics as part of the “local organization” (p. 431), and mathematics embedded in the work-contexts, in the sense that both the “activities” include a notion of relating ideas to practice. The present study shows that although mathematics in work-context related activities remains implicit and hidden, it is hands-on and uses construction in varied ways. Though mostly routine and fragmented in nature, some of the work-contexts entail on-the-spot decision making and optimatisation that goes beyond “ready made mathematics” prescribed in schools and comes closer to “mathematics as an activity”. Aided with such real life experience of dealing with mathematics in their work-contexts, some students possessed rich potential resources for gaining deeper understanding in the course of formal learning than what they currently acquire.

Similar to the above, mathematics as embedded *in* the culture is distinct from “mathematics *as* a culture” (Subramaniam, 2012, p. 111). For example, the researcher in this study noted that building connections between school mathematics learning and culture would involve exploration and unpacking of the implicit mathematics embedded in the cultural practices (read here: work-context related), while mathematics in school emphasizes its distinctness as a form of knowledge. Any curricular debate that recognises and addresses the value of work related knowledge for school education needs to take into account these distinctions. This study contends that an approach that seeks to connect mathematics learning in schools with out-of-school mathematics embedded in the cultural practices or work-contexts can adopt the approach of an archaeology of the concretised mathematics in the artefacts or in the practices as ready-made systems. Such an approach goes beyond the “guided re-invention” proposed in the Realistic Mathematics Education (Gravemeijer & Doorman, 1999, p. 116), which is about experiencing the development of mathematics knowledge by discovering it similar to the ways it has progressed. At present the ready-made mathematics that comes as school mathematics or the mathematics that is taught at the higher levels is just the opposite of how mathematical knowledge developed and progressed as a body of knowledge, a process which Freudenthal aptly calls as “anti didactical inversion” (Freudenthal, 2002, pp. x, 305).

The work-contexts as other everyday practices made use of the underlying concepts not as teaching or learning objects but as working tools. Use of the embedded concepts in the artefacts was the focus rather than conceptual attainment or understanding (Freudenthal, 2002). We observed from our students' interviews that those engaged in the work practices in the neighbourhood took responsibilities as workers in doing tasks that came by; some students at times were required to take decisions, or to manage the entire work affair and carry out the tasks themselves. Execution of the responsibility and task completion as doers were considered as important traits in the work contexts. However, during our classroom observation (during regular lessons) the researcher noticed that the same students played different roles, their roles were reversed – in the classroom most students functioned as passive learners in contrast to being active doers in their work-contexts. Such changes in the roles are indicative of the "transitions" (Civil and Andrade, 2002) from one context to the other. For instance, in this study, transitions occurred from everyday work-context to the classroom context and vice-versa when the teacher co-constructed the classroom norms and encouraged enabling of the "shifts". Therefore, we looked at transition in terms of negotiating the shifts and identities during the teaching intervention.

The results and the findings of this study are discussed in the following sections together with their implications for school mathematics learning. This is followed by a discussion of future directions, recommendations and the researcher's personal postscript.

8.1 The findings: Answering the research questions

This study recorded that children were often involved in complex everyday activities that can potentially be linked to school mathematics such as, counting and transacting currency notes, weighing and counting products, calculating proportions, optimisation processes, decision making, measurement skills, and so on. However, in the mathematics classrooms in both the schools, there was little effort to connect such out-of-school mathematical knowledge with school mathematics or to merge students' identities. The major findings of this study are presented in this section in the form of answers to each research question (see Chapter 3), informed by the data and the discussion presented in the earlier chapters.

8.1.1 Research Question 1 (Nature and extent of out-of-school mathematics knowledge):

The study has noted that the middle graders' out-of-school mathematical knowledge was largely gained from their exposure and/or experience in the work-contexts, and other everyday practices such as shopping and economic transactions. Often such knowledge revealed students' familiarity with the "concretised" mathematics in the tools and artefacts used in the work-contexts. Students' knowledge gained through the use of such tools and artefacts show competence in their applications, but as discussed before, the underlying mathematical constructions remained implicit and hidden. The arithmetic tasks in the inteviews showed flexible competence of the students in the contexual problem tasks, and better levels of competence in handling currency or in doing calculations when currency was given as a cue. Although students had wide and diverse experiences of measurement, their knowledge of concepts associated with measurement was fragmentary and partial.

The arithmetic tasks of the interviews revealed students' propensity towards using their own situation-specific competencies and often these out-of-school learnt strategies were amalgamated with elements of school mathematics, for example, the use of school learnt algorithms and multiplication tables. As evidenced from the students' answers, their use of multiplication tables reflected instances of convenient decomposition, for instance, in the case of solving a contextual problem with 16×7, the multiplicand was decomposed into the addends 10 and 6 and both were multiplied by 7 separately and the results obtained were added together to get the final answer. One student mentally calculated and correctly obtained the partial products – 70 and 42, and added them on the paper and obtained "742" as the answer. She realised the error and added mentally to arrive at 112. The interesting point here is the hybrid nature of students' mathematical knowledge which is a mix of school as well as out-of-school mathematical knowledge and driven by convenience. In fact, the distinction in both the forms of mathematical knowledge has persisted in the literature, but in our study, has emerged as blurred and overlapping. Thus although we applied the distinction framework (presented in Chapter 2) in the study to distinguish out-of-school mathematical knowledge from school knowledge of mathematics, the study showed the limitations of this distinction and revealed students' mathematical knowledge to be a hybrid of what they learnt in different contexts. As explained in the discussion on students' solution strategies of the arithmetic tasks (Section 4.7.5), it was noticed that commonly taught school techniques such as the rule of "multiplication by ten" or "cross-multiplication" were not made use of. While some students were able to use convenient decomposition, there were a few students like E_6 who did a cumbersome addition of ten 35s to solve 35 × 10 (see Fig. 8.1 below).

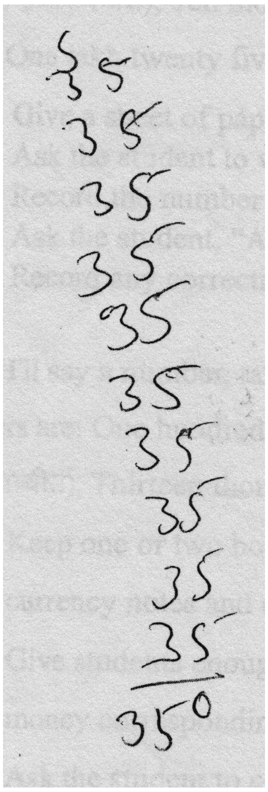


Fig. 8.1 E_6's computation of 35×10

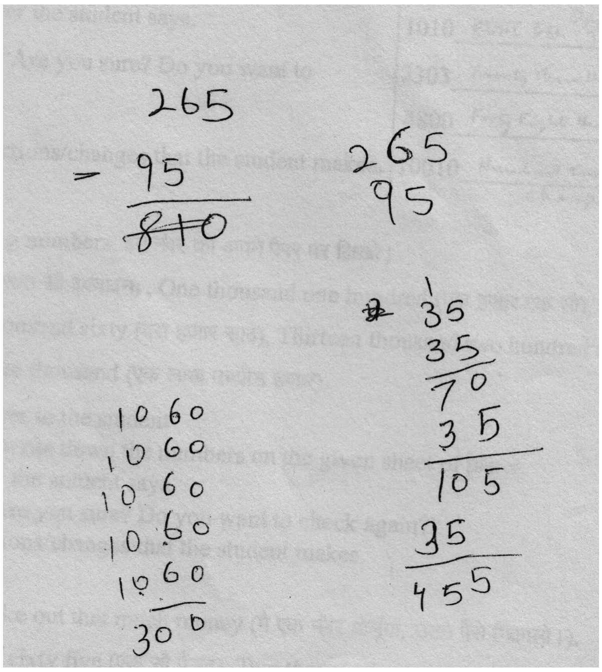


Fig. 8.2 Computation strategies used by E_8

The extent of students' mathematical knowledge varied between school learnt mathematics and techniques on the one hand, and wide ranging convenience-based strategies used in out-of-school contexts on the other hand. The students chose to use the situation specific techniques and strategies that they were competent in using, frequently moving between a variety of techniques till an acceptable form of answer or solution to the given task was obtained. For instance, the Fig. 8.2 in the previous page shows that E_8 used different operations and techniques to solve the tasks often abandoning the attempt midway after he realised the error and felt stuck. These techniques revealed that students lacked conceptual understanding of the decimal place value and made errors, but still had a wide range of different strategies at their disposal for using. Since most students were equipped with out-of-school mathematical knowledge, they could figure out if they obtained answers were absurd and changed strategies when required.

Features of the out-of-school mathematics knowledge of the sample students were similar to the main characteristics highlighted in Table 2.1 (Distinction table). Students' strategies during arithmetic task interviews reflected evidence of their metacognitive awareness of the stage of calculation while solving a maths problem. For instance, in ascertaining the number of partial products added and how many more remained to be added was visible from E_5's method of solving 16 × 7 who added 32 (= 16 × 2) three times and reflected back that six 16s had been added and one more remained. Such mid-way looking back strategies was noted in other students' maths problem solving techniques as well.

The study has shown that students' use of out-of-school mathematics knowledge came with justification for their strategies. These justifications at times were correct and sometimes erroneous, but revealed a clear grasp of the principles underlying the calculation. In contrast, in the case of using school mathematics there was a lack of such clarity. Students often struggled to figure out which operation to apply. Though they were aware of the school algorithms and how to begin, often they got stuck midway and could not proceed, and eventually fell back on the convenience strategies. Some students opted for out-of-school strategies at the outset instead of school algorithms despite being good at school mathematics (for example, E_{16}). The researcher noted that the sample students had lack of faith in school learnt algorithms since it was not clear to them why the algorithms worked or why or when they did not, unlike the out-of-school strategies for which the students had justification and a sense of "control". The researcher noticed that this was a distinction in the nature of students' knowledge of school and out-of-school mathematics.

The extent of students' out-of-school mathematical knowledge was confined to the apects within the work-contexts that the students were familiar with. While orality and mental computations were among the preferred techniques, discussion in Section 4.7.5 has shown that for many students reading and writing numbers with more than four and five digits were erroneous. Despite such constraints, as the researcher's interactions with the teachers and his own observation revealed, students were able to tackle the maths problems given in the textbooks after each lesson. Evidently, although many students had difficulty in following the decimal representations in case of bigger numbers, but while solving the textbook word problems one just needed to follow the routine procedure either shown and explained by the teacher or needed to copy from similar solved exercises given in the textbook. Understanding the numbers and working with them were different. This is an evidence of how without the conceptual understanding of the place values, students were still able to manage the mathematical tasks set in the classroom.

8.1.2 Research Question 2 (Everyday contexts and situations that provide opportunities to gain and use maths knowledge):

As described in Chapter 3, the low-income settlement – the field of study was dotted with micro enterprises either as house-hold based work practices or as workshop based enterprises. The descriptions given in Chapters 4, 5 and 6 have explicated the work practices in these enterprises and have unpacked the mathematics used both implicitly as well as explicitly in varied ways in these work-contexts. Students participating in these work practices acquire knowledge related to the practices and they also get to learn about other work-contexts in the vicinity or in which their friends are engaged. These diverse work-contexts and everyday settings create opportunities and affordances for gathering mathematical knowledge that are often different in nature as compared to school mathematics (explicated in Chapter 5).

This study reports that children living in the low-income settlement have wide access to the funds of knowledge available in the community. Access to funds of knowledge ensures knowledge about different work procedures, work requirements, raw materials, profit margins, wage calculations depending upon the kinds of work, duration of work and so on. Funds of knowledge about work-contexts are also shared between friends who often help each other in arranging jobs and in supplementing earning (viz., E_5's narrative discussed in Section 4.3). Building on one's funds of knowledge is seen as helpful in finding better job avenues in future. It was with this kind of requirement in mind that U_{21} had approached the researcher to teach him division since he needed it at work.

The researcher noted that the low-income settlement has diverse communities of work practices and consequently the funds of knowledge that children acquire is diverse. Indications of such a knowledge base emerged during the discussion with students and adults about the work-contexts and the details that they shared. As mentioned in an earlier section, these funds of knowledge are not seen as tools for teaching or for giving instructions but as aids for working. This study shows that students' access to such knowledge base creates opportunities for furthering their knowledge including mathematical knowledge. One such example is the familiarity that all study participants had about the *inch tape* or about the work-context of tailoring or about different binary fractions. Students gathered such mathematical knowledge not because they were taught in their communities but by sharing of the rich funds of knowledge available within the community.

Diverse work contexts: potentially rich resources for learning

The low-income settlement presents a diverse setting in terms of work-contexts, everyday practices and varied occupations of its residents. The characterisation of the work-contexts as presented in Chapter 5 identifies opportunities for learning as dependent on certain features of the work context such as (i) diversity in the nature of work, the types of materials handled, the relationships encountered, (ii) opportunities for optimising resources and earnings, and decision making, and (iii) awareness of the linkages in the production network and sense of ownership. These features interacted to determine the extent of the available opportunities for acquiring mathematical knowledge. The extent of the opportunities depended on how the particular task one gets to handle reflected these features. For examples, there were work-contexts where the workers had opportunities to handle diverse goods or parts thereof, both branded and locally made, complex optimatisation tasks such as quoting a price keeping profit in mind, optimal purchase and use of raw materials and, decision making while offering discounts or in deciding where to buy raw materials or whom to sell the product and at what price and maintaining accounts of all this. Diversity of work practices comes with diversity of situations, which entails diverse possibilities of mathematical application both implicitly or explicitly. Diversity also comes in terms of opportunities to handle situations of optimatisation and decision making. These opportunities emerged in those work-contexts where students were exposed to a wider linkage on the production chain and possessed a sense of ownership. Thus the nature of working conditions and environment aided or constrained mathematics learning opportunities. Diversity on the whole emerged as a potentially powerful and a rich resource of learning. Chapter 5 of the thesis has discussed this research question on the opportunities by drawing on the data mainly from the students' interviews and the researcher's observation of the community.

We do not claim that specific features of work-contexts caused our study participants to acquire better mathematical knowledge. The study was not designed to test this aspect, since it is likely that many other variables determine the acquiring of mathematical knowledge by an individual student. Moreover, it is not clear whether students acquire richer mathematical knowledge because of a richer work-context, or whether they obtain access to richer work-contexts because they are already more proficient in terms of their knowledge and understanding. What we observed was the rich detail that participants shared about their work, which included mathematical aspects implicitly or explicitly. Such details were richer when the work context had features as described above. We have tried to illustrate in specific ways how the features of the work-context created opportunities for learning mathematics. In broad terms, our observations were consistent with the claims made by other researchers such as Khan (2004), that diversity in work-context is associated with better mathematical knowledge. More importantly, we have attempted to show how out-of-school mathematical knowledge has connections to school learning even though it may be implicit, partial and fragmented.

It must be noted that many work-contexts that the researcher became familiar with were compartmentalised and distributed among different groups of workers and called for only routine and repetitive kinds of skills. It was possibly the compartmentalised and simplified nature of the tasks that attracted large cohorts of people and also allowed children to participate in the work. Such distributed nature of the work made it possible for workers to be substituted easily, because of the availability of cheap labour. These work features and working conditions ensured that unfair deals and unequal distribution were accepted and perhaps went unchallenged, as indicated in some of the students' interviews (viz., U_{13}, E_{16}, E_{15}, U_{2}). Thus, as it emerged from the interviews, it was a challenge for some of our participants (such as U_{21}), to find work-contexts that allowed for meaningful and useful learning, as opposed to work contexts, which called largely for unskilled labour (discussed in Chapter 4).

8.1.3 Research Question 3 (Overlap and differences between out-of-school and school maths knowledge):

In concurrence with the recent studies done in the areas of out-of-school or everyday mathematics, our study indicates the overlapping nature of students' school and out-of-school math knowledge. The forms of mathematical knowledge were not distinct and students drew from both. Our data from students' interviews on arithmetic tasks support this claim (discussed in Chapter 4). This is despite the prevalent classroom culture and the beliefs that many children hold, which tend to reinforce the separation of the two forms of knowledge.

The researcher noted during the interviews that students displayed reasoning ability while explaining their problem solving strategies that were typically different from those taught at the school. Barring a few, most students were able to justify the procedure that they used and how they arrived at the answer. This was unlike the case of the school learnt strategies which remain unclear and students do not understand why they work. One such instance was the *burfi* problem [20 *burfis* cost Rs 42, then how much will 25 *burfis* cost?] (Section 4.7.5). This problem-task presented an occasion where some students used the "halving" strategy to arrive at the solution, rounded off the answer and presented a practical solution (Rs 53 instead of Rs 52.50). As described before, some students tried to employ the unitary method at the outset but upon getting stuck or when the calculations became complex, turned to the alternative convenient method. This task showed that students were able to switch between their out-of-school and school mathematical knowledge which supports our contention about the hybridised form of mathematical knowledge drawing elements from both the domains. It was interesting to see that some students brought in the reality perspective from their everyday experience of rounding off that is a common practice in economic transaction and trade these days and that customers are charged Rs 53 and not Rs 52.50. It appeared that students like E_5, and E_{16} and a few others had the flexibility to use either of school maths knowledge or out-of-school techniques, though clearly the preference was for the out-of-school techniques. The important point here is the advantage that such students had in terms of the problem solving strategies since they had multiple techniques at their disposal which came from their experience of the diverse everyday contexts.

Overlap and differences

We have indicated above the overlapping nature of school and out-of-school mathematical knowledge. However, both the domains of knowledge employ repetitive processes that are skill driven. While school maths learning aims to cultivate skills that are generalisable, skill development in out-of-school mathematics is specific in nature and linked with the task at hand. Therefore, as the literature suggests (for intance, Nunes, Carraher and Schliemann, 1993), competencies developed in the out-of-school contexts are often situation-specific and hence not generalisable.

Students began the solution procedures by using either of school learnt techniques or out-of-school strategies. There was however, preference among many students in the sample to use their own methods that we have referred to as out-of-school mathematical knowledge, over school mathematics. Approximation and estimation were valued more than accuracy. Approximation and estimation were used in judging the validity of the answers or in ascertaining the possible range of the answers (for example, U_{21}'s solution of 981 ÷ 9 discussed in section 4.7.5 or the solution discussed in Fig. 8.2 above). This approach was different from school mathematics whose hallmark is accuracy and not estimation or approximation. Students could gauge the approximate value that a computation must lead to and any major deviation was indicative of a flaw. This was in contrast to Carraher *et* *al*.'s finding (1985) that school students often fail to realise that the obtained results were absurd and untenable, which suggested a sharper separation of out-of-school and school mathematical knowledge.

Another major difference between these two forms of mathematical knowledge lies in the task procedures used in different work-contexts, such as the notion of construction. School mathematics as pointed out before, offers "ready made" mathematics and learning of its application is stressed upon, while in the out-of-school work-contexts, "construction" comes as an essential part of the work process. As discussed in Chapter 6, construction is also an important aspect of out-of-school measurement learning, which we discuss below. We revisit some features of the distinction between school and out-of-school mathematical knowledge in the context of measurement learning when we discuss Research Question 4.

8.1.4 Research Question 4 (Out-of-school measurement knowledge and implications for school maths curriculum):

One of the findings of the study was the richness and diversity of measurement related experiences that our study participants had. We have not come across any account of such diverse exposure and knowledge possessed by students in a single classroom. We analysed the measurement experience and knowledge gained from out-of-school contexts under the rubrics of (i) comparison, estimation, quantification and construction and (ii) diversity of objects, measurement instruments and units. This indicated that students have exposure, although in implicit ways, to many of the conceptual constructions underlying measurement. They also are aware of the diverse ways in which measurement occurs in the real world, using a variety of tools for a range of purposes.

Prior research on the learning of measurement in school include Post-Piagetian studies that have highlighted the importance of concepts such as conservation, transitivity, equi-partitioning, displacement, iterative covering, accumulation of distance and additivity and the role of the origin on scales. From the point of view of the diversity of out-of-school experience, we need to go beyond these to include critical concepts such as construction of units and templates, equi-partitioning and chunking of measures and unit, construction of measuring scales, design of convenient measuring instruments and units. Further aspects critical to the understanding of measurement that have not been adequately addressed in the curriculum include the extensive use of comparison and estimation in real life contexts, the use of the body as a measuring instrument, the trade offs between convenience and accuracy, the variety of purposes of measurement, the variety of modes of quantification and the limits of informal quantification, and the cultural-historical origins of units and systems of units. These aspects, with the exception of estimation, have not received the attention of mathematics education researchers. The diversity of measurement experiences in out-of-school work contexts can be drawn upon to illustrate each of these concepts and ideas, and also for understanding the difference between comparison, estimation and measurement and their purposes.

We elaborate on the implications of our analysis with a specific example. In the school context, length measurement is taught using an inexpensive plastic scale that has both inches and centimeters marked on either side of it. The researcher's classroom observation revealed that students often confuse between these units from two different systems and they are also not clear about the distinction. Interestingly, in the work-contexts, particularly in the context of tailoring, plastic tapes (often referred to as *inchi tape*) are used. Students learn to use inch and foot in their everyday contexts and learn to carry out measurement of acceptable accuracy by reading off the lengths from the tape. As we have argued in the Section 6.2.1, although such measurements are fully quantified, the quantification remains unclear to the students. However, it was also noted that some students' familiarity with a particular unit (for e.g., *inch*) resulted in an embodied skill to estimate lengths without the aid of any measuring instruments. The estimations were mostly accurate. Similarly, in the case of weight measurement, estimation was in the range of a few kilograms or fractions of a kilogram. This is an example of how an artefact in the form of a measuring tool is integrated with bodily proprioception to create a form of embodied knowledge or skill. However, this does not necessarily imply that children are aware of how weight or length is quantified.

The instances of measurement knowledge that we found in our participants indicated, on the one hand familiarity with many aspects that are relevant to the conceptual understanding of measurement. However, in work-contexts, participants merely focus on using measurement tools or practices, and not on conceptual understanding. We note that measurement instruments and practices embody conceptual mathematical knowledge in relation to measurement and thus the contexts that came up in this study in which measurement knowledge is applied are instances where archaeological explorations are possible. The teaching intervention in the vacation course indicated that the familiar measuring instrument like an *inch tape* as an artefact has many possibilities structured around it that can be explored. These are the concepts of a unit length, requirement of partitioning a unit, construction of newer units, sub-units, notion of iterative covering, use of fractional and decimal notations – all in the activity of measurement that remains implicit and students' knowledge about these embedded mathematics remains unexplored and disconnected with school mathematics. At the end of the vacation course, as revealed from the students' discussions, students reflected on and appreciated their deeper understanding of the artefact *inch tape* that was otherwise so familiar to them. Such archaeological explorations had the potential of encouraging the students to also discover more such instances where notions of mathematics are being implicitly used. Such explorations also create opportunities for the students to connect and build on the fragmented and partially obscure form of mathematical knowledge that they gain from the work-contexts (Subramaniam, 2012).

The experiences of measurement in out-of-school contexts are characterised by diversity as well as structural differences from the school mathematical treatment of measurement. The outcome that school education aims for is distinct from the knowledge that is acquired in out-of-school settings. A central aspect of such knowledge is its generality, of its not being tightly bound to particular contexts. Specialised knowledge is context-bound, well-practiced and embodied in individuals, and leads to expertise and efficient action in limited domains and situations. Generalised forms of knowledge may not lead to efficiency and expertise at particular tasks, but have generality and wide applicability (Sfard & Cole, 2003). From the standpoint of valuing such generality as an aspect of school learning, it is the diversity of out-of-school experiences that creates the context for school learning. Thus, from our perspective, it is incorrect to claim that work practice already reflects mathematical thinking or understanding. Mathematical aspects are only present in hybridised and opaque embeddings. It is also incorrect to expect school learning to illuminate or strengthen a single kind of practice in a particular work context. It is the diversity of practices taken together that formal mathematical learning can illuminate. It strengthens understanding, not practice.

8.1.5 Research Question 5 (Integration of everyday and work-context maths knowledge with school learning):

One of the challenges before the teacher or the instructional designer is to imagine connections between school and out-of-school knowledge that can produce powerful learning. What should be the goals of a pedagogical intervention that aims at building connections between out-of-school knowledge and school learning? What forms of participation could one expect to see in a classroom implementing these goals? These questions were addressed in the pedagogical intervention in the form of a teaching design experiment discussed in Chapter 7. The goals also included conceptual aspects as well as the setting up of a classroom culture that valued making such connections.

Pedagogical Implications

This study has explored how exposure to out-of-school and work-contexts influence possibilities of mathematics learning at different sites – out-of-school settings, work practices and school learning. The insights gained from this study about the middle-graders' context-bound mathematical competencies and techniques provide a perspective that challenges the common interpretation of these students as unfit and underprepared for succeeding in school learning due to the perceivably high demands that schools are seen to be putting forth and the middle-graders' low performance or ability to excel during formal mathematics learning. The study noted that though there were multiple factors that significantly constrained the possibilities and opportunities of further learning for children from low-income families, the impact of the multiple factors also have influence on possibilities and affordances for learning in varied ways.

This research study proposes that learning of mathematics is aimed at acquiring conceptual understanding and insight and not at practical training. This study proposes a pedagogic approach that connects out-of-school and school mathematics learning by enabling a series of shifts. We contend that these shifts in the classroom norms were essential for building connections between between students' out-of-school knowledge resources and mathematics learning within their classroom setting. These shifts encouraged merging of students' identities as knowers, doers and learners which facilitated drawing on what they knew from their exposure and experience in the work practices and used such knowledge during school mathematics learning.

Enabling of shifts

The enacted goals of the teaching design experiment as part of the vacation course sought to negotiate classroom norms by enabling a series of shifts in the norms. The teaching design explicitly attended to the shifts as a way of bringing together out-of-school knowledge and school learning. These shifts were from *oral* to *written* mathematics, from *knowledge about use* of tools and artefacts to *understanding,* towards *building the identities* of participants as a *mathematically focused discursive community,* shift towards *shared, public expressions,* and shift to *identities that are connected*. These five shifts in the classroom norms broadly reflected two kinds of movements – connecting out-of-school practice with classroom practice (embedded in the first three shifts) and movement within the classroom practices (embedded in the rest two shifts). One of the features of the shifts worth highlighting was to support moving from "helping interactions" towards a more discursive culture in which reasoning is central, where statements are listened to with attention, are challenged, elaborated and justified. Building of such mathematically discursive practices is one of the objectives for enabling the shifts in the classroom norms. As mentioned earlier, the objective of the study was to look for ways that facilitated conceptual understanding rather than helping or promoting practice.

8.1.6 The role of identity in integrating out-of-school and school learning

The ethnographic and case studies phases of the study have shown through interactions with students and their accounts of the work-contexts, that out-of-school contexts contain opportunities that afford students to co-construct their identifies. Exposure and experience in the work-context often shape students' identities as doers – that they can handle and accomplish tasks assigned to them, carry out responsibilities and view themselves as agents capable of doing a work. In addition, we also noticed that out-of-school experience also helped students to construct their identities as knowers as someone who is knowledgable, possessing knowledge about resources, tools and instruments, work processes, and as someone who can share knowledge with others. In the out-of-school work practices, the study noted that students often shared adult space during the work-contexts and at times handled complex and difficult situations that called for taking decisions in the absence of adults or seniors who can guide. One needs to keep in mind here that in work-contexts, a flaw in calculation or in a strategy results in loss and therefore errors are consciously avoided unlike school situation where errors and flaws are not welcome, but they do not lead to monetary loss.

The study further noted that students readily shared what they knew and made efforts to add to their knowledge corpus that would be helpful to them for future work prospects. We have discussed instances in Chapters 4 and 5 of how some students approached the researcher to fix certain mathematical difficulties since they required such knowledge in the work-context. Most students knew what they knew and what they lacked. It was their identities as knowers that encouraged them to find and fix what they lacked. The work-contexts also create opportunities for doers to learn about tasks and about other work requirements. Such instances at work-contexts and role of students in the classrooms co-construct their identifies as learners. However, as has been claimed before, in the formal, regular classroom settings, students' identities remain disconnected and even un-acknowledged. The present study has shown through the teaching design experiment that merging of students' identities allowed for transfer of knowledge from one domain to the other while engaging in mathematical activity and thereby supported the integration of out-of-school knowledge with school learning. Merger of students' identities is a pedagogic approach that acknowledges and values students' prior knowledge, their knowledge resource, and encourages co-construction of their identities as active learners. The series of shifts in the classroom norms described in Chapter 5 facilitates merger of identities and therefore makes possible the integration of students' out-of-school mathematical knowledge with school maths learning.

8.2 Curricular implications

The study focused specifically on the topic of measurement, with the goal of articulating the curricular implications of out-of-school mathematical knowledge. As mentioned above, the curriculum needs to go beyond the aspects emphasised so far in the research on the learning of measurement or the aspects that have been included in school textbooks, viz., by including concepts that connect with and illuminate the diversity of measurement related practice encountered in work and everyday contexts. It needs to focus on the idea that quantification is achieved in different ways for different attributes and for different purposes. The curriculum needs to develop an appreciation of the difference between scientific measurement and measurement in the everyday world. With this study, we therefore argue for the inclusion of conceptual aspects that have so far not been included either in the curriculum or in the research on measurement learning. The diversity of measurement experiences in work contexts and everyday settings justify inclusion of these aspects in the curriculum and the knowledge that children bring into the classroom from out-of-school contexts supports learning of these ideas.

Existing curricula and teaching practices, in contrast to policy documents, serve to reinforce the separation of the everyday from formal school learning. Underpinning this may be an implicit awareness of the structural differences between these forms of knowledge and learning. This may combine with an anxiety about the potential distractions caused by the contextual details of the everyday. Thus teaching practice typically keeps the everyday out of the classroom and creates school mathematics as a culture and practice that is not only distinct, but also disconnected. The topic of measurement reflects this separation even more than arithmetic, which has some overlaps and connections with everyday calculation. The textbook treatment of measurement makes no references to the everyday world, and everyday experiences of measurement do not enter classroom discussion. A reflection of this is the fact that the textbooks faithfully implement the government directive of including only metric units and banning inches from textbooks, while out-of-school contexts happily ignore such directives. A consequence is that the school mathematics topic of measurement produces disconnects between itself and measurement in the real world.

The separation of school and everyday learning is also internalised by students, who do not expect that what they learn in school will be related to the knowledge that they acquire from out-of-school contexts. Learning skills and acquiring knowledge through participation in work is valorised in the community that we studied, although a few families discourage their children from participating in work because they think it would affect their studies. School learning too is valued, although for different reasons and as a different kind of learning. It has aspirational value, and the community believes that education is the route to social and economic mobility. However, it is self-defeating for an education system to merely aim to produce the trappings of social class, while depriving learners of knowledge that has power because it illuminates aspects of life. Students from deprived backgrounds enter the classroom with their own rich complement of experiences. In the case of measurement, we see that such experience is diverse and incorporates familiarity and intimate knowledge of measures, of measuring instruments and embodied skills. Our perspective is that education that shuts this rich resource out of the classroom is a recipe for failure.

The mathematics textbooks around a century ago contained a different arithmetical treatment compared to the present textbooks. These old textbooks aimed to develop extensive arithmetical skills that were required and seen as essential in the everyday world of finance and commerce around that time (Subramaniam & Bose, 2012). These textbooks contained detailed exercises involving conversion of units that were mostly non-decimal, for instance, conversion into currency denominations that were in practice in those days followed base-4 and base-12 conversion that made the arithmetical tasks cumbersome and complex. Around eight decades ago, the "new arithmetic" introduced in the 1930's completely did away with the base-4 fractions and related compound operations (Subramaniam & Bose, 2012). Even then, the complex currency conversion exercise existed in the arithmetic textbooks and so were the exercises related to the inter-conversion between old British units that were still in practice in the Indian economy. Conversion tables of complex and extensive system of different measures were an important part of the arithmetic textbooks at the elementary levels of schooling. Similarly, in older arithmetic curricula in the country, extensive treatment was given to the binary fractions and fractional tables. The use of mnemonic tables and fractional tables of *quarter*, *half*, *three-quarter*, *five-quarters*, *one-and-a-half*, and *two-and-a-half* were in practice in some old cultures till recently and are gradually fading away from the cultural practices of those communities (Bose & Kantha, 2014). Although, the specific topics present in older curricula or textbooks may no longer be relevant, the contrast in approach is remarkable. While older textbooks strongly focus on connections with the world of work, the contemporary textbooks do not reflect similar connections with the prevalent out-of-school contexts even though the major curricular document for school education, The NCF-2005, explicitly recommends building connections with the outside world.

8.2.1 Archaeology of artefacts

Another aspect of out-of-school knowledge that makes for potentially powerful connections with school learning is the fact that artefacts and practices from everyday settings represent a crystallised and embodied form of mathematics (Chevellard, 2007). The measuring tape embodies the processes of unit construction, unit iteration and counting and partitioning of units into sub-units. These processes are however hidden from view and are opaque. The redundant inclusion of a second system of units in the form of inches and feet on the measuring tape crystallises a part of historical reality, and highlights the arbitrariness of the choice of the basic unit of length. The purpose of such embodiment is precisely to make the mathematical thought and processes behind the construction of the measuring scale unnecessary, and to reduce the practice of measurement to the simple act of reading off the scale. This is the general phenomenon of demathematisation described by Chevellard (2007) and Jablonka and Gellert (2007) where material artefacts and embed increasingly sophisticated mathematical ideas, while rendering the user's knowledge of such mathematics unnecessary. As long as we treat the learning of measurement as merely the learning of a skill, unpacking the mathematical ideas that are embodied in artefacts will remain unnecessary. However, if we view the learning of measurement as conceptual understanding, then such material artefacts present an opportunity for archaeological investigation. Such "archaeology" aims to uncover the generally hidden and "black boxed" aspects of mathematics crystallised in artefacts and practices (Subramaniam, 2012). Archaeology as a pedagogical mode may have an important place in providing opportunities to learn powerful mathematics that illuminates the diverse aspects of everyday experience.

8.2.2 Pedagogic role of work in education

The population of the economically marginalised children in India is around half of the entire child population of the entire country (age group of 5-14 years) (Census Report, 2011). Often economically marginalised students are seen to be associated with work practices of various forms and in different capacities. In such situation where one's subsistence is at stake, relevance and meaning of school education emerges even though it carries a value in the community and in the society. The disconnected school curriculum with the outside real world creates a gap between these two domains. The prevalent gap reflected in the school curriculum between school and out-of-school contexts is, according to the *Position Paper of the National Focus Group on Work and Education* (NCERT, 2007), an "artificially instituted dichotomy" between work and knowledge (p. iii). There is a need to bridge the gap and the make the school curriculum a springboard for promotion of conceptual understanding and clarity not just securing a body of knowledge. The study has underlined the pedagogic role of work that can help integrate school learning with out-of-school knowledge contexts.

8.3 Moving ahead

Mathematics education is a relatively less explored area of research in India and there is a need of examining systematically the effects of larger context and socio-economic background of learners in Indian conditions which are in many ways distinctive and different from developed countries and most of the other developing nations. One may hypothesise that more than individual aspirations or even learning abilities or potential, contextual factors like social environment, awareness and engagement in diverse work-contexts, neighbourhood with its socio-cultural features, funds of knowledge and students' identities, and school education are likely to determine the trajectory of learning in general and mathematics learning in particular. These circumstances are potential avenues for educational researchers to offset the educational disadvantage that stems from low socio-economic conditions, build connections with students' knowledge resource and also to counter the prevalent culture of rote learning in school education (Subramaniam, 2012). Rigorous studies and more of them can uncover and show linkages between development of mathematical cognition and conceptual understanding.

It requires sensitivity to follow how children accomplished the tasks and arrived at the results by means that could be often seen as anything but mathematics. Student's problem-solving procedures often reflected use of the underlying knowledge of mathematics that Freudenthal would term as “mathematics as an activity”. Detailed study focused on the notion of “archaeology” can uncover more instances where mathematics remain hidden and implicit which can illuminate better mathematical understanding.

There is an emergent possibility arising from the present study to inquire into the connection between language, culture and mathematical cognition. Below is a list of possible areas in language and communication that can be explored in similar studies which could not be pursued in the present study:

- How are communicative activity and mathematical thinking linked and in what ways does language negotiation support (or not) such activities in work-contexts and in mathematics classrooms?

- Different representations of mathematical concepts in different languages and their connection in building mathematical understanding drawing from out-of-school mathematics learning in multicultural and multilingual settings.

- Language negotiation at the interface of knowledge drawn from cultural embeddings and formal, academic knowledge – mutual impact on different sites of mathematical learning.

- A look at the curriculum and policy planning taking on board (or not) the connection between out-of-school mathematics learning and language diversity and ways in which such integration can be achieved.

8.4 Personal postscript

The study entailed handling of sensitive issues of social and ethical nature, for example, the issue of child labour, difficult and oppressive work conditions, prolonged working hours, unfair wage pattern, social stereotypes (gender, caste and others), and so on. As a researcher, it was challenging to tackle and address such issues during interviews or during social interactions. The dissertation journey has brought to me a platter of learning and training to prepare myself to carry forward similar research and also to embark upon new research on other social issues. As a researcher, I feel better able now. The social relationship with the community developed during the prolonged engagement with the field will remain as an asset for me.

Reflecting over the data and over myself since I undertook this research study, and how the study has been effective in affecting my professional practice, I realise that the study has given me tools to see things which I was unaware of or at the most vaguely aware of. The research study has benefited hugely from the participation and support of students and teachers, but I am sceptical whether this study gave them back some tools to judge and tackle the equity, fairness and other social issues that they face through learning of mathematics. As most studies on social issues, the respondents and participants (especially the sample students) did not directly get the benefit of the study outcomes. One can only hope that, as an after-effect of the study, future batches of students will get some benefit.

Revisiting the study in its entirety indicates to me that as a researcher, I felt perhaps more sensitivity is required towards handling social issues arising out of low SES, work requirement, aspirations and child labour than what I had. Researchers embarking upon similar studies need to be more cautious with such issues.

The low-income settlement where the study was conducted was truly a site with high economic output but the residents (workers) were economically poor. There was no dearth of jobs yet availability of cheap human labour in abundance did not let the wage to go up. Ironically, interactions with a few adults revealed that they understand that the wage paid to them for their work was less, but it also ensured a continuity, that they at least had something to do. Back home in their native places, there were hardly any work opportunities for many of the residents. It is a negotiation between work and wage and a balance between them that one needs to look at. The working conditions as was revealed during the study was found to be taxing for the students, a few of whom felt tired to focus duing the classroom lessons. Though I envision education drawing from knowledge centred around work-contexts, in no way can such practices of over-work or over-engagement in work practices be acceptable that has cascading ill effect on students' health and studies.

Bibliography

ASER (2012). *Annual status of education report rural 2012*. New Delhi: ASER Centre. http://img.asercentre.org/docs/Publications/ ASER %20Reports/ASER_2012/fullaser2012report.pdf. Accessed 10 June 2014.

ASER (2013). *Annual status of education report rural 2013*. New Delhi: ASER Centre. http://img.asercentre.org/docs/Publications/ASER %20Reports/ASER_2013/ASER2013_report%20sections/aser2013fullreportenglish.pdf Accessed 10 June 2014.

Anderson, J. R., Reder, L. M. & Simon, H. A. (1996). Situated learning and education. *Educational Researcher, 25*(4), 5-11.

Anderson, J. R., Reder, L. M. & Simon, H. A. (1997). Situative versus cognitive perspectives: From versus substance. Rejoinder. *Educational Researcher, 26*(1), 18-21.

Andrews, J., Yee, W. C., Greenhough, P., Hughes, M., & Winter, J. (2005). Teachers' funds of knowledge and the teaching and learning of mathematics in multi-ethnic primary schools: Two teachers' views of linking home and school. *ZDM – The International Journal on Mathematics Education, 37*(2), 72-80.

Arcavi, A. (2002). The Everyday and the Academic in Mathematics. *Journal for Research in Mathematics Education. Monograph, 11 (Everyday and Academic Mathematics in the Classroom),* 12-29.

Atweh, B., Forgasz, H. & Nebres, B. (Eds.). (2001). *Sociocultural Research on Mathematics Education: An International Perspective*. Mahwah, NJ: Lawrence Erlbaum Associates Publishers.

Atweh, B., Graven, M., Secada, W., & Valero, P. (Eds.). (2011). *Mapping Equity and Quality in Mathematics Education*. Dordrecht: Springer.

Banerjee, R. (2008). *Developing a learning sequence for transiting from arithmetic to elementary algebra.* Unpublished PhD Dissertation Thesis. Mumbai: Homi Bhabha Centre for Science Education, TIFR.

Banerjee, R. (2012). Mathematics education research in India: issues and challenges. In R. Ramanujam & K. Subramaniam (Eds.), *Mathematics education in India: Status and outlook*. Mumbai: HBCSE.

Banerji, R. (1997a). Why don't children complete primary school? A case study of a low-income neighbourhood in Delhi. *Economic & Political Weekly*, 32(32), 2053-2063.

Banerji, R. (1997b). Child labour and Children's Schooling. *Economic & Political Weekly*, 32(39), 2458-2459.

Barton, A. C. & Tan, E. (2009). Funds of knowledge and discourses and hybrid spaces. *Journal of Research in Science Teaching, 46*(1), 50-73.

Barton, B. (2007). *The language of mathematics: Telling mathematical tales*. New York: Springer Science + Business Media, LLC.

Barwell, R. (2005). Integrating Language and Content: Issues from the Mathematics Classrooms. *Linguistics and education 16*, 205-218.

Basu, D. D. (2008). *Introduction to the Constitution of India.* (1st Edition: 1960). Nagpur, India: LexisNexis Butterworths Wadhwa.

Basu, K. (1999). Child labour: Cause, consequences, and cure, with remarks on International Labor Standards. *Journal of Economic Literature, 37*(3), 1083-1119.

Basu, K. (2003). International labour standards and child labour. In N. Kabeer, G. B. Nambissan and R. Subrahmanian (Eds.). *Child labour and the right to education in South Asia,* (pp. 95-106). New Delhi: Sage Publications.

Basu, K. (2006). Gender and Say: a Model of Household Behaviour with Endogenously Determined Balance of Power*. *The Economic Journal, 116*(511), 558-580.

Basu, K. & Van, P. H. (1998). The economics of child labour. *The American Economic Review, 88*(3), 412-427.

Bhadke, S. (2011). Mathematical ideas of illiterate vegetables vendors, Hoshangabad, Madhya Pradesh, India. Paper presented at the *National Initiative on Mathematics Education Eastern Regional Conference.* Patna: NIME.

Bhowmik, S. (Ed.). (2010). *Street vendors in the global urban economy.* New Delhi: Routledge.

Bishop, A. J. (1988a). *Mathematical Enculturation: A Cultural Perspective on Mathematics Education*. Dordrecht: Kluwer Academic Publishers.

Bishop, A. J. (1988b). Mathematics Education in its Cultural Context. *Educational Studies in Mathematics, 19*, 179-191.

Bissell, S. (2003). The social construction of childhood: a perspective from Bangladesh. In N. Kabeer, G. B. Nambissan and R. Subrahmanian (Eds.). *Child labour and the right to education in South Asia*, (pp. 47-72). New Delhi: Sage Publications.

Boaler, J. (1993). The Role of Contexts in the Mathematics Classroom: Do They Make Mathematics More "Real"? *For the Learning of Mathematics, 13*(2), pp. 12-17.

Boaler, J. (1999). Participation, knowledge and beliefs: A community perspective on mathematics learning. *Educational Studies in Mathematics*, *40*(3), 259-281.

Boaler, J. (2002). Exploring the nature of mathematical activity: Using theory, research andworking hypotheses' to broaden conceptions of mathematics knowing. *Educational Studies in Mathematics*, *51*(1-2), 3-21.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning*, (pp. 171-200). Westport, CT: Greenwood Publishing Group, Inc.

Bose, A. (2009). Mathematical Riddles among the Mushars: Linked to a Historical Tradition? In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.). *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education, Vol. 5*, p. 439. Thessaloniki, Greece: PME.

Bose, A. (2011). Knowing the World Better through Mathematics: Bringing together 'Critical Mathematics Education' and 'Everyday Mathematics'. *The Mathematics Teacher*, *47*(3 & 4), 169-186.

Bose, A. (2014). Revisiting transfer of learning in mathematics: Insights from an urban low-income settlement. In P. Liljedahl, C. Nicol, S. Oesterle & D. Allan (Eds.). *Proceedings of the Joint Meeting of PME 38 and PME-NA 36*, Vol. 2, pp. 177-184. Vancouver, Canada: PME.

Bose, A. & Choudhury, M. (2010). Language Negotiation in a Multilingual Mathematics Classroom: An Analysis. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd Conference of the Mathematics Education Research Group of Australasia, Inc.*, pp. 93-100. Fremantle, Australia: MERGA.

Bose, A. & Clarkson, P. (*forthcoming*). Relating multilingual registers: An example of the socio-cultural role of language in multilingual classrooms. In A. Halai & P. Clarkson (Eds.). *Teaching and learning mathematics in multilingual classrooms: Issues for policy and practice*. Rotterdam, The Netherlands: Sense Publishers.

Bose, A. & Kantha, V. K. (2014). Influence of socio-economic background and cultural practices on mathematics education in India: A contemporary overview in historical perspective. *ZDM – The International Journal on Mathematics Education*, *46*(7).

Bose, A. & Subramaniam, K. (2011a). Exploring school children's out of school mathematics. In Ubuz, B. (Ed.). *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, pp. 177-184, Ankara, Turkey: PME.

Bose, A. & Subramaniam, K. (2011b). Exploring school children's 'everyday' mathematical knowledge. Poster Presentation. In Ubuz, B. (Ed.). *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education, Vol. 1*, p. 477, Ankara, Turkey: PME.

Bose, A. & Subramaniam, K. (2012). Profile of students' arithmetical knowledge acquired in and outside school. *Proceedings of the 12th International Congress on Mathematical Education (ICME-12),* p. 7386, Seoul: Korea.

Bose, A. & Subramaniam, K. (2013). Characterising work-contexts from a mathematics learning perspective. In Nagarjuna, G., A. Jamakhandi & E. M. Sam (Eds.). *Proceedings of epiSTEME-5: Third International Conference to Review Research on Science, Technology and Mathematics Education.* pp. 173-179, Margao: CinnamonTeal.

Bransford, J. D. & Schwartz, D. L. (2001). Rethinking transfer: A simple proposal with multiple implications. *Review of Research in Education, 24*, 61-100.

Brenner, M. E. (1985). The practice of Arithmetics in Liberian schools. *Anthropology & Educational Quarterly*, *16*(3), 177-186.

Brenner, M. E. (1998). Meaning and Money. *Educational Studies in Mathematics, 36*(2), 123-155.

Brenner, M. E. (2002). Chapter 5: Everyday Problem Solving and Curriculum Implementation: An Invitation to Try Pizza. *Journal for research in mathematics education. Monograph, 11 (Everyday and Academic Mathematics in the Classroom)*, 63-92.

Brenner, M. E. & Moschkovich, J. N. (Eds.). (2002). Everyday and academic mathematics in the classroom. *Journal for Research in Mathematics Education, Monograph No.* 11. Reston: National Council of Teachers of Mathematics.

Brown, J. S., & Burton, R. R. (1978). Diagnostic Models for Procedural Bugs in Basic Mathematical Skills*. *Cognitive science*, *2*(2), 155-192.

Burra, N. (1995). *Born to work: Child labour in India*. Delhi, India: Oxford University Press.

Burra, N. (2003). Rights versus needs: is it in the ‘best interest’ of the child? In N. Kabeer, G. B. Nambissan and R. Subrahmanian (Eds.). *Child labour and the right to education in South Asia,* (pp. 73-94). New Delhi: Sage Publications.

Burra, N. (2005). Crusading for children in India's informal economy. *Economic and Political Weekly, 40*(49), 5199-5208.

Campana, J. (Ed.). (2013). *Dharavi: the city within*. Noida: HarperCollins Publishers India.

Carraher, T. N. (1986). From drawings to buildings; working with mathematical scales. *International Journal of Behavioral Development, 9*, 527-544.

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the streets and in schools. *British journal of developmental psychology*, *3*(1), 21-29.

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1987). Written and Oral Mathematics. Journal for Research in Mathematics Education, 18 (2), 83-97.

Carraher, D. W., Nemirovsky, R., & Schliemann, A. D. (1995). Situated generalization. *Proceedings of the 19th annual meeting of the International Group for the Psychology of Mathematics Education,* Vol. 1, p. 234. Universidade Federal de Pernambus, Recife, Brazil: PME.

Carraher, T. N., & Schliemann, A. D. (1985). Computation routines prescribed by schools: Help or hindrance?. *Journal for Research in Mathematics Education*, 37-44.

Carraher, D. W. & Schliemann, A. D. (2002a). Is everyday mathematics truly relevant to mathematics education? *Journal for Research in Mathematics Education. Monograph, 11 (Everyday and Academic Mathematics in the Classroom),* 131-153.

Carraher, D. W. & Schliemann, A. D. (2002b). The transfer dilemma. *The Journal of the learning sciences*, *11*(1), 1-24.

Chevellard, Y. (2007). Implicit mathematics. In U. Gellert & E. Jablonka (Eds.). *Mathematisation and demathematisation: Social, philosophical and educational ramifications.*Rotterdam: Sense Publishers.

Choudhury, M. & Bose, A. (2011). An Investigation of the Role of Language-negotiations in a Multilingual Mathematics Classroom. In M. Setati, T. Nkambule, & L. Goosen (Eds.). *Mathematics and language diversity*: Proceedings of the ICMI Study 21 Conference, pp. 28-37, Sào Paulo, Brazil.

Civil, M. (1995). Everyday Mathematics," Mathematicians' Mathematics," and School Mathematics: Can We (Should We) Bring These Three Cultures Together? In the proceedings of the symposium of the Annual Meeting of the American Educational Research Association, "Communities of Practice in MathematicsClassrooms: Reconciling Everyday and Mathematicians'Mathematics?". San Francisco: AERA.

Civil, M. (2002). Culture and mathematics: a community approach. *Journal of Intercultural Studies, 23*(2), 133-148.

Civil, M. (2014). STEM learning research through a funds of knowledge lens. *Cultural Studies of Science Education*, 1-19.

Civil, M. & Andrade, R. (2002). Transitions between home and school mathematics: rays of hope amidst the passing clouds. In G. de Abreu, A. J. Bishop & N. Presmeg (Eds.). *Transitions between contexts of mathematical practices* (pp. 149-169). Dordrecht: Springer Science + Business Media B. V.

Cobb, P. & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher, 28*(2), 4-15.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(9), 9-13.

Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education*, *40*(1), 40-68.

Cohen, L., Manion, L., & Morrison, K. (2013). *Research methods in education*. London: Routledge.

Cole, M. & Scribner, S. (1974). *Culture and thought: A psychological introduction.* New York: John Wiley & Sons.

Collins, A., Joseph, D., & Bielaczyc, K. (2004). Design research: Theoretical and methodological issues. *The Journal of the learning sciences*, *13*(1), 15-42.

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches*. Los Angeles: Sage.

D'Ambrosio, U. (2010). Mathematics Education and Survival with Dignity. In H. Alro, O. Ravn, & P. Valero, (Eds.). *Critical Mathematics Education: Past, Present and Future: Festschrift for Ole Skovsmose*. Rotterdam: Sense Publishers.

Dasgupta, U. (2008). Tagore’s ideas of social action and the Sriniketan experiment of rural reconstruction, 1922–41. *University of Toronto Quarterly, 77*(4), 992–1004.

de Abreu, G. (2008). From mathematics learning out-of-school to multicultural classrooms: A cultural psychology perspective. In L. D. English (Ed.). *Handbook of International Research in Mathematics Education*. 2nd ed., 352-384. NY: Routledge.

de Abreu, G., & Cline, T. (2007). Social valorisation of mathematical practices: The implications for learners in multicultural schools. In Nasir, N. & Cobb, P. (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 118-131). New York, NY: Teachers College Press.

Delpit, L., (2006). *Other People's Children: Cultural Conflict in the Classroom*. New York: New Press.

Delpit, L. (2008). The Silenced Dialogue: Power and Pedagogy in Educating other People's Children. *Harvard Educational Review, 58*, 280-298.

Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths/pedagogic texts*. London: The Falmer Press.

Evans, P. (Ed.). (2002). *Livable Cities? Urban Struggles for Livelihood and Sustainability*. Berkeley: University of California Press.

Fagg, H. (2002). *Back to the Sources: A Study of Gandhi's Basic Education*. New Delhi: National Book Trust.

Flavell, J. H., Miller, H. M., & Miller, S. A. (2002). *Cognitive development.* Pearson.

Flick, U. (2009). *Introducing research methodology: A beginner's guide to doing a research project*. Sage.

Frankenstein, M. (2010). Developing a Critical Mathematical Numeracy through real Real-life Word Problems. In U. Gellert, E. Jablonka, & C. Morgan (Eds.). *Proceedings of the Sixth International Mathematics Education and Society (MES) Conference, Vol. I*, pp. 248-257. Berlin: Freie Universitat.

Freudenthal, H. (1971). Geometry between the devil and the deep sea. *Educational Studies in Mathematics, 3*, 413-435.

Freudenthal, H. (2002). *Didactical phenomenology of mathematical structures.* New York: Kluwer Academic Publishers.

Gandhi, M. K. (1927). *An autobiography or the story of my experiments with truth*. Ahmedabad: Navjivan Publishing House.

Gandhi, M. K. (1951). *Basic education.* Ahmedabad: Navjivan Publications.

Gandhi, M. K. (1953). B. Kumarappa (Ed.). *Towards new education*. Ahmedabad: Navjivan Publishing House.

Gay, J., & Cole, M. (1967). *The new mathematics and an old culture: A study of learning among the Kpelle of Liberia*. New York: Holt, Rinehart, and Winston.

Gee, J. P. (1999). *An introduction to discourse analysis: theory and method*. London: Routledge.

Gee, J. P. (2001). Reading as situated language: A sociocognitive perspective. *Journal of adolescent & adult Literacy*, *44*(8), 714-725.

Gellert, U. & Jablonka, E. (2007). *Mathematisation and demathematisation: Social, philosophical and educational ramifications.*Rotterdam: Sense Publishers.

González, N. (2005). Beyond culture: The hybridity of funds of knowledge. In N. González, L. C. Moll & C. Amanti (Eds.). *Funds of knowledge: Theorizing practices in households, communities, and classrooms*, pp. 29-46. Mahwah, NJ: Lawrence Erlbaum.

González, N., Moll, L. C., & Amanti, C. (Eds.). (2005). *Funds of knowledge: Theorizing practices in households, communities, and classrooms*. Mahwah, NJ: Lawrence Erlbaum.

González, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk*, *6*(1-2), 115-132.

Govt. of India (1999). *The collected works of Mahatma Gandhi.* (Electronic book, 98 volumes). New Delhi: Publications Division, Govt. of India. Available at: http://www.gandhiashramsevagram.org/mkgandhi/cwmg/cwmg.html

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational studies in mathematics*, *39*(1-3), 111-129.

Greeno, J. G. (1997). On claims that answer the wrong questions. *Educational Researcher, 26*(1), 5-17.

Greeno, J. G., Smith, D. R., & Moore, J. L. (1993). Transfer of situated learning. In D. K. Detterman, & R. J. Sternberg (Eds.), *Transfer on trial: Intelligence, cognition, and instruction* (pp. 99–167). Norwood, NJ: Ablex Publishing Corp.

Greiffenhagen, C., & Sharrock, W. (2008). School Mathematics and its Everyday Other? Revisiting Lave's 'Cognition in Practice'. *Educational Studies in Mathematics, 69*, 1-21.

Groen, G., & Resnick, L. B. (1977). Can preschool children invent addition algorithms?. *Journal of Educational Psychology*, *69*(6), 645.

Guberman, S. R. (1996). The Development of Everyday Mathematics in Brazilian Children with Limited Formal Education. *Child Development, 67*, 1609-1623.

Guberman, S. R. (2004). A comparative study of children's out-of-school activities and arithmetical achievements. *Journal for Research in Mathematics Education*, *35*(2), 117-150.

Gurupadaswami, M. S. (1979). *Committee on child labour, 1979: A report.* New Delhi: Ministry of Labour, Govt. of India.

Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education, 44*(1), 37-68.

Gutstein, E. & Peterson, B. (Eds.). (2006). *Rethinking Mathematics – Teaching Social Justice by the Numbers*. Milwaukee, Wisconsin: Rethinking Schools.

Halstead, J. M. (1996). Values and values education in schools. In J. M. Halstead & J. T. Taylor (Eds.). *Values in education and education in values.* London: RoutledgeFalmer.

Halstead, J. M. & Taylor, J. T. (Eds.). (1996). *Values in education and education in values.* London: RoutledgeFalmer.

Hardy, G. H. (1940/1967). *A Mathematician's Apology*. Cambridge: Cambridge University Press.

Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom: on learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research, 51*, 128-145.

Hobbs, S., McKechnie, J. & Lavalette, M. (1999). Child labour: A world history companion. Santa Barbara, CA: ABC-CLIO, Inc.

ILO, UN. http://www.ilo.org/legacy/english/regions/asro/newdelhi/ipec/responses/india/national.htm Accessed on 27 November 2012, 1900 IST.

ILO (Convention 182). http://www.ilo.org/public/english/standards/relm/ilc/ilc87/com-chic.htm Accessed on 27 November 2012, 2234 IST.

Jablonka, E. & Gellert, U. (2007). Mathematisation-Demathematisation. In U. Gellert & E. Jablonka (Eds.). *Mathematization and de-mathematization: Social, philosophical and educational ramifications* (pp. 1-18). Rotterdam: Sense Publishers.

Jha, J. (2004). [Review of the book *Child Labour and the Right to Education in South Asia: Needs versus Rights?*]. *Contemporary Education Dialogue, 2*(1), 287-291.

Joram, E., Gabriele, A. J., Bertheau, M., Gelman, R., & Subrahmanyam, K. (2005). Children's use of the reference point strategy for measurement estimation. *Journal for Research in Mathematics Education, 36*(1), 4-23.

Kabeer, N., Nambissan, G. B., & Subrahmanian, R. (Eds.). (2003). *Child Labour and the Right to Education in South Asia*. New Delhi: SAGE Publications.

Kantha, V. K. (2009). *Exclusion in education: A preliminary enquiry into elementary education of Dalit children in Bihar*. Patna: Centre for Study of Social Exclusion and Inclusive Policy.

Keitel, C., Kotzmann, E., & Skovsmose, O. (1993). Beyond the tunnel vision: analysing the relationship between mathematics, society and technology. In C. Keitel and K. Ruthven (Eds.), *Learning from Computers: Mathematics Education and Technology*, (pp. 243-279). Berlin: Springer Verlag.

Kent, P., Bakker, A., Hoyles, C., & Noss, R. (2011). Measurement in the workplace: the case of process improvement in manufacturing industry. *ZDM – The International Journal on Mathematics Education, 43*(5), 747-758.

Khan, F. A. (2004). Living, Learning and Doing Mathematics: A Study of Working Class Children in Delhi. *Contemporary Education Dialogue*, *2*, 199-227.

Knijnik, G. (1993). An ethnomathematical approach in mathematical education: A matter of political power. *For the learning of mathematics*, *13*(2), 23-25.

Knijnik, G. (1998). Ethnomathematics and political struggles. *ZDM – The International Journal on Mathematics Education, 98*(6), 188-194.

Kothari, D. S. (1966). *Report of the Education Commission 1964–66*. New Delhi: Government of India.

Kumar, K. (1993). Literacy and primary education in India. In P. Freebody & A. R. Welch (Eds.). *Knowledge, culture and power: International perspectives on literacy as policy and practice*, (pp. 102-113). London: Falmer Press.

Kumar, K. (2005). *Political agenda of education: A study of colonialist and nationalist ideas*. New Delhi: Sage Publications India Pvt. Ltd.

Kumar, K. (2006). Childhood in a globalising world. *Economic and Political Weekly, XLI* (38), 4030-4034.

Kumar, K. (2008). *A Pedagogue's Romance: Reflections on Schooling*. New Delhi: Oxford University Press.

Labour & Employment Ministry, Govt. of India. http://www.labour.nic.in/content/division/about-child-labour.php Accessed on 27 November 2012, 1924 IST.

Lave, J. (1977). Cognitive consequences of traditional apprenticeship training in West Africa. *Anthropology & Education Quarterly*, *8*(3), 177-180.

Lave, J. (1985). 'Introduction: situationally Specific Practice'. *Anthropology and Education Quarterly, 16*(3), 171-176.

Lave, J. (1988). *Cognition in Practice: Mind, mathematics and culture in everyday life*. Cambridge, UK: Cambridge University Press.

Lave, J. (1991). Situated learning in communities of practice. In L. B. Resnick, J. M. Levine & S. D. Teasley (Eds.). *Perspectives on Socially Shared Cognition*. Washington, DC: American Psychological Association, pp. 63–82.

Lave, J., & Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*. Cambridge: Cambridge University Press.

Leontiev, A. N. (1978). *Activity, consciousness, and personality*. Englewood Cliffs, NJ: Prentice Hall. (Original work published 1975)

Lipka, J. (1991). Toward a culturally based pedagogy: A case study of one Yupi'k Eskimo Teacher. *Anthropology & Education Quarterly, 22*(3), 203-223.

Maharashtra Textbook Bureau. (2006). *Mathematics Standard 5, 6, 7*. Pune: Maharashtra State Bureau of Textbook Production and Curriculum Research.

Masingila, J. O., & DeSilva, R. (2001). Teaching and Learning School Mathematics by Building on Students' Out-of-School Mathematical Practice. In B. Atweh, H. Forgasz, & B. Nebres (Eds.). *Sociocultural Research on Mathematics Education: An International Perspective*. pp. 329-344. Mahwah, NJ: Lawrence Erlbaum Associates Publishers.

Masingila, J. O., Davidenko, S., & Prus-Wisniowska, E. (1996). Mathematics Learning and Practice in and out of School: A Framework for Connecting These Experiences. Educational Studies in Mathematics, 31(1/2), 175-200.

Masingila, J. O. (1993). Learning from Mathematics Practice from Out-of-school Situations. For the Learning of Mathematics, 13(2), 18-22.

Menninger, K. (1958/1969). *Number words and number symbols: A cultural history of numbers*. Original German Edition. Cambridge, MA: The MIT Press.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook.* 2nd edition. Thousand Oaks: Sage.

Millroy, W. L. (1992). An Ethnographic Study of the Mathematical Ideas of a Group of Carpenters. *Journal for Research in Mathematics Education*. Monograph, Vol. 5, pp. i-210. NCTM.

Moll, L. C., Amanti, C., Neff, D., & Gonzalez, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory into practice, 31*(2), 132-141.

Moschkovich, J. N. (2002). An Introduction to Examining Everyday and Academic Mathematical Practices. *Journal for Research in Mathematics Education. Monograph, 11 (Everyday and Academic Mathematics in the Classroom)*, 1-11.

Moschkovich, J. N. & Brenner, M. E. (2002). Preface. *Journal for Research in Mathematics Education. Monograph, 11 (Everyday and Academic Mathematics in the Classroom)*, v-x.

Mukhopadhyay, S. (2011). Making Visible: Mathematics of Cultural Practices. Talk delivered at the Conference on the History and Cultural Aspects of Mathematics Education, Indira Gandhi National Open University, New Delhi.

Mukhopadhyay, S. (2013). The mathematical practices of those without power. In *Mathematics Education and Society 7th International Conference.* Plenary talk. Cape Town: South Africa.

Mukhopadhyayay, S. & Roth, W-M. (Eds.). (2012). *Alternative forms of knowing (in) Mathematics.* Rotterdam: Sense Publishers.

Mukund, K. (1988). The Hoshangabad science teaching programme. *Economic & Political Weekly, 23*(42), 2147–2150.

Naik, J. P. (1982). *The education commission and after*. New Delhi: Allied Publishers.

Nambissan, G. B. (2003). Social exclusion, children's work and education: A view from the margins. In N. Kabeer, G. B. Nambissan and R. Subrahmanian (Eds.). *Child labour and the right to education in South Asia,* (pp. 109-142). New Delhi: Sage Publications.

Nambissan, G. B. & Rao, S. S. (Eds.). (2013). *Sociology of Education in India: Changing Contours and Emerging Concerns*. New Delhi: Oxford University Press.

Naresh, N. (2009). Interplay between School Mathematics and Work Place Mathematics. In K.Subramniam & A. Mazumdar (Eds.), *Proceedings of epiSTEME-3: Third International Conference to Review Research on Science, Technology and Mathematics Education.* Mumbai, India: Macmillan.

Naresh, N., & Chahine, I. (2013). Reconceptualizing Research on Workplace mathematics: Negotiations Grounded in Personal Practical Experiences. *REDIMAT-Journal of Research in Mathematics Education*, *2*(3), 316-342.

Nasir, N. S. (2002). Identity, goals, and learning: Mathematics in cultural practice. *Mathematical thinking and learning*, *4*(2&3), 213-247.

Nasir, N. S., & de Royston, M. M. (2013). Power, identity, and mathematical practices outside and inside school. *Journal for Research in Mathematics Education*, *44*(1), 264-287.

Nasir, N. S., Hand, V. & Taylor, E. (2008). Culture and Mathematics in School: Boundaries between "Cultural" and "Domain" Knowledge in the Mathematics Classroom and Beyond. *Review of Research in Education, 32*, 187-240.

National Council of Educational Research and Training. (2005). *The National Curriculum Framework*. New Delhi: NCERT.

National Council of Educational Research and Training. (2006a). *Position Paper of National Focus Group on Teaching of Mathematics.* New Delhi: NCERT.

National Council of Educational Research and Training. (2006b). *Math-Magic 5*. Textbook for Grade 5. New Delhi: NCERT.

National Council of Educational Research and Training. (2006c). *Mathematics*. Textbook for Grades 6, 7. New Delhi: NCERT.

National Council of Educational Research and Training. (2007). *Position Paper National Focus Group on Work and Education.* New Delhi: NCERT.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Sample Survey Office. *NSSO estimate of child labour in major Indian states, 2004-2005 (Age group 5-14 years).* http://labour.gov.in/upload/uploadfiles/files/Divisions/childlabour/NSSOEstimateofChildLabourinMajorIndianStates.pdf Accessed 28 December 2012.

Nunes, T. (1999). Mathematics learning as the socialization of the mind. *Mind, Culture, and Activity*, *6*(1), 33-52.

Nunes, T. (1992). Ethnomathematics and Everyday Cognition. In D. A. Grouws (ed.). *Handbook of Research on Mathematics Teaching and Learning*. Reston: NCTM, 557-574.

Nunes, T., & Bryant, P. (1996). *Children Doing Mathematics*. Cambridge, MA: Blackwell Publishers Inc.

Nunes, T. & Bryant, P. (Eds.). (1997). Learning and Teaching Mathematics. Hove: Psychology Press.

Nunes, T., Light, P., & Mason, J. (1993). Tools for thought: the measurement of length and area. *Learning and Instruction, 3*, 39-54.

Nunes, T., Light, P., & Mason, J. (1995). Measurement as a social process. *Cognition and Instruction, 13*(4), 585-587.

Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). *Street Mathematics and School Mathematics*. New York: Cambridge University Press.

Nunes, T. C., & Schliemann, A. D. (1985). Computation Routines Prescribed by Schools: Help or Hindrance? *Journal for Research in Mathematics Education, 16*(1), 37-44.

Pais, J. (2006). Wages and Earnings in Leather Accessories Manufacture in India: An Analysis of the Industry in Mumbai. *The Indian Journal of Labour Economics*, *49*(4), 697-714.

Panda, M. (2007). Saora Culture, As-if Discourse and Mathematics Learning. In G. Zheng, K. Leung & J. Adair (Eds.). *Perspectives and progress in contemporary cross-cultural psychology*. Beijing: China Light Industry Press.

Pande, P. & Ramadas, J. (2013). Students' measurement experiences and responses to length comparison, seriation and proportioning tasks. In Nagarjuna, G., A. Jamakhandi & E. M. Sam (Eds.). *Proceedings of epiSTEME-5: 5th International Conference to Review Research on Science, Technology and Mathematics Education.* pp. 145-150, Margao: CinnamonTeal.

Patel, S., & Arputham, J. (2007). An offer of partnership or a promise of conflict in Dharavi, Mumbai?. *Environment and Urbanization, 19*(2), 501-508.

Piaget, J., Inhelder, B., & Szeminska, A. (1960). *The child's conception of geometry* (E. A. Lunzer Trans.). London, England: Routledge & Kegan Paul.

Prediger, S. Clarkson, P. & Bose, A. (*in press*). Purposefully relating multilingual registers: Building theory and teaching strategies for bilingual learners based on an integration of three traditions. In R. Barwell, P. Clarkson, & J. Moschkovich (Eds.). *Multilingual contexts for teaching and learning mathematics* (ICMI Study 21 Report). *New ICMI Study Series* (NISS). Dordrecht: Springer.

PROBE (1999). *Public report on basic education in India.* Chennai: The PROBE Team in association with Centre for Development Economics.

Rahaman, J., Subramaniam, K., & Chandrasekharan, S. (2012). Exploring the connection between multiplicative thinking and the measurement of area. In *Proceedings of the 12th International Congress on Mathematical Education- 12*, (pp. 1964-1973). Seoul, South Korea: ICME.

Raman, V. (2000). Politics of childhood: perspectives from the South. *Economic and Political Weekly, XXXV* (46), 4055-4064.

Rampal, A., Ramanujam, R., & Saraswati, L. S. (1998). *Numeracy Counts!*, Mussoorie, India: LBS National Academy of Administration.

Rampal, A. (1991). Deliverance from the 'delivery' metaphor: curriculum innovation in India. J*ournal of Education for Teaching: International Research and Pedagogy, 17*(3), 237–244.

Rampal, A. (2003a). Counting on Everyday Mathematics. In T.S. Saraswathi (ed.) *Cross-cultural Perspectives in Human Development: Theory, research and practice*. New Delhi: Sage Publications.

Rampal, A. (2003b). Indian Market Women and Their Mathematics. In J. Solomon (Ed.), *The Passion to Learn: An Inquiry into Autodidactism*. Routledge Falmer.

Rampal, A. (2007). Ducked or bulldozed? Education of deprived urban children in India. In W. T. Pink and G. W. Noblit (Eds.), *International Handbook of Urban Education*, pp. 285-304. Dordrecht, The Netherlands: Springer.

Reed, H. J., & Lave, J. (1979). Arithmetic as a tool for investigating relations between culture and cognition. *American Ethnologist, 6*(3), 568-582.

Resnick, L. B. (1995). Inventing arithmetic: Making children's intuition work in school. In C. A. Nelson (Ed.). *Basic and Applied Perspectives on Learning, Cognition, and Development: The Minnesota Symposia on Child Psychology, 28*, pp. 75-102.

Resnick, L. B. (1991). Shared cognition: Thinking as social practice. In L. B. Resnick, J. M. Levine and S. D. Teasley (Eds.). *Perspectives on Socially Shared Cognition*. Washington, DC: American Psychological Association, pp. 1–22.

Resnick, L. B. (1987). Learning In School and Out. *Educational Researcher, 16*(9), 13-20.

Resnick, L. B., Levine, J. M., & Teasley, S. D. (Eds.). (1991). *Perspectives on socially shared cognition*. Washington DC: American Psychological Association.

Resnick, L. B., & Ford, W. W. (1981). *The Psychology of Mathematics for Instruction*. Hillsdale, NJ: Erlbaum.

Rogoff, B. (1994). Developing understanding of the idea of communities of learners. *Mind, culture, and activity, 1*(4), 209-229.

Rogoff, B. (1997). Foreword. In E. Tobach, R. J. Falmagne, M. B. Parlee, L. M. W. Martin & A. S. Kapelman (Eds.). *Mind and social practice: selected writings of Sylvia Scribner*. Cambridge, UK: Cambridge University Press.

Sarama, J., & Clements, D. H. (2009). *Early Childhood Mathematics Education Research: Learning Trajectories for Young Children (Studies in Mathematics Thinking and Learning)*. New York: Routledge.

Sarangapani, P. (2003). *Constructing school knowledge: An ethnography of learning in an Indian village*. New Delhi: Sage Publications.

Saraswathi, L. S. (1988). Practices in identifying (reckoning), measuring and planning for utilization of time in rural Tamil Nadu. *Journal of Education and Social Change, 2*(3), 125-140.

Saraswathi, L. S. (1989). Practices in linear measurements in rural Tamil Nadu: implications for adult education programmes. *Journal of Education and Social Change, 3*(1), 29-46.

Saxe, G. B. (1988a). The Mathematics of Child Street Vendors. *Child Development, 59* (5), 1415-1425.

Saxe, G. B. (1988b). Candy Selling and Math Learning. *Educational Researcher, 17*(6), 14-21.

Saxe, G. B. (1992). Studying Children's Learning in Context: Problems and Prospects. *The Journal of Learning Sciences*, *2*(2), 215-234.

Saxe, G. B. (2002). Children's Developing Mathematics in Collective Practices: A Framework for Analysis. *The Journal of the Learning Sciences*, 11 (2&3), 275-300.

Saxe, G. B. (2004). Practices of Quantification from a Socio-cultural Perspective. In A. Demetriou and A. Raftopoulos (Eds.). *Cognitive Developmental Change: Theories, Models and Measurement*. pp. 241-263. Cambridge: Cambridge University Press.

Saxe, G. B. (2012). *Cultural development of mathematical ideas: Papua New Guinea studies*. NY: CUP.

Saxe, G. B., & Bermudez, T. (1996). Emergent mathematical environments in children's games. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.). *Theories of Mathematical Learning*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Saxe, G. B., & Guberman, S. R. (1998). Emergent arithmetical environments in the context of distributed problem Solving: Analyses of children playing an educational game. In J.G. Greeno and S. V. Goldman (Eds.). *Thinking Practices in Mathematics and Science Learning*. pp. 237-2 . Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Saxe, G. B., Guberman, S. R., Gearhart, M., Gelman, R., Massey, C. M., & Rogoff, B. (1987). Social processes in early number development. *Monographs of the society for research in child development*, *52*(2), i-162.

Saxe, G. B., & Moylan, T. (1982). The Development of Measurement Operations among the Oksapmin of Papua New-Guinea. Child Development, 53, 1242-1248.

Schliemann, A. D. (1998). Logic of meanings and situated cognition. *Learning and instruction*, *8*(6), 549-560.

Schliemann, A. D., & Acioly, N. M. (1989). Mathematical knowledge developed at work: The contribution of practice versus the contribution of schooling. *Cognition and Instruction*, *6*(3), 185-221.

Schunk, D. H. & Zimmerman, B. (Eds.). (2012). *Motivation and self-regulated learning: Theory, research and applications.* New York: Routledge.

Scribner, S. (1984). Studying working intelligence. In B. Rogoff & J. Lave (Eds.). *Everyday cognition: Its development in social contexts* (pp. 9-40). Cambridge, MA: Harvard University Press.

Scribner, S. (1986). Thinking in action: Some characteristics of practical thought. In R. J. Sternberg & R. K. Wagner (Eds.). *Practical intelligence: Nature and origins of competence in the everyday world*, (pp. 13-30). Cambridge: Cambridge University Press.

Scribner, S., & Cole, M. (1978). Literacy without schooling: Testing for intellectual effects. *Harvard Educational Review*, *48*(4), 448-461.

Scribner, S. & Cole, M. (1981). *The psychology of literacy* (Vol. 198, No. 1). Cambridge, MA: Harvard University Press.

Sengupta, A. (2007). Report on conditions of work and promotion of livelihoods in the unorganised sector. New Delhi: National Commission for Enterprises in the Unorganised Sector, Government of India.

Sfard, A., Cole, M. (2002). Literate mathematical discourse: What it is and why should we care? Retrieved December 2013 from http://communication.ucsd.edu/lchc/vegas.htm

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14-22.

Sharma, K. (2000). *Rediscovering Dharavi: Stories from Asia's largest slum*. New Delhi: Penguin Books.

Shavelson, R. J., Phillips, D. C., Towne, L., & Feuer, M. J. (2003). On the science of education design studies. *Educational researcher, 32*(1), 25-28.

Sitabkhan, Y. (2009). The Use of Convenient Value Strategies among Young Train Vendors in Mumbai, India. In K. Subramaniam & A. Mazumdar (Eds.). *Proceedings of epiSTEME-3: An International Conference to Review Research on Science, Technology and Mathematics Education*, pp. 114-118. Mumbai, India: Macmillan.

Skovsmose, O. (2009). *In Doubt – about Language, Mathematics, Knowledge and Life-Worlds*. Rotterdam: Sense Publishers.

Skovsmose, O. (2005). *Travelling Through Education: Uncertainty, Mathematics, Responsibility*. Rotterdam: Sense Publishers.

Smith III, J. P., Van den Heuvel-Panhuizen, M., & Teppo, A. R. (2011). Learning, teaching, and using measurement: introduction to the issue . Editorial. *ZDM The International Journal on Mathematics Education, 43*(5), 617–620 .

Srivastava, J. (2006). Review of the book. *Child Labour and the Right to Education in South Asia: Needs versus Rights? Social Scientist*, *34*(3/4), 90-93.

State Council of Educational Research and Training. (2008). *Bihar curriculum framework*. Patna: SCERT.

Stephan, M. (2003). Conclusion. In M. Stephan, J. Bowers & P. Cobb (Eds.). Supporting students' development of measuring conceptions: Analysing students learning in social contexts. *Journal for Research in Mathematics Education, Monograph, 12,* 123-130.

Steffe, L. P. (2003). Fractional Commensurate, Composition, and Adding Schemes Learning Trajectories of Jason and Laura: Grade 5. *Journal of Mathematical Behaviour, 22*, 237-295.

Streefland, L. (1985). Search for the Roots of Ratio: Some Thoughts on the Long Term Learning Process (Towards... A Theory). *Educational Studies in Mathematics, 16*, 74-95.

Subramaniam, K. (2010). Culture in the learning of mathematics. *Learning Curve, 14*(2), 25-28.

Subramaniam, K. (2012). Does participation in household based work create opportunities for learning mathematics? *In Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 107 – 112. Taipei, Taiwan: PME.

Subramaniam, K. & Bose, A. (2012). Measurement units and modes: the Indian context. In the *Proceedings of the 12th International Congress on Mathematical Education (ICME-12)*, pp. 1974-1983, Seoul: Korea.

Swaminathan, M. (1995). Aspects of urban poverty in Bombay. *Environment and Urbanisation*, *7*, 133-143. DOI: 10.1177/095624789500700117.

Sykes, M. (1988). *The story of nai talim: fifty years of education at Sevagram, 1937-1987: a record of reflections*. Sevagram: Nai Talim Samiti. Available at: http://www.swaraj.org/shikshantar/naitalimmarjoriesykes.htm

Szilagyi, J., Clements, D., & Sarama, J. (2013). Young children's understandings of length measurement: evaluating a learning trajectory. *Journal for Research in Mathematics Education, 44*(3), 581-620.

Talib, M. (2003). Modes of learning-labour relations: Educational strategies and child labour. In N. Kabeer, G. B. Nambissan and R. Subrahmanian (Eds.). *Child labour and the right to education in South Asia: Needs vs. rights?* New Delhi: Sage.

Taylor, E. V. (2009). The purchasing practice of low-income students: The relationship to Mathematical Development. *The Journal of the Learning Sciences, 18*, 370-415.

Taylor, E. V. (2012). Supporting children's mathematical understanding: professional development focussed on out-of-school practices. *Journal of Mathematics Teacher Education, 15*(4), pp. 271-291.

Thorndike, E. L. & Woodworth, R. S. (1901). The influence of improvement in one mental function upon the efficiency of other functions. *Psychological Review, 8,* 247-261.

Triantafillou, C., & Potari, D. (2014). Revisiting the place value concept in the workplace context: the issue of transfer development. *Educational Studies in Mathematics*, *86*(3), 337-358.

UK National Archives. http://www.nationalarchives.gov.uk/pathways/citizenship/struggle_democracy/childlabour.htm Accessed 04 December 2012, 1925 IST.

UK Child Employment Guide. https://www.gov.uk/child-employment/minimum-ages-children-can-work Accessed 04 December 2012, 1922 IST.

UNICEF (CRC). http://www.unicef.org/crc/index_30197.html accessed 03 December 2012, 2135 IST.

UNICEF (2002). http://www.unicef.org/india/children_3220.htm accessed 03 December 2012, 2134 IST.

UNICEF (1997). *State of the World's Children*. New York: Oxford University Press.

United Nations Treaty Collection. https://treaties.un.org/Pages/ViewDetails.aspx?mtdsg_no=IV-11&chapter=4&lang=en#EndDec accessed 25 November 2014, 0757 IST.

Valero, P. (2004). Socio-political perspectives on mathematics education. In P. Valero and R. Zevenbergen (Eds.). *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology,* 5-23. New York: Springer Science + Business Media.

Van den Heuvel-Panhuizen, M., & Elia, I. (2011). Kindergartners' performance in length measurement and the effect of picture book reading. *ZDM The International Journal on Mathematics Education*, *43*(5) , 621-635.

Van Maanen, J. (2011). *Tales of the Field: On Writing Ethnography.* Chicago: The University of Chicago Press.

Vasanta, D. (2004). Childhood, Work and Schooling: Some Reflections. *Contemporary Education Dialogue*, *2*(1), 5-29.

Vélez-Ibáñez, C., & Greenberg, J. (2005). Formation and transformation of funds of knowledge. *Funds of knowledge: Theorizing practices in households, communities, and classrooms*, 47-70.

Vergnaud, G. (1994). Multiplicative Conceptual Field: What and Why? In G. Harel and J. Confrey (Eds.). *The Development of Multiplicative Reasoning in the Learning of Mathematics*. Albany: State University of New York Press.

Vygotsky, L. S. (1978). *Mind and society: The development of higher mental processes*. Cambridge, MA: Harvard University Press.

Wegner, E. (1998). *Communities of Practice: Learning, Meaning and Identity*. Cambridge: Cambridge University Press.

Weiner, M. (1991). *The Child and the State in India*. Oxford: Oxford University Press.

Weiner, M. (1995). Foreword. In N. Burra, *Born to work: Child labour in India* (pp. ix-xvi). Delhi, India: Oxford University Press.

Weiner, M., Burra, N., & Bajpai, A. (2006). *Born Unfree: Child Labour, Education, and the State in India*. New Delhi: Oxford University Press.

Zaslavsky, C. (1973). *Africa counts: Number and pattern in African cultures*. Chicago: Lawrence Hill Books.

Zevenbergen, R. (1996). Constructivism as a liberal bourgeois discourse. *Educational Studies in Mathematics, 31*, 95–113.

List of Publications

Bose, A. & Kantha, V. K. (2014). Influence of socio-economic background and cultural practices on mathematics education in India: A contemporary overview in historical perspective. *ZDM – The International Journal on Mathematics Education, 46*(7), 1073-1084. DOI: 10.1007/s11858-014-0607-x. ISSN: 1863-9690 (Print) 1863-9704 (Online).

Bose, A. (2014). Revisiting transfer of learning in mathematics: Insights from an urban low-income settlement. In P. Liljedahl, C. Nicol, S. Oesterle & D. Allan (Eds.). *Proceedings of the Joint Meeting of PME 38 and PME-NA 36*, Vol. 2, pp. 177-184. Vancouver, Canada: PME.

Bose, A. & Subramaniam, K. (2013). Characterising work-contexts from a mathematics learning perspective. In Nagarjuna, G., A. Jamakhandi & E. M. Sam (Eds.). *Proceedings of epiSTEME-5: International conference to review research on Science, TEchnology and Mathematics Education,* pp. 173-179, Margao, India: CinnamonTeal Publishing. ISBN: 978-93-81542-83-5.

Prediger, S., Clarkson, P. C. & Bose, A. (2012). A way forward for teaching in multilingual contexts: Purposefully relating multilingual registers. *Proceedings of the 12th International Congress on Mathematical Education (ICME-12)*, pp. 6213-6222, Seoul, Korea: ICME.

Subramaniam, K. & Bose, A. (2012). Measurement units and modes: The Indian context. *Proceedings of the 12th International Congress on Mathematical Education (ICME-12)*, pp. 1974-1983, Seoul, Korea: ICME.

Bose, A. & Subramaniam, K. (2011). Exploring school children's out of school mathematics. In Ubuz, B. (Ed.). *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 2, pp. 177-184, Ankara, Turkey: PME. ISSN: 0771-100X, ISBN: 978-975-429-296-1.

Choudhury, M. & Bose, A. (2011). An Investigation of the Role of Language-negotiations in a Multilingual Mathematics Classroom. In M. Setati, T. Nkambule, & L. Goosen (Eds.). *Mathematics and language diversity*: *Proceedings of the ICMI Study 21 Conference*, pp. 28-37, Sào Paulo, Brazil. ISBN: 978-1-86888-682-1.

Bose, A. & Choudhury, M. (2010). Language Negotiation in a Multilingual Mathematics Classroom: An Analysis. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd Conference of the Mathematics Education Research Group of Australasia, Inc*., Fremantle, Australia: MERGA. (pp. 93-100). ISBN: 978-1-920846-25-1.

Book Chapters

Prediger, S. Clarkson, P. & Bose, A. (*in press*). Purposefully relating multilingual registers: Building theory and teaching strategies for bilingual learners based on an integration of three traditions. In R. Barwell, P. Clarkson, & J. Moschkovich (Eds.). *Multilingual contexts for teaching and learning mathematics* (ICMI Study 21 Report). *New ICMI Study Series* (NISS). Dordrecht: Springer.

Bose, A. & Clarkson, P. (*forthcoming*). Relating multilingual registers: An example of the socio-cultural role of language in multilingual classrooms. In A. Halai & P. Clarkson (Eds.). *Teaching and learning mathematics in multilingual classrooms: Issues for policy and practice*. Rotterdam, The Netherlands: Sense Publishers.

Reports

Atweh, B., Bose, A., Graven, M., Subramanian, J., & Venkat, H. (2014). *Teaching numeracy in pre-school and early grades in low-income countries.* Bonn: Deutsche Gesellschaft für Internationale Zusammenarbeit (GIZ) GmbH , German Federal Ministry for Economic Cooperation and Development (BMZ) .

APPENDIX – A

Phase-II, Part-I

Interview Protocol – Section I

Name: ____

Class: __________________

School: ____

Date: __________________

Q. 1 Do you live in?

rented house ☐ own ☐ house

Q. 2a Who all live in your home?

Relation	Age	Occupation	You help them in what kind of work?

Q.2b Father's Religion ______________ Mother's Religion_____________

Festivals celebrated ____

____

Prayer/Puja done at home: ____

How frequently? ____

Q. 3 Do you have any of the following at home? (Put √ in the right boxes).

Mobile Phone ☐ ☐ How many?

TV ☐ ☐ Fridge

Music system/deck/radio ☐ ☐

Two-wheeler (Motorbike/Scooter)

☐

Three/Four-wheeler (Auto-rickshaw/car/van) ☐

Washing machine? ☐

Q. 4 Do you bring tiffin to school?

Yes ☐ ☐ No

If yes, what do you generally bring for tiffins?

Q. 5 Do you take the 'khichdi' served in your school during tiffin breaks?

Q. 6 Do you get money from your parents? How much? How frequent?

Q. 7 What kind of food do you (why prefer to) eat at home?

Q. 8 Do you celebrate your birthdays? In what ways?

Q. 9 When do you get/buy new dresses? Who buys for you?

Q.10a Do you go out with your parents/friends? Where do you like going?

Q.10b Do you go to movie/theatre halls? How often? Do you go out to eat?

Q.11 Do you go for any tuition/coaching classes?

If yes, then, on which days?

____

Subjects taught?

____

____

Timing?

____

Fees?

____

Q.12 Do you help your younger siblings in their studies?

Q.13 Does your family/parents buy ration (provisions) in bulk? How often do they buy? (Put √ in the right box).

once a month ☐ twice a month ☐ ☐ every week

once in two days ☐ everyday ☐

Q.14 Which language do you speak in:

when at home?

in the school – with your teachers,
with your friends?

at workplaces or while working?

while playing?

when in the shops/markets:
with shopkeepers, with vendors?

Q.15 What TV programmes do you watch?

Q.16 Do you like reading story-books or magazines? Where do you get them from? How often do you buy them?

Q.17 Do you buy newspaper at home? What do you read or see in the paper?

Q.18 Do you use 'gullak'? How often do you put money in 'gullak'? How much each time?

Q.19 Do your parents visit banks? Do you accompany them to a bank? Do you or your parents have a bank-account?

Q.20 Which calendar do you use? What information does the calendar have? Can you read the calendar?

APPENDIX – B

Phase-II, Part-I

Interview Protocol – Section II

Name: ____

Class: _________________

School: ____

Date: _________________

Q.1 What games do you play? (Indoor/outdoor, name of the games, with whom) How long do you play in a day?

Q. 2 What do you do at home at leisure?

Q. 3 In what ways do you assist your parents at home?

Q. 4 Are you involved in any work at home? How much time do you spend in the work?

Q. 5 Do you assist your parents in keeping *hisaab* (accounts)? How do you do that? Who taught you how to keep accounts?

Q. 6 Do you visit local shops/market? What do you usually buy from there?

Q. 7 Do you make the payment for any purchase in cash or make an entry in the record-book?

Q. 8 How do you calculate how much to pay to the shopkeeper? How did you learn this?

Q. 9 Do you read/check the measurements? How did you learn this?

Q. 10 Do you count back the balance amount received?

Q. 11   Do the shopkeepers sell goods on credit?

Q. 12   Do you know anyone in your neighbourhood who lends/borrows money? How much can you borrow? How much interest?

APPENDIX – C

Phase-II, Part-I

Interview Protocol, Section-III (Arithmetic Knowledge)

Name: ____

Class: ______________ **School:** ________________________

Date: _________________

Q.1 Can you read these numbers? बताओ ये कौन से नंबर हैं ?

279, 607, 1010, 2303, 4800, 10010

• Show a number card for each of these numbers and ask the student to read the number.	279________________ 607________________
• Record the number the student says.	1010_______________
• Ask the student, “Are you sure? Do you want to check again?”	2303_______________ 4800_______________
• Record any corrections/changes that the student makes	10010_______________

Q.2 Write the following numbers. इन नंबर को अपने पेपर पर लिखो।

Seven hundred fifty one (सात सौ इक्यावन), One thousand one hundred (एक हजार एक सौ), Two thousand fifty (दो हजार पचास), Ten thousand sixty (दस हजार साठ), Thirteen thousand two hundred six (तेरह हजार दो सौ छः), One lakh twenty five thousand (एक लाख पच्चीस हजार).

- Give a sheet of paper to the student
- Ask the student to write down the numbers on the given sheet of paper.
- Record the number the student says.
- Ask the student, “Are you sure? Do you want to check again?”
- Record any corrections/changes that the student makes.

Q.3 I'll say a number, take out that much money (मैं एक नंबर बोलूंगा, उतने पैसे निकालो।).

Numbers are: One hundred sixty five (एक सौ पैंसठ), Two thousand seven hundred twenty five (दो हजार सात सौ पच्चीस), Thirteen thousand two hundred six (तेरह हजार दो सौ छः).

- Keep one or two boxes with different currency notes and coins:
- Give students enough time to take out money corresponding to the given number.
- Ask the student to count the notes that (s)he took out from the box and note the count.

Count on

Q.4(a) Show the number-card '88' and ask, "which number is this?" "can you count on from here?" ("यह कौन सा नंबर है? इसके आगे गिनती करो।")

- Ask the student to count on from 88, stop when (s)he reaches 105.

Q.4(b) Show the number-card '995' ask to read it. Then ask, “count the numbers one by one 995 onwards”. (“995 से आगे गिनती करो।”).

- Ask the student to count by ones starting from 995, stop when they reach 1012.

Q.4(c) (To be done only if student is not able to do 4(b)). Show the number-card '595'. Then ask, “Count the numbers one by one starting from 595. (“595 से आगे के नंबर एक-एक कर बताओ।”).

- Ask the student to count by ones starting from 595, stop when they reach 612.

<u>'Count on' using money</u>

Q.5(a) "Here is an envelope with some money in it. Count and tell how much money is there in it? Now, I put one more rupee in it. How much is the amount now? " (यहाँ एक लिफ़ाफा है जिसमें कुछ रुपये हैं। गिनकर बताओ कुल कितने रुपये हैं। मैंने उसमें एक रुपया डाला।तो अब कुल कितने रुपये हैं?) (*repeat 8 times*).

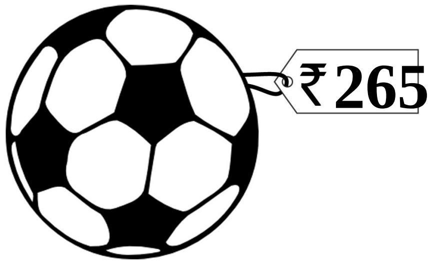
- Keep different envelopes with different amounts in them: "Rs 995", "Rs 80", "Rs 72", "Rs 970", "Rs 275", "Rs 800" and "Rs 9700".
- Keep Rs 995 in the first envelope. Ask her/him to count the money and put it back in the envelope.
- Check whether the student can add the amount without recounting all over again.

Q.5(b) "Here is an envelope with some money in it. Now I put a 10-rupee note in it. How much is the amount now?" (यहाँ एक लिफ़ाफा है जिसमें कुछ रुपये हैं। गिनकर बताओ कुल कितने रुपये हैं। मैंने उसमें दस रुपये डाले। तो अब कुल कितने रुपये हैं?) *(repeat the task 2 more times)*.

- Keep Rs 80 in this envelope and repeat the above task.
- Repeat the same task of adding 10 rupees for the envelopes with "Rs 72" and "Rs 970" till the respective amounts become Rs 92 and Rs 1020.
- Repeat the same task of adding 100-rupees for the envelopes having "Rs 275", "Rs 800" and "Rs 9700" till the respective amounts become Rs 375, Rs 1200 and Rs 10200.

Q.6(a) Look at these two pictures. Which one is costlier? How do you know? Costlier by how much? (इन दोनों चित्रों को देखो।इनमें कौन सा ज्यादा महँगा है? कितना ज्यादा? आपको कैसे पता चला?)

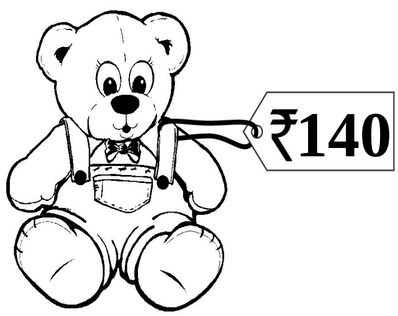
- Keep two pictures of two different dresses with price-tags on them. Show the pictures to the student while asking the question. Read out the prices mentioned in the tags.



Q.6(b) How much would you have to pay if you were to buy both the articles? (आपको कितनी कीमत देनी होगी अगर आप ये दोनों सामान खरीदते हैं?)

Q.6(c) Repeat the above question for the next set of two pictures.

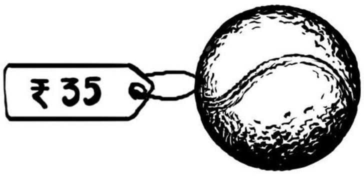
Look at these two pictures. Which one is costlier? How do you know? Costlier by how much? (इन दोनों चित्रों को देखो। इनमें कौन सा ज्यादा महँगा है? कितना ज्यादा? आपको कैसे पता चला?)

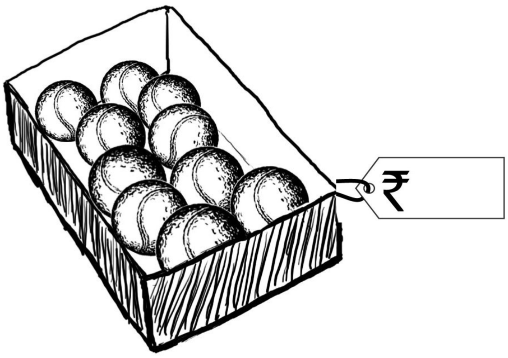


Q.6(d) How much would you have to pay if you were to buy both the articles? (आपको कितनी कीमत देनी होगी अगर आप ये दोनों सामान खरीदते हैं?)

Q.6(e) Suppose if you pay two 500-rupee notes to buy all the four articles, how much balance should you get back? (मान लीजिए, आपने चारों सामान खरीदने के लिए 500 रुपये के दो नोट दिये, तो आपको कितने रुपये वापस मिलेंगे?)

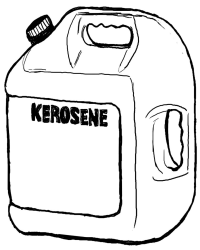
Q.7(a) If the price of a tennis ball is Rs 35, then how much would 10 tennis balls cost? (एक टेनिस बॉल की कीमत 35 रुपये है, तो दस बॉल की कीमत कितनी होगी?)





Q.7(b) One can of kerosene oil is enough for 16 days of cooking, then how many days' cooking can be done with 7 cans? (खाना बनाने के लिए किरासन तेल का एक डिबबा सोलह दिन चलता है, तो 7 डिबबे कितने दिन चलेंगे?)

Guess the answer (अंदाज़ से बताओ।)

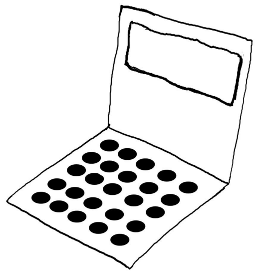


Q.8(a) 5 families together buy one large can of oil for Rs 315 from the wholesale market, and share it equally. How much money should each family pay? (5 परिवार-वाले एक होल-सेल मार्केट जाकर 315 रुपये में तेल का एक डिबबा खरीदा फिर आपस में बाँट लिया। अब हर एक परिवार को कितने रुपये देने पड़ेंगे?)

Guess the answer (अंदाज़ से बताओ।)

Q.8(b) बिंदी की एक पत्ती बनाने में 25 बिंदी लगती है, तो 400 बिंदियो से कितनी पत्तियाँ बन सकती है?

Guess the answer (अंदाज़ से बताओ।)



- Record student's verbal responses, if any.
- Then, ask her/him to do the sum on the sheet.

Q.9 Once during a festival you went to buy 'burfi'. The shopkeeper said, “20 burfis cost 42”. At this rate how much would you require to pay for 25 burfis? (किसी त्योहार में आप कुछ बर्फियाँ खरीदने मिठाई की दुकान पर जाते हैं। दुकानदार बर्फी की कीमत बताता है, “बीस बर्फियों के बयालीस", तो आपको पच्चीस बर्फियों के लिये कितने रुपये देने पड़ेंगे?)

APPENDIX – D

Protocol For Knowing Students' Knowledge About Their Work-Contexts

Guiding Questions: Children working at home/outside

Begin by chatting with them about the work that they do now. Have done in the past, etc.

Ask general questions:

- What do you like about the work?
- What is the skill needed?
- What do you need to know to do the work well?
- What are the other jobs associated with the work that you do (e.g. buying things)?
- Have you learnt anything useful from your work? How will it be useful?

[*Go over the different processes in the work. Get an idea of who does which part, what is done outside? What the adults do? What is given to the children, etc.?*]

- Is it possible for you to do what the adults do? Why or why not?
- What knowledge/ skill/ understanding do you lack for that job?
- What are the different steps in what you do? [*Now probe to get some detail of each part, what knowledge or skill is required? Do you need to figure out anything (hisaab or andaaz)? How much raw material, how much time?*]

Fairness questions: How much do you get paid in a day? Is it fair? How do you know it is fair? [*see what comparison the child does*]

Buying groceries, toffees, going to restaurants, etc. What kind of figuring (hisaab) out do you have to do? Do you feel that you get cheated sometimes? On such occasions, how do you take care so that you are not cheated?

Additional questions:

1. Do you need to weigh or measure the material that you use for the work? How do you do that? Use any scale, measuring units?
2. Who makes the drawings? How did you learn the job?
3. Do you buy materials for work, for home? When the money falls short, how do you decide which item to buy and how much?
4. Which time of the year does the workload increase? How do you meet the target?
5. How much do you get for the work? How often do you get money? What all do you do with it?
6. Do you get to use anything learnt at school in your work?
7. How much time do you spend working? Do you help others in working? How long have you been doing this work?
8. Did you do any other work in the past?

Appendix E

To
Shri Abasaheb Jadhav
Education Officer,
Municipal Corporation of Greater Mumbai,
Dadar (E), Mumbai – 14

HBCSE-TIFR,
Mumbai,
21 September 2010

Subject: Seeking permission for visiting municipal schools for collecting data for research study

Dear Sir,

I am a research scholar in Homi Bhabha Centre for Science Education (TIFR), pursuing my doctoral studies in Mathematics Education. HBCSE is a national centre of TIFR (Deemed University) and offers PhD programme in Science and Mathematics Education and is also the nodal centre of all science, mathematics olympiad activities in the country.

The data collection for my research study involves visits to schools and observe classrooms where students come from low socio-economic background as well as have an exposure to household based economic activities and other income generating practices. For this purpose, I have identified the following municipal schools which I would like to visit for the data collection:

1. [redacted] Municipal School, [redacted]
2. [redacted] Municipal School, [redacted]
3. [redacted] Municipal School, [redacted]

For my research study, I shall be interacting with the children of grades IV, V, VI and VII involving around 50 (fifty) children altogether and I would like to visit the Urdu and English medium sections.

It is for this purpose that I request you to kindly grant me permission to visit the above-mentioned schools and allow me to carry out my research study. I shall ensure that the routine and activities of the schools are not disturbed in any manner whatsoever because of my visit.

For this I shall remain grateful to you,

Thanking you,

Faithfully Yours,

(Arindam Bose)
Research Scholar (PhD Student),
Homi Bhabha Centre for Science Education (TIFR),
V N Purav Marg, Mankhurd,
Mumbai - 400 088
Email: arindam@hbcse.tifr.res.in
Ph: 0 98699 85183

Encl.
1. Letter from my research-guide
2. Synopsis of my research proposal

Appendix F

होमी भाभा विज्ञान शिक्षा केन्द्र
टाटा मूलभूत अनुसन्धान संस्थान
वि. न. पुरव मार्ग, मुम्बई 400 088

<u>सहमति-पत्र/इक़रार-नामा</u>

होमी भाभा विज्ञान शिक्षा केन्द्र (एच.बी.सी.एस.ई), 'टाटा मूलभूत अनुसन्धान संस्थान' (टि.आय.एफ.आर) मुम्बई का एक राष्ट्रीय केन्द्र है। इसका ख़ास मक़्सद प्राथमिक स्कूलों से लेकर ग्रेजुएशन तक विज्ञान और गणित की पढ़ाई में क़ाबलियत को बढ़ावा देना, देश (मुल्क़) में विज्ञान-शिक्षा की तरक़्की और समाजी तरक़्की को बढ़ावा देना, और खोज (तहकीकी) और सामग्री विकास करना शामिल है। पिछले डेढ़ सालों से हम [redacted] उर्दू और अंग्रेजी स्कूल में बच्चों के 'स्कूल के बाहरी ज़िन्दगी' से गणित के अलग-अलग पहलू सीखने के तरीकों को समझने की कोशिश कर रहे हैं। इसी सिलसिले में उर्दू और अंग्रेजी स्कूल के छठीं और सातवीं क्लास के बच्चों के लिए दो हफ़्तों का रियाज़ी पर स्पेशल क्लास (छुट्टी-कैम्प) रखा गया है जो 12 अप्रैल से 28 अप्रैल 2012 तक चलेगा। इन स्पेशल क्लासों का विडियो-रेकॉर्डिंग किया जा रहा है जिनका इस्तेमाल सिर्फ़ अनुसन्धान-कार्य (तहकीकी) और भविष्य के कैम्पों का ख़ाका तैयार करने में किया जाएगा। साथ ही साथ, इससे बच्चों की हुनर और ज़रूरतों का भी पता चलेगा।

इस कार्यक्रम में आपकी सहमति (रज़ामन्दी) ज़रुरी है। आपकी दी गई जानकारी गुप्त (खुफ़िया) रखी जाएगी।

तालीब इल्म का नाम : ______________________________

क्लास : ______________________________

पता : ______________________________

____

फोन नम्बर: ______________________________

वालिदेन का दस्तख़त : ______________________________

तारीख़ : ______________________________

English Translation of Appendix F

Homi Bhabha Centre for Science Education

Tata Institute of Fundamental Research

V. N. Purav Marg, Mumbai 400 088

<u>Consent form</u>

Homi Bhabha Centre for Science Education (HBCSE) is a national centre of the Tata Institute of Fundamental Research (TIFR). The broad goals of the Centre are to promote equity and excellence in science and mathematics education from primary school to undergraduate college level, and encourage the growth of scientific literacy in the country. For the past one-and-a-half year, we have been trying to understand the different aspects of children's acquisition of out-of-school mathematics in the [redacted] English and Urdu medium schools. In this connection, we have arranged a 2-week long special class (vacation course) on mathematics for students of Grades 6 and 7 which will be held between 12 and 28 April 2012. We have planned to video-record these special classes and the recordings will be solely used for research purposes and for planning future camps. In addition, this will help us know about the students' skills as well as their requirements.

We need your consent for this programme. Information provided by you will be kept confidential.

Name of the student: ________________________________

Grade: ________________________________

Address: ________________________________

____

Phone number: ________________________________

Parent's/Guardian's Signature ________________________

Date: ________________________

Appendix G

Work, Knowledge and Identity
Implications for school learning of out-of-school mathematical knowledge

Arindam Bose

Synopsis of Ph.D. Thesis
submitted in partial fulfilment of the requirements for the degree
of
Doctor of Philosophy
in
Science Education

Homi Bhabha Centre for Science Education

Tata Institute of Fundamental Research
(*A Deemed University*)

Thesis advisor: Prof K. Subramaniam

Chapter 1 : Introduction

1.1 Rationale and motivation for the study

The study was motivated by our observations during initial field visits and interaction with school students and non school going children from low-income settlements. We observed that children from such backgrounds often experience difficulties in learning mathematics in schools, face failure and in many cases drop out. We also observed that they gained considerable informal knowledge from outside of school which remains unacknowledged in the classroom. Researchers and philosophers have recommended the use of such informal knowledge to support classroom learning (NCERT, 2005; 2007). The present study explores the prevalence of out-of-school mathematical knowledge among students from the low SES backgrounds and the implications for teaching and learning mathematics in school.

Research studies on out-of-school mathematical knowledge of children and adults have been carried out in many cultures since the 1960s. These studies have explored the contours of out of school mathematical knowledge, the ways in which it is acquired, and how it is different from mathematics learnt at school. While such studies initially communicated a promise of reshaping school math education based on what was known about out of school knowledge, there is still a lack of clarity about the implications of such studies for school learning. In this study, we have explored the nature and extent of everyday mathematical knowledge possessed by middle grade school students living in an urban low-income settlement that has embedded in it a thriving micro-enterprise economy. Children living in this settlement either have exposure to the diverse work-contexts prevalent in the neighbourhood or participate in and contribute to the production and income generation right from an early age. In the course of our exploration of the nature and extent of the opportunities available to the middle graders to gather everyday mathematical knowledge, we have characterised the work-contexts from a mathematics learning perspective. Our purpose is to unpack and document the connections between students' mathematical knowledge, work practices and identity formation, and inquire into the implications of these connections for school learning.

1.2 Need for drawing on out-of-school math knowledge during formal math learning

In mathematics education research (henceforth, MER), it is increasingly felt that learning mathematics can be helpful for students if the classroom teaching involves familiar contexts and methods. The major educational policy document that is currently followed in India, the *National Curriculum Framework* (NCERT, 2005) points out that "learning takes place both within school and outside school" and that "learning is enriched if the two arenas interact with each other" (p. 15). The Framework gives importance to connecting school learning with the child's lived experience, "not only because the local environment and the child's own experiences are the best entry points into the study of disciplines of knowledge, but more so because the aim of knowledge is to connect with the world" (p. 30). Connecting with the child's environment also has a role to play in creating an educational culture that is equitable. "Our children need to feel that each one of them, their homes, communities, languages and cultures, are valuable as resources for experience to be analysed and enquired into at school; that their diverse capabilities are accepted" (p. 14). The position paper of the Focus Group on the teaching of Mathematics expresses the same concern and emphasises the use of "experience and prior knowledge" to construct new knowledge in school mathematics (NCERT, 2006, p. 8). It is therefore of importance to a community of mathematics educators to investigate the kind of mathematics children draw from the outside world and the possible bearing that such knowledge might have on their learning of school mathematics.

1.3 Organisation of thesis

The thesis is organised into eight chapters. Chapter 1 gives an introduction to the thesis, places the background and the context in which the research was undertaken by presenting the motivation and the rationale behind the study and its scope and limitations. Chapter 2 presents the relevant literature in the domain of out-of-school mathematics, mathematics in work-places and also takes a look at the curricular documents currently followed in India. Chapter 3 discusses the research questions and the research design, location of the study and its significance, sample, methods and ethical considerations that informed the study. Preliminary findings that function and set the context of the study are presented as well. Chapter 4 presents the work profiles of the sample students and their parents to give a picture of the diverse work-contexts that children are immersed in. Descriptive reports of four cases of students who participate in work contexts are presented, focusing on the mathematical elements embedded in their work practices. Chapter 5 analyses the opportunities and affordances available in diverse work-contexts and in everyday settings (drawn from case studies and supplementary data) for school learning of mathematics.

Aspects of the participants' identities as learners in out-of-school contexts are analysed here. Chapter 6 unpacks the measurement knowledge embedded in various work-contexts emergent from the students' semi-structured interviews and discusses implications for the school learning of this topic. Chapter 7 presents an analysis of a teaching design experiment focusing on integrating students' out-of-school measurement knowledge. Chapter 8 concludes the thesis by discussing the results and findings of the study and presents possible curricular and pedagogic implications and future road-maps.

1.4 Limitations

The interaction with students and conducting interviews with them was a challenge. Though the researcher conducted the interviews after building a personal rapport with the students, there could be many invisible factors that might not have emerged in the responses. In such an eclectic exploration, there was also an ethical consideration about how deep one can probe about family and work details. At times, respondents might have felt uncomfortable to respond despite best of efforts by the researcher to create a comfortable environment. These affective factors could have limited the scope of the interviews. The vacation course following the teaching design experiment though conducted in the actual classroom set-up was a short, two-week long experiment. The implications drawn are thus indicative and need to be explored through more extensive, long-term work.

Chapter 2: Setting the Context

2.1 Out-of-school and School Math

Out-of-school mathematical knowledge (also termed as "everyday" math, "street" math and "informal" math) of children has been studied extensively beginning with the pioneering work of Nunes, Carraher and Schliemann (1985) and other authors in the Latin American and African countries (Lave, 1988; Saxe, 1988). Most of these studies referred to "out-of-school mathematics" as the form of mathematics that people make use of in everyday settings while engaging in contextually embedded practices, viz., work-contexts, shopping, house-hold activities, games and so on. These studies have focused on the work-place activities of street vendors, carpenters, fishermen, farmers, construction site foremen, tailors, carpet-weavers, grocery-shoppers, and provided a systematic comparison between the "everyday" and "school" mathematics primarily considering them as two forms of activities based on different cultural practices but on the same mathematical principles (Nunes, Schliemann & Carraher, 1993). Table 2.1 below summarises the distinction between school and everyday math that emerged in the literature.

Difference	Out-of-school Mathematics	School Mathematics
Basic feature	-Based on shared cognition (Resnick, 1987) -Manipulations are carried out using quantities -Use of group work and division of labour (Resnick, 1987) -Use of tool manipulations	-Based on individual cognition (Resnick, 1987) -Manipulations are carried out using symbols (Resnick, 1987) -Individual, independent work -Use of pure mentation
Goal	-Situation specific competencies	-Generalised learning, power of transfer (Resnick, 1987)
Difference in numeration/procedure	-Orality -Use of multiple units and operations (Saxe, 1988) -Use of contextualised reasoning (Resnick, 1987) -Use of decompostion and repeated groupings (Carraher et al., 1987) -Use of convenient numbers (Nunes, et al., 1985)	-Written -Use of symbols -Use of formal reasoning -Use of formal algorithms taught in schools
Mechanisms of acquiring knowledge	-Communication, Sharing, Legitimate Peripheral Participation (Lave and Wenger, 1991) -Learning from one-another, Circulates in communication, Role of artifacts and language (Carraher et al., 1987)	-Knowledge acquisition and knowledge building is textbook based -Based on individual thinking, group-work is not always encouraged
Meta-cognitive awareness	-Confidence in procedures, meaningfulness of obtained results (Nunes et. *al.* 1985; Saxe, 1988) -Continuous monitoring ('where they are' in the middle of calculations) (Carraher et al., 1987)	-Heavy use of algorithms, lack of meaningfulness and relevance -Continuous monitoring usually not possible
Test of the acquired knowledge	-No formal examination -Tested by seniors/experts through observations	-Use of formal examinations, consisting of mostly written tests

Table 2.1 Distinction between Everyday Math and School Math

Research on out-of-school mathematics has highlighted instances of meaning making and reasoning as embedded in work-contexts (Carraher & Schliemann, 2002; Nasir, Hand & Taylor, 2008). While solving problems in everyday contexts, participants operated meaningfully with quantities, made intermediate checks if the numbers obtained were reasonable, and used flexible procedures that were based on sound mathematical principles. Problem-solving strategies in everyday contexts were in stark contrast to the symbol pushing, mechanical implementation of procedures and tolerance of absurd solutions that characterised school mathematical performance (Khan, 2004; Nunes, Carraher & Schliemann, 1985; Saxe, 1988). This led to researchers exploring how to integrate out-of-school mathematical knowledge with school mathematics. Subsequently, researchers also raised doubts about the usefulness of everyday mathematics for school learning by pointing to the very different ways in which mathematical knowledge is acquired within and outside of school, and the very nature of the enterprise of school mathematics (Carraher & Schliemann, 2002; Dowling, 1998). It was argued that the goals of both the domains are different.

More recent studies have contested the distinction perspective adopted earlier between out-of-school and school math and claimed that they may not be mutually exclusive or dichotomous (Nasir, Hand & Taylor, 2008). It has been argued that there exists shared relationships between them hinting at the hybridised nature of mathematics that students gather. However, the nature of hybridity of mathematical knowledge has not emerged explicitly through empirical findings though there are claims about students constructing knowledge from their experience in different settings. From the cultural perspectives, school mathematical learning is also a cultural form. Pedagogical approaches informed by such perspectives seek to blur the boundary between culture and domain knowledge and allow multiple points of connection to form a body of knowledge that has overlaps of different forms of mathematics (Abreu, 2008). Further, school education and mathematics teaching is “not only about building on what students are familiar with... but also about introducing new ideas, concepts and sensibilities” (Nasir, Hand & Taylor, 2008, p. 220).

Indian studies of mathematics in diverse work-contexts

Studies done in India have highlighted that different procedures and strategies adopted in work-contexts were often governed by the situation-specific requirements depending upon the diversity of goods handled and requirement of varied calculation. Khan's (2004) study of the *paan* (betel leaf) vendors in Delhi or Naresh's study of bus conductors in Chennai (Naresh & Chahine, 2013) or Sitabkhan's (2009) study of child vendors in Mumbai's suburban trains indicated that diversity of goods handled helped the doers acquire greater proficiency with computations and often determined their problem-solving strategies that were distinctly different from the regular school procedures. Similarly, studies that looked at the development and use of measurement knowledge, viz., those by Mukhopadhyay (2013) and Saraswathi (1989) in their respective work on boat making and agricultural labour, emphasised that spatial visualisation and estimation skills often shaped the measurement knowledge and proportional reasoning in work-contexts. Though these studies underlined that diversity of work-contexts creates affordances for innovating newer, context specific problem-solving strategies, the possible pedagogic implications remained elusive. To our knowledge, there are few studies focused on children's out-of-school knowledge of mathematics in Mumbai (other than Sitabkhan's), although it has a large population living in low-income settlements, which are often economically active centres of house-hold based micro-enterprise.

Role of Work and Education

The participation of children in work is a complex issue, enmeshed in questions about the notion of childhood, the role of education and the exploitation of children. In India, debates about child labour as a form of exploitation are a central part of the debate on the right to education. While it is undeniably the case that many children suffer economic and other forms of exploitation, it is important to recognize that conceptions of childhood can be different for different cultures and for different communities (NCERT, 2007; Vasanta, 2004). In particular, for children from low socio-economic background, work is a part of the experience of childhood and a site for learning. We feel that school education should not drive a wedge between such experiences and classroom learning, as is often the case. The recommendations of the NCF, that the knowledge children gain from work contexts should be seen as a means of connecting school learning with out-of-school experience, are hence an important corrective to the “bookish” knowledge dispensed in schools in India.

2.2 Analysing learning through the lens of identity

In the recent years, in MER, the notion of *identity* has emerged as an important construct in understanding how out-of-school experiences can influence classroom learning (Nasir, Hand & Taylor, 2008). The notion captures the growing belief in the MER community that interpersonal and affective relationships have a bearing on learning (for example, Boaler & Greeno, 2000; Cobb, Gresalfi & Hodge, 2009). However, there are not many studies in MER with analyses using this notion as a tool. None of the above studies adopted any particular operational definition of *identity*, instead they commonly drew the notion from the narratives of or about individuals (Heyd-Metzuyanim & Sfard, 2012). In MER, studies have typically used the lens of *identity* in two ways: constitution of normative identity as learners of mathematics have been explored in the classroom, and affective factors like emotional hue have been analysed by looking at the narratives of identifying and subjectifying the students themselves or between them or the teacher. Current educational discourse on *identity* seeks to replace the widely used motivational notions of *beliefs* and *attitudes* which are seen as discourse-independent (Sfard & Prusak, 2005).

2.3 Funds of Knowledge

It is widely seen that children in low-income conglomerations are often bound in social relationships and work practices from an early age and the broad features of their learning develop at their home as well as in their surroundings. Households and their surroundings contain resources of knowledge and cultural insights that anthropologists have termed as *funds of knowledge* (FoK) (Gonzalez, Moll & Amanti, 2005; Velez-Ibanez & Greenberg, 2005). The "funds of knowledge" perspective brings to mathematics education research insights that emphasise the hybridity of cultures and the notion of "practice" as "what people do and what they say about what they do" (Gonzalez, 2005, p. 40). The perspective also opens up possibilities of teachers drawing on such funds of knowledge and relating it to the work of the classroom (Moll *et. al*, 1992). When FoK are not readily available within households, then they are drawn from the networks in the community. The perspective thus emphasises social inter-dependence and shows children in households to be active participants, not passive by-standers.

We use the notion of "funds of knowledge" to inform the analysis of work contexts that students are exposed to, and in illuminating the nature and extent of everyday mathematical knowledge available within the community of the classroom. We look at FoK as a resource pool that emerges from people's life experiences and is available to the members of the group which could be households, communities or neighbourhoods. In a situation where people frequently change jobs and look for better wages and possibilities, community members need to possess a wide range of complex knowledge and skills to cope with and adapt to the changing circumstances and work contexts and to avoid reliance and dependence on experts or specialists.

2.4 Funds of knowledge and pedagogical implications

Socio-cultural studies in mathematics and science education have argued that cultural resources and *funds of knowledge* (Gonzalez, Andrade, Civil & Moll, 2001) of people from non-dominant and underprivileged backgrounds are often not leveraged (Barton & Tan, 2009) in school teaching and learning practices. Neither is their knowledge from everyday life experience valorised (Abreu, 2008) and built upon in the classrooms nor is their identity acknowledged.

Educational thinkers in the developing world, and particularly in India, have recognized the value of work experience for education conceived in a broad sense. Educational philosophers, such as Gandhi developed a vision of education centred around productive work and different from the traditional education in the crafts. The aim of his educational philosophy *Basic Education* or *Nai Talim* was not training in a particular craft, but a "well rounded education of the mind, the body and the heart" (Gandhi as quoted in Fagg, 2002). Gandhi argued that "the proposition of imparting the whole of education through the medium of trades (crafts) was not considered [in earlier days]. A trade (craft) was taught only from the standpoint of a trade (craft). We aim at *developing the intellect also with the aid of a trade or a handicraft...* we may… educate the children entirely through them" (NCERT, 2007, p. 4, italics in original).

In the context of a developing society like India, in contrast to societies with advanced economies, participation of school children from low socio-economic backgrounds in work either within the household or in the neighbourhood allows integration of children into social networks that generate *funds of knowledge*, and makes this knowledge present and available in the classroom. Taking on board this insight, the current policy document in India, the National Curriculum Framework urges educators to draw on work experiences as a resource for learning. It points out that "productive work can become an effective pedagogic medium for (a) connecting classroom knowledge to the life experiences of children; (b) allowing children from marginalised sections of society, having knowledge and skills related to work, to gain a definite edge and respect among their peers from privileged sections; and (c) facilitating an appreciation of cumulative human experience, knowledge and theories by building rationally upon the contextual experiences" (NCERT, 2005, p. 6).

Chapter 3: The Study, Setting and Style of the Research Study

3.1 Research questions

The main research objective of the study is to explore the implications of everyday mathematical knowledge prevalent among the low income students exposed to work contexts for learning school mathematics. This has been elaborated in the form of specific research questions as below.

Q.1 What is the nature and extent of out-of-school knowledge of mathematics prevalent among middle graders from urban, low SES backgrounds?

Q.2 What are the everyday contexts and situations in which school going children of 10-12 years of age have opportunity to gain and use mathematical knowledge?

Q.3 What are the overlap and differences between the out-of-school and school mathematical knowledge?

Q.4 In the topic of measurement specifically, what out-of-school knowledge do students gain and what are the implications for the school mathematics curriculum?

Q.5 How can mathematical knowledge gained from everyday and work-contexts be integrated with school learning so as to enhance students' conceptual understanding of mathematics?

3.2 Location of the study

The study was located in central Mumbai's large, densely populated low-income settlement which has a vibrant economy in the form of micro and small enterprises dispersed in house-hold based workshops and manufacturing, trade and service units with high economic output. The entire neighbourhood generates huge employment opportunities. Being an old and established settlement, this low-income area attracts skilled and unskilled workers from all parts of India who come to the financial hub of Mumbai in search of livelihood. Generally the single-room, small and low-height dwellings are used for dual purposes – as workshops and as living room for the family and the workers. The settlement is thus a co-location of workplace and home for most of its residents. Practically every house-hold here is involved in income-generating work and children start taking part in them when they are young. Even such children who do not participate in work also develop fair knowledge and reality perspective about the activities and diverse work-contexts around them by virtue of the high levels of social interaction prevalent in the neighbourhood. The settlement is multilingual, multi-religious and multi-ethnic. Common house-hold occupations include embroidery, *zari* (needle work with sequins), garment stitching, making plastic bags, leather goods, textile printing (dyeing), recycling work, pottery, food cooking and delivery and so on. The goods produced in this locality are not only sold in Mumbai but also exported.

Two government schools located in the settlement and run by the local civic body were chosen for the study - an English and an Urdu medium school co-located in a five-floor school building which also houses three other schools with different languages of instruction. All these schools draw students from the neighbourhood.

3.2.1 Significance of the location

The low-income settlement is economically active with resource-rich, diverse work places and communities of work practice which create varied opportunities for school going children to gather everyday mathematical knowledge. Learning sites for children of this settlement apart from the regular schools are house-holds engaged in work, diverse work-contexts, tuition classes, shopping and house-hold chores. People in this settlement maintain strong social connections and are well networked with their employers, middle-men, distributors, shopkeepers, friends and relatives. From our interactions with the community members, it appeared that the social relationships are mostly economy driven. The entire neighbourhood creates opportunities that expose children living here to the *funds of knowledge* available within the community.

3.2.2 Socio-cultural and socio-economic scenario

Participants in the study belonged to immigrant families living in low socio-economic conditions. Most women in the settlement are engaged in some house-hold based micro enterprise in the locality, while men either run their own workshops or small business, or are employed in one. We noted that parents of the students had varying years of schooling including no schooling. The settlement consists of heterogeneous groups of residents belonging to different ethnicity, language, religion and socio-cultural background.

3.3 Style of the research study

Establishing access to the field

The research study done over a period of two years and a half, was conducted in phases. Access to the field was established with the help of the key informant – a 74 year old resident of the settlement and a community leader, well regarded for his social work in the settlement. He guided the researcher in getting necessary permissions from the civic body (Municipal Corporation) for the study and introduced him to the English and the Urdu school authorities to begin classroom observation and interaction with the teachers. The researcher started visiting English and Urdu medium schools daily and began to observe mathematics lessons in Grades 5 and 7. Such visits helped in building rapport with the teachers, students and other people on the staff. The researcher held informal discussions with students, teachers and visited students' homes and workplaces. Discussions with the students helped to get a broad picture of the nature of their daily activities that have aspects of mathematics and the nature and extent of their everyday mathematical knowledge.

3.3.1 Research design and methodology

The research study followed a blend of ethnographic, case study and teaching design experiment methods in broadly three overlapping phases discussed in the following table:

Phase		Objective	Sample	Method	Data Sources
I (Ethnographic)		Exploration of the children's life-world and opportunities	Classroom observation: Grade 5 (Urdu); Grade 7 (English)	Building a rapport with the students, teachers and community members; classroom observation; visits to field, workshops, students' home, shops.	Field notes Photographs
II (Case-studies)	Part-I	To understand family background, SES, students' outdoor activities, engagement in work, parental	Randomly selected representative sample of 31 students	Semi-structured interviews	Audio records Transcripts Students' worksheets

		occupation			
	Part-II	To understand students' basic arithmetical knowledge	30 students from the previous sample	Interviews based on structured questionnaire	Photocopies of students' “tuition” notebooks
	Part-III	To focus on students' knowledge about their work	Sub-sample of 10 students + 7 additional students who volunteered	Semi-structured interviews	
III (Teaching Design Experiment)		Connecting students' out-of-school math knowledge & school learning Exploring role of students' knowledge and identity	About 25 Grade 6 & 7 students of Urdu school and 3 Grade 6 students of English school	12-days' Teaching intervention camp	Classroom videos & logs Transcripts (first 2 lessons)

Table 3.1 Research Design & Methodology

Phase-I (ethnographic exploration) had an overlap with *Phase-II* (case studies) and *Phase-III* and continued till the end.

3.3.2 Strategies and Instruments for Data Collection and analysis

- Classroom observation
- Teaching as a participant observer
- Prolonged engagement of around three years with the field (visits & recordings)
- Exploratory interaction
- In-depth interview

Data transcripts & Coding scheme

All the students' interviews about work-contexts and the two lessons from the teaching intervention in Phase-III were fully transcribed for coding and further analysis. Written logs of the lessons in Phase-III supplemented the transcripts of the lessons. The interview transcripts were coded at first and second levels to review what they indicated about the nature of students' work and their knowledge about aspects of the work. The coding was done separately by the researcher and his colleague and coded for 10 different categories: work, learning, everyday math, affect, foreground, personal, and others, and the differences were reconciled through discussion. Lesson transcripts from Phase-III were read together with logs by three researchers. Segments of the transcripts relevant to the research questions were identified and carefully reviewed.

3.4 Ethical considerations

The researcher ensured that respondents' and informants' anonymity was protected. The researcher conveyed to the participants the purpose of the study in broad terms and did not engage in deception of any kind about the study. In this dissertation thesis, the researcher has not included such information that participants shared with him on “good faith” or to be kept “off the record”. Such information has not been made part of the analysis while ensuring that the analysis is not inconsistent with such information.

3.5 Overview findings: Children's life-world

This section discusses findings based on visits and interaction with students and the community, from the classroom observations, from the records of students' work in the tuition classes and from other learning sites (not included in synopsis).

Chapter 4: Participation in work-contexts and mathematical knowledge: case studies

This chapter presents detailed findings from the interview data of Phase-II of the study concerning students’ immersion in work contexts and their mathematical knowledge. A total of 31 students from the two grades were interviewed to obtain basic information about family background and participation in work. We observed that 30 of the 31 students were engaged in some work practice. We summarise the students' and their parents' work-profiles in Tables (not included in the synopsis). An in-depth semi-structured interview was conducted with a sub-sample of 10 students and 7 additional students about their knowledge of work-contexts and mathematical elements embedded in work-contexts. This is analysed through the case study approach and we discuss four case studies chosen from the reduced sample of 10 students in detail in this chapter. We use information from the remaining interviews to round-off the findings of the case studies to arrive at a reasonably comprehensive picture of the knowledge of work-contexts and of mathematics of our participants.

4.1 Case Studies

In this section, four representative cases are reported, two from the English medium and two from the Urdu medium schools (boys – E_5 and U_2, girls – E_{16} and U_{13}), with a focus on their exposure to work-contexts, their knowledge of arithmetic and measurement and the mathematics related to earning. The four cases described were chosen since together they reflected four different kinds of work-contexts with a range of opportunities for mathematics learning, viz., independent collection of material for recycling (E_5), an employee in tailoring work who goes through different learning stages (U_2), a girl with exposure to diverse kinds of stone-fixing work on jewellery done at home and of running a stationary shop (E_{16}), and a girl with relatively limited exposure to work-contexts (U_{13}) (Bose & Subramaniam, 2013). An embedded analysis of the cases has been preferred over a holistic analysis as the intent of conducting case study was to understand the extent of mathematical elements embedded in each work-context and students' knowledge of them. Hence, an analysis of themes (viz., knowledge of work context, mathematical knowledge, measurement knowledge, mathematics of earning) is presented focusing on the aspects of mathematics emergent from each work-context.

4.2 Drawing the cases together

Connections between out-of-school and school mathematics

From a viewpoint of looking at the overlaps between out-of-school and school mathematics, it was observed that use of inch scales and other standard and non-standard units/scales are more common in work practices than in schools. Some students showed more reliance over formal algorithms and used them in maintaining accounts or doing calculations. On the other hand, students like E_{16} preferred to use their own convenient and situation-specific strategies despite having learnt formal algorithms well. E_5 and U_2 had similar reliance on their own strategies for computations. Such characteristic features of students' work-contexts and everyday experience indicate the hybrid nature of mathematical knowledge prevalent among children and they draw from both school and everyday mathematical experience. Most mathematical procedures that the students used show inter-penetration of both school and out-of-school mathematics.

Most of the students that the researcher came across spoke about the work-contexts happening around them with confidence showing access to *funds of knowledge* which included not only the kinds of work that they themselves participated in, but also about other work-contexts that occur in the settlement. This phenomenon was interesting since within a single class, students had peers who were engaged in diverse work practices and created opportunities for learning about them.

Issues of fairness

Fairness is seldom taken into consideration in the world of work, which is governed far more by possibilities and bargains. For poor children in the metropolis, fairness is not easy to grasp. To cite an example, when the researcher discussed with U_{13} whether she was satisfied with the wage for making *Rakhi* (decorative wrist-bands) she answered in affirmative. On asking she could only tell the retail price of one dozen *Rakhi* – at least Rs 60 (one *rakhi* is sold for Rs 5; 1 USD = Rs 60 approx.), whereas for making one gross (12 dozen) *rakhi*, she gets Rs 15 or less. The researcher helped her calculate the retail price of one gross *rakhi* – Rs 720 and compared it with her wage (Rs 15 or less), but the discussion did not trigger any concern about fairness of wages in the student. Here is an occasion where knowledge of mathematics can possibly lend power to call for fairness and justice (Bose & Kantha, forthcoming).

Gender aspects of work-context

In some work-contexts, especially those which are typically done by women and girls at their home as in the case of U_{13} and E_{16}, the opportunities to use diverse goods or raw materials or awareness about the linkages that their work has with other tasks on the production network are largely constrained. Women in the community and school going girls like E_{16} or U_{13} are mostly involved in those kinds of work which require working at home. In the case of U_{13}, she did only a small chunk of the entire *rakhi* work or *garment manufacturing* work. Although her work was large in terms of quantity of output, there was little diversity in the work.

The examples emergent through case-studies underline our claims that the whole gamut of everyday experiences including diversity of cultural and work practices shape students' everyday mathematical knowledge and has structural difference with school mathematics. However, the inter-penetration between everyday and school mathematics indicates that learning in one domain has relevance for the other which remains to be unpacked.

Chapter 5: Learning, mathematical knowledge and identity in out-of-school contexts

In this chapter, we analyse the data from the case-studies to show how opportunities for learning in general and learning mathematics in particular arise in work contexts. We discuss aspects of the mathematical knowledge gained in out-of-school contexts and its relation to school mathematics. We also discuss how work-contexts shape the identities of participants in our study as learners. Finally we draw some implications from these analyses for the teaching and learning of school mathematics aimed at making connections with out-of-school knowledge.

5.1 How do work contexts create opportunities for learning?

Features of work contexts and the degree of students' engagement in them shape the learning experience of students who participate in the work-contexts and the richness of the knowledge that they acquire. From an analysis of the data, we discuss how opportunities arise for learning in work contexts under three rubrics (Bose & Subramaniam, 2013):

- Diversity
- Making decisions in relation to work; optimising resources and earnings
- Involvement in the work; awareness of linkages

Handling of diversity of goods and requirement of the tasks, control over and extent of decision making, need for optimisation, knowledge of backward and forward linkages are strongly related to the sense of ownership that participants have about their work. Study participants whose close relatives, friends or families own businesses have a stronger sense of ownership of the work, in comparison to those who work merely for wages. Such involvement creates greater opportunities to gather and use mathematical knowledge. For example, in the case of E_5 (garment recycling), U_{22} (mobile repairing), U_{23} (textile printing) and U_{24} (ready-made garment selling), we noticed that these students had a sense of ownership and were aware of diverse aspects of their work as well as the forward and backward linkages that the work had. Except for E_5, these students did not participate in the work primarily for the income, but rather also to learn something and to pick up useful skills which are valued as they are perceived as securing opportunities to get future employment. For example, U_{22} took pride in knowing about both kinds of work – mobile phone repairing as well as garment stitching work. His father runs a shirt stitching workshop where three other workers are employed and U_{22} does not particularly need to earn to support family as is the case with several other children. In the case of E_5, U_{22}, U_{23}, and U_{24}, where the sense of ownership and control over decisions was strong, frequent references were made to decisions over deals. In the case of some students, we noticed a reluctance to use mathematical calculation to engage with questions of fairness of income, and in some cases inappropriate use of calculation.

5.2 Features of participants' mathematical knowledge in relation to out-of-school context

We identify knowledge that students demonstrate as out-of-school knowledge on the basis of features identified in the literature. These include the form in which the task is presented and accompanying contextual details which elicit student knowledge, the presence of oral computation strategies and the reference to mathematical entities that do not appear in the school curriculum such as words for binary fractions. In the interview, arithmetical tasks that were both context-rich and presented in purely symbolic forms were used. Students' performance in context-rich forms was slightly better. Tasks which were formulated in rich contextual detail usually elicited oral computation strategies and factoring in of reality perspective. On several occasions participants used school math knowledge in the form of formal algorithms like unitary method as well as oral computation strategies. Although the tasks included prompts located in the out-of-school contexts, participants' responses often began with using school learnt method, subsequently falling back on their out-of-school math knowledge. Thus on multiple occasions, we found students using methods that they had learnt at school together with those that were likely not taught explicitly at school.

Students' response to arithmetical tasks allowed us to get a sense of the kinds of mathematical elements embedded in out of school contexts, and also the nature of such knowledge. We describe aspects of such out-of-school mathematical knowledge of the study participants.

Limited combinations and fragmented knowledge: A feature that we noticed about the mathematical aspects embedded in work contexts was that variation was limited to what the context itself included. Thus the mathematical experience of students was constrained and limited in terms of variations and possibilities, the exploration of which is an essential part of mathematical abstraction. For example, E_{16}'s stone-fixing work required her to make arrays of the finished jewellery pieces in only limited arrangements: 6×4 or 12×2 arrays on a card so that six such cards put together can make one gross (144 units). Though she knew about such combinations reflecting familiarity and ease of calculation, it was not clear from her responses whether she considered other combinations or total quantities other than 144.

Similarly, students displayed familiarity with inch and metre for length measurement, but were unaware of the connection. In the neighbourhood shops, small quantities of milk are sold by weight as well as by volume (interchanging of volume and weight units: “kilo” instead of “litre”), but our participants did not show awareness of the relation between these two measures. Thus everyday contexts give rise to pieces of mathematical knowledge that may be intimately familiar to students but may be unconnected to other mathematically related pieces of knowledge. The familiarity and confidence that students display about what they know suggests however that even such fragmented knowledge can be a potential resource for classroom learning.

Knowledge for use rather than conceptual knowledge: In the work-contexts, non-transparent mathematical artefacts are used which are familiar to the users in practice but the conceptual underpinnings are blurred. For example, inch tape is used for quantification of length, but the principles underlying its construction remains unclear to the users. Students may be aware that length can be measured (quantified) by iteratively covering with a unit, but may not be aware that this principle is the basis for constructing the length units (as was revealed in the teaching intervention discussed in Chapter 7). Similarly, construction of small weight measures like small stone markers or sub units marked on a scale remain unclear. Students may know about the use of length dimensions for designating “size” (area), such as in frame sizes, but the connection between length and area remains opaque. To take another example, it remains unclear as to how the numbers or labels designating the garment-sizes are arrived at and what they actually signify. In the teaching interaction phase of our study, we attempted to address this issue and the participants (middle graders) who took measures of different parts of the garments given to them as part of the activity, could not actually see the relation between the “size number” and any of the measures. We therefore argue that in most everyday or work contexts, knowledge and mathematical artefacts though frequently used, remain opaque and non-transparent when it comes to the conceptual knowledge associated with those artefacts.

The features discussed above have implications for school teaching and learning which aims at making connections with out-of-school knowledge of mathematics. The implications are explored in the context of a particular topic area, that of measurement, in chapter 7.

5.3 How does out-of-school knowledge shape learners' identities?

Our interaction with the community members indicated that learning of work skills as well as learning at school – both are valued in the community. Though we came across many children during the study who dropped out of school for different reasons, most parents and elders we came across seemed to be concerned about their children's school learning while at the same time wished their children learnt hand skills in their spare time.

Value of learning hand skills (*haath ka kaam sikhna*)

Learning *haath ka kaam* (hand skill) is seen as laying a foundation and making it easier to get *acchha kaam* (good job) in the future. This view was frequently echoed by students. Getting involved in “any kind of work” (*koi bhi kaam*) is valorised in the community since it builds networks with people including *seth* (workshop owners who provide jobs), helps in learning hand skill and utilising time in a better way. It is believed in the community that learning hand skill early would be “useful later on” (*aage kaam aayega)* to “learn something different” (*kuchh alag seekhne ko milega*) and also to earn more. “*Time barbad nahin karna”* (not to idle away time), “*khali nahin baithta”* (not to sit idle) and “*samay ka sahi istemal”* (proper use of time) are other phrases that students frequently used. This could be a reason why such children also work whose families do not need to supplement their income.

Value of school learning

As described before, school learning is valorised in the community and seen as a gateway to future opportunities. Graduating from school is taken as a benchmark and parents often urge their children to complete schooling. The excerpt presented below highlights how E_5's parents and relatives suggest that he continue his studies so that he can be placed in a job:

456	S	meri ammi boli barawi padh le, mera babora bola chaudha padh lega to tere ko police ki naukri mein daal dunga/	My *ammi* (mother) is asking me to study till twelfth, my uncle said if you study till fourteenth then I will put you in the police's job/

In the excerpt below, U_2 reiterates his interest in learning mathematics. On earlier occasions too, he expressed his concern to the researcher that he is not good at math and he wants to learn it well.

573	S	Main bus sir math padhna chahta hoon/ math achhi se koi padha de na...	Sir, I just want to study math/ if only someone can teach me math well...

Identities in work-contexts

Students identify themselves as knowledgeable, with skills and capabilities in work contexts. They often described their work with pride. For example, E_5 mentioned his record collection, when he once collected 95 kilos of *chindhi* and earned Rs 640. This functioned as a benchmark and helped him to decide how much time to spend on a given visit. Most students described their work arithmetically and with many precise details. Students often invited the researcher to their workplaces/workshops and explained their work and the related work processes. The researcher was able to visit U_{21}'s garment stitching workshop and U_{25}'s *zari* workshop several times on their invitations. During these visits, the researcher was invited to learn a few skills like holding the needle used in *zari* work and picking the sequins. In the process, the researcher's action was evaluated and often corrected. The researcher came across many students outside the sample who were keen to talk about their work-contexts. This included the seven students who volunteered to take part in the in-depth interviews as well as other students.

5.4 Implications

Work-contexts have rich resources and opportunities for mathematical learning though such contexts often extend only limited possibilities of using mathematics in the form of “use knowledge” devoid of the requirement of conceptual understanding. However, the affordances created by the work-contexts can scaffold school learning towards conceptual development. The case-studies indicated the formation of identities among the participants through their out-of-school knowledge. These implications are also important for school experience which as of now tends to reinforce the disconnect between out-of-school and school learning.

Chapter 6: Opportunities and Affordances for Measurement Learning

This chapter elaborates on the implications for school mathematics learning of out-of-school knowledge of our participants by focusing on the topic of measurement. The discussion begins with the research literature on measurement knowledge in out-of-school and in school learning contexts.

6.1 Measurement in the everyday context

Previous research on measurement within work-contexts or in other everyday settings was carried out alongside or within the research on out-of-school mathematics, with a particular focus on the alternative ways of thinking in different everyday contexts. Such research provided evidence of how mathematical ideas were developed and framed within work-contexts.

Most of these research studies focused on participants' measurement knowledge involving adults in their singular work-contexts. We have not come across studies that looked at the varied contexts in the everyday settings that students from low socio-economic backgrounds are exposed to and the affordances of these settings for school learning about measurement. The literature mentioned above has led to a cumulative understanding of the skills, procedures and strategies based on mathematical principles that are acquired in out of school work contexts. The focus has been on oral computation strategies, proportional reasoning strategies, visuo-spatial and geometric reasoning and estimation skills and strategies. In our study, we restrict focus to the topic of measurement, but take a broader view of not only what our participants know or can do, but also what they have observed and are familiar with even if the mathematical knowledge associated with these aspects is partial and fragmented. Our perspective is to explore what aspects can serve as starting points or building blocks for mathematical exploration in the classroom. We are also interested in how mathematical learning can strengthen the understanding of measurement practices in the real world.

6.2 Research on the learning of measurement as a school curriculum topic

Current Indian mathematics textbooks do not adequately cover many of the key ideas underlying measurement learning such as conservation, transitivity, equi-partitioning, iterative covering, additivity and role of scales. A look at textbooks prescribed by the central and state governments (followed by the vast majority of students in India) reveals that the dominant emphasis is on acquiring measurement skills and on knowledge of the international system of units (for example, Maharashtra Math Textbooks 5, 6, 7, 2006; NCERT Math textbooks 5, 6, 7, 2006). Conceptual issues are dealt with briefly under the rubrics of "use of non-standard units" and "need for standard units", before the treatment moves over wholly to the development of skills. These include familiarity with common measurement instruments, use of standard measurement procedures, inter-converting between smaller and larger international units and computing with units. Observations of the classroom teaching in the schools that formed part of the study revealed that there is even greater emphasis on paper and pencil computation skills with very little treatment of either conceptual matters or even of practical measurement.

6.3 Measurement related experience

The diversity of everyday and work settings discussed in the case studies (chapter 4) give rise to diverse experiences of measurement. In this section, an analysis of diversity is presented highlighting the inherent richness of concepts implicated in such experiences, which can help connecting such knowledge with classroom learning. These aspects are discussed under two rubrics:

6.3.1 Comparison, estimation, quantification and construction in relation to measurement

Measurement in everyday contexts including work and domestic settings is different from measurement in the scientific world. Precision and accuracy are not as important as convenience. In many situations approximate measurements suffice. Comparison at times draws on embodied kinesthetic knowledge, as in *zari* or *latkan* making work, while judging the suitability of a decorative piece within a defined space. Bag or purse-making commonly use congruence and similarity of shapes and designs. These features lead to a diversity of measurement modes that are used in everyday contexts. In contrast, the school curriculum emphasises scientific measurement based on full quantification using a system of units, with well-defined relationships between sub-units and between fundamental and derived units.

Comparison between school and everyday measurement experience

In our study, it was observed that measurement experience in the everyday context is richer and more sophisticated than measurement experience that arise in the classroom context. This is due to the diversity of measurement modes and aspects of construction of units and tools that are often encountered in everyday contexts. In work-contexts, construction of convenient units or templates derived from standard units are conceptually rich actions since they involve partitioning, combining or manipulating a standard unit and quantification. Students learn to use inch and foot in their everyday contexts and learn to carry out measurement of acceptable accuracy by reading off the lengths from the tape. Such measurement is fully quantified, but the quantification is opaque and the measurement itself depends critically on the integrity of the artefact. However, most children are unaware of how weight or length is quantified. Table 6.1 below presents a summary of the diversity of measurement modes, units, processes and attributes in a few illustrative contexts related to work, school and shopping.

Contexts	***Objects & Attributes measured***	***Measuring instruments***	***Measurement units***	***Measurement modes: Quantification, Estimation, Construction***
Tailoring, leather work	Length of cloth, area of leather pieces	24” steel scale, 1m steel bar, 60” plastic tape *tedha* scale (bent scale used in tailoring)	Old British units (inch, foot, *guj* or yard) International units (metre, centimetre) Non-standard units (cubit, finger-band) Indigenous units (*desi*, *waar*, *kattha*, *bigha*)	Construction of Templates (*farma*), iterating to measure length and area Estimation Comparison
Recycling	Grading of plastic sheets (recycling work) Weight of *chindhi*	Weighing hook/beam scale (*kaanta*)	Standard units (kg)	Ordinal numbers (grades of plastic sheets) Estimation Comparison

Shopping	Weight of provisions, goods length volume counts of discrete objects sizes of garments, shoes (denoted by number)	Balances of different kinds, meter scales, volume measures	Old British units (dozen, gross, ream) International units (metre, centimetre) *Mutthi* (fistful) *cutting* (tea)	Construction of standard units by partitioning construction of convenient units
School	Length Weight	Standard ruler (6" or 12") None	International units (metre, cm, mm) (Kg, g)	Measurement by reading from a scale

Table 6.1 Diversity of measurement related experience

6.3.2 Diversity of object, measurement units and tools

As summarised in Table 6.1, this section discusses the variety of objects that are measured and the variety of measurement units, tools and modes that students encounter in their everyday settings.

- Objects and attributes measured
- Use of different measuring instruments
- Use of different measurement modes
- Use of different measurement units

6.3.3 School and work-context math: different requirements

We noted that the abstraction entailed in work-contexts vary and are different from abstractions handled at schools. For example, diverse measurement work-contexts implicitly use abstract notions like construction of units and sub-units, chunking of measures, partitioning, unit iteration, covering, use of convenient units and modes (like templates) which are available to students as part of the everyday mathematical knowledge. The school curriculum, in contrast, treats learning of measurement as a skill development and then moves towards abstraction without building on the knowledge resource already available to the children from the work-contexts. The abstractions available to students are in implicit form through their exposure and experience in work-contexts are potentially rich resources for building on measurement knowledge in the classrooms. Similarly, conservation of attributes, transitivity and seriation that are the foundation of comparison thence backbone for developing critical understanding of measurement are not sufficiently emphasised while handling abstractions in the school context. Thus, although experiences in work-contexts or in the cultural practices can help in broadening children's learning, they are not leveraged in the formal learning situation.

We highlight below some of the ways in which the measurement knowledge gathered from out-of-school contexts discussed in this chapter may be used as learning resources in the mathematics classroom.

Construction of templates and units:

Familiarity of students to construction of templates for purposes of comparison and measurement by iteration (e.g., *farma* in leather work for required size or for iteration to optimize use large leather piece) is valuable for measurement learning. Templates have fixed measures, while units can be chunked or partitioned to obtain larger or smaller units and generalisable beyond the immediate context of application. Participants' familiarity with the construction of templates to measure length, area and weight can give rise to questions that can lead to fruitful mathematical work in the classroom: why is the construction of units or templates needed; how do we construct new templates or units from given templates; in what contexts are units partitioned to yield smaller units; what quantities can be measured with a given combination of templates.

Measurement of area:

Area as seen in the examples above was frequently specified using a rectangular template. Templates can serve as a unit for iteration and give rise to discussion about the variations in the area measurement, use of different shaped units to measure area, their relationships and equivalences.

Opaque quantification & Archaeology of measurement tools:

Our study-participants were familiar with some quantifications but their origin was obscure, as in garment sizes. Similarly, they used common measuring tools such as the inch tape, but were unclear about the meaning and construction of the markings on the tape. This can gain be a powerful starting point for archaeological exploration that can lead to learning about length measurement and its uses.

Prevalence of different units and systems:

Students in our study used different kinds of units: international units, old Indian units, old British units and non-standard units. Besides the idea that units are purely conventional creations and are embedded in cultural and political histories, such diverse knowledge is useful in exploring the relation and differences between different systems. Questions that can be fruitfully explored for example are, why do we need unit systems rather than just units; what are the different principles of subdivision and the advantages and disadvantages of the binary and decimal systems.

Quantification of various attributes:

Drawing on their familiarity with the range of objects and attributes that are quantified, students can explore questions such as what is common and what is different in how we quantify different attributes; how is an abstract attribute like monetary (exchange) value quantified; how do we quantify different aspects of labour such as time, effort and expertise.

Chapter 7: The teaching intervention

Phase III of the study consisted of a teaching design experiment (Cobb, *et al*., 2003) in the form of a two-week long summer vacation course conducted for sixth and seventh graders, aimed at drawing curricular and pedagogic implications of connecting everyday and school math knowledge. The classes were conducted by the researcher's senior colleague from HBCSE for one hour and a half everyday for 12 days. All the lessons were recorded on video.

This chapter reports the analysis of the teaching design experiment that was focused on integrating students' out-of-school measurement knowledge with formal teaching aimed at building conceptual understanding among the participants. The design experiment was intended to explore the possibilities and limits of connecting everyday mathematical knowledge with school learning. The emphasis was on exploration and establishing feasibility, rather than effectiveness of instruction in terms of learning outcomes.

A major purpose of the analysis was to elaborate the enacted goals of the teaching design experiment. This analysis is important to answer the question "what should be the goals of an approach to teaching that attempts to connect out of school knowledge with school mathematical learning?" The analysis focuses broadly on two aspects (i) conceptual connections between everyday mathematical knowledge and school mathematical knowledge with a focus on the topic of measurement and (ii) agency and identity formation in the classroom in relation to the connection between out of school and school learning.

7.1 Goals and objectives of the vacation course

The broad goals of the vacation course were to

1. make connections between out-of-school mathematical knowledge of school children and learning of school mathematics
    1.1. By using their out-of-school knowledge to organise and build conceptual learning of school mathematical topics,
    1.2. By using school mathematical learning to illuminate aspects of out-of-school knowledge.
2. foster identities that allow connections to be made between out-of-school and school math knowledge and to align students' identities as learners of mathematics and as experienced and knowledgeable persons in everyday contexts
    2.1. By legitimising the sharing of everyday knowledge in the classroom,
    2.2. By encouraging explanations that connect everyday and school knowledge,
    2.3. By building a culture of shared learning in the classroom.

The specific instructional objectives of the vacation course were formulated in the light of the broad goals. The enacted objectives focused on two mathematical topics (i) length measurement and (ii) fractions and decimals. On the topic of measurement, the objectives included drawing on students' out-of-school knowledge of length measurement to deepen conceptual understanding of units and sub-units in measurement by connecting them with out-of-school contexts known to children such as tailoring. The second objective was to connect students' out-of-school knowledge of fractions and proportional reasoning to the school topic of fractions and decimals by (i) strengthening and extending students' understanding of binary fractions gained from everyday contexts, (ii) making connections between binary and decimal fractions, and (iii) building students' understanding of decimal fractions.

The instructional goals described above spell out how classroom teaching might draw on and connect with out of school knowledge that students bring to the classroom. However, school learning is not the same as out-of-school learning and the goals need to acknowledge the complementary dimension of the differences between out-of-school and school learning. This complementary dimension of the instructional objectives could be viewed through the lens of enabling a series of shifts:

- Shift from oral to written math
- Shift from knowledge about use (tool/artefact) to understanding the tool (e.g., measuring tape, numeral sizes of garments)
- Shift from individual expression in private to shared, public expression
- Shift from co-operation to a mathematically focused discourse community (e.g., shift from making assertions to providing clarifications, justifications, explanations; moving from "helpful" interactions to a discursive culture)
- Shift from identities that are disconnected (or reinforce disconnection) to identities that are connected

7.2 Making conceptual connections

A second focus of the analysis of the teaching design experiment was on the conceptual connections between out of school and school mathematical knowledge with specific reference to the topics covered in the lessons of measurement and fractions. Classroom interaction in the course of the design experiment supported some findings concerning the mathematical knowledge of students from earlier phases of the study and elaborated on others. The fragmented nature of knowledge from out of school contexts was reinforced on multiple occasions.

Although students were familiar with words for binary fractions and made connections between them, the written notation was elusive. This indicates the fragmentary and tenuous connections that exist in students' minds about the fraction notation for binary fractions or the equivalent decimal fractions. Students often used proportional reasoning and convenient decompositions in computing answers when they were encouraged to do so in the classroom. An example was the task of finding out ½, ¼, 1½, 2½ and 3½ times a given number. Students explained the various strategies that they used to compute these multiples to their peers, which indicated that they had a robust and confident awareness about decompositions of fractions (twice 1½ is the same as three; half of 1½ is equal to ¾, half of 2½ is equal to 1¼, etc.).

7.3 Student participation: constructions of identity

A third major purpose of the analysis was to arrive at an understanding of students' receptivity to the instructional goals from the nature of participation in the classroom. The analysis draws on the notion of "normative identity" as a construct illuminating the participation structure in the classroom (Cobb, Gresalfi & Hodge 2009, see chapter 2). This refers to the set of norms co-constructed by teacher and students in the classroom that determine expectations about how students should ideally participate in the classroom. With regard to mathematics, the normative identity refers to what is considered appropriate mathematical engagement on the part of a student. Individual students may accept the normative identity, merely co-operate without accepting the identity, or actively resist the identity.

From the transcript of the lessons, we reviewed episodes that revealed the teacher's and learners' engagement in (i) requesting and sharing out-of-school knowledge, and (ii) asking for and providing explanations, clarifications and justifications. The teacher's invitations to students to share what they knew about work contexts was a striking feature of the initial teaching episodes, where setting up of norms is a primary goal. Students readily participated in such interaction suggesting acceptance of the norm of sharing knowledge about work and other out of school contexts. Another device implemented by the teacher was to bring artefacts from work contexts into the classroom, setting up a difference from a typical school classroom. For example, shirts and kurtas of different sizes as well as the measuring tape were introduced in the classroom followed by a non-standard but fixed unit (a paper-strip made from A4 paper). These moves by the teacher elicited enthusiastic participation from the students. The students worked in groups and quickly taught each other the correct ways of taking the measurements of a shirt. Simple calculator commonly used by shopkeepers and familiar to students was introduced eliciting similar enthusiastic participation from the students.

The teachers' questioning frequently focused on providing explanations, clarifications and justifications. Three kinds of sources of justification were accepted in resolving "how do we know" questions. One was the invoking of authority which was done in cases where information was to be shared, or conventions about symbols needed to be cited. A common source of authority was the teacher himself. Another source of authority was an artefact (e.g., calculator). Computations done on the calculator were frequently invoked to judge the correct decimal representation of a known fraction.

A second source of justification was prior knowledge of mathematics. For example, a relation between fractions may be justified using a computation procedure. A specific example is to explain that 1/10 is the same as 0.1 because adding 0.1 ten times using the vertical addition algorithm gives 1.0. A justification of this kind is lengthy and not mathematically elegant. For a justification such as this to be accepted, such procedures needed to be part of the shared knowledge of several students in the classroom. Prior knowledge was typically restricted to math procedures learnt at school.

A third important source of justification was experiential knowledge, which typically was from out-of-school contexts. The use of binary units and fractions was frequently invoked, as was proportionality reasoning and convenient decompositions. Knowledge about units and relations between units were sometimes cited. One student frequently justified his oral computation strategies using money as a convenient representation for quantity. That such justifications were accepted by other students indicated a level of shared knowledge drawn from out-of-school contexts.

Chapter 8

Conclusions, implications and future directions

This chapter attempts to bring together the results and findings of the study in terms of out-of-school knowledge of children immersed in work-contexts and its curricular and pedagogic implications under a unifying perspective. Future directions and road-maps are also discussed. The research study proposes that learning of mathematics is aimed at acquiring conceptual understanding and insight and not at practical training.

The funds of knowledge perspective illuminates how the connectedness of social networks gives rise to diverse and rich knowledge and experience that can be drawn on for the purposes of school learning. In our study, which is set in an urban, developing world context, we found that students often directly participate in work, or are closely aware of work contexts and practices. Experiences and knowledge of measurement drawn from such contexts are intimately familiar and present in the classroom. Such diversity of experience, within a school community hence presents potentially rich opportunity for learning that has been largely ignored in formal school education.

The findings

Overlapping school and out-of-school (everyday) math knowledge

In concurrence with the recent studies done in the areas of out-of-school or everyday mathematics, our study indicates the overlapping nature of students' school and out-of-school math knowledge, i.e., the forms of mathematical knowledge were not distinct but drew from each other as well as from other nodes on students' social and work network. Our data from from students' interviews on arithmetic tasks support this claim (discussed in Chap. 4). This is despite the prevalent classroom culture and the beliefs that many children hold, which tend to reinforce the separation of two forms of knowledge.

Diverse work contexts: potentially rich resources for learning

The diversity of work practices prevalent in the low-income settlement presents potentially rich resources and opportunities for gathering mathematical knowledge. For example, using funds of knowledge perspective, we observed that our student participants have varying degree of measurement knowledge derived from the work-contexts around them. Student participants are likely to know some elementary notions such as transitivity, conservation, partitioning and unit iteration through their exposure to work-contexts, although they may be unclear about how these notions form the basis for common measurement procedures, tools and conventions.

School & out-of-school math: difference in structure, goals & requirements

The experiences of measurement in out-of-school contexts are characterised by diversity as well as structural differences from the school mathematical treatment of measurement. A central aspect of school knowledge is its generality, of its not being tightly bound to particular contexts. Specialized knowledge is context-bound, well-practiced and embodied in individuals, and leads to expertise and efficient action in limited domains and situations in contrast to generalisability and wide applicability. From the standpoint of valuing such generality as an aspect of school learning, it is the diversity of out-of-school experiences that creates the context for school learning. Thus, from our perspective, it is incorrect to claim that work practice already reflects mathematical thinking or understanding. Mathematical aspects are only present in hybridised and opaque embeddings. It is also incorrect to expect school learning to illuminate or strengthen a single kind of practice in a particular work context. It is the diversity of practices taken together that formal mathematical learning can illuminate. It strengthens understanding, not practice.

Learning skills and acquiring knowledge through participation in work is valorised in the community that we studied, although some families discourage their children from participating in work because they think it would affect their studies. School learning too is valued, although for different reasons and as a different kind of learning. It has aspirational value, and the community believes that education is the route to social and economic mobility. However, it is self-defeating for an education system to merely aim to produce the trappings of social class, while depriving learners of knowledge that has power because it illuminates aspects of life. Students from deprived backgrounds enter the classroom with their own rich complement of experiences. Our perspective is that education that shuts this rich resource out of the classroom is a recipe for failure.

Conceptual underpinnings

The thesis focuses on the topic of measurement in the school curriculum to draw specific implications for curriculum and pedagogy. Post-Piagetian research studies have highlighted the importance of concepts such as conservation, transitivity, equi-partitioning, displacement, iterative covering, accumulation of distance and additivity and the role of the origin on scales. From the point of view of the diversity of out-of-school experience, we need to go beyond these critical concepts to include construction of units and templates, equi-partitioning and chunking of measures and unit, construction of measuring scales, design of convenient measuring instruments and units. Further aspects critical to the understanding of measurement that have not been adequately addressed in the curriculum include the extensive use of comparison and estimation in real life contexts, the use of the body as a measuring instrument, the trade offs between convenience and accuracy, the variety of purposes of measurement, the variety of modes of quantification and the limits of informal quantification, and the cultural-historical origins of units and systems of units. These aspects, with the exception of estimation, have also not received the attention of mathematics education researchers. The diversity of measurement experiences in out-of-school work contexts can be drawn upon to illustrate each of these concepts and ideas, and also for understanding the difference between comparison, estimation and measurement and their purposes.

Archaeology of artefacts

A second aspect of out-of-school knowledge that makes for potentially powerful connections with school learning is the fact that artefacts and practices from everyday settings represent a sedimented and embodied form of mathematics (Chevellard, 2007). The measuring tape embodies the processes of unit construction, unit iteration and counting and partitioning of units into sub-units. These processes are however hidden from view and are opaque. The redundant inclusion of a second system of units in the form of inches and feet on the measuring tape captures an aspect of history and highlights the arbitrariness of the choice of the basic unit of length. The purpose of such embodiment is precisely to make the mathematical thought and processes behind the construction of the measuring scale unnecessary, and to reduce the practice of measurement to the simple act of reading off the scale. This is the general phenomenon of demathematisation described by Chevellard (2007) and Gellert and Jablonka (2007) where material artefacts embed increasingly sophisticated mathematical ideas, while rendering the user's knowledge of such mathematics unnecessary. As long as we treat the learning of measurement as merely the learning of a skill, unpacking the mathematical ideas that are embodied in artefacts will remain unnecessary. However, if we view the learning of measurement as conceptual understanding, then such material artefacts present an opportunity for archaeological investigation. Such "archaeology" or "unpacking" aims to uncover the generally hidden and "black boxed" aspects of mathematics sedimented in artefacts and practices (Subramaniam, 2012). Archaeology as a pedagogical mode may have an important place in providing opportunities to learn powerful mathematics that illuminates the diverse aspects of everyday experience.

Pedagogical Implications

One of the challenges before the teacher or the instructional designer is to imagine connections between school and out-of-school knowledge that can produce powerful learning. What should be the goals of a pedagogical intervention that aims at building connections between out-of-school knowledge and school learning? What forms of participation could one expect to see in a classroom implementing these goals? These questions are addressed in the pedagogical intervention discussed in chapter 7. The goals must include conceptual aspects as well as the setting up of a classroom culture that values making connections. It must also explicitly attend to the shifts that are needed in bringing out-of-school knowledge with school learning in terms of shifts *oral* to *written* mathematics, from *knowledge about use* of tools and artefacts to *understanding* and building the identities of participants as a mathematically focused discursive community. The latter shift involves moving from helping interactions to a more discursive culture in which reasoning is central, where statements are listened to with attention, are challenged, elaborated and justified.

8.2 Personal postscript

The study entailed handling of sensitive issues of social and ethical nature, for example, the issues of child labour, difficult and oppressive work conditions, unfair wage pattern, social stereotypes (gender, caste and others), and so on. As a researcher, it was challenging to tackle and address such issues during interviews or during social interactions. The dissertation journey has brought to me a platter of learning and training to prepare myself to carry forward similar research and also to embark upon new research on other social issues. As a researcher, I feel better able now. The social relationship with the community developed during the prolonged engagement with the field will remain as an asset for me. Reflecting over the data and over myself since I undertook this research study, I realise that the study has given me tools to see things which I was unaware of or at the most vaguely aware of. The research study was hugely benefited by the students and teachers' support and their participation in it, but I am sceptical whether this study gave them back some tools to judge and tackle the equity, fairness and other social issues through learning of mathematics.

Revisiting the study in its entirety indicates to me that as a researcher, I felt, perhaps more sensitivity is required towards handling social issues arising out of low SES, work requirement, aspirations and child labour than what I had. Researchers embarking upon similar studies need to be more cautious with such issues.

References

Abreu, G. de (2008). From mathematics learning out-of-school to multicultural classrooms: A cultural psychology perspective. In L. D. English (Ed.). *Handbook of International Research in Mathematics Education*. 2nd ed., 352-384. NY: Routledge.

Barton, A. C. & Tan, E. (2009). Funds of knowledge and discourses and hybrid spaces. *Journal of Research in Science Teaching, 46*(1), 50-73.

Bishop, A. J. (1988). Mathematical Enculturation: A Cultural Perspective on Mathematics Education. Dordrecht: Kluwer Academic Publishers.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. *Multiple perspectives on mathematics teaching and learning*, 171-200.

Bose, A. & Kantha, V. K. (Accepted). Influence of socio-economic background and cultural practices on mathematics education in India: A contemporary overview in historical perspective. *ZDM – The International Journal on Mathematics Education, 46*(7).

Bose, A. & Subramaniam, K. (2013). Characterising work-contexts from a mathematics learning perspective. In Nagarjuna, G., A. Jamakhandi & E. M. Sam (Eds.). *Proceedings of epiSTEME-5: Third International Conference to Review Research on Science, Technology and Mathematics Education.* pp. 173-179, Margao, India: CinnamonTeal Publishing.

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1987). Written and Oral Mathematics. *Journal for Research in Mathematics Education, 18*(2), 83-97.

Carraher, D. W. & Schliemann, A. D. (2002). Is everyday mathematics truly relevant to mathematics education? *Journal for Research in Mathematics Education. Monograph, 11 (Everyday and Academic Mathematics in the Classroom)*, 131-153.

Chevellard, Y. (2007). Implicit mathematics. In U. Gellert & E. Jablonka (Eds.). *Mathematisation and demathematisation: Social, philosophical and educational ramifications.*Rotterdam: Sense Publishers.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schuble, L. (2003). Design experiment in educational research. *Educational Researcher*, 32(1), pp. 9-13.

Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education*, *40*(1), 40-68.

Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths/pedagogic texts*. London: The Falmer Press.

Fagg, H. (2002). *Back to the Sources: A Study of Gandhi's Basic Education*. New Delhi: National Book Trust.

Gellert, U. & Jablonka, E. (2007). *Mathematisation and demathematisation: Social, philosophical and educational ramifications.*Rotterdam: Sense Publishers.

González, N. (2005). Beyond culture: The hybridity of funds of knowledge. In N. González, L. C. Moll & C. Amanti (Eds.). *Funds of knowledge: Theorizing practices in households, communities, and classrooms*, pp. 29-46. Mahwah, NJ: Lawrence Erlbaum.

González, N., Moll, L. C., & Amanti, C. (Eds.). (2005). *Funds of knowledge: Theorizing practices in households, communities, and classrooms*. Mahwah, NJ: Lawrence Erlbaum.

González, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk*, *6*(1-2), 115-132.

Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom: on learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research*, *51*, 128-145.

Khan, F. A. (2004). Living, Learning and Doing Mathematics: A Study of Working Class Children in Delhi. *Contemporary Education Dialogue*, *2*, 199-227.

Lave, J. (1988). *Cognition in Practice*. Cambridge, UK: Cambridge University Press.

Lave, J., & Wenger, E. (1991). Situated Learning: Legitimate Peripheral Participation. Cambridge: Cambridge University Press.

Maharashtra Math Textbook 5, 6, 7. (2006). *Mathematics Standard 5, 6, 7*. Pune: Maharashtra State Bureau of Textbook Production and Curriculum Research.

Moll, L. C., Amanti, C., Neff, D., & Gonzalez, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory into practice*, *31*(2), 132-141.

Mukhopadhyay, S. (2013). The mathematical practices of those without power. In *Mathematics Education and Society 7th International Conference.* Plenary talk. Cape Town: South Africa.

NCERT (2005). *The National Curriculum Framework*. New Delhi: National Council of Educational Research and Training.

NCERT (2006). *National Curriculum Framework 2005 Position Paper of National Focus Group on Teaching of Mathematics*. New Delhi: National Council of Educational Research and Training.

NCERT (2007). *Position Paper National Focus Group on Work and Education.* New Delhi: National Council of Educational Research and Training.

NCERT Math Textbook 5. (2006). *Math-Magic 5*. New Delhi: NCERT.

NCERT Math Textbook 6, 7. (2006). *Mathematics*. New Delhi: NCERT.

Naresh, N., & Chahine, I. (2013). Reconceptualizing Research on Workplace mathematics: Negotiations Grounded in Personal Practical Experiences. *REDIMAT-Journal of Research in Mathematics Education*, *2*(3), 316-342.

Nasir, N. S., Hand, V. & Taylor, E. (2008). Culture and Mathematics in School: Boundaries between "Cultural" and "Domain" Knowledge in the Mathematics Classroom and Beyond. *Review of Research in Education, 32*, 187-240.

Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). *Street Mathematics and School Mathematics.* New York: Cambridge University Press.

Nunes, T. C., Carraher, D.W., & Schliemann, A. D. (1985). Mathematics in the Streets and in Schools. British Journal of Developmental Psychology, 3, 21-29.

Resnick, L. B. (1987). Learning In School and Out. *Educational Researcher, 16*(9), 13-20.

Saraswathi, L. S. (1989). Practices in linear measurements in rural Tamil Nadu: implications for adult education programmes. *Journal of Education and Social Change, 3*(1), 29-46.

Saxe, G. B. (1988). Candy Selling and Math Learning. Educational Researcher, 17(6), 14-21.

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the society for research in child development*.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14-22.

Sitabkhan, Y. (2009). The Use of Convenient Value Strategies among Young Train Vendors in Mumbai, India. In K. Subramaniam & A. Mazumdar (Eds.). *Proceedings of epiSTEME-3: An International Conference to Review Research on Science, Technology and Mathematics Education*, pp. 114-118. Mumbai, India: Macmillan.

Subramaniam, K. (2012). Does participation in household based work create opportunities for learning mathematics? In *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, (Vol. 1, pp. 107 – 112). Taipei, Taiwan: PME.

Vasanta, D. (2004). Childhood, Work and Schooling: Some Reflections. *Contemporary Education Dialogue*, *2*(1), 5-29.

Vélez-Ibáñez, C., & Greenberg, J. (2005). Formation and transformation of funds of knowledge. *Funds of knowledge: Theorizing practices in households, communities, and classrooms*, 47-70.