Graphicacy issues in school textbooks and designing learning contexts to address them

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Declaration

This thesis is a presentation of my original research work. Wherever contributions of others are involved, every effort is made to indicate this clearly, with due reference to the literature, and acknowledgement of collaborative research and discussions. The work was done under the guidance of Dr. Nagarjuna G., of the Homi Bhabha Centre for Science Education, Tata Institute of Fundamental Research, Mumbai.

Amit Ramesh Dhakulkar
28 March 2018

In my capacity as supervisor of the candidate's thesis, I certify that the above statements are true to the best of my knowledge.

Dr. Nagarjuna G.
28 March 2018

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To the children of the world.

Many objects are obscure to us not because our perceptions are poor, but simply because these objects are outside of the realm of our conceptions.

Kosma Prutkov

Preface

Motivation for this study came from my own experience while learning physics during the master's course. In general, in the subjects that we were taught, use of graphs was peripheral. But as I became exposed to aspects of doing research in physics, graphs seemed to be omnipresent. Most experimental data could be and was often summarised by graphs, and the rest of the paper tried to explain or understand the meaning that one can derive from the graphs. As is often the case with physics research, the researchers compare experimental data to those predicted through theoretical models. The theoretical models offer equations depicting a given phenomenon and often an accompanying graph visualises the equation in two-dimensional space. Indeed this is an act of translation wherein the equation is in algebraic form, while the graph is in geometric form. During a class, we were told to plot the graph of $\ln(x)$ vs x. None of us could draw the graph. What could be wrong? Perhaps, we never had opportunities to explore the graphical domain meaningfully.

Another experience in this regard came in when I was asked to explain the electronic energy levels (Figure 1) in an interview. I was confident that I had understood the concept and was explaining how the levels on the graph correspond to the energy levels of the electrons. Then a panellist on the interview asked me, "the energy is on the Y-axis of the graph, but can you tell us what is on the X-axis?" I was totally blank. I thought I had grasped the concept well, till this question was asked. I had never thought about this, though I had seen and drawn this graph many times. These experiences left a deep impact on me and made me consciously and deliberately think and work through graphs and motivated me to take it up as a topic of research.

Organization of the Thesis: The present work is about graphs and their use in the context of science education. Graphs are a very powerful form of inscrip-

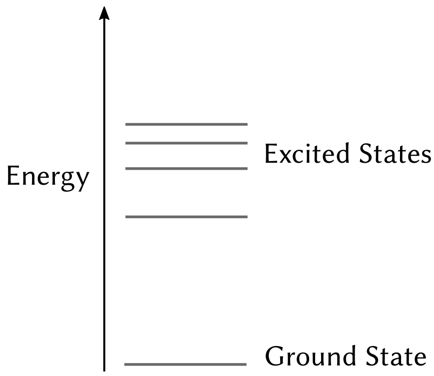

Figure 1: A typical energy level diagram depicting the ground state and excited states. The Y-axis depicts the energy, while X-axis is usually not labelled in most diagrams.

tion, which enables communication, visualisation and analysis of data. Education researchers report that the ability to work with graphical representations is underdeveloped among learners. Surprisingly, despite the centrality of graphical representations across different subjects, there has not been a comprehensive monograph on graphs that identifies the current theme in science and mathematics education, specifically keeping the Indian context in mind. We carried out an inquiry to assess the use of graphs in NCERT textbooks for science, maths and other subjects. Based on the analysis of this preliminary study, we first provide a critique of the shortcomings of the use of graphs in the textbooks, using various established criteria commonly used by educational researchers, and then devise and test pedagogic strategies to determine good practices for educating young learners about graphs. We address this problem along two themes in our work: What is currently being done to enhance the ability to engage with graphs successfully? What kind of experiences will provide the learners with the ability to use graphs to navigate between experimental data and theory? These two overarching questions form the basis of the two parts presented here. The body of the main thesis is presented in two parts, followed by the Appendix.

Part 1: Graphicacy and Its Problems introduces the concept of graphicacy and its importance. This part builds a case for graphicacy as a core skill and provides the rationale for the present work. *Chapter 1* discusses the importance of graphs in science and science education, and defined the research problems for the current work. *Chapter 2* reports a critical review of literature for problems of comprehension and construction of graphs and various models of graph comprehension. *Chapter 3* reviews textbooks for graphical practices in the Indian context. We conclude the first part with problems of graphicacy and their possible solutions.

Part 2: Learning Contexts presents our attempts at developing activities aimed at addressing some of the concerns raised in the first part. The design of these activities follows from the findings from the textbook analysis and their implication as understood from the literature. The activities developed from this framework, their field testing and analysis are detailed (*Chapter 4*). *Chapters 5, 6*, and *7* provide the description and the analysis of the three field studies presented here. The last chapter discusses the major outcomes of the work, with its limitations and scope for future work (*Chapter 8*).

The three Appendices contain material pertaining to the current work, not included in the main text. *Appendix A* provides the quantitative and qualitative data from the textbook analysis. *Appendix B* is a short history of the conceptual development of the use of the graphical method in displaying quantitative data. *Appendix C* contains the Student Handbook for the activities, as well as the Pre- and Post-Test Questionnaires used in the study.

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In due course of the work many people helped in completing the work by providing support both in the form of the facilities and the academic and moral support in completing the thesis.

I would like to thank facilities provided by Homi Bhabha Centre for Science Education, Mumbai for aiding me in the completion of the degree. I thank my guide, Prof. Nagarjuna G., under whose guidance this dissertation has been completed. Friends at Gnowledge Lab, HBCSE helped in various ways for my work, Rafikh Shaikh helped in data analysis. I have been working at Tata Institute of Social Sciences during the last few years. I am especially grateful to Prof. Padma Sarangapani and Dr. Ajay Singh for providing support and motivation towards completion of the thesis while working at TISS. I am especially thankful to Dr. Arvind Jamkhandi for providing critical and insightful comments on the drafts of the dissertation which made the thesis more readable.

I am thankful to Inter-University Centre for Astronomy and Astrophysics, Pune for the facilities provided to me for conducting the field studies. In particular, many parts of this work would not have been possible without help and cooperation from my old friend Samir Dhurde. I am also thankful to Maharudra Mate for helping with the conducting of activities. The presence of Arvind Gupta (Guptaji) and his colleagues Ashok Rupner, Vidula Mhaiskar and Shivaji Mane at Muktangan Vidnyan Shodhika in IUCAA was always inspirational during my stay there. The thesis was created using XƎLaTeX, and during the typesetting process, I faced several problems and challenges. I am thankful to the users on the forum tex.stackexchange.com who gave insightful and innovative solutions to these problems.

I am also thankful to my mother Kalindi Dhakulkar and sister Shrutika Bagul for always encouraging me and providing moral support throughout this journey. Last, but not the least, I am thankful to my wife Smita Patil for coping with me during all this time, supporting and encouraging me to complete the thesis.

Part I

Graphicacy and its Problems

Alice was beginning to get very tired of sitting by her sister on the bank, and of having nothing to do: once or twice she had peeped into the book her sister was reading, but it had no pictures or conversations in it, and what is the use of a book, thought Alice, without pictures or conversations?

Lewis Carroll *Alice's Adventures In Wonderland*

1

A Case For Graphicacy: An Indispensable Skill For All Citizens

1.1 Situating graphicacy

The quote of Alice at the start of the chapter in a way captures the central theme of the work in this thesis. This work is about pictures and conversations about the pictures. Graphics of various kind permeate our world. By graphics here we mean various forms of non-textual representations which include but are not limited to photographs, sketches, illustrations, diagrams, charts, plans, maps, graphs. Both the popular and the academic media profusely use images to convey powerful messages. It is even claimed that we live in a *visual culture* (Jenks, 1995; Mirzoeff, 1999). What are the reasons for graphics to hold so much power? Graphics can be superior to a verbal description for solving problems as they can provide computational advantages to the viewer (Larkin & Simon, 1987). Also, graphics can communicate faster than text as Mason (2006) describes:

> Written text is linear. To gain control of its meanings you have to follow the order of words from beginning to end. An image by contrast communicates `all at once' as it were. You can start anywhere and as you gather information from one point on the image you can cross over and pick up other lines of meaning. This is because an image contains a vast amount of information in a compact space. An image does not have to explain itself as it goes along as prose text has to (as you can see in the text you are reading now). An image makes instantaneous claims on the eye and the mind ? it draws both into its visually constructed internal logic. (p. 11)

The review by (Purchase, 2014) looks at the historical evolution of the work and the current state of the field related to research on diagrams which is defined as: "a diagram is taken to mean a composite set of marks (visual elements) on a two- dimensional plane that, when taken together, represent a concept or object in the mind of the viewer" (p. 59). This definition includes most of the categories of graphics that we have considered here in the previous paragraph except perhaps photographs.

John Tukey, one of the pioneers in developing new techniques for graphical visualisations of data in the last century, summarised the uses of graphics as under (Tukey, 1993):

1. **Graphics are for the qualitative/descriptive** - conceivably the semiquantitative never for the carefully quantitative (tables do that better).
2. **Graphics are for comparison** - comparison of one kind or another - not for access to individual amounts.
3. **Graphics are for impact** - interocular impact if possible, swinging - finger impact if that is the best one can do, or impact for the unexpected as a minimum - but almost never for something that has to be worked at hard to be perceived.
4. **Finally, graphics should report the results of careful data analysis** - rather than be an attempt to replace it. Exploration-to guide data analysis - can make essential interim use of graphics, but unless we are describing the exploration process rather than its results, the final graphic should build on the data analysis rather than the reverse. (p. 2)

Bertin (2011) in his work on looking at graphics with a semiological perspective enumerates three functions of graphic representations concerning memorability and comprehensibility:

1. **Recording information:** creating a storage mechanism which avoids the

effort of memorisation. The graphic utilised for this purpose must be comprehensive and may be non-memorisable in its totality.

2. **Communicating information:** creating a memorisable image which will inscribe the information in the viewer's mind. The graphic used here must be memorisable and may be non-comprehensive. The image should be a simple one.

3. **Processing information:** furnishing the drawings which permit a simplification and its justification. The graphic should be memorisable (for comparisons) and comprehensive (for choices). (p. 14)

By doing so, Bertin is also putting forward a perspective of recording, communicating and processing of information.

In today's information-centric digital world, we live with graphics or images across media types. In the ever-expanding multi-media culture we find popular media, such as, television, newspapers, magazines, making use of graphics widely. In the context of science, it is almost impossible to find a journal or a textbook on science which does not have a variety of graphics. The graphics in popular media are designed to be attention-grabbing (Mason, 2006):

> The images of popular culture are created to grab attention and communicate information as quickly as possible, and they must be novel and entertaining at the same time. ...They cannot ask the viewer to process too much information. The result is that popular images tend to be eye-catching and easily understood but without internal complexity. They ask for no more than a simple and superficial understanding. (p. 11)

Mason says that we might intuitively learn to read these images through simple repetition, but the understanding might not be profound. Processing such graphical information has become an essential element of our lives.

In our work, we look at a specific category of graphics: *graphs*. For the purpose of this work graphs can be defined as *two-dimensional representations which enable us to display a relationship between two or more variables*. We have a variety of graphs that allow us to visualise data, for example, line graphs, scatter plots, bar graphs, pie charts. In sciences, the line graphs are found to be particularly useful to find patterns and trends. According to Anscombe (1973), graphs can have varied functions such as:

(i) to let us perceive and appreciate some broad features of the data,

(ii) to let us look behind those broad features and see what else is there

Graphs are a very significant form of pictures used in almost all fields. In case of quantitative data, graphs epitomise the phrase "A picture is worth a thousand words." A quote from Henry Hubbard in the introduction to W. C. Brinton's *Graphic Presentation* (Brinton, 1939), reflects this notion and importance of graphs as a parsimonious and efficient means of communicating ideas very well.

> There is a magic in graphs. The profile of a curve reveals in a flash a whole situation - the life history of an epidemic, a panic, or an era of prosperity. The curve informs the mind, awakens the imagination, convinces . . . Graphs are all inclusive. No fact is too slight or too great to plot to a scale suited to the eye. Graphs may record the path of an ion or the orbit of the sun, the rise of a civilization, or the acceleration of a bullet, the climate of a century or the varying pressure of a heart beat, the growth of a business, or the nerve reactions of a child . . . Graphs are dynamic, dramatic. They may epitomize an epoch, each dot a fact, each slope an event, each curve a history. Wherever there are data to record, inferences to draw, or facts to tell graphs furnish the unrivalled means whose power we are just beginning to realize and to apply. (p. 2)

Hubbard's quote is from an era (c. 1939) when graphs had just begun to proliferate the presentation of information in all spheres of communication. The optimism expressed in the quote arises from the fact that the graphical method *is* indeed a powerful and trans-disciplinary method able to display, communicate and analyse data. Tufte (2001) claims that:

> Graphics *reveal* data. Indeed graphics can be more precise and revealing than conventional statistical computations. (emphasis in original, p.10)

The power of graphs in revealing the patterns in data can be seen in a famous example by Anscombe (1973). Anscombe gives an example of discriminating power of graphs on a set of similar data. In his example, the statistical parameters of the data are same, but the graphical display shows how varied the data sets are. These data sets, also known as *Anscombe's quatret* (Figure 1.1), are exemplary in showing the power of graphical display in finding out patterns and contrasting data sets which are statistically similar. The result of regression analysis of these four data sets gives us the *same* numbers for many parameters like means, regression coefficients, the sum of squares. The example shows us the immense power of visual display in distinguishing between data sets, finding patterns which are not visible

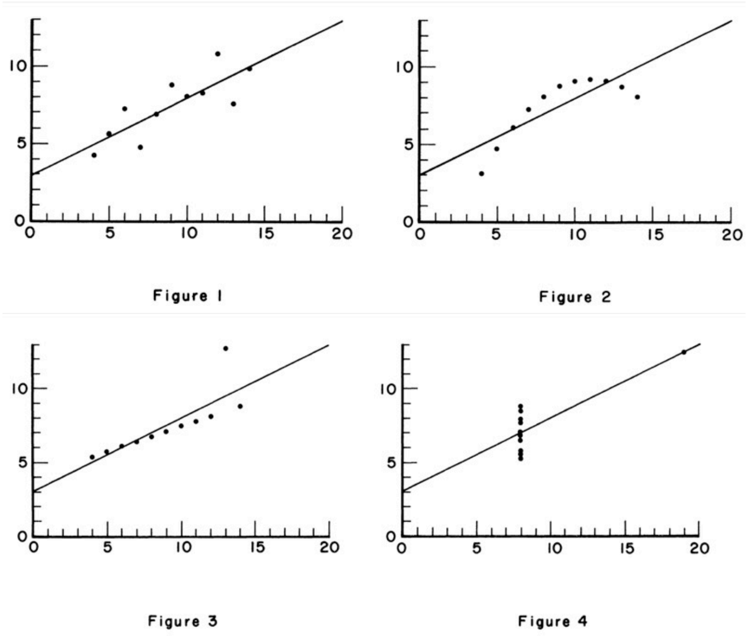


Figure 1.1: The graphical representation of Anscombe's quartet. The data sets in these four graphs have same statistics like means, regression coefficients, the sum of squares from the regression analysis. From Anscombe (1973) (p. 19-20).

by only statistical calculations. This example brings home the point that graphical pattern reveals more than calculations. In the current era, graphs are an integral part of the everyday and scientific discourse. Presence of graphs is ubiquitous in both mass-media and scientific communication. Due to the ubiquitous nature of graphs, the ability to interpret and create them is an indispensable skill for all.

The modality of reading graphs is different from that of reading texts. In our everyday life, there is a complex interaction between our different abilities which help us to make meaning of the information in the world around us and communicate with others. These are generic cognitive skills and do not pertain to any one subject, though emphasis might be different for each subject. For each of the skill, we have a name which tells us that the person can deal with: *Literacy* stands for being able to understand and communicate in verbal form while *numeracy* stands for being able to understand and perform operations with numbers. These two notions are well known and find their place in the present curricula. Apart from these two, we have two more notions corresponding to the social and the visuospatial abilities, which are not commonly used. The ability to perform social interaction has been termed as *articulacy* derived from the already existing adjective 'articulate', but the visuospatial ability did not have a name for a while. The visuospatial mode includes sketches, maps, diagrams, figures, graphs, photographs, graphic arts, plans, charts and others. This ability to understand and to be able to communicate in terms of graphics or being graphically literate has been termed as *graphicacy*. As Balchin & Coleman (1966) who coined the term have put:

> Graphs, photographs, cartography, the graphic arts . . . The syllable ``graph'' which is common to all these names for visual aids can be used as a root to coin the word ``graphicate'' by analogy with literate, numerate and articulate. . . . It (graphicacy) is the communication of relationships that cannot be successfully communicated by words or mathematical notation alone. . . . graphicacy spills over into literacy on the one hand and numeracy on the other without being more than marginally absorbed in either. It is inter-disciplinary without losing its own identity as a distinct medium of communication. (p. 25)

In another article titled *Graphicacy* Balchin (1972), further elaborates on this topic:

> In the choice of a word to denote the educated counterpart of visual-spatial ability one must first ask the question what exactly does this form of communication involve. It is fundamentally the communication of spatial information that cannot be conveyed adequately by verbal or numerical means, e.g. the plan of a town, the pattern of a drainage network or a picture of a distant place - in other words the whole field of the graphic arts and much of geography cartography, computer-graphics, photography, itself. All of these words contain the syllable ``graph'' which seemed a logical stem for ``graphicacy'' which was completed by analogy with literacy, numeracy and articulacy. (p. 185)

Balchin primarily had the subject of geography education in mind when he coined the word and the concept of graphicacy. However, the term was readily adapted for other subjects, apart from the root of its origin. Aldrich & Sheppard (2000) define graphicacy as:

> the ability to understand and present information in the form of sketches, photographs, diagrams, maps, plans, charts, graphs and other non-textual, two-dimensional formats. (p. 8)

After a definition of the term *graphicacy*, we now look at what does it mean to be *graphicate*. Balchin & Coleman (1966) notes:

> Graphs, photographs, cartography, the graphic arts . . . The syllable ``graph'' which is common to all these names for visual aids can be used as a root to coin the word ``graphicate'' by analogy with literate, numerate and articulate. (p. 27)

After a definition of the term *graphicacy*, we now look at what does it mean to be *graphicate*. Balchin & Coleman (1966) notes: Further elaborating on this notion, to be graphicate is to: (a) understand and describe the phenomenon behind the graph, (b) construct a graph to describe a phenomenon, (c) interpret and construct the meaning of various features and nuances of the graph, (d) extrapolate and interpolate the data using the graph, (e) predict unknown quantities or situations based on the graph, (f) make hypotheses and models describing the phenomenon, and, (g) ask critical questions about the graph and the phenomenon it represents.

The review work by (Zacks, Levy, Tversky & Schiano, 2002) looks the graph usage in academic as well as popular media in the years 1985-1994. They note an increase in the graph usage in their survey is also reflected in parallel in the manner in which software tools developed to produce graphics. They note in the survey that in the time they studied the number of graphs per scientific journal nearly doubled and most of the graphs they saw were simple depicting relationship between two variables.

In contrast to the graphics in popular media, Mason (2006) elaborates that the images in scientific textbooks require particular attention and background knowledge.

> . . . the images in the book are not dedicated to grabbing attention and entertaining the reader. The book assumes that the reader is already interested and will be giving full attention to the subject. Scientific images are similar to popular images in that they aim to communicate a specific idea or concept but they offer themselves as vehicles for analytical thought and extended interpretation. Scientific images are exceptionally rich in content because the concepts they carry are meaningful only within the context of the network of scientific principles and procedures which have brought the concept into being in the first place. (p. 12)

Thus the graphs are richer than the text and the way scientific graphics are produced and consumed, at least in principle, differ in the popular and scientific media. We encounter different types of graphics like photographs, illustrations, graphs, paintings, sketches, maps, diagrams among others. Each of these types has its potential and performs a set of functions. A particular graphic may perform more than one function, and in some cases, it is almost impossible to achieve the same result without the graphic. Processing information in the present society which is full of technological artefacts is highly dependent on the reader's ability to comprehend graphs (Curcio, 1987).

However, making sense of graphics, particularly those used in science, does not happen spontaneously. For each of the types, we have mentioned there is a way of understanding them, performing operations and also making them. This skill of understanding graphics has to be taught explicitly. The term *graphicacy*, coined by (Balchin & Coleman, 1966) carries this connotation - the skill to understand graphics in general.

In our work, we focus on graphs which represent quantitative data that help build models. Friel, Curcio & Bright (2001) use the term *graph sense* in a similar spirit to concepts of number sense and symbol sense. They suggest that:

> Graph sense develops gradually as a result of one's creating graphs and using already designed graphs in a variety of problem contexts that require making sense of data. (p. 145)

They further provide a list of behaviours associated with the idea of graph sense, this list comprehensively covers many aspects of using a graph:

> 1. To recognize the components of graphs, the interrelationships among these components, and the effect of these components on the presentation of information in graph.
> 2. To speak the language of specific graphs when reasoning about information displayed in graphical form.
> 3. To understand the relationships among a table, a graph, and the data being analyzed.
> 4. To respond to different levels of questions associated with graph comprehension or, more generally, to interpret information displayed in graphs.
> 5. To recognize when one graph is more useful than another on the basis of the judgment tasks involved and the kind(s) of data being represented.
> 6. To be aware of one's relationship to the context of the graph, with the goal of interpretation to make sense of what is presented by the data in the graph and avoid personalization of the data. (p. 146)

In this section, we have seen how various authors have positioned graphs and their uses in communicating ideas, finding patterns. We have also looked at the definition of the term *graphicacy* and the notion of skills this definition conveys. In the next section, we focus on the importance of graphs in the process of doing science.

1.2 Graphs in science

Graphs play a crucial role in the discourse of scientific practices. There is a ubiquitous presence of graphs in all forms of scientific literature like journal papers, books, presentations and posters. It is virtually impossible to find a scientific textbook or a scientific journal without graphs and diagrams (Roth & Bowen, 2003). Among the various graphical representations in the scientific practice, graphs are found to be the dominant ones (Roth, Bowen & McGinn, 1999). Particularly in the case of science, graphs serve as the tools for understanding and analysing the phenomenon under investigation (Cleveland, 1984; Krohn, 1991; Larkin & Simon, 1987). The graphical representations, in turn, inform and influence fact construction, theory testing, and the intermediate process of theory formation (L. D. Smith, Best, Stubbs, Johnston & Archibald, 2000). Furthermore, among the various graphical representations found in the scientific discourse, graphs constitute the dominant form (Roth et al., 1999).

What makes graphs so central and essential in sciences? Graphs provide several affordances to those who can read them thoroughly. A well-trained scientist is adept at grasping the underlying phenomenon while engaging with abstracted representations. Arguably, this ability influences multiple activities that a working scientist engages with namely, making a hypothesis, correlating different observations, designing experiments, and drawing inferences. There are several instances in history of science where graphs have played a significant role, leading to discoveries and better theories. For example, the discovery of dark matter in the galaxies from galactic rotation curves. In *Graphical Methods for Data Analysis* (Chambers, Cleveland, Beat & Tukey, 1983), the authors say this about the power of graphs:

> There is no single statistical tool that is as powerful as a well-chosen graph . . . An enormous amount of quantitative information can be conveyed by graphs; our eye-brain system can summarise vast information quickly and extract salient features, but it is also capable of focusing on detail. Even for small sets of data, there are many patterns and relationships that are considerably easier to discern in graphical displays that by any other data analytic method. (p. 1)

Chambers et al. (1983) cite three main objectives for graphical display (a) to record and store data compactly, (b) to communicate information to other people, and, (c) to analyse a set of data to learn more about its structure. Roth et al. (1999) points to three main purposes for graphs in science and engineering: "graphs are semiotic objects that constitute and re-present (and reify) other aspects of reality; graphs serve a rhetorical function in scientific communication; and graphs act as conscription devices that mediate collective scientific activities (talking, constructing facts)." (p. 97) In fact, it has been argued that for experts in the field, the graphs and variables themselves become transparent, and experts can see the phenomena behind the graph directly (Roth, Pozzer-Ardenghi & Han, 2005). By practice, graphs can themselves become *iconic*, representing the underlying phenomena.

The centrality and the pervasiveness of graphs in science led Latour (Latour & Woolgar, 1986) to conclude that scientists exhibit a "graphical obsession", and to suggest that, in fact, *the use of graphs is what distinguishes science from non-science.* Perhaps this misperception that any discourse containing graphs is scientific leads to many commercial advertisements to exploit a 'scientific' basis for selling. Studies show that use of graphs makes advertisements more convincing (Tal & Wansink, 2016). There are several instances of graphs used to obfuscate data, make misleading claims on many issues and matters relating to public (Wainer, 1984). This social reality further underscores the need to ensure the education of the general public in understanding graphs. Given that graphs are part of the standard curriculum in our existing school system, it would be highly desirable to inculcate a critical understanding of graphs from early learning stages.

Given the preference for graphs in scientific discourse, we collate, from various published research articles, a comprehensive set of functions a graph may satisfy. We can broadly classify the uses of graphs in the following categories:

§ **Communication and Visualisation of Data:** To allow us to present the data in a way which can be understood by the intended audience. Often, particularly when the data sets are large, graphs are the only parsimonious way to visualise and communicate the data. We 'see' the data and many times the phenomenon itself through graphs.

§ **Analysis of Data:** To analyse data for trends and patterns. Visualising data in the form of graphs reveal patterns or peculiarities which might stay hidden otherwise. The ability of graphs to visualise data is perhaps the most powerful use of graphs in scientific practice.

§ **Modeling:** To create and test mathematical models for explaining data and phenomena. The mathematical models of various phenomena are compared with the actual data points. The "visual fit" between the theoretical curve representing the mathematical model and the actual data points can tell us about the accuracy and the applicability region of the model. On the other hand, when the observational data does not "fit" the predicted curve from the theoretical model, interesting results can arise. In this context, the graphs can be seen as mediators between the theoretical model and the phenomena which the model describes. It is usually through graphs that a model and the empirical data "meet" each other.

§ **Rhetorical Device:** To argue for a particular conclusion. Graphs are routinely used for arguing for a particular conclusion, based on a particular feature on the graph.

In this section, we saw the centrality of graphs in the scientific discourse. In the next section, we see graphs in the context of science education and define the research questions being addressed in this work.

1.3 The problem statement

With graphs being so vital in science, one would expect that in science and mathematics education there is a specific emphasis on developing the skill of graphicacy. However, as various studies reveal (for details, see Chapter 3), the students (from school level to undergraduate level) either do poorly in graph related tasks or are not taught graphicacy as a separate skill. In *Critical Graphicacy*, Roth et al. (2005) discuss graphs and their usage in textbooks. They found both qualitative and quantitative differences between the uses of graphs in textbooks and scientific journals. The number of Cartesian graphs that were used in the textbooks was found to be low as compared to that of journals. In textbooks, only a few of the graphs had data from actual experiments. Frequently, the graphs lacked basic features like units, scales and captions and many graphs did not have any mention in the main text. All these issues made the interpretation of the graphs difficult not only for the students but also for the experts. In contrast, scientific journals provide detailed help to understand the meaning of the graph and the graphs are usually very well integrated with the main text. The authors of the textbooks assume knowledge about the graphical representation of the phenomenon to be presented. So the problem for the students, according to Roth et al., becomes *two-fold*: on the one hand, the literature suggests that the skills required to comprehend and construct graphs were lacking among students, and on the other hand the textbooks do not provide them with enough resources and opportunities to read and interpret graphs. Little is currently being done to address this directly issue (Aldrich & Sheppard, 2000; Paoletti, 2007; Peden & Hausmann, 2000). When present, the tasks on graphs, with little emphasis on qualitative, investigative or critical questions (Brasell & Rowe, 1993; Leinhardt, Zaslavsky & Stein, 1990; Padilla, McKenzie & Shaw, 1986). This discrepancy between the crucial role of graphs in the process of doing science and perceived lack of emphasis on teaching graphing skills to students sets the basic premise of our work.

We thus see that there is a clear incongruity between the needs (both general and science-specific) pertaining to an understanding of graphs, and way this need is addressed in the curriculum. Keeping this in mind, and absence of any systemic study of these issues in the Indian context sets the context for the problems addressed in the first part of the thesis. The first part has research questions to describe state of graphicacy in the Indian context. To address this aspect, we look at the most influential curriculum framework in India (NCF 2005) and the textbooks resulting from it. with a focus on the graphical practices. The result of this analysis coupled with studies from the literature, point us to a direction that a holistic approach to the issue of graphicacy is needed which is considered in the second part. The second part is developmental in nature and looks at what are the best ways in which to teach graphicacy keeping in mind the organic perspective. In this regard we conjecture a learning design arrived at by literature and our own analysis of textbooks. This way we mark the two major objectives of this work: first to establish and map the state of graphical practices in the Indian context, and second to establish a holistic framework for designing activities with embedded graphical practices.

Our work has two major parts. In the first part, we look at how Indian textbooks make use of graphs. In the Indian education system, textbooks are central to the discourse in the classroom, and for many students, they remain the only source of knowledge. The students formally encounter graphs via their textbooks. In the schools, the three categories of textbooks which make use of graphs are science, mathematics and social sciences. Seen with this viewpoint, we felt the need to understand the way in which textbooks expose the students to graphs and help them in making meaning from them. These notions form the theme of the basic idea that we explore in *Part 1: Graphicacy and its problems*. We have two research questions in Part 1, which pertain to the presence of graphs in the textbooks.

The first question was to find out (a) how *frequently* graphs occur and, (b) what *types* of graphs students encounter in the textbooks. We think of graphicacy as an interdisciplinary, generic skill. Hence we looked at how textbooks of different subjects represented graphs. To also know whether there is any progress in the frequency of the graphs appearing in the textbooks as the classes increase, we also included the textbooks of different classes. These two themes further informed two sub-questions: (a) how are graphs placed in textbooks of different subjects, and, (b) whether the frequency of graphs increases with classes. This led to the formulation of our first research question:

Part 1: Research Question 1

① How are graphs placed in the Indian school textbooks in different subjects and different classes?

(a) What are the different types of graphs that are present in the textbooks?

(b) How frequently do graphs occur in the textbooks?

(c) How do graphs compare across subjects?

(d) How do graphs compare across classes?

Research question ① is addressed by performing a quantitative analysis of textbook contents and is presented in the first part of Chapter 3. Answering research question ① could give us a bird's-eye view of the presence of graphs in the textbooks. To understand the use of graphs in the meaning-making process, we need to understand what function do they play in the textbooks. In this case, we particularly focus on the science textbooks. We felt a need to take a detailed look at the function of graphs in the narrative of scientific textbooks. In particular, we were interested to look at (a) role that the graphs play, (b) the scientific concepts associated with them, (c) data handling associated with the graph, and finally, (d) the design aspects of the graphs. These sub-questions emerge from the usage of graphs in scientific discourse and pedagogical studies related to graphs in the context of teaching and learning. Graphs play a variety of roles in supporting and many times being central in the narrative of scientific processes and communication. Depending on the context the graphs can play a variety of roles, including, but not limited to, (a) display of data, (b) finding information and patterns, (c) interpolating or extrapolating data, (d) making inferences and predictions, and,

(e) making and testing mathematical models. Each of these roles of graphs can be used to enhance the presentation of a given concept in the textbooks. We looked at which of these roles do the science textbooks employ.

Some scientific concepts in the textbooks are in general more amenable to the use of graphical representations. Notably, the concepts in which mathematical models are well established, and quantitative data is available (for example, the concept of motion) have a substantial number of graphs associated with them than other concepts. This analysis would give us an opportunity to critically look at the relation of graphs to various concepts presented in the textbooks. Also, this would enable us to find "missed opportunities" where graphs could have been used to enhance and reinforce conceptual learning.

When the graphs are presented in the textbooks, the associated data in the form of a table can be crucial for informing the learners about the various aspects of the graph. Particularly knowing how the data was collected (devices and procedures) and made amenable to plotting, makes the learner aware of the physical situations representing the data. The aspect of data handling is seen as a norm in scientific journals. We wanted to know whether the same practice is used in the scientific textbooks too. Particularly for the learners, the presence of data in the form of multiple representations like tabular, algebraic and graphical along with verbal descriptions of how to read and make sense of data is significant. The ability to move between multiple representations of the data has established advantages (Even, 1998; Friel et al., 2001; Mosenthal & Kirsch, 1990a; 1990b).

Finally, we need to look at the various design aspects of graphs presented in the textbooks. The design of the graphs can have a significant impact on the way they are perceived and comprehended. As shown in various studies in the area of graphical perception and meaning-making, the design details of the different features on the graphs (like units, labels, legends, colour) can impede or enhance the comprehension of graphs.

The points discussed above led to the formulation of our next research question:

Part 1: Research Question 2

(2) What kind of opportunities do Indian school science textbooks offer to the learners to engage with graphs meaningfully?

(a) What is the role graphs play in the Science textbooks?

(b) Which are the topics where graphs are used?

(c) How is the data associated with the graph presented?

(d) What are the missed opportunities where graphs could be used effectively?

(e) How are the graphs designed to aid their comprehension?

To answer research question (2) we did a detailed analysis of graphs in Science textbooks from Grade 5 to 10. We present our findings on research question (2) in the second part of Chapter 3.

In Part 1 of the work, via research questions (1) and (2) we establish how well are the graphs situated in the Indian Science textbooks. We also reviewed the literature (Chapter 2) for the various problems that arise in the context of constructing and comprehending graphs in various disciplines.

In *Part 2: Developing and testing the activities* guided by the inputs from the literature and by our analysis of the textbooks, we want to develop a constructionist framework for designing activities which will address some of the concerns raised. In particular, we want to establish a comprehensive framework for understanding meaning making from graphs in varied contexts. While creating this framework, we did not see graphicacy as an ability in isolation but situated in a broader context of comprehending the world around us. As we have seen in the last section, the use of graphs in science is for specific purposes, and the graphs are set in the broader context of the topic where they appear. We believe that from a learners' perspective, this point should be exemplified by real-world experience. The multitude of connections that the graphs make to other concepts and variety of roles that the graphs have to be brought to the cognisance of the students. These issues form the basis of our next research question.

Part 2: Development Objective 1

(1) What design principles of a learning activity could comprehensively address the issues of comprehending and constructing graphs informed by the literature situated within a constructionist framework?

Chapter 4 presents the design principles. To check the implementability of the learning framework resulting from development objective (3) we developed and piloted a few activities through the field studies. This forms the part of the fourth research question.

Part 2: Development Objective 2

(2) What are the required experiences and technological tools that support the comprehension and construction of graphs informed by the design principles and whether they can be implemented?

The development objective (2) is addressed by designing, developing and deploying three activities reported in Chapters 5, 6 and 7 respectively.

The research questions (1) and (2) are investigated by quantitative and qualitative analysis of textbooks. For conducting the qualitative analysis, we developed a rubric. Development objective (1) is addressed by drawing from the literature review and our textbook analysis presented in Part 1, the overarching theoretical framework that we have used is that based on constructionist pedagogy. Development objective (2) is addressed by exploratory study leading to the development of activities situated in a real-world problem-solving contexts.

In the next chapter, we start with a review of literature which informs us about various issues pertaining to construction and comprehension of graphs in different contexts.

*Evidence presentations are seen here from both sides: how to **produce** them and how to **consume** them. As teachers know, a good way to learn something is to teach it. The partial symmetry of producers and consumers is a consequence of analytical design, which is based on the premise that the **point of evidence displays is to assist the thinking of producer and consumer alike**. (emphasis in original)*

Edward Tufte, *Beautiful Evidence, 2006*

2

Problems of Reading and Making graphs

2.1 Introduction

In this chapter, we present a review of literature about the difficulties that are faced while constructing and comprehending graphs. The importance of constructing and comprehending graphs as a core skill for science and mathematics is well established (Gallagher, 1979). The *Benchmarks for Science Literacy* (American Association for the Advancement of Science [AAAS], 1994), consider making and understanding graphs as an important skill. The *Learning Indicators* of NCERT mention at each stage the requirement of reading and constructing graphs (NCERT, 2014). The NCF 2005 position papers on *Science Education* (2006b) and *Mathematics Education* (2006a) also mentions graphing as an important component. Particularly the position paper on Mathematics Education notes:

> Children learn to draw graphs of functional relationships between data but fail to think of such a graph when encountering equations in physics or chemistry. That

> algebra offers a language for succinct substitutable statements in science needs underlining and can serve as motivation for many children. Eugene Wigner once spoke of the unreasonable effectiveness of mathematics in the sciences. Our children need to appreciate the fact that mathematics is an effective instrument in science. (p.10)

However, the learners face many challenges in both the comprehension as well as the construction of graphs. The lack of meaningful opportunities in the textbooks compounds this problem. We review the main outcomes from earlier work done in the area of constructing and comprehending graphs. Earlier reviews in this area include studies by Friel et al. (2001), Glazer (2011), Leinhardt et al. (1990), Shah & Hoeffner (2002).

Graph comprehension or interpretation is a complex activity. There are many factors involved in the correct reading a graph. The correct reading of a graph would indicate the reader extracting meaning, mathematical relationship or making sense of a situation or a phenomenon depicted. Some of the major categories of research in this area include (a) prior domain knowledge of the learners, (b) prior experience of reading graphs, (c) the design of the graph, (d) the contextual setting of the graphs, and, (e) models (cognitive and social) for the understanding of graphs. Thus a successful comprehension of a graph could involve an interaction of many factors.

A graph comprehension or interpretation can be said to be successful if the reader or interpreter can answer a specific set of questions based on the graph. Depending on the nature of these questions and the processing required to answer them, they can be categorised by levels of comprehension as suggested by Bertin (cited in Wainer (1980) and also in Bertin (2011)). These categories are:

1. **Elementary:** these type of questions involve extraction of exact information, usually involving a single element of a component for answering. For example, what was the amount of rainfall on Wednesday?
2. **Intermediate:** these type of questions involve a group of elements in the component for answering questions related to detection of trends or patterns. For example, did the average temperature rise in the first four days of the week?
3. **Comprehensive:** these type of questions involve comparison of whole structures in the graph. For example, how is the dependent variable related to the

independent variable?

Curcio (1987) puts these three categories as *read the data*, *read between the data*, and *read beyond the data*. These three levels are in the increasing level of complexity of the reading tasks. A study by Pereira-Mendoza & Mellor (1990) shows that the literal reading of graphs by grade four students (elementary level questions) presents a minor challenge. On the other hand, the students face more difficulties in solving the intermediate level questions which may involve interpretation, computations and reading of both text and numbers. We next look at various studies in the domains of science and mathematics education with a focus on graphs. The approach of the science and mathematics education towards graphs has different foci, addressing the needs of the field.

2.2 Misconception studies

In general, the studies in the area which look at aspects of graph construction and comprehension report that students face difficulties in dealing with both these activities.

Students do have some *intuitive* understanding of graphs and graphing. By intuitive, it is meant that the understanding that the students have regarding graphs without any explicit teaching. This is particularly true when the students are looking at graphs or constructing graphs in a context they are familiar and understandable. Ainley (1995) follows this *intuitive* approach to graphing, in which the primary grade learners first make sense of graph intuitively and then with the help of computers to construct graphs in familiar contexts. In this study, the students were provided with continuous and immediate access to computers. The study is set in the context of the growth of human body, and the class had been researching about their body measurements and how they have changed over the years. A simple table showing the age and the height of the children from birth to three years of age and a graph depicting it was plotted in front of the class using a spreadsheet. The children were then asked to comment on what they saw. What emerged was that the children were able to make sense of information intuitively. The children were then provided data showing heights of four imaginary children, and the task was set to find an unknown piece of data using interpolation.

The results of the study indicated that the children were able to respond intuitively to the idea of growth using graphs and were successful in plotting, reading values from the graph and interpolating them. Some of the features that helped in this regard were: the meaningful and familiar work; presentation of a holistic image; working with many similar graphs; use of computers to create many types of graphs with different scales dynamically; a set of purposeful tasks.

Motion: The concept of motion presents several opportunities to introduce the idea of graphs. In the case of physical sciences, the topic of motion usually has several graphs introducing the students to the graphical representation. The topic of motion has several studies related to graphs. Lillian McDermott's and her colleagues' work with graphs in the context of motion gives particular examples of students' mistakes and alternative conceptions (McDermott, Rosenquist & van Zee, 1987; Trowbridge & McDermott, 1980; 1981). A summary of the major findings from their work is as under: (a) Confusing between slope and height: students typically find it difficult to determine if the relevant information should be extracted from the height (of the line intercept) or slope of the graph. (b) Interpreting changes in slope and height: this mainly relates to graphs which show a change in the direction of motion. The change in slope is confused with the change in height, leading to incorrect conclusions regarding motion depicted by the graph. (c) Relating different types of graphs: students mainly are unable to create graphs of physical concepts which are related to each other. For example, distance vs time and velocity vs time graphs. (d) Matching narrative information with relevant features of the graph: Students cannot make use of the relevant information given in the text, and rather rely on memorised algorithms. (e) Interpreting area under the graph: Students are unable to understand the physical significance of the area under the curve for a given graph.

They further note that the simple questions, solved by relying on memory, present little difficulties to the students. In contrast to this, the questions about interpretation which are unsolvable by using memorised algorithms, for example, the questions which would fall in *intermediate* and *comprehensive* categories of Bertin (Bertin, 2011), are challenging. The movement between the physical phenomena and the graph representing it is not natural and presents comprehension difficulties. In the second part of the article McDermott et al. (1987), note the problems students have while making connections between graphs and real-world examples.

(a) Representing continuous motion by a continuous line: in this case, students are not able to use the data given to plot the graph correctly. They often ignore that the graphs should be smooth and continuous without kinks. (b) Separating the shape of a graph from the path of the motion: in this case, the students confuse between the shape of the actual path of the object and the graph depicting it. This is same as the "Graph as a picture" error. (c) Representing a negative velocity on a v vs t graph: students often fail to represent negative velocity as a reversal of the graph itself. (d) Representing constant acceleration on a a vs t graph: students often fail to understand what constant acceleration means, sometimes confusing its representation with other variables like speed and displacement. (e) Distinguishing among different types of motion graphs: students are typically unable to look at graphs and depict the kind of motion that would be represented by these graphs.

Kinematics: Other studies in the area of motion are on kinematics, for example, that of Beichner (1994) also report similar problems. Beichner developed and analysed a test to look at graphing related issues in the area of kinematics. This test, called as Test of Understanding Graphs in Kinematics (TUG-K), was administered to high school and college students after instruction on the topic. The mean score on the test was 40%. The analysis of the test reports trends similar to the study of McDermott et al. The misconceptions reported include: (a) misunderstanding graph as a picture, (b) slope and height confusion, (c) variable confusion, (d) inability to understand area under the curve and (e) non-origin slope error. In the last error type, that is the non-slope origin error, the students can attempt a task correctly if the line is passing through the origin, but in case the line is not passing through the origin, they are not able to do so. Another study exploring motion graphs in the context of kinematics is the work by Eshach (2014). This work explores the framework of *intuitive rule theory* to deepen the understanding of difficulties in creating and reading kinematic graphs. This article argues that "explicit teaching in the context of graphs may trigger the learner to engage in active thinking and to foster a deep understanding of graphs. In the case of graphs, explicit teaching should include knowledge about intuitive rules and their impact on graph reading and that teaching should explicitly refer to the pitfalls those intuitive rules may cause." (p. 7)

Distance-time graphs are one of the first instances that students encounter the graphical representation with mathematical modelling in the context of physical sciences. In case of distance-time graphs, the concrete experience of moving from one place to another can be represented using graphs. In our review of textbooks presented in the next chapter (Chapter 3) we find that majority of the graphs in the science textbooks are related to motion. Wemyss & van Kampen (2013) investigated responses of university students' difficulties in dealing with problems involving distance-time graphs. Another aspect of their study was to find the dependence on qualitative aspects of numerical time-distance graphs to answer problems based on them correctly. Moreover, finally, they investigated if this reasoning is transferable to another setting (water-level vs time graphs). The study found that students have difficulty in answering questions about speed from the numerical value. In cases where numerical answers are expected, the students do not use the graphs but switch to a formula. The researchers also found an improvement in student responses in graphs with the context-free settings. One of the conclusions that they draw is that:

> Before and after instruction, students answered qualitative questions on water level versus time graphs better than questions on distance-time graphs; differences in answering the numerical question, in terms both of quality and of prevalence, disappeared. This suggests to us that the school experience with the distance-time graphs negatively impacted on the students' problem solving strategy and their likeliness to engage with the question. (p. 15)

This outcome has important ramifications on how motion graphs are introduced to students in school textbooks. In some studies, the focus is both on graph comprehension and construction skills. For example, Brasell & Rowe (1993) looked at both graph comprehension and construction skills of high-school students with a focus on physics. This study found that students face difficulty in comprehending graphs of derived quantities than directly measured quantities such as length. They also found that students had errors in linking graphs with verbal descriptions of events or description of variables. They further found that the students had a very superfluous understanding of the meaning associated with graphs and were unaware (as measured by a self-test) of their deficiencies with graphing.

Computers and study of motion: The idea of using computers to aid understanding of graphs in the context of motion is explored well by several studies. Many works explore the idea of using a computer-based laboratory to learn the concepts of motion (Kozhevnikov & Thornton, 2006; Mokros & Tinker, 1987; Thornton, 1987; Thornton & Sokoloff, 1990). The common thematic elements in these studies are that students can explore graphs in real-time (or almost immediately) by trying out different experiments with the setup. For each experiment performed, they get a corresponding graph so that the students can make the connection between the physical phenomenon and its graphical representation.

In the study by Thornton & Sokoloff (1990), the students collected data in *real-time* about the motion of objects using a motion detector attached to a computer. The data from the measurements was displayed in a graphical form on the computer. Students could then transform and analyse the data, print or save the data in the form of tables or graphs for later analysis. The setting of these experiments was a specially designed curriculum which guided the students to examine the appropriate phenomena. They found a substantial improvement in student's understanding of concepts associated with Newton's laws of motion.

The study by Kozhevnikov & Thornton (2006) looks at relations between spatial visualisation ability and graphical performance. Spatial ability can be defined as an "ability to generate, retain, and manipulate abstract visual images" (Lohman, 1979, p. 188). The tasks involved in measuring spatial ability typically involve performing mental rotations, visualising paper folding or "tasks that require the subjects to generate, maintain, and coordinate information during spatial transformations"(Kozhevnikov & Thornton, 2006, p. 112). Studies on spatial ability and spatial visualisation in science form a sub-discipline of their own and have many implications on how learning happens. This study found that spatial ability is related to graph interpretation tasks in the context of graphs of motion and force. In their study, they found that solutions involving motion of only one object do not require much spatial ability. They found that students with high-spatial ability face lesser difficulties particularly in solving multi-dimensional problems. The spatial ability is found linked to "graph as a picture" error: low-spatial ability students were more susceptible to "graph-as-picture" interpretations compared to high-spatial ability students. The study found that use of computers improved spatial visualisation skills by experience with visual graphical representations.

Computers and graphs: Another theme in the context of the study for the use of computers is to collect data using hardware probes and to plot graphs using the computers. Nachmias & Linn (1987) designed a computer-based laboratory course, which involved critical evaluation of scientific data collection and analysis in the context of heating and cooling. They note that the students initially looked at the computer generated graphs in an uncritical way, but during the intervention, various aspects that might affect the interpretation of graphs like scaling and hardware issues were brought to focus. This addition helped the students to develop a deeper understanding of the data and link it to the knowledge about heating and cooling they had.

In the context of chemistry, a study by Dori & Sasson (2008) points that the use of computer-based intervention was most helpful to low-performing students. This study explores graphing in the context of computer-based chemistry learning environment. They also looked at the bi-directional transfer by students between the textual representations and computer-generated graphical representations. They found that use of computers had a positive impact on students' ability to sketch predictive curves based on the textual information. Adams & Shrum (1990) discuss the effects of a computer-based intervention on construction and interpretation of line graphs of grade 10 students in the context of biology. They found plotting line graphs by hand had a better impact on graph construction tasks than plotting graphs by using a computer. In the case of graph interpretation tasks, they found that "microcomputer-based laboratory exercises that collect and present experimental data to students as 'real-time' graphs result in educationally significant achievement on graph-interpretation tasks." (p. 785)

The studies with computers based on interpretation and construction of graphs in different contexts and at different levels show positive learning experience for the participants. We explore this idea in one of our field studies in the context of electromagnetic induction in Chapter 7.

The work by (Lai et al., 2016) presents a graph inventory to measure graph comprehension based on the knowledge integration (KI) framework (seen in next section). They found that these items were reliable as per item response theory and showed that most students had problems in making connections between scientific concepts and graphs especially when asked to critique or construct graphs. They also found differences in student performance related to first language and use of computers. They recommend that graphs should be better integrated to science teaching.

Roth et al. (1999) did a comparative study of university biology students and research scientists for graph interpretation. Scientists and students differed in their approach towards interpreting graphs. Tairab, Khalaf & Ali (2004) found that students found it more challenging to construct graphs than to interpret them. The study also indicates that students find the global interpretation of graphs challenging, particularly when asked to plot qualitative graphs explaining what the graph represents. One of the implications of the study is that the students should be provided experience in interpreting qualitative, quantitative tasks and also in constructing graphs.

Many studies look at graphs in the context of statistics. The works by Cleveland and Wainer primarily focus on the statistical relevance of graphs (Anscombe, 1973; Cleveland, 1987; Cleveland & McGill, 1984; D. R. Cook & Weisberg, 1999; Wainer, 1984). The focus in these studies is on understanding the meaning present in the graphs and possible problems in the context of statistics.

2.3 Studies with focus on mathematics

In mathematics education, the focus in the context of graphs is primarily on understanding the relations between multiple representations. The graphs (usually of algebraic functions) are seen in relation to other representations like textual, algebraic and tabular. Multiple representations is an area of mathematics education research, and a significant number of studies in this domain deal with graphical representations. Another aspect of mathematics education studies focuses on technologies like graphing calculators and dynamic mathematics applications, in constructing and comprehending graphs (Cavanagh & Mitchelmore, 2000; Demana, Schoen & Waits, 1993; Demana & Waits, 1990).

Leinhardt et al. (1990) reviewed students' ideas and problems on mathematical functions and their representation using graphs. According to them, the focus on functions and graphs is crucial for the following reasons: (a) Functions and graphs is a topic that does not appear until upper elementary grades. (b) Functions and graphs represent one of the earliest points in mathematics in which student uses one symbolic system to expand and understand the other. (c) Graphing can be seen as one of the "critical moments" in mathematics. They define a *moment* as "site within a discipline when the opportunity for powerful learning - different from other episodes may take place." Graphs present this opportunity for powerful learning by enabling the visualisation of data and algebraic form. This review article focuses on the difficulties that students have regarding the comprehension of graphs with mathematical functions as the background. The study classified students' difficulties in this area into four kinds of categories, which included difficulties in both constructing and comprehending the graph: (a) Confusing the slope and the height: typically students confuse the height of the graph for its slope when two intersecting graphs are involved. This is similar to the error reported earlier in the context of physics. (b) Confusing an interval and a point: students tend to confuse the idea of an interval for a function to one single point. (c) Considering a graph as a picture or a map: students consider the graphs typically as a picture representing a situation. This is similar to conceiving the graph as a path of motion seen in the context of physics. (d) Conceiving a graph as constructed of discrete points: Students do not see the graph as the whole unit. As we can see, some errors in the context of graph interpretation and constructions are common across domains. Errors are also reported in the context of the construction of graphs in studies with a mathematical focus. The study by Mevarech & Kramasky (1997) presents students' alternative conceptions related to construction of graphs. They report three major issues in this regard: (a) Constructing an entire graph as one single point: students will construct just one point corresponding to one situation while constructing graphs. (b) Constructing a series of graphs, each representing one factor from the relevant data: instead of integrating data pertaining to the situation, student construct separate graphs for each point. (c) Conserving the form of an increasing function under all conditions: students use the linear form even while describing situations which are non-linear.

Studies by Janvier and colleagues focus on the ability of students to represent situations using graphs (Bell & Janvier, 1981; Janvier, 1981; Preece & Janvier, 1992). Janvier and colleagues explore the idea of varied situations and the way in which students create and interpret graphical representations of these situations. Janvier (1981) notes "Situations in graphical interpretation and in mathematics are used as *mental images* which are brought to consciousness by memory" (p. 121, emphasis in original). These situations include using a situation of correlating racing tracks with speed-time graphs of cars (1981). In another study, they looked at graphs based on the contexts of orchards and sewage (Preece & Janvier, 1992). This study points that the students need features from familiar contexts to relate to features of a graph. In familiar contexts, the students find it easy to understand the features of the graph. The students approach a graph with background knowledge, most of which is irrelevant, and while interpreting the student has to decide the relevant information. The variables in the graph and its shape play a role in the selection of this relevant information. The shape of the graph affects the students in two ways: (a) a prominent feature will draw the pupil's attention and (b) "the form of the graph in relation to its context will lead the pupil to select knowledge and to make up an initial story about the variables." (1992, p. 305)

Once the particular knowledge is selected the student "will gradually build up the story by integrating the graph with its context more closely". Thus an understanding of graphs is inherently tied to the context in which the students situate the graph. When graphs are in unfamiliar context, the students tend to give a graphical interpretation in which he or she describes the graphical features in detail with little reference to context." (p. 305)

In the implications of their work, they point out that " [students] need to be given practice in interpreting trends in graphs as well as reading values, constructing graphs, and doing more mathematical tasks, such as calculating the area under curves. etc." (1992, p. 305). In another study, Bell & Janvier (1981) also notice the importance of tables of data values being present in graph interpretation tasks :

> ``As mentioned before, the use of tables proved a powerful tool to study `how variables change'. The results conclusively show that the table approach certainly spelt out many ideas to the extent of making possible transfers from tables to graphs. Consequently, results suggest that the use of tables should be included in our graph teaching scheme.'' (p. 41)

We think this is an important point, and refer to it in our analysis of textbooks (Chapter 3), our design framework (Chapter 4) and analysis of activities (Chapters 5, 6, 7)

Linear functions are the most basic functions that the students encounter in mathematics. Functions of the form $y = mx + b$ can be used to represent a variety of situations from real-life. There are many studies in mathematics education which explore the ideas of students in the context of linear equations. Typically these studies explore the interpretation of slope m and intercept b in a variety of situations, for example (Baltus, 2010; Edwards & Chelst, 1999; Johnsen & Wilkerson, 2003; Nagle & Moore-Russo, 2013; Newburgh, 2001; T. M. Smith et al., 2013; Wagener, 2009). In different situations, the two parameters of slope and intercept have different physical meaning. Even though the linear function is considered to be simple and elementary, studies indicate that there are systemic problems that students encounter. Schoenfeld, Smith & Arcavi (1993) provide a very detailed case study of one student set in the context of graphs with a focus on conceptual structure and change. They focused on the concept of the slope-intercept schema for straight lines and used an interactive graphing environment (Grapher) to situate the study. They found that the student had conceptual difficulties concerning (a) Slope: For example, the sign of the slope depends on where the origin of the line: negative slope means it is coming from negative side (of the coordinate quadrants) and magnitude of the slope is dependent on the location of the graph. (b) Intercept: For example, initially the student believed that three parameters, namely slope of the line, x-intercept and y-intercept are required to determine a line. They further found that y-intercept to be highly context-dependent, and slope and intercept are not independent. Chiu, Kessel, Moschkovich & Muñoz-Nuñez (2001) present a case study of one learner's conceptions about the intercept in case of linear functions. They particularly look at the change in conception regarding the vertical movement of the linear function $y = mx + b$, (where m is the slope, and b is the intercept) when b is changed.

In our work, we look at two examples of modelling with linear functions. Both of these contexts involve indirect measurements and are situated well within the school curriculum. The first example involves measuring the average diameter of mustard seeds (Chapter 5) and the second example involves measuring the distance-diameter ratio of the Sun (Chapter 6). In particular, we look at how situations in these two cases concretely represent the slope.

Some of these difficulties could be addressed using the interactive graphing environment. The use of dynamic mathematics tools opens up a whole new segment of approaching functions graphically. By using dynamic mathematics tools, students can explore the functions by varying different parameters. The option of zooming in and out, automatic calculation of values, all adds to helping students develop a better understanding of the concepts involved. Arcavi & Hadas (2000) describe the use of a dynamic geometry software to explore the areas of triangles graphically. The situations and graphs representing them are made available to the learners *before* the algebraic representations. In Arcavi (2008) the variation of the area of geometrical figures by keeping one side constant is used to create graphical, mathematical models of the variation. Moschkovich, Schoenfeld & Arcavi (1993) consider the *process* and the *object* perspectives when looking at symbolic representations of algebraic, graphical and tabular in the context of functions. The process perspective perceives a function as linking of x and y values, while in the case of the object perspective, a function or any of its representations are seen as entities. In this framework, the competence in the handling of functions can be seen as comfortably moving between the two perspectives and the three symbolic representations (tabular, graphical, and algebraic) as required. Hitt (1998) identifies five different levels of understanding of the concept of function. This study looks at the problems faced by mathematics teachers regarding the concept of a function. Some of the major issues that are identified are: the teachers are not able to discriminate if a given curve is a function, if an algebraic equation exists (not able to use the vertical line rule effectively), definition of a function related to concept of variable is not used, instead preferred choice is the rule of correspondence or the ordered set. The teachers also had problems in constructing graphs from a function and representing physical situations using graphs. Asiala, Cottrill, Dubinsky & Schwingendorf (1997) provide a detailed deconstruction of epistemological concepts linked to the derivative with focus on graphs based on the action-process-object-schema (APOS) framework.

Ben-Zvi & Arcavi (2001) present a case study of two grade seven students performing Exploratory Data Analysis (EDA) using interactive technological tools with a focus on evolving global view of data. They provide analysis from both cognitive and socio-cultural perspectives. The study notes that local point-wise view of data can sometimes limit the students from having a global look, but in other occasions, it can provide a basis for a global vision of the data and global view may indicate different meanings for students and experts in given contexts.

Work with teachers: Another area of research has been the work with teachers and their ideas about graphs. Even (1993) looked at the pedagogical content knowledge regarding functions and graphs in a sample of prospective teachers. This study found that many of the prospective teachers do not hold a modern conception of a function. Bowen & Roth (2005) discuss the understanding of graphs in practice by pre-service teachers, and they found that these teachers seem ill-prepared to teach data collection and analysis. Even (1998) discusses the issues involved in linking multiple representations involved in case of the basic functions. It was noted that the participants, prospective secondary teachers, had difficulties with linking multiple representations. The participants could not connect the role of parameters in functions in different representations.

An interesting project, RiskLiteracy.org, looks at how graphs can help adults to make informed decisions about variety of risks (Ybarra et al., 2017). They have developed a *Graph Literacy Tutor*, an adaptive programme based on the framework of cognitive theories of graph comprehension and *Skilled Decision Theory*. This work provides a summary on the Graph Literacy Tutor and its efficacy.

2.4 Graph comprehension and construction

Summarising the studies seen in the last section, we can include the following broad skills or abilities for comprehending graphs:

1. **Understand the different legends and information present in the graph.** For example, to be able to see information in the form of different types of points, colours and other design features of the graph.

2. **Understand the physical or mathematical situation depicted in the graph by making use of domain knowledge in that field.** For example, to be able to understand which the variables plotted, what does the graph represent?

3. **Understand the physical significance of features on the graph.** For example, what is the significance of slope of the line, what is the significance of point of intersection.

4. **Draw predictions, correlations, conclusions or inferences from the graph.** For example, what was the distance covered during the first one hour of the travel?

5. **Check accuracy of the depicted data.** For example, if the points are correctly plotted, or if the scales chosen are proper.

In the process of reading graphs, readers often make systematic errors. These errors might have their origin in the prior experience or the lack of it. While reading a graph, domain knowledge about the phenomena depicted by the graph, design of the graph or possessing different models about features of the graph play an important role in its comprehension. There are cognitive models which attempt to explain these errors. For example, Physics Education Research (PER) provides us with some of the common misconceptions that students have while reading graphs in the context of physics as discussed in the previous section. The common problems or misconceptions that students have while reading graphs are shown in Table 2.1.

Similarly, we look at the broad set of skills or abilities needed for constructing the graphs. In the construction of graphs, the learner has to make sense of data

1. Confusing slope with height to arrive at wrong conclusions.
2. Unable to interpreting changes in height and changes in slope.
3. Difficulty in interpreting negative values of variables.
4. Unable to relating one type of graph to another.
5. Matching narrative information with relevant features.
6. Not able to interpret the area under a graph.
7. Conceiving a graph as a picture or a map.
8. Confusing an interval and a point on the graph.
9. Conceiving a graph as constructed of discrete points.
10. Misreading the scale and variables on them.
11. Confusing between variables when scales on graphs are changed.
12. Being able to find the slope of lines passing through the origin but not otherwise.
13. Looking for information that cannot be obtained from the graph.
14. Lack of knowledge about interpretive sources and familiarity.

Table 2.1: Some of the problems reported in the literature on comprehending graphs.

and choose an appropriate way of displaying the data. In this process, the learner has to create new structures which have a relationship to the graphs and represent the data graphically given in different formats. (Leinhardt et al., 1990).

> Construction refers to building a graph or plotting points from data (or from a function rule or a table) or to building an algebraic function for a graph. In its fullest sense, construction involves going from raw data (or abstract function) through the process of selection and labeling of axes, selection of scale, identification of unit, and plotting. ((1990, p. 12))

In science, constructing graphs is a means of selecting the relevant information. The construction of graphs involves the prior knowledge of the reader about the topic of graphs, the nature of graphs, and design of graphs as important requirements. McKenzie & Padilla (1986) note that:

> Graphing ability is an often overlooked skill that should be addressed in all science programs. Failure to provide graphing instruction or remediation will undoubtedly place students at a disadvantage in comprehending many science concepts. (p. 578)

Experience in constructing graphs in a familiar context helps the comprehension of graphs. Thus the construction of graphs is a more involved process than comprehension of graphs. Table 2.2 shows some of the common problems pertaining to the construction of graphs reported in the literature. Below, we summarise from the review, the broad set of skills or abilities needed in the process of constructing graphs (for both hand-drawn and computer-drawn graphs):

1. **Prepare raw data so that it becomes amenable for drawing a graph.** For example, to be able to form tabular data from verbose text to plot graphs, this may include cleaning, averaging, ordering the data.

2. **Choose appropriate graphic format.** For example, whether a line graph or a histogram will provide the best way to achieve the purpose of plotting the data.

3. **Correctly depict the data on the graph.** For example, incorrectly plotting points on the graph.

4. **Reasonably interpolate or extrapolate the lines or curves from existing data points.** For example, to draw a 'reasonably' good fit straight line through given data points.

5. **Correctly choose the scales so that the salient features of the data stand out.** For example, to choose the scales of the graph so that salient features stand out.

6. **Providing necessary information about reading the graph.** For example, providing proper keys and legends in the graph and providing an explanation of the graph in the caption and the associated text.

1. Inability to correctly plot the points on a coordinate grid.
2. Inability to choose the correct scales on the axes.
3. Providing additional information to read graph, legends, labels.
4. Constructing an entire graph as one single point.
5. Drawing iconic presentation of data.
6. Constructing a series of graphs, each representing one factor from the relevant data.
7. Conserving the form of linear function in depicting a non-linear data.
8. Difficulty in graphing slope and intercept.
9. Inability to construct graphs to depict situations.

Table 2.2: Problems reported in the literature on constructing graphs.

Some of these points are redundant, like correctly depicting data on the graph, interpolating or extrapolating lines or curves (via using best-fit sub-routines when plotting the data with computers). Constructing graphs by hand has its own set of problems, for example, choosing a correct scale, being time-consuming when there are a large number of points to plot. Also, in this case, students might not choose to provide necessary information for reading the graph. For example, labelling of axes or providing scale maybe missing in the graphs made by the students.

The use of computers in constructing graphs and reasoning using graphs has been in general found to be beneficial for learning at various levels. The use of computers has two significant advantages. The first advantage is that the students can save time from the laborious plotting of points on paper and then draw lines through it. Typically when large data-sets are concerned, use of computers becomes imperative. The second advantage is that, once the data is fed in, it allows the learners to 'play' with the data. They can explore the data by changing the scale of the axes, or zoom in into a particular feature of the graph. The dynamic nature of the shape of the graph and immediate and interactive feedback concerning changing scales, zooming in and out can empower the learner to explore the graph in various aspects (Ainley, 2000). Anything similar to this is very inconvenient to do in a hand-drawn graph. Emphasising the skill of using computers for constructing graphs is crucial as most professionals in all fields, including scientists, use computers for constructing graphs.

Constructing graphs with computers might present problems of their own. The automatic features in many popular graph plotting software can lead to poorly designed graphs. Particularly use of 3-D graphics in case of bar graphs or pie-charts can lead to problems when reading the graphs. For example, see the discussion on this topic in *Creating Effective Graphics* by Robbins (2012), Section 2.2 *Charts With A Three-Dimensional Effect* deals with how using 3-D effects obscure data and make the reading of the graph difficult. We see an example of this in our textbook analysis of how presenting an essentially 2-D data with 3-D graphics can cause misreading of information (see the discussion on the graph on page 97).

Leinhardt et al. (1990) differentiate construction of graphs from interpretation in the following manner:

> Whereas interpretation relies on and requires reaction to a given piece of data (e.g., a graph, an equation, or a dataset), construction requires generating new parts that are not given? (p. 12)

The "new parts" that are mentioned in the quote above may include plotting points, drawing lines or curves. Studies indicate that the understanding of graphs is better when students construct graphs from the data collected in a familiar context. In the previous sections, we looked at graphs from a pedagogical perspective. In the next section, we look at how graphs are understood in both cognitive and social frameworks.

2.5 Models of graph comprehension

There are several models of graph comprehension which explain how the comprehension of a graph happens. Some of them focus on cognitive processes in the individual reading the graph (for example, that of Pinker (1990), Shah, Freedman & Vekiri (2005)) while others see comprehension of graphs as a form of social interaction (for example that of Roth & Bowen (2001)). The studies in psychological and cognitive research try to find which types of graphs are more suited for making inferences. In some approaches, the overall factors which influence graphing are taken into consideration. The model of graph comprehension that one uses will largely determine the approach that one will take in addressing the problems raised.

2.5.1 Cognitive focus

The main point of focus in cognitive theories seems to be the features in graphs which help in their comprehension. A certain 'encoding' of the data happens when graphs are made from data. The graph fails in its objective unless the readers of the graphs can 'decode' the graph.

> When a graph is constructed, quantitative and categorical information is encoded, chiefly through position, shape, size, symbols and color. When a person looks at a graph, the information is visually decoded by the person's visual system. A graphical method is successful only if the decoding is effective. No matter how clever and how technologically impressive the encoding, it fails if the decoding process fails. (Cleveland & McGill, 1985, p. 828).

Many of the studies with cognitive focus look at the speed and accuracy with which *elementary perceptual tasks* are performed in the context of a graph. Table 2.3 shows the elementary perceptual tasks. These are called the *elementary perceptual* *tasks* because the viewer performs one or more of these visual mental tasks to extract the values of variables represented on most graphs.

- § Position on common scale
- § Position on non-aligned scales
- § Length
- § Direction
- § Angle
- § Area
- § Volume
- § Curvature
- § Shading
- § Color Saturation

Table 2.3: A list of elementary perceptual tasks for graph comprehension from Cleveland & McGill (1985).

The studies on the graph perception imply that certain graphical designs are perceived more easily than others. A typical study in this regard is done by Culbertson & Powers (1959), where the researchers look at various graph types and try to find efficacy of correlations of graphical performance with other competencies such as verbal. Stock & Behrens (1991) look at the efficacy of display characteristics on box, line and midgap plots. Cleveland & McGill (1984, 1985) provide a list of the most relevant perceptual features in the reading of graphs. These include, in order of accuracy, (1) Position along a common scale (2) Positions along nonaligned scales (3) Length, direction, angle (4) Area (5) Volume, curvature (6) Shading, color saturation (1984, p. 536)

Cleveland (1993) presents a model for understanding graphical perception and the process of visual decoding. According to this model, the information on the graph can be categorised into *quantitative* information (numerical values) and *categorical* information. Both types of information are represented in two ways on a graph: As *scale* information and as *physical* information. An example of scale information would be the data graphed in units for quantitative information and the names of categories for categorical information. The physical information can be thought of as information without the tick labels (no numbers) and names (no categorical information). Thus we have four classes of information resulting from these dichotomies: categorical-physical, quantitative-physical, categorical-scale, and quantitative-scale.

The visual processing of the display is posited to happen in two ways: *pattern perception* (mostly about the physical information, for example, finding trends or peculiar features) and *table look-up* (decoding of the scale information). The two visual processing categories are further divided into sub-tasks. The main point of the model is that effectiveness of the display methods can be explored by studying the speed and accuracy of these visual processing operations. Then the model is applied to graphical perception with use of grid lines in the background and aspect ratio of the graph. Both these play a significant role in interpreting the graphs.

Simkin & Hastie (1987) provide insights from Information-Processing perspective on graph perception with regards to speed and accuracy of elementary perceptual tasks. They propose a three-step process to understand graph perception. The first step is establishing a mental representation by vision, and the second step is to operate on this representation for making inferences or finding peculiarities and finally the third step is to integrate the knowledge about the context and the mental representation to give the required response.

Kosslyn (1989) and Pinker (1990) provide a theory of graph comprehension based on perceptual and cognitive theory. Pinker (1990) argues that graph comprehension should result from effectively using general cognitive and perceptual mechanisms, and is not accomplished by a special purpose mental faculty. He starts with Bertin's characterisation of graph reading which happens in three steps. The first is "external identification", which is to identify the conceptual or real-world referents about which the graph conveys information. The second step is "internal identification", in this the reader identifies the relevant dimensions of variation in the graph's pictorial content, to determine the correspondence between visual dimensions and conceptual scale or variables. Finally, the third step is "perception of correspondence", in this the reader uses the particular levels of each visual dimensions to derive conclusions about particular levels of each conceptual scale.

To accomplish these three steps, a graph reader has to do two things. The first step is to mentally represent the objects in the graph only in a particular way. Second, the reader has to make a correspondence between the aspects of the visual constituents of the graph and mathematical scales that the graph is trying to communicate. In Pinker's theory, these two forms are claimed to be embodied in two types of mental representations: *visual description* and *graph schema*. Figure 2.1 diagrammatically shows the flow of information and process of reading a graph, where the graph schema plays the role of mediation between the graph and the required information. The visual description is the encoding of the visual information on the graph like symbols regarding their physical dimensions. The graph schema, on the other hand, relates to the mapping of the physical dimensions on the appropriate mathematical scales. The terms *conceptual questions* and *concep-*

tual message are used for the information that a reader wants to extract from the graph and actually takes away from the graph respectively.

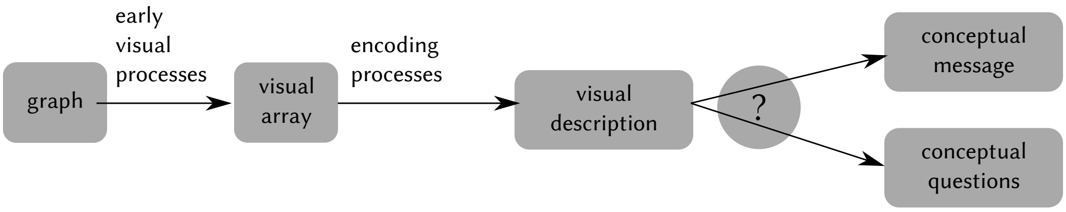


Figure 2.1: The flow of information in Pinker's theory of graph comprehension. The "?" mark represents the *graph schema*, which (a) specifies how to translate information from visual description into conceptual message, (b) specifies the translation of a conceptual question into a process that accesses the relevant parts of the visual description and (c) recognize which type of graph is currently being viewed.

Pinker provides a detailed step-by-step analysis of how these two forms, namely visual description and graph schema interact to give the reader the comprehension of a graph. Thus, the inferential and encoding processes determine the problems with comprehension of graphs, which is summarised by the *Graph Difficulty Principle*:

> A particular type of information will be harder to extract from a given graph to the extent that inferential processes and top-down encoding processes, as opposed to conceptual message look-up, must be used. (p. 108)

The graph schema plays a vital role in this, and problems with the schema can lead to problems with the interpretation of the graph.

Kosslyn (1989) provides a broad framework for evaluating both graphs and charts for conveying information effectively. He categorises basic-level graph constituents into four components: *background, framework, the specifier* and *the labels*. Figure 2.2 shows these constituents in the context of a graph.

The *background* is an object on which the graph is overlaid, in the above example, the background is the white space. The *framework* specifies the related entities in the graph, and it functions to organise the graph into a meaningful whole. The *specifier* conveys particular information about the entities specified by the framework. For example, in case of a scatterplot or a line graph, the actual line would be a specifier. Moreover, finally, the *labels* composed of words, symbols and numbers provide an interpretation of the specifiers and the framework.

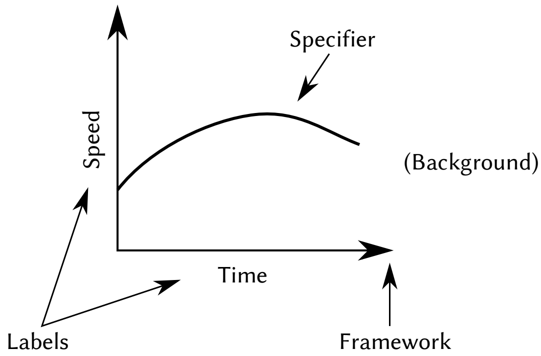


Figure 2.2: The four basic-level components of a graph according to Kosslyn.

These four basic-level constituents (Figure 2.2) and their interrelations are described at three levels of analysis: *syntactic, semantic* and *pragmatic*. The *syntactic* analysis focuses on the perceptual properties of the graph such as properties of the lines or areas, possible groupings. The *semantic* level is about the meanings of configurations of lines and their significance, and it is the literal reading or meaning of the components of the graph and their relations. Reading beyond the literal meaning, as in the semantic level, is the focus of the *pragmatic* level. This level also considers the connection between information in a graph and the information needed by the reader.

The basic-level constituents and the three levels of analysis are further seen in the light of information processing perspective and symbol systems for evaluation. The information processing in the context of graphs starts with the perceptual properties of the graph; its transfer to short-term memory; encoding of the features for transfer to long-term memory, and use of knowledge from the long-term memory. The symbol system (derived from the theory of symbols by Goodman (1968)) in this context deals with how the marks on the graph can serve as symbols for various concepts. In this case, it is assumed that there is a unique mapping between a symbol and its interpretation, and the symbol itself should be distinguishable from others.

Based on the above parameters, a fine-grained analysis of a graph is then performed. The problems with graph comprehension can occur at the perceptual or conceptual levels. Within this theory, the problems that a graph reader faces, can be due to the capacity of the short-term memory, the complexity of the graphical display and the lack of knowledge in the long-term memory. Also concerning design issues, if the discrimination between symbols is not clear, then there can be an ambiguity in the interpretation of the symbols and their intended meaning.

Wavering (1989) classifies graph construction abilities into nine categories, ranging from no attempt to make graphs to making a complete graph with statements about correlation. A correlation is proposed between the said nine categories with Piagetian stages. Friel et al. (2001) provide a review of graph comprehension from various perspectives. They identify three main components of graph comprehension going from local to global features of a graph.

The review work by Shah et al. (2005), Shah & Hoeffner (2002) describes the influence of display and design characteristics, for example, line graphs vs bar graphs (Shah & Freedman, 2011), data complexity, and task on the graph interpretation. In their study, they ask these two overarching questions:

> Why are some graphs relatively easy for viewers to comprehend for a particular task, and other graphs more difficult? How do individual differences in graph reading skill and domain knowledge influence the kinds of interpretations that viewers give to graphs presented in texts? (p. 49)

They categorise the factors into three main headings:

Visual characteristics of the display: these include formats of graphs like line graphs, bar graphs and pie charts and their contextual usefulness. Other characteristics of the display which influence the interpretation include various visual dimensions to represent information accurately. Position on a common scale was ranked first and shading and colour saturation the last (Cleveland & McGill, 1984). Also considered are design features such as colour, 3-dimensional displays, legends, labels, aspect.

Knowledge about graphs: the prior knowledge and experience of dealing with graphs aids in making the correct interpretation of the graphs. This skill might not be transferable to newer domains.

Knowledge about content: the specific background knowledge regarding the content in which the graph appears is crucial for understanding many aspects of a graph. The content knowledge can have substantial implications particularly for young learners and particularly where higher level evaluation is involved.

The above three categories are the interacting factors in the Construction Integration (CI) model (Figure 2.3) to understand graph comprehension proposed by Freedman & Shah (2002), Shah & Hoeffner (2002). The perceptual organisation of data can have a substantial effect on the comprehension, even in case of familiar contexts and complex tasks (Shah, Mayer & Hegarty, 1999).

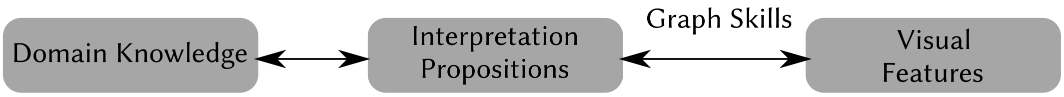


Figure 2.3: The basic blocks of the construction-Integration (CI) model for a framework to understand graph comprehension.

The work by Canham & Hegarty (2010), Hegarty, Canham & Fabrikant (2010) explores the factors influencing comprehension of complex graphics which display meteorological information. In their study, they infer that (1) learners can use newly acquired declarative domain knowledge to aid in making inferences from graphics, and (2) design of the graphic plays an important role in learners ability to infer. Particularly they note that design artefacts which do not directly correspond to the task at hand might actually impede the inference tasks.

M. P. Cook (2006) presents instructional design considerations for designing visual representations for learning. This work takes into account the cognitive load that a visual requires and ways in which this load can be reduced to maximise its potential. The study also acknowledges that each learner will have a different response to a given visual representation, depending on their prior knowledge. The work by (Riechelmann & Huestegge, 2018) looks at the effect of multiple graphs presented simultaneously (instead of a single graph) on the performance tasks related to the graphs. They found that graph comprehension is more effective when there is a compatibility in the data-legends relation especially in the case of complex graphs.

An interesting angle is given by the work of Garcia Moreno-Esteva, White, Wood, Black et al. (2017). They look at *eye-tracking* data to identify cognitive processes involved in graph interpretation. This study involved studying eye-movements of students while reading a bar graph. Some of the main inferences that are drawn from this study are that (1) order of fixations while looking at the graph play no role in learners ability to interpret graphs, (2) the amount of time that learners spend on looking at crucial features of the graph as compared to non-crucial features plays an important role in ability to interpret the graph correctly. This has a direct implication for classroom teaching, indicating that while instructing learners about graphs special emphasis should be given to features of the graphs which are crucial for the interpretation tasks.

The work by Michal & Franconeri (2017), Michal, Uttal, Shah & Franconeri (2016) also looks at eye-tracking data to explore the visual mechanism in processing of data from the graph. They propose that *embodied visual routines* deployed during the reading of a graph. This study used bar graphs and the studied eye-tracking movement of the subjects to understand the responses. The implications that they suggest for the learning processes are (1) learners should focus on comparisons which are relevant to the current task; (2) one task-relevant anchor point should be chosen while interpreting graphs in an open-ended way. They further suggest that "By attending to graphs using a task- relevant visual routine and implementing that routine systematically, students may be able to improve their ability to focus on relevant graph comparisons." (p. 9)

2.5.2 Sociological focus

In the cognitive approaches seen in the earlier section, the "meaning" of a graph and its interpretation lies *in* the graph itself. This meaning is then *uniquely* extracted by the reader from the graph, using perceptual and cognitive processes. In case the reader fails to read the graph correctly, the problem of interpretation of the graph lies within the reader and her mental processes. The sociological approach argues against this way of understanding graphs (Roth, 2004). In the sociological perspective, graphing is looked at as a *practice*; it focuses attention on the competence of students and rhetorical purposes, and on the affordances of graphs to collective sense-making (Roth et al., 1999). Roth supports the claims that:

> ...a graph is meaningful to the extent that an individual can integrate it into his or her existential and embodied understanding of how the world works in general, and here, the workplace and work processes, in particular. Graphs do not have or get meaning. Rather, graphs accrue to meaning that has its source in our existential understanding. (Roth, 2004, p. 79)

As in the cognitive approach, the sociological approach does not take isolated individual and her associated cognitive processes as the unit of analysis. The focus here is shifted from representation as a mental activity to a social activity. The production of graphs is seen associated inherently with its meaning:

> Graphs are produced as something for some something (specific purposes, e.g., convincing evidence in publication), and this is an integral part of what they ``mean.'' [p. 88](2004)

According to (2004), there is a dialectical relationship between the graph and the reference to the graph:

> The graph exists as a concrete object instantly available to perception conventionally associated with making reference to some thing. But this something cannot be grasped in one instance: considerable text is required to elaborate both sense and reference of the graph. This is the first dialectic, the instant presence as an object and the requirement for a temporally unfolding articulation of sense and reference. The second dialectic exists in the contrast between the real presence of the graph and verbal explanation, on the one hand, and the impossibility to articulate the necessary background that permeates graph and language and into which both fade. p. 90

The relationship between a phenomenon and its representation, which the cognitive models consider as an *inherent* property of the inscription is seen in the sociological model as a matter of *convention.* The problems that the learners face are seen as arising due to inexperience with conventions, rather than mental deficiencies (Roth & McGinn, 1998). The emphasis that most cognitive models place on a prerequisite of the formal operational stage in the learner for construction and comprehension of graphs is questioned. The sociological approach sees graphing as a *practice* focusing on learner competence, rhetorical perspective and affordances of graphs to collective sensemaking, and hence does not need to be explained in terms of the cognitive deficits (Roth & Bowen, 2000; Roth & McGinn, 1997). The problems that the students face can be seen in the light of differences in resources and practices that are different from those of experts.

Practice in this context refers to the actual working processes and the conventions followed in the domain under consideration. The acquisition of this practice happens by relevant experience and exposure to various opportunities of dealing with data. According to Bowen & Roth (1998), the interpretation of a graph according to this approach does not lie in "understanding the representation itself as a static object but rather in understanding the social actions through which the graph was originally constructed". The emphasis here is on the notion of graphing as a practice. In this framework, the mathematical graph related experience is linked with experience in the world (Roth & Bowen, 2001).

We feel that the sociological focus provides a deeper understanding of graphing as a practice embedded in the contexts in which graphs are produced and consumed. Looking at graphing as a social practice also helps us understand the contextual aspect of graphical comprehension and construction very well. It is only by understanding the contexts for which the graphs are to be constructed, and the way data was obtained, that a fuller understanding of the graphs will be achieved. We think that these two points concerning (a) the need for constructing a graph (its function), and, (b) the way the data was obtained, are particularly germane for presenting graphs in school textbooks. We consider these points while analysing graph in textbooks in Chapter 3.

2.6 Reflections

The review sets a framework for addressing various concerns brought out by different approaches. Students have various difficulties in constructing and comprehending graphs. What is clear from the literature review and our work presented in the following chapters, is that graph comprehension is not easy, and it does not come naturally but has to be taught *explicitly*. In introducing graphs to the learners, emphasis must be on the presentation of same data in different representations. Our primary emphasis has been to look at various approaches towards addressing these difficulties. Without experience in dealing with contextual data and graphs representing them, interpreting graphs can be tricky. The context and the setting of the graph turned out to be an essential element in interpreting graphs. Without sufficient prior background knowledge of the concerned topic, it is difficult to understand the subtleties of the graph. The decoding of relevant information from a graph is almost impossible if the context of the graph is not clear. This point is particularly important in constructing a graph from a given situation. The mapping that the graphs provide between the abstract two-dimensional representation and the concrete reality that they represent can be made more accessible to comprehend by providing the learners with different situations in which similar principles apply.

Some of the difficulties in interpreting graphs have their origin in the design of the graphs. As many studies in the cognitive framework have shown, certain graphical formats help in understanding graphs much better than others. For example, line graphs are better suited when we are looking for trends in the data. The design of the graphs can have a significant impact on the way people decode the graph, and at times it can lead to an incorrect reading of the graph. Conveying incorrect conclusions using persuasive and deceptive design elements seems to be common practice in popular media, if not in the scientific communications.

We look at some of the issues that are raised here again in Chapter 4 when we look at the design framework of the activities that we have reported in this work. The implications of the work done in the area of graph comprehension and construction are seen in the context of designing activities.

Given all these difficulties, how are we helping our students to overcome them? In the Indian context, textbooks, for most students, remain the only source of knowledge (NCERT, 2006b). For school students, textbooks are meant to provide the students with the relevant information and experiences. How well do the school textbooks provide the experience dealing with graph construction and interpretation? A detailed analysis was conducted to understand the extent to which the textbooks support graphicacy. The next chapter presents the results of this analysis, addressing research question (2).

" Scientists who have more training in reading graphs are provided with more resources in the journals for the same, while students are left clueless about the same in the textbooks that they read."

Roth et al, *Critical Graphicacy, 2005*

3

The poverty of graphicacy in school textbooks

In this chapter, we look at how the graphs are presented in National Council for Educational Research and Training (NCERT) textbooks. An analysis was done on the graphs in the textbooks for the common problems associated with graphicacy as identified and listed in the previous chapter. This analysis has both quantitative and qualitative components. The quantitative analysis gave the spread of graphs across grades and subjects. In qualitative analysis, we assessed graphs in science textbooks in detail for the affordances they provide for the learners including the design aspects. We discuss the significant results from this analysis and implications for graph design in the textbooks.

3.1 The importance of textbooks

According to Kuhn (1961) "textbooks are the sole source of most people's firsthand acquaintance with physical science." This statement finds resonance in many other writings in the Indian context. For example, the Position Paper on *National Focus Group on Teaching of Science* National Curriculum Framework of 2005 (NCERT, 2006b) notes that “In India, for the great majority of school-going children, as also for their teachers, the textbook is the only accessible and affordable curriculum resource.” (p. 19). The textbooks form an integral part of a schooling system in India. In the Indian educational system, the State prescribes textbooks for use in the classroom. The historical origins for such an important role given to textbooks in the Indian context are discussed by Krishna Kumar in *Origins of India’s “Textbook Culture”* (Kumar, 1988). In this article, Krishna Kumar’s use of the term “textbook culture” to have certain standard features in the Indian context:

1. Teaching in all subjects is based on the textbook prescribed by State authorities.

2. The teacher has no freedom to choose what to teach. She must complete the prescribed syllabus with the help of the prescribed textbook.

3. Resources other than the textbook are not available in the majority of schools, and where they are available they are seldom used. Fear of damage to such resources (e.g., play or science equipment) and the poor chances of repair or replacement discourage the teacher from using them.

4. Assessments made during the year and end-of-year examinations are based on the textbook. (p. 455)

Apart from the above four obvious features, the teacher is the least powerful in the hierarchy of the educational system. Just the presence of the textbook in the classroom is, according to Krishna Kumar, “a symbol of bureaucratic control”, a reminder for the teachers using the textbook about the authority of the State.

The conditions in which such a culture arose has roots deeply embedded in the colonial past of India. The British, over decades, systematically dismantled the old educational system, and established the new system in which earlier local knowledge was neglected. One of the outcomes of this was the relation of production and consumption of knowledge to the learners.

> Colonial education meant that its beneficiaries would begin to perceive themselves and their society as consumers of the knowledge supplied by the colonizer and would cease to see themselves as people capable of producing new knowledge (1988, p. 454).

Centralised examinations in which a person other than the teacher, who taught the students, examined the students played an essential role in establishing the textbook culture.

> The official function of the examination system was to evolve uniform standards for promotion, scholarship, and employment and to thereby consolidate government control. In the social context, the examination system served the purpose of instilling in the public mind the faith that colonial rule was fair and free of prejudice. It imparted this faith by being impersonal, hence nondiscriminatory in appearance, and by being so wrapped up in secrecy (1988, p. 458).

The examination conducted thus was intricately linked to the textbooks, and "students were examined on their study of specific texts, not on their understanding of concepts or problems." (p. 458). This mode of examining students gave very high prominence to the *reproducing* the text from memory, rather than *understanding* the content of the text. The standard examination included essay type written answers, and all other modes which could be used for examination were left out of the system. In this regard, practical and vocational skills suffered, and subjects like science which depended on these suffered too.

Another problem with the textbook culture was complete neglect of local relevance of the curriculum for the students. Thus any meaning-making processes while the reading of texts, and they could only be memorised. The main aim of the student-teacher interaction in the classroom was to pass in the examination.

> When the main concern of both the teacher and student was to prevent failure at the examination, the best possible use of classroom teaching could only be to prepare students as meticulously as possible for the event, and this was done by confining teaching to the content of the prescribed textbook (1988, p. 461).

Apart from this primacy of English language and the lucrative jobs after passing the exams (in which the failure rates were kept high), consolidated the examination-textbook linkage, with textbooks assuming the central and skills assuming a peripheral role in the Indian educational system.

After Indian independence, the textbook culture with all its systemic problems continued. The textbook has the central authority concerning the content of the exam and the planned out the structure for the delivery of the curriculum rule supreme in the classroom. The formation of National Council of Educational Research and Training (NCERT) moreover, various other state boards are in a sense continuation of the colonial tradition of a central bureaucracy deciding the educational curriculum, which keeps the teachers on the field as “meek dictators” (2015).

Krishna Kumar finally notes that the content quality of the textbook material is essential, but the textbook culture did not arise because of the poor quality of textbooks. So just providing the students with well-written textbooks alone will not solve the problem at hand.

> One fact that might help us reflect on this question is that the origins of the textbook culture had nothing to do with poor production of textbooks. There always were some good textbooks (limited though they were by the state of knowledge at the time) as there are now. But they could not transcend or alter the norms of teacher-pupil interaction, shaped, as we have seen, by larger socioeconomic and cultural conditions. Within the narrower context of the education system, the textbook culture was linked to teacher preparation and evaluation. Even a dramatic improvement in the quality of individual textbooks cannot be expected to alter the textbook culture if these corners of the system remain unattended (1988, p. 463).

With the above mentioned central role that the textbooks play in the educational system, it becomes imperative that the textbooks be well structured in their content and delivery. In the scenario when the textbooks themselves fail in providing the necessary knowledge regarding a particular topic the students will be at a significant disadvantage.

Seen with this background, we wanted to assess whether the Indian textbooks allow the students enough opportunities to make and read graphs and whether they allow the students to become graphically literate. Studies, in general, show a poor record of textbooks helping students learning graphicacy explicitly. Clement (1985) found that the students rarely question graphs in textbooks although they often misinterpret them. Roth et al. (2005) analysed biology textbooks for various graphical practices and found that they lacked resources for the students to understand the graphs, which are usually available to a scientist reading a journal. An analysis of Indian school textbooks was fone to determine the state of graphs present in them. The analysis of the textbooks was performed both qualitatively and quantitatively. In the next two sections, we describe the analysis, its results and implications.

3.2 Sample textbooks

In India the National Council for Educational Research and Training (NCERT) is the highest body which publishes and prescribes the curriculum. All other state and other boards, though autonomous, follow the guidelines produced by the NCERT. Most of the textbooks produced by the different state boards derive their form and content from the NCERT textbooks. Thus to have comprehensive coverage of the curriculum in India, we analysed the NCERT textbooks. For this reason, we have chosen the NCERT textbooks as a sample in our analysis. We have covered the textbooks for science, mathematics and social sciences except the language textbooks ranging from Class 5 to 10. Table 3.1 presents the list of books used in the analysis. These books were made after the recommendation of the National Curriculum Framework (NCF) of 2005 (NCERT, 2005).

The Syllabus based on NCF 2005 (2006d) has put understanding information from graphs as one of the objectives of learning for Classes 9-10. Regarding the topic of graphs, it says "to transcode information from a graph/chart to a description/report" (p. 155). We look at what kind of opportunities do the students get to exercise this learning objective.

We have chosen all the subjects, except the languages, as we consider the ability to write and interpret graphs as inter-disciplinary skill, not limited to one particular subject. We have already seen in the introductory chapters the importance of graphs in different fields of inquiry. We wanted to see the way graphs are used *across* the grades and *across* the subjects. For the analysis, we searched the textbooks for *graphical practices*. By graphical practices, we mean at least one or more of the following criteria:

§ A figure or illustration containing a graph.

§ An activity which has or leads to a graph.

§ Discussions, problem and exercises about graphs.

The analysis used both qualitative and quantitative techniques. Quantitative techniques are mainly used concerning space and frequency as reported in (Pingle, 1999). For example, it can be a quantification of *how many times* a particular word appears in the text, or what was the space allocated for a particular theme, event or topic. Quantitative methods are best suited for analysing large samples. With these methods, we can cover a large area, as Pingle says:

> [quantitative methods tell] us a great deal about where the emphasis lies, about selection criteria, but nothing [in themselves] about values and interpretation (p. 38)

For the values and interpretation, we need to have a component of qualitative research. The qualitative analysis looks at the contents of the textbook in terms of their *intended purpose* or *function* and *conceptual standing*. In our analysis, we used both quantitative and qualitative techniques to bring out the broad picture of usage of graphs in the textbooks. The quantitative and qualitative the analysis complement each other, in the sense that the former provides a broad picture of the number of graphs present in the textbooks, while the latter provides us with their functional information. In the next section, we discuss the quantitative aspects of graphs in the textbooks. In qualitative research, the analysis tends to be deeper regarding the structure of the textbook and affordances the unit of analysis provides to the learner.

§ **Class 5**

- » Mathematics: Mathemagic
- » Social Science: Environmental Science

§ **Class 6**

- » Mathematics: Mathematics
- » Science: Science
- » Social Science: Social & Politcal Life
- » Social Science: History - Our Past
- » Social Science: The Earth - Our Habitat

§ **Class 7**

- » Mathematics: Mathematics
- » Science: Science
- » Social Science: Environment
- » Social Science: Social Political Life
- » Social Science: History

§ **Class 8**

- » Science: Science
- » Mathematics: Mathematics
- » Social Science: Political Science
- » Social Science: History
- » Social Science: Geography Resources and Development

§ **Class 9**

- » Mathematics: Mathematics
- » Science: Science
- » Social Science: Geography - Contemporary India
- » Social Science: Introducing Sociology
- » Social Science: Political Science
- » Social Science: History

§ **Class 10**

- » Mathematics: Mathematics
- » Science: Science
- » Social Science: Geography
- » Social Science: Democratic Politics
- » Social Science: Understanding Economic Development
- » Social Science: History

Table 3.1: The sample of the textbooks used in the analysis. Published by the NCERT, New Delhi, after NCF 2005.

3.3 Quantitative analysis

The main aim of this part of the analysis was to get a trend of the presence of graphs in the textbooks. This part largely addresses Research Question (1).Therefore the analysis was conducted to address the following questions.

Part 1: Research Question 1

(1) How are graphs placed in the Indian school textbooks in different subjects and different classes?

(a) What are the different types of graphs that are present in the textbooks?

(b) How frequently do graphs occur in the textbooks?

(c) How do graphs compare across subjects?

(d) How do graphs compare across classes?

The sub-questions look at the relative presence of graphs in the textbooks. The questions also set up a stage for further detailed analysis of graphs. To answer the questions raised above the textbooks in different subjects were scrutinised for graphs and graphical practices. The researcher thoroughly went through the textbooks looking for the required material. Appendix A.1 has the database that was created for this analysis. Table 3.2 shows the information that we have collected during the survey.

Parameter	Description
Class	The class in which the graph appears.
Subject	The subject textbook in which the graph appears.
Page Number	The page number on which the graph appears.
Figure Number	The figure number for the graph, if applicable.
Legend	The legend of the graph, if applicable.
Caption	The caption of the graph, if applicable.
Graph Type	Type of graph, namely, Line, Bar, Pie or Other.
Description	Description of the graph in the text.
Data	Whether data for the graph and its source is provided, or whether the students are to collect the data.
Comments	Our comments on the design and use of the graph.

Table 3.2: List of variables collected during the quantitative survey of textbooks for graphs.

We have categorised the subjects into three major groups: *Science, Mathematics* and *Social Sciences*. The social science group includes Geography, Environmental

Science, Political Science and Sociology in class 8 and above. Table 3.3 shows the total of number graphs in each class. We calculated the frequency with which graphs appear in each of these subjects, across all the classes under consideration.

	Subject				*Graph Type*			
Class	Science	Social	Maths	**Total**	Line	Bar	Pie	Other
5	0	1	5	**6**	1	3	2	0
6	0	1	42	**43**	23	9	1	10
7	8	4	19	**31**	19	9	1	2
8	3	3	39	**45**	22	11	11	1
9	14	11	41	**66**	47	9	4	6
10	4	21	35	**60**	23	8	13	16
Total	29	41	181	**251**	135	49	32	35

Table 3.3: A table showing total number and type of graphs in the textbook in different classes and subjects. The data in this table is used to plot the graphs in this section.

Figure 3.3 shows the general trend of the total number of graphs as a function of the class in which they appear. Particular topics mark the increase in the number of graphs in each class and subject. The increase in class 6 is due to the number-line graphs, while in case of class 9 graphs on the topic of motion and Cartesian coordinate system are introduced in mathematics.

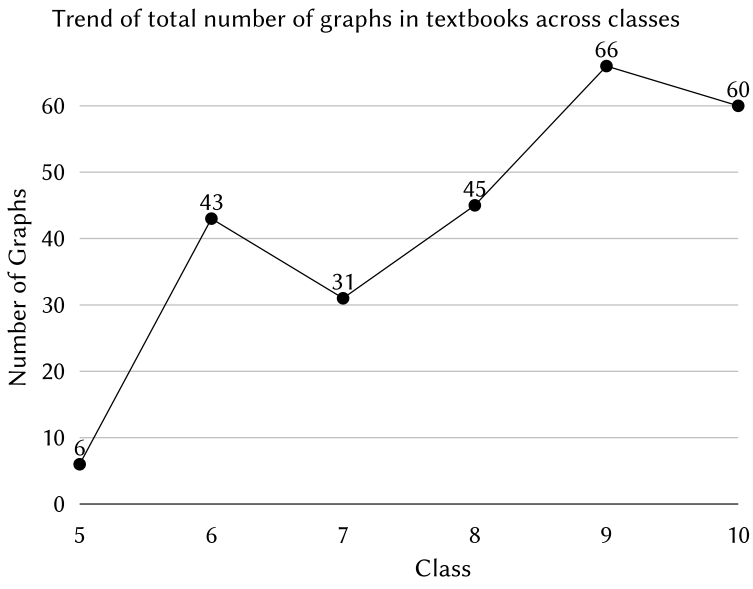


Figure 3.1: The graph showing the total number of graphs in all textbooks for each class.

We further explore the distribution of the graphs according to the types of graphs in Figure 3.2. We classified the graphs present in the textbook into following three categories: *line graphs, bar graphs* and *pie charts*. By a line graph, we mean a Cartesian graph. When a bar graph and a line graph are simultaneously

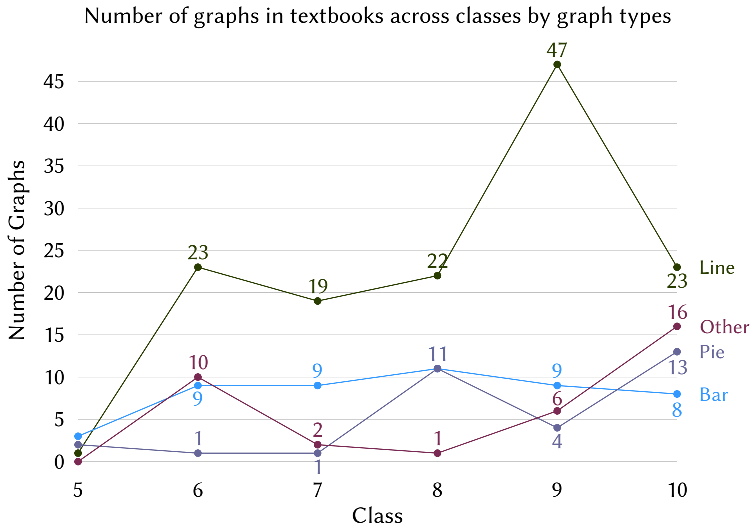


Figure 3.2: The graph showing the distribution of types of graphs in different subjects across classes.

present in a figure, we have included them in the line graph category. The category "Others" includes graphs which could not be sorted in the above listed three categories. Typical examples of these categories are presented in Figure 3.3.

Figure 3.4 shows the number of graphs in different subjects across the classes. Mathematics has the largest share, with the topics of the number-line and the Cartesian coordinate system contributing significantly to these numbers. Following the mathematics textbooks, are the social science textbooks, which include many statistical graphs. In case of the science textbooks, the increase in class 6 is due to the number-line graphs, while in case of class 9 graphs on the topic of motion and the Cartesian coordinate system are introduced.

One would expect that the total number of graphs in the textbooks would increase with the classes, that is, higher classes will have a more substantial number of graphs. The reason for such an expectation is that as the students progress through the classes, they would require more opportunities to explore and engage with graphs. Just as in case of verbal literacy, complex and increasing amount of text is provided, we think a similar trend should be seen in the context of graphicacy. This inconsistency in the number of graphs can also be seen as an indication that graphicacy *per se* is not recognised as an essential skill in the curriculum. There is no explicit or systemic planning we could find to build graphicacy across classes or subjects. Even a cursory search of documents related to NCF 2005 (including position papers on Science and Mathematics) reveals that graphs are underrepresented. Another issue is that mere presence of graphs in the textbook is

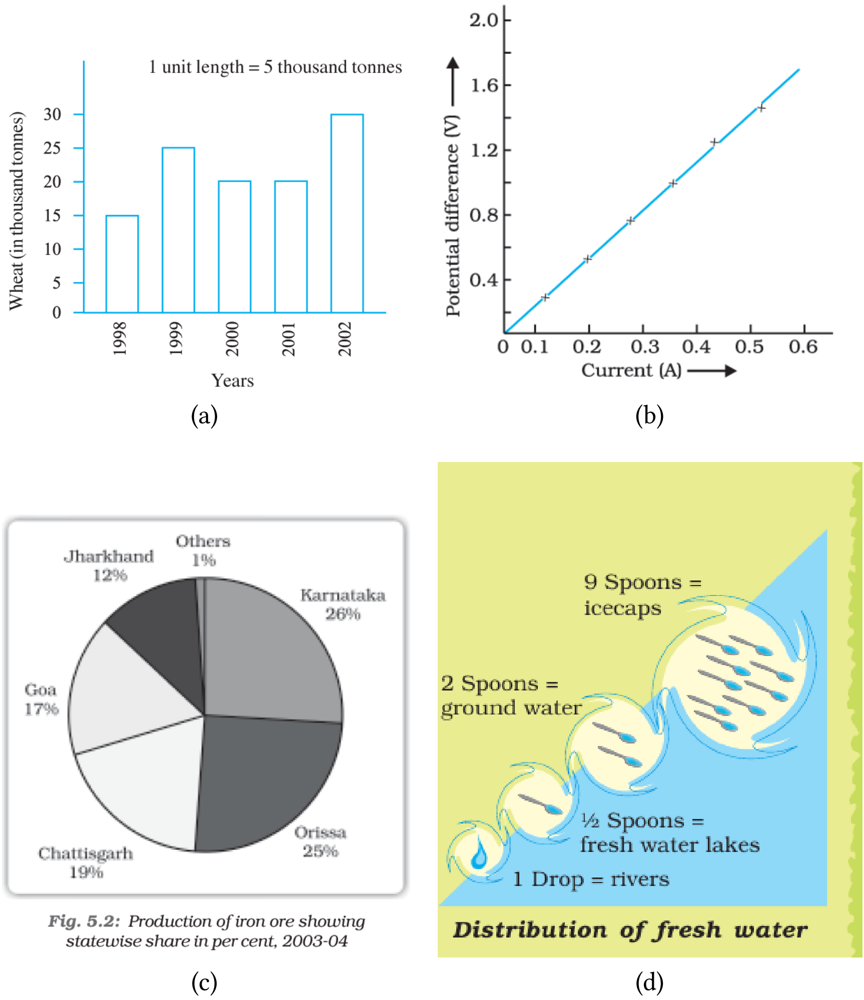


Figure 3.3: Examples of different types of graphs in the textbooks. (a) An example of a bar chart/histogram type graph. (b) An example of a line graph. (c) An example of a pie chart. (d) An example of "Other" category of graphs.

not justified unless it is appropriately related to the subject matter and fulfils the goal for which it was introduced. As the overall integration with the narrative, the context in which the graphs appear and the design of the graphs, are also crucial in maximising the impact of graphs. The qualitative component of the analysis in the next section inquires how these issues are addressed in the textbooks. Cleveland (1984) make a note germane to our discussion in the context of their survey of scientific journals:

> The fraction of space a journal uses in printing graphs does not measure the effectiveness or the quality of the graphs. It is certainly possible that a journal with less of its space devoted to graphs than another journal might be communicating more effectively with its graphs. But the fraction of space used is a measure of the im-

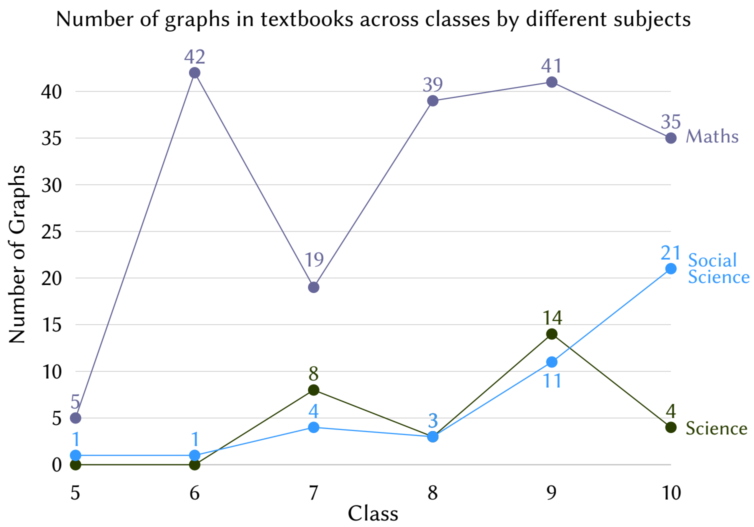


Figure 3.4: A graph showing the distribution of different types of graphs across classes and subjects.

> portance that the journal and the authors attach to graphical communication. (p. 265)

Similarly, in our case, the mere presence of graphs does not warrant any bearing on the communication that a given graph might make. The mere presence of graphs in the textbook does not justify its presence unless it is appropriately related to the subject matter. As Brasell & Rowe (1993) point out:

> Many textbooks include graphs but, because they fail to discuss them adequately in the text, the graphs may be treated as superfluous adornment. Students need to have repeated experience with a wide variety of graphs used as an integral part of communicating information in many courses and contexts. (p. 69)

However, the presence or absence of graphs does surely indicate the importance that the textbook writers ascribe to the graphs in the context of learning. Graphs in each of the three disciplines that we have considered have been treated *independently*. There is no coordinated effort to build upon the skills and conceptual understanding that the other subjects offer. In the science textbooks, the students are taught to draw graphs, without any reference to what they have already learned in Mathematics. Similarly, in case of Mathematics, any reference to science textbooks or relevant topics is absent, and graphs are again taught independently. Social sciences also follow a similar, without building upon any skills that the students might have gained in the other two subjects and vice versa. Though there is no problem if a topic is repeated, for the benefit of the learners, to completely overlook what similar topics the students might be learning the other subjects is another matter.

Trends across subjects

In this section, we look at the distribution of the graphs in different subjects and classes. For the science textbooks to visualise the total number of graphs as a function of classes, we plotted the two. Figure 3.5 shows these trends for the science textbooks. Similar analysis was done for other subject categories, social sciences (Figure 3.6), and for Mathematics (Figure 3.7). The total number of graphs, among

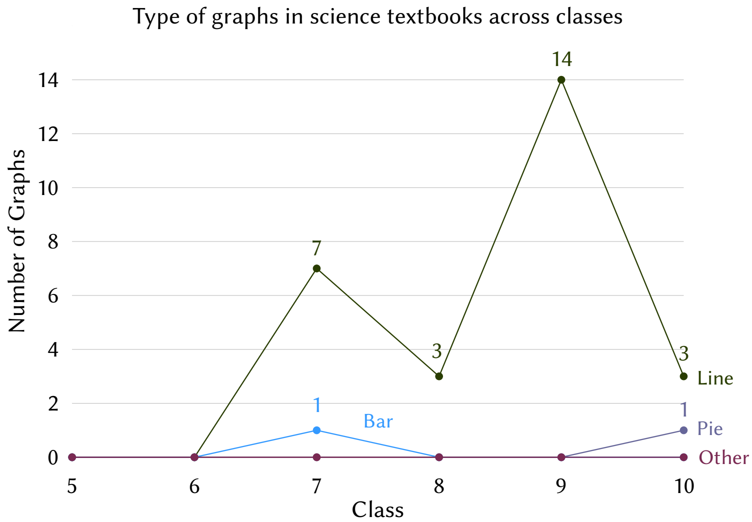


Figure 3.5: A graph showing the variation of the total number of graphs with classes for the science textbooks.

the subjects that we have considered, is the *least*, in case of science, totalling to 29 graphs. In case of social studies and mathematics, this number is 41 and 181 respectively. These numbers are one indication that the school science textbooks need content which would address the issue of handling graphs. Out of these 29 graphs, 27 are line graphs, and the other two are a bar graph and a pie chart. Thus we see a trend that science textbooks mostly use line graphs, even if used sparingly. We see that in the science textbooks class 9 has the maximum number (14) of graphs. Most of the line graphs appear in the context of motion in classes 7 and 9.

In the social science textbooks (Figure 3.6), we see almost an exponential increase in the total number of graphs as the classes increase. Here we notice that the bulk of the graphs are statistical: out of the 41 graphs 35 are bar graphs and

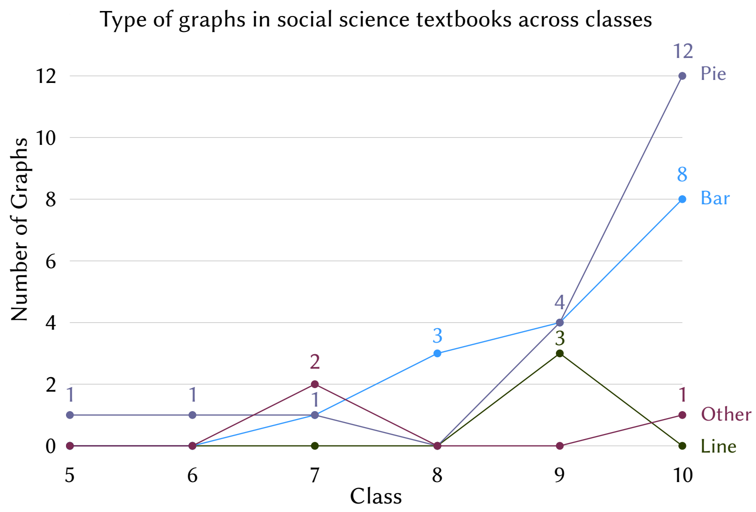


Figure 3.6: A graph showing the variation of the total number of graphs with classes for the social science textbooks.

pie charts. In social studies mostly the graphs have been used to display various types of statistical data.

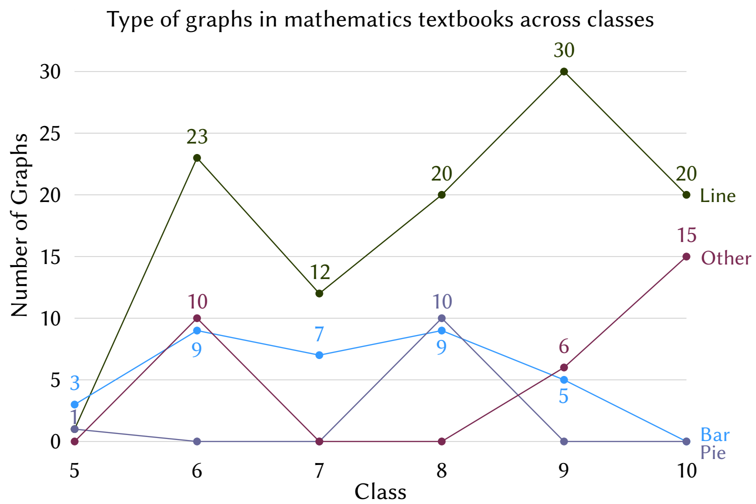


Figure 3.7: A graph showing the variation of the total number of graphs with classes for the mathematics textbooks

In the mathematics textbooks, line graphs are the most frequent ones to appear, 106 out of total 181 graphs are line graphs. Most of them are concerned with either the arithmetic operations using the number line or developing concepts in the Cartesian coordinate system. Especially in classes 6 and 7, the number line has been used extensively to teach the concepts of addition, subtraction and multiplication of real numbers and integers. Our category of line graphs includes the number line. Class 10 introduces the concept of functions using graphs. In the mathematics textbooks, there are chapters on 'Data Handling', which talk about statistical graphs like pie charts, bar graphs, histograms.

In case of the social sciences, we have a perceptible increase in the number of graphs with the classes. Whereas for science and mathematics textbooks this increase in the number of graphs is not seen. In fact, in the science textbooks, the total number of graphs substantially decreases in class 10. In mathematics textbooks, on the other hand, the total number of graphs do not vary this much, at least in the last three classes.

We think that there is a significant requirement for reconsidering how the graphs are represented and referred to in the science textbooks. Given the existence of a "textbook culture" in India, and the importance of graphs for all subjects, the trends that we have observed in the textbooks are worrisome. A proper recommendation for integrating graphical content could then be framed considering the need for graphicacy in Science and Mathematics education.

We have noticed that in the science textbooks, the presence of graphs is insufficient. Reading, writing and understanding graphs being an important skill in science along with other subjects, this trend needs to be changed. We see that there exists a tremendous opportunity to explore and utilise this particular aspect of graphicacy in the sample of textbooks that we have studied. Curcio (1987) puts this idea effectively as: "Elementary school children should be actively involved in collecting "real-world" data to construct their simple graphs". The qualitative study of the context in which graphs appear and the way they are utilised in the textbooks forms the next section.

This part of the work was presented as "**An Analysis of graphs in school textbooks**" in epiSTEME 4 Conference, in Mumbai (Dhakulkar & Nagarjuna, 2011)

3.4 Qualitative analysis

As we have noted in the previous section, the mere presence of graphs does not guarantee their understanding. As Brasell & Rowe (1993) note:

> Graphs should not just be present in the curriculum but should become cognitively available to students. Room must be made somewhere in our already overcrowded curriculum to teach adequate graphing skills. (p. 69)

In this section, we look at the graphs in the textbooks in detail. We chose to analyse science textbooks in detail, as it was the main theme of our study. The analysis was undertaken to understand the presentation of the graphs in the textbooks concerning their function and usability. Table 3.4 contains the categories used in critically examining the nature of graphs and the learning they might support. The framework used by (Roth et al., 2005) in *Critical Graphicacy* to analyse graphs in textbooks was used to arrive at some of the categories, while for other categories literature relevant to the design of the graphs was used.

Category	Description
Function	What function does the graph serve in the textbook? Whether the function is narrative, organisational, analytical or metaphorical representation.
Reference	Whether the graphs are referred to in the main text? If they are, what is the manner and frequency of reference?
Integration	How well are they integrated into the overall text? How do they go with the flow of the narrative?
Data Used	What is the data used in making the graphs? Is the data provided in a tabular form, is the source of the data provided? Is real data used in making the graphs.
Legend and Axes	Is the graph with key and labels to the axes? Are the variables on the axes with units and labels?
Close-to-life	Does the graph link to any everyday experience of the students?
Design aspects	Is the graph well designed? Does it have unnecessary decorative elements?

Table 3.4: The categories used for qualitative analysis of science textbooks and their description.

An analysis of each of the graphs that appears in the textbooks was performed in the context of these categories. The categories in Table 3.4 provided a deeper understanding of the use of graphs in the science textbooks. As seen in Chapter 2, the experience of learners (in both handling graphs and content) does play a significant role in the reading of graphs, the design of the graph itself is also at times detrimental to the understanding. A poorly designed graph might make it unreadable or even lead to an erroneous conclusion being drawn. Works by Tufte and Wainer contain many such examples of poorly designed graphs. Design considerations while constructing a graph can help the readers of the graph positively. Some authors provide a step-by-step guide for graph creation. For example, Deacon (1999) gives a five-step guide for drawing graphs from experimental data in physics. The first step is to be clear about what is to be plotted? The second step is about the choice of units and axes. Step three is regarding joining the points either in a linear or a polynomial and finding coefficients. Step four considers if there are alternative plotting schemes, e.g. could changing the axes to logarithms help. The final step, which pertains to comprehending the graph in the context of the experiment is to relate the graph to the experiment or phenomena and make predictions if possible.

The presentation of graphs in the textbooks is important as the students encounter these graphs while learning. Shah et al. (1999) furnish guidelines for presenting graphs in textbooks. This work provides the design principles to help understand the graphs easily keeping in mind the visual encoding that is done by the readers of the graph:

1. Line graphs emphasize $x - y$ trends. If there are three or more variables in a data set, then the most important relationship should be plotted as a function of the $x-$ and $y-$axes.

2. Bar graphs emphasize comparisons that are closer together on the display. If there are three or more variables, the most relevant trends should be plotted closer together along the axes when using bar graphs.

3. Line graphs are more biasing (emphasizing the $x - y$ relations), whereas bar graphs are more neutral; thus, if two independent variables are equally important, bar graphs should be used. If a particular trend is the most important information, then line graphs should be used.

4. The scale should reflect whether the goal is to understand relative or absolute information, because people have difficulty translating between different graphic scales. (p. 701)

The text with which the graphs appear are crucial to aid in the comprehension of graphs which relates to the two parameters, namely Reference and Integration in Table 3.4. In *Critical Graphicacy* Roth et al. (2005) elaborate this point well:

> Main texts provide the story line in which graphs with their captions are embedded. They provide the story lines that set up a frame for graphs and their captions. Main texts may supply readings of the graph, and therefore instructions for how particular graphs can and should be read; they put graphs or aspects of graphs into relief and therefore set readers up for the interplay between the multimodal nature of concepts. Finally, main texts provide interpretations that integrate the graph into a theoretical framework. If graphs are not referenced from within the main text, they become ancillary to the text?s argument, and there may be little reason for readers to attend to this additional material.? (p. 33)

Cleveland (1984) analysed in detail all the graphs in a volume of the scientific journal *Science*. Errors were found in more than 30% of the graphs even in a highly rated journal like Science. This study reported four types of errors. This included errors in construction (mistakes while plotting the graph, $\sim$ 6%), reproduction (of the graph while printing, $\sim$ 6.5%), (design aspects of the graph render reading of certain parts incomprehensible, $\sim$ 10%) and discrimination explanation (some aspects of graph are unexplained, $\sim$ 16%). The study also investigated the space allotted to graphs in different journals from a sample of 57 journals from 14 different subject areas ranging from physical sciences, geology to psychology and education. In each of the journal 50 articles from the period, 1980-1981 were sampled, and the results analysed. The results indicate that the highest amount of space (about one-third of the total space) given to graphs was by *Journal of Geophysical Research*, while *The Journal of Social Psychology* has no graphs in this sample. They particularly note the discrepancy of lack of graphs in social sciences and provide this explanation:

> The lesser use of graphs in the mathematical sciences than in the natural sciences quite likely stems from the mathematics, computing, and statistics journals' having far less observational data to present; they focus mostly on mathematics and methodology. Many of the social science journals, however, have much data yet make very little use of graphs. (p. 265)

Furthermore, the article makes a note that just the presence of graphs (the quantity) is not an indication of effectiveness or the quality of the graphs.

> The fraction of space a journal uses in printing graphs does not measure the effectiveness or the quality of the graphs. It is certainly possible that a journal with less of its space devoted to graphs than another journal might be communicating more effectively with its graphs. But the fraction of space used is a measure of the importance that the journal and the authors attach to graphical communication. (p. 265)

On similar lines, we looked at the possible function the graph plays in the textbooks that we analysed. Just the presence of graphs is not sufficient. Cleveland also provides a set of guidelines for authors and editors of academic journals. The guidelines for authors provides a list of do's and don'ts concerning design and usage of graphs.

1. Graphs must be visually clear and capable of withstanding reduction.
2. Graphs must be clearly described. The combined information of the figure legend and the text of the body of the paper should provide a clear and complete description of everything that is on the graph. Detailed figure legends can often be of great help to the reader. First describe completely what is graphed in the display, then draw the reader's attention to salient features of the display, and then briefly state the importance of these features.
3. When feasible, put important conclusions into graphical form. Not everyone reads an entire article from beginning to end. When readers skim a paper they are drawn toward graphs. Try to make the graphs and their legends tell the story of your article.
4. Make the quantitative information that is graphed stand out. Be sure that different items on a graph can be easily visually distinguished.
5. Avoid cluttering graphical displays. For example, too much writing on the plotting region can interfere with the viewer's perception of geometric patterns. Put as much of the writing as possible-for example, a key for symbol types-outside of the plotting region, unless you are certain the writing will not interfere.
6. Proofread graphs. (p. 268)

In case of authors coming from an *information design* perspective like Edward Tufte, a good graph should have what he terms "graphical excellence" (Tufte, 2001). The term "graphical excellence" covers various aspects of the graph including its design, parsimony and ability to communicate ideas. In his works, Tufte provides many exemplars of "graphical excellence" from various subjects. Though Tufte's work is not limited to Cartesian graphs alone, his set of points for "graphical excellence" provide a good set of principles for the design of the graphs. Tufte's list shown below includes points which cover the fundamental reasons (for showing a graph) and possible insights that one can get from a graph.

§ Graphical excellence is the well-designed presentation of interesting data - a matter of substance, of statistics, and of design.

§ Graphical excellence consists of complex ideas communicated with clarity, precision, and efficiency.

§ Graphical excellence is that which gives to the viewer the greatest number of ideas in the shortest time with the least ink in the smallest space.

§ Graphical excellence is nearly always multivariate.

§ And graphical excellence requires telling the truth about the data (p. 51).

Thus, we see that there are sound design principles for creating graphs. These aspects are particularly important for the students who are just learning to make and read graphs. An exemplary graph might provide the students with a learning experience that will help them understand and appreciate the meaning of graphing much better. The parameter of *close-to-life* derives from many studies which claim that the context of the graph should be meaningful and familiar to the learners, for example, Ainley (1995). The category of the design of the graphs is crucial for graph comprehension, (Tufte, 2001; Wainer, 2007). We evaluate each graph in the science textbooks in the context of these categories.

To provide possible inter-connections with other subjects, we have also added and analysed few graphs from the mathematics and social science textbooks when necessary. Tufte and Wainer well elaborate the concepts of data-ink ratio and chart junk in their books on information design (Tufte, 1997; 2001; 2005; 2006; Wainer, 2007). In the rest of the section, we give a few examples of analysis of the graphs the science textbooks concerning the parameters mentioned Table 3.4. Appendix A.2 provides a detailed analysis of all the graphs in the science textbooks. At many places in this analysis, we present alternative options of either designing or presenting the graph and additional activities which may enhance learning. This section addresses research question (2). To understand the nature of graphical practices in the textbooks and to recognise the affordances given to the students by the textbooks to develop graphicacy.

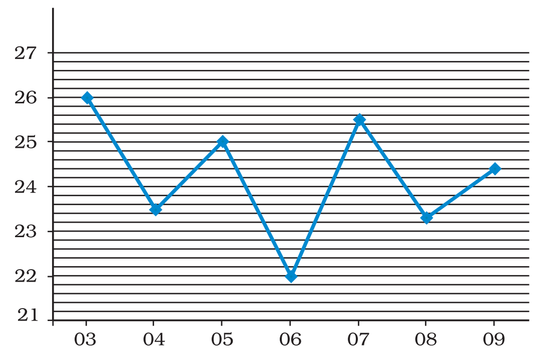
3.5 Some sample graphs from science textbooks

In this section, we present some sample graphs in the NCERT science textbooks. We have created information boxes for each graph with a unique number for the graph (the title of the box). Each box for the graph contains the particulars of the parameters used (Table 3.4) in the qualitative analysis.

Class 7

In this section, we some examples of the graphs appearing in Class 7 science textbook (NCERT, 2007b). There are a total of eight graphs appearing in this Class 7. Seven of these appear in Chapter 13 *Motion and Time.* Graphs are seen here mostly in the context of motion, though as we find there are many places where we can use graphs for improving the understanding of concepts at hand. The graph which does not appear in the context of motion is in the context of weather (Figure 3.8).

Example 1



Date	Maximum temperature
03-08-06	26.0°C
04-08-06	23.5°C
05-08-06	25.0°C
06-08-06	22.0°C
07-08-06	25.5°C
08-08-06	23.3°C
09-08-06	24.4°C

Fig. 7.2 *Graph showing the variation of maximum temperature during 03 to 09 August 2006*

Figure 3.8: A sample graph from the science textbook. This graph appears in Class 7, Chapter 7, on Page 70.

Function

The first graph in Class 7 textbook, (Figure 3.8 here) shows the variation of maximum temperature for the seven days in August for the city of Shillong, in the state of Meghalaya. Here the graph is introduced in a way under the assumption that the students already know about graphs.

Reference

The graph is referred to in the main text with the figure number, but there is no elaboration on the nature of the graph or its interpretation.

> Look at the graph given below which shows the maximum temperature recorded during 03 August 2006 to 09 August 2006 at Shillong, Meghalaya (Fig. 7.2).

Data

The table next to the graph shows the data used, the source for this data is not provided. Though the data has a 0.1 degree accuracy, the plot has only five divisions per degree hence 0.2 degree accuracy.

Legend, Axes

The labelling on the axes is missing. The scales or units are not mentioned anywhere in the graph.

Close-to-life

The situation depicted in the graph is close-to-life, as the variation in daily temperature, is a phenomenon which is experienced by us every day.

Design Aspects

The graph has too many horizontal lines, which might be useful in finding the temperature. However, the number of grid lines can be reduced.

Being the first graph, the introductory aspects related to reading of the graph are entirely missing. Information on how to read the graph, what are its salient features, how to interpret the variation in the graph and how to infer from the graph is completely missing in the main text. The graph itself has data for maximum temperature from many days and does not show any particular trend. Neither are any particular questions asked about the graph. The next paragraph that follows mentions maximum and minimum temperature, and also the probable times during a day when these temperatures might be reached. A graph depicting the variation of temperature over the course of a single day, and the supporting text would have done a better job and made the point about the variation of temperature during the day. We think that the poor treatment given to this graph is a missed opportunity to introduce graphs in a close-to-life context like the daily variation of temperature.

Figure 3.9 shows an alternative graph that we offer to illustrate the points for daily maximum and minimum temperature. Table 3.5 shows the data which was used to draw this graph. For the variation of the maximum temperature on the annual scale, Figure 3.11 illustrates how the minimum and maximum temperature varies through the year. The data for this figure is from Tables 7.2 and 7.3 in Class 7 science textbook, (pg. 71) (Table 3.10 here).

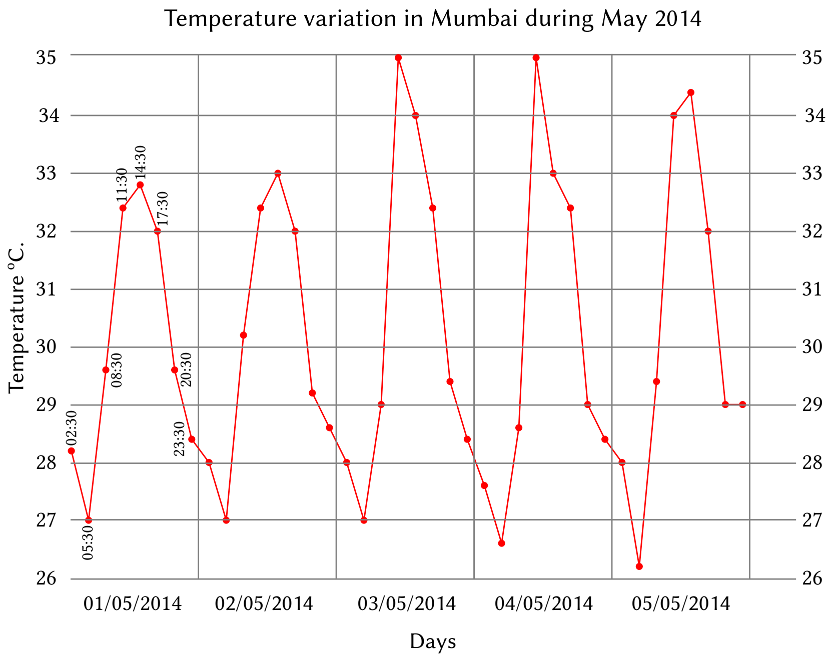


Figure 3.9: A graph showing the temperature variation in Mumbai in the first week of May 2014. The observations were taken eight times every day with 3 hour intervals. The readings were taken at 02:30, 05:30, 08:30, 11:30, 14:30, 17:30, 20:30 and 23:30 hours. Data from http://rp5.in

Date/Time	02:30	05:30	08:30	11:30	14:30	17:30	20:30	23:30
01/05/14	28.2	27.0	29.6	32.4	32.8	32.0	29.6	28.4
02/05/14	28.0	27.0	30.2	32.4	33.0	32.0	29.2	28.6
03/05/14	28.0	27.0	29.0	35.0	34.0	32.4	29.4	28.4
04/05/14	27.6	26.6	28.6	35.0	33.0	32.4	29.0	28.4
05/05/14	28.0	26.2	29.4	34.0	34.4	32.0	29.0	29.0
06/05/14	28.2	28.0	29.6	33.0	34.2	32.0	30.0	30.2

Table 3.5: Variation of temperature during the day in Mumbai for the first week in May 2014. The temperature is in collected eight times a day in three-hour intervals. Figure 3.9 uses this data.

In the textbook, the students are told to analyse the data given in Tables 7.2 and 7.3 (shown in Figure 3.10 here). The textbook has following to say about the use of tables:

> In Table 7.2 and 7.3, we have given the climatic condition at two places in India. The mean temperature for a given month is found in two steps. First, we find the average of the temperatures recorded during the month. Second, we calculate the average of such average temperatures over many years. That gives the mean temperature. The two places are: Srinagar in Jammu and Kashmir, and Thiruvananthapuram in Kerala.

This paragraph explains the concept of mean temperature in the tables. Then in the next paragraph, the textbook says:

> By looking at Tables 7.2 and 7.3 we can *easily see* the difference in the climate of Jammu & Kashmir and Kerala. We can *see* that Kerala is very hot and wet in comparison to Jammu & Kashmir, which has a moderately hot and wet climate for a part of the year. (emphasis added)

It is not clear by looking at the tables how the difference in the climate can be *easily* seen. Perhaps one is reminded of the quote from Farquhar and Farquhar (quote at the beginning of Appendix B) when this claim is made.

When the same data given in the two tables are plotted (Figure 3.10), the difference in the climate of the two places stands out.

We provide the analysis of Figure 3.11 as follows. In general, the trend that we observe is that the range of temperature in Thiruvananthapuram is moderate between 22 to 32 °C throughout the year. Srinagar on the other hand has significant variation in temperature through the year.

We now provide a month-by-month description of how the graph is to be read.

Jan-Feb

Starting in January we see that the temperature in Thiruvanathapuram is in the range of 22 to 32 °C, while at the same time in Srinagar the temperature is much lower in the range of −5 to 5 °C. In the case of rainfall during these two months Srinagar experiences *more* rainfall than Thiruvananthapuram.

March-April

During March and April the temperature rises in Srinagar, moving from sub-zero to

Table 7.2 Srinagar (Jammu & Kashmir)

Information about climate

Month	Mean temperature °C		Mean total rainfall (mm)
	Daily minimum	Daily maximum	
Jan	-2.3	4.7	57
Feb	-0.6	7.8	65
Mar	3.8	13.6	99
Apr	7.7	19.4	88
May	10.7	23.8	72
Jun	14.7	29.2	37
July	8.2	30.0	49
Aug	17.5	29.7	70
Sep	12.9	27.8	33
Oct	6.1	21.9	36
Nov	0.9	14.7	27
Dec	-1.6	8.2	43

Table 7.3 Thiruvananthapuram (Kerala)

Information about climate

Month	Mean temperature °C		Mean total rainfall (mm)
	Daily minimum	Daily maximum	
Jan	22.2	31.5	23
Feb	22.8	31.9	24
Mar	24.1	32.6	40
Apr	24.9	32.6	117
May	24.7	31.6	230
Jun	23.5	29.7	321
July	23.1	29.2	227
Aug	23.2	29.4	138
Sep	23.3	30.0	175
Oct	23.3	29.9	282
Nov	23.1	30.3	185
Dec	22.6	31.0	66

Figure 3.10: Tables 7.2 and 7.3 for Figure 3.11 from pg 71. Chapter 7, Class 7, science textbook.

positive. While in Thiruvananthapuram the minimum temperatures rise slightly. In case of rainfall in March Srinagar still has more rainfall than Thiruvananthapuram. However, in April Thiruvananthapuram has more rain than Srinagar.

May-June

During May-June the temperature continues to rise in Srinagar with the maximum crossing 20 °C and almost reaching 30 °C in June. While in Thiruvananthapuram there is a drop in temperature by a couple of degrees, (this is perhaps related to the rainfall due to the onset of monsoon). In May Thiruvananthapuram gets second highest rainfall in the year (∼ 230 mm), while Srinagar gets lower rainfall (snowfall?) than the previous months. In June monsoon is at the peak in Thiruvanathapuram the rainfall is above 300 mm, but at the same time in Srinagar, the rainfall is modest, while the temperature is on the rise.

July-August

During July the rainfall is lower than in June in Thiruvananthapuram. The mean temperature difference is minimum in Thiruvananthapuram, while it is maximum in Srinagar (8.2 to 30 °C). Also in July Srinagar experiences the highest mean maximum temperature, making it the hottest month of the year. In August the rainfall drops significantly in Thiruvananthapuram, while in Srinagar it increases. During August the daily minimum temperature in Srinagar is highest.

Sept-Oct

With the onset of September the mean temperatures begin to fall in Srinagar, while in Thiruvananthapuram the temperature is almost the same as previous months. The rainfall increases in Thrivananthapuram, while in Srinagar it decreases. In October, the general decline in mean temperatures continues in Srinagar while in Thiruvananthapuram there is not much change in the temperature. The minimum temperature goes below 10 °C in October in Srinagar, which indicates that the winter is setting in. The second monsoon peaks during October in Thiruvananthapuram, while Srinagar registers little rainfall.

Nov-Dec

During November - December the temperatures in Thiruvananthapuram rises slightly from the months before. In Srinagar the winter has set in, the mean minimum temperature going close to 0 °C in November, while going to negative in December. The rainfall in Thiruvanantahpuram decreases sharply in December while in November it still has rainfall above 150 mm after the October monsoon peak. In Sringar the rainfall is lowest during November and increases in December and following months.

We provided this detailed exposition to show how to read graphs such as these, especially when presenting them in the textbooks. Expositions like this in which the narrative have the following advantages:

1. Provides a way of reading graphs to the learner.
2. Integrates the text and the graph with the data provided.
3. Provides help on reading salient features from the data and their meaning.
4. Provides connections to the physical meaning of the various features of the graph.
5. Poses questions that can be asked and answered from the graph.

Work by (Bell & Janvier, 1981), discussed earlier on page 29, shows the importance of using tables in teaching graphs.

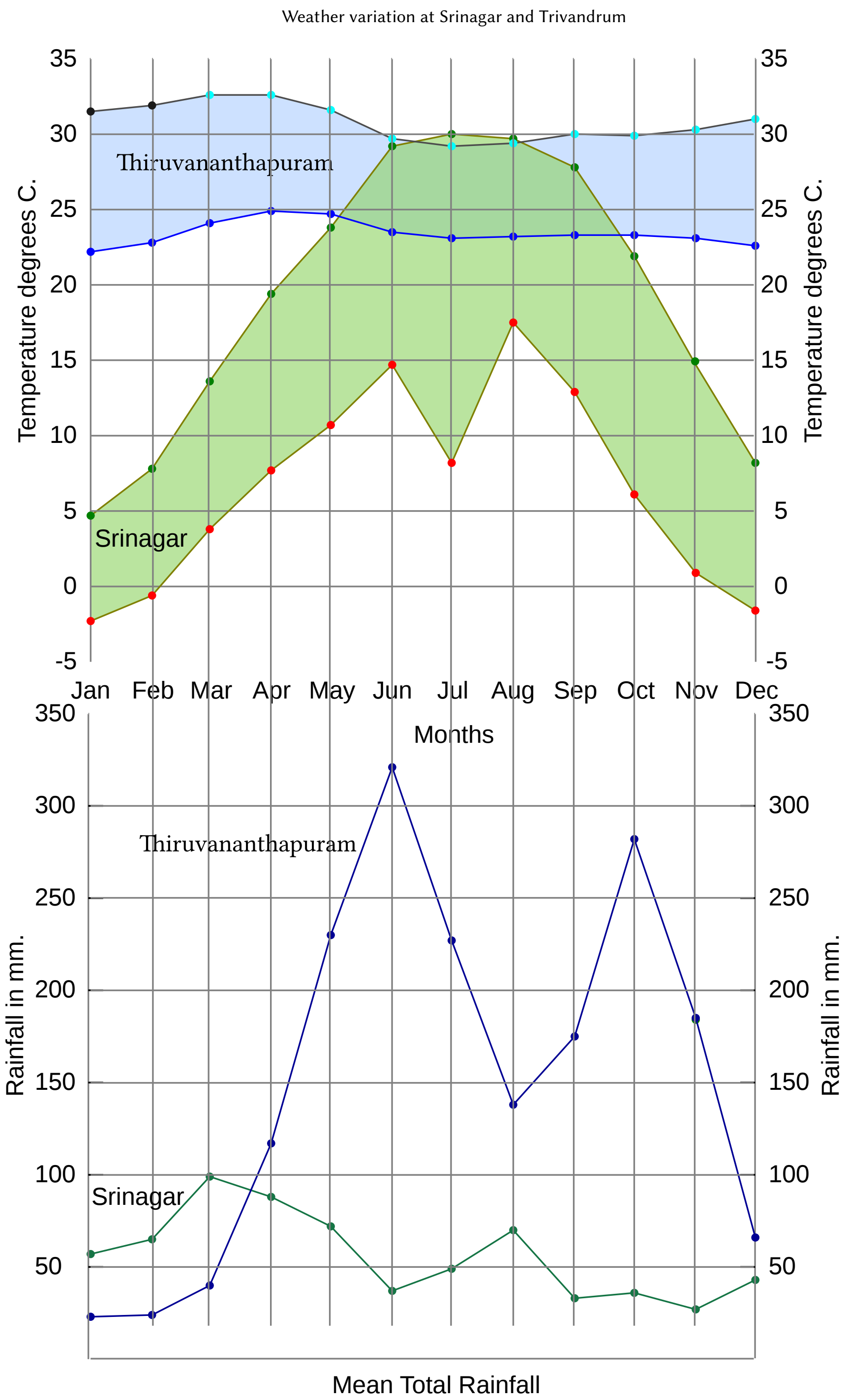


Figure 3.11: A graph showing the annual mean rainfall and maximum and minimum temperatures in Srinagar and Thiruvananthapuram. The data for this graph is in Figure 3.10.

Data table for Graphs

Table 13.5 Odometer reading at different times of the journey

Time (AM)	Odometer reading	Distance from the starting point
8:00 AM	36540 km	0 km
8:30 AM	36560 km	20 km
9:00 AM	36580 km	40 km
9:30 AM	36600 km	60 km
10:00 AM	36620 km	80 km

Figure 3.12: A table containing readings from odometer of a car for analysing. From Class 7 science textbook p.151.

Function

The Chapter on *Motion and Time* in Class 7 in science textbooks introduces graphs to the students in the context of solving a problem.

Reference

The problem involves a travelling bus, with odometer readings available for specific times.

> Looking at the Table, Boojho teased Paheli whether she can tell how far they would have travelled till 9:45 AM.

The question posed is to find the distance travelled by 09:45 AM.

Data

A table (Figure A.17) gives the data used for the problem. The textbook does not elaborate the method for obtaining the data.

Legend, Axes

Not applicable.

Close-to-life

This is a close-to-life context as people travel every day by various means of transport.

Design Aspects

Table can be made more readable by use of alternating coloured rows. The units for entries in the columns can be given at the column heads and not repeated with every entry. Vertical lines should be avoided while drawing a table. Mori (2007) gives excellent guidelines for drawing tables, and the first point is "never use vertical lines ".

To answer the question "Looking at the Table, Boojho teased Paheli whether she can tell how far they would have travelled till 9:45 AM." A line graph should be constructed, as per the instructions. In the three graphs that follow, (Figures 3.13, 3.15, 3.17 here), we see that the students are introduced to different *types* of graphs namely bar graph, pie-chart and line graph.

Example 2

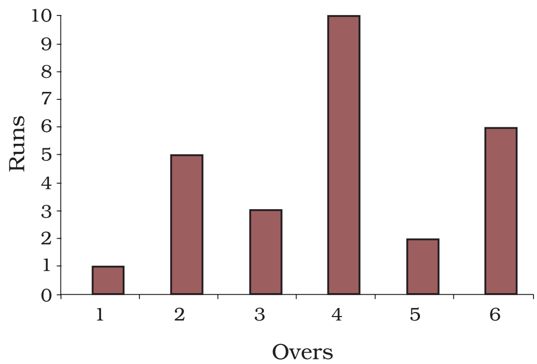


Fig. 13.8 *A bar graph showing runs scored by a team in each over*

Figure 3.13: An example of a bar graph, showing runs scored in an over. From Class 7 science textbook p.151.

Function Presented as an example of a bar graph.

Reference The bar graph is only referenced in the main text once as:

> The type of graph shown in Fig. 13.8 is known as a bar graph.

Data The data used in the graph is not given.

Legend, Axes The axes are labelled and have units.

Close-to-life The data presented has close-to-life context in form of score of a cricket match score.

Design Aspects

The graph presented is free from chart-junk.

The concept of graphs is introduced as if the students have not encountered graphs before. There is no effort to link it to the study of graphs that the students have already done in other subjects like mathematics and the social sciences (Geography). For example, in case of mathematics, we see bar charts introduced previously in Class 5 (Figure 3.14). There are also questions asked about these bar graphs which require that the students study these graphs and answer them: (1) Which city is hottest on June 1? (2) Which city is coldest on December 1? (3) Which city shows little change in temperature on the two days - 1 June and 1 December? (pg. 165, Class 5, Mathematics textbook.)

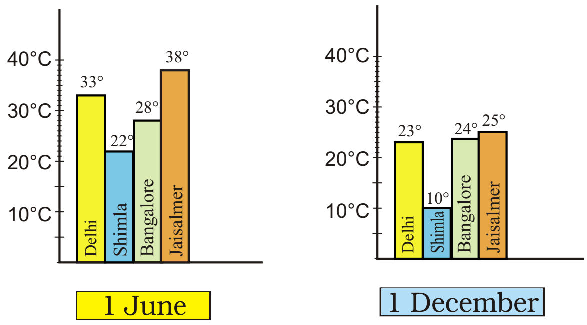


Figure 3.14: Figure from Class 5, Chapter 12, page number 165 in Mathematics textbook. The two bar graphs show temperatures in four different cities, namely Delhi, Shimla, Bangalore and Jaisalmer at two different times in the year. Students are asked questions to be answered by studying the graphs.

There is also an associated activity for the students to collect data about the temperature of a city from TV or newspapers and plot it."On any one day, choose any three cities and record their temperatures from the TV or newspaper. Make a bar chart in your notebook and ask your friends a few questions about it. See if they understand your chart! (pg. 165, Class 5, Mathematics textbook.)"

However, in the bar chart in science textbook (Figure 3.13) no such attempt is made. There is no further information or elaboration for the inclusion of this graph here. Neither the text details how to construct one. Sometimes, it is good to repeat a point more than once, but it is immensely beneficial if a reference is made to the earlier presence of the topic. For example, Roth et al. (2005) make a pertinent point regarding the use of repetition in the contest of captions of a graph and how it helps in constructing meaning from the graphs: " . . . *caption* illustrates that scientists do not leave uncertain how to read a line graph. It repeats information that might be gleaned from the graph alone. The effect of this redundancy, however, is to guide readers to a congruent construction of graph and text. (emphasis in original, p. 36)"

So, it is a little strange that the science textbook is not building upon the mathematical knowledge that the students have been already exposed to, but instead treats it as an entirely new concept. Even when treating as a new idea, no effort is seen to engage the students about how such graphs are made, or how to read them or what is the use case for each one of them. This exercise would have been an excellent opportunity to show the idea that how the same type of graphs can be used to depict different situations, across disciplines. We find this approach towards graphs lacking coherence across subjects.

Example 3

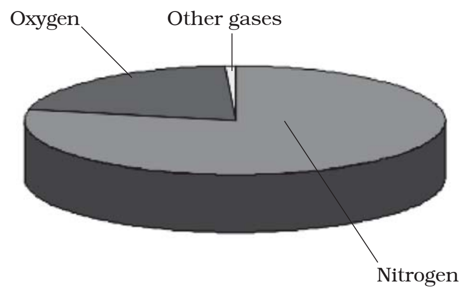


Fig. 13.9 *A pie chart showing composition of air*

Figure 3.15: A pie chart showing components of air. Figure from Class 7, Chapter 13, page number 152 in the science textbook.

Function In Figure 3.15 we see a 3-D pie chart showing the different component gases of air.

Reference The only reference to this pie chart in the textbook is this:

> Another type of graphical representation is a pie chart (Fig. 13.9).

This graph presented here adds no value to the context or discussion.

Data Data for the graph is not given. If data was given, the students could at least redraw the chart if required.

Legend, Axes Component gases are labelled in the marked areas.

Close-to-life The air components are close-to-life.

Design Aspects

Presenting the pie chart in 3-D for essentially a 2-D data can be misleading. It adds to the difficulty in reading the graph and might lead to incorrect display/reading of the values, especially when the not providing corresponding data. We provide a detailed analysis of one such graph later in this section (Figure3.28).

Another graph (Figure 3.16) for the composition of air appears in the Class 7 Environmental science book in Chapter 4, page 20. The percentage values of the components are provided, making it better than the previous graph. The pie-chart is presented in the form of a musical instrument *dafali* held by one of the characters in the textbook.

A pie chart from EVS book showing components of the air.

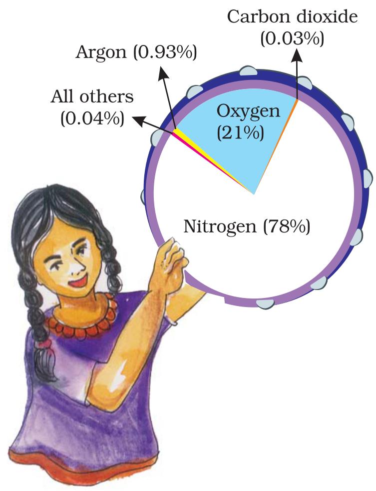


Fig. 4.1 *Constituents of Air.*

Figure 3.16: A Pie chart showing components of the air. Figure from Class 7, Chapter 04, page number 20 in EVS textbook.

Function

The pie chart serves to show the components of air. But, the aim of presenting this chart is not clear from the textual narrative in the accompanying text.

Reference

The graph is referred in the main text in a single sentence. “The pie chart gives you the percentage of different constituents of air (Fig. 4.1).” No further elaboration is provided.

Data

The data source is not given.

Legend, Axes

The components are colour coded.

Close-to-life

This is a close-to-life context.

Design Aspects

The pie chart is shown in the form of a musical instrument, the *dafali*, in hand of a girl. The design of the graph is unnecessarily complicated by this. This graph qualifies as chart-junk as the added elements have no value either to aid understanding of the graph or to connect it to the context of its presentation.

The graph in EVS book has a reference in the text, which explains the different component gases and their relevance. However, there is no mention of the presence of water vapour! Neither it is explained that these percentage values of the gases are for *dry* air. Water vapour is an important component of life and weather should have a mention in the chart.

The pie chart gives you the percentage of different constituents of air (Fig. 4.1).

When presenting a graph, the data used for creating the graph should be made available in the form of a table. The presence of data tables is helpful for the students learning to read and make graphs. Presenting the data in multiple representations aids the students to move between representations. Bell & Janvier (1981) argue that: " . . . the use of tables proved a powerful tool to study "how variables change". The results conclusively show that the table approach certainly spelt out many ideas to the extent of making possible transfers from tables to graphs. Consequently, results suggest that the use of tables should be included in our graph teaching scheme." (p. 41)

After just showing the pie chart and the bar graph, a line graph is shown in the textbook (Figure 3.17 here).

Example 4

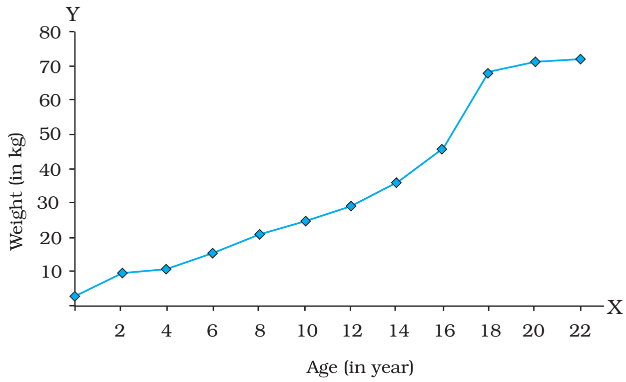


***Fig. 13.10** A line graph showing change in weight of a man with age*

Figure 3.17: Graph showing weight of man as a function of age. Figure from Class 7, Chapter 13, page number 152 in science textbook.

Function

Graph shown in Figure 3.17 is presented as an example of a line graph.

Reference

This is an interesting graph which shows the weight of a man as a function of his age. The only mention in the text that this graph finds is:

> The graph shown in Fig. 13.10 is an example of a line graph.

Data

The data is not provided.

Legend, Axes

The axes are labelled and have units.

Close-to-life

The data presented has close-to-life context in the growth of humans.

Design Aspects

The graph presented is free from chart-junk. Grid lines (both X and Y) would have helped to improve readability.

Many interesting questions and observations are present in this graph, but as is the case with the previous two graphs, no effort is made to engage the students with the graph. Some of the interesting questions that can be asked and answered for the graph in Figure 3.17 are:

- § During which two years does the weight gain is maximum?
- § During which two years the weight gain is least?
- § What is the total percentage gain in weight till 10 years, 20 years?

Two activities on the same theme, with a good scope for asking and answering questions, using real-world data are in class 8 (Figure 3.19 here).

However, line graphs are not new to the students. In fact, just a few pages prior to this graph, in the same textbook, we have a line graph already shown (Figure 3.8). This example, as we have shown, is another illustration of graphs appearing without any context in the text. We see this as another lost opportunity in the context of graphicacy.

We would also like bring to notice that mathematics textbooks also teach graphs in the previous classes. In mathematics, line graphs make their first appearance in Class 5. There is an activity centred around data collected from growing seeds (pg. 168-169, Class 5, Mathematics). The activity also asks questions which require the students to study the graph (Figure 3.18a). Though this graph is not from the science textbooks, we consider it here to display the disparity and missing links to other parts of the curriculum concerning the treatment of graphs in the science textbooks.

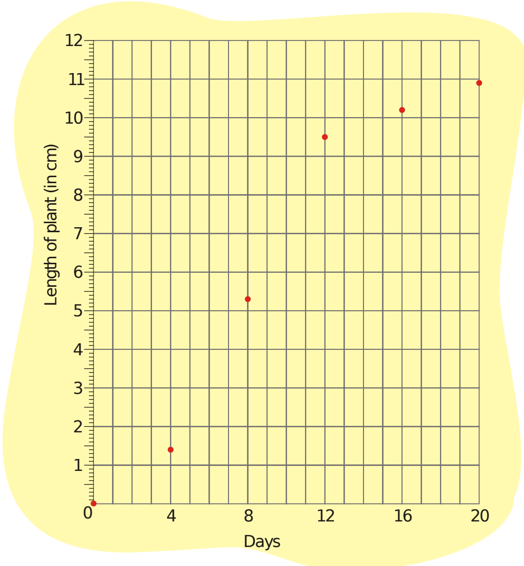
Graph from Class 5 Mathematics Textbook

Function

Data for the growth of plant shown in graphical representation along with questions that can be asked and answered. (Figure 3.18 here.)

Reference

Activity centred around data collected from growing seeds (pg. 168-169, Class 5, Mathematics). The activity also asks questions which require the students to study the graph (Figure 3.18a here). There are follow up tasks after the activity on similar lines.



(a)

Day	Length of the plant (in cm)
0	0
4	1.4
8	5.3
12	9.5
16	10.2
20	10.9

(b)

Find out from the growth chart

a) Between which days did the length of the plant change the most?

i) 0-4 ii) 4-8 iii) 8-12 iv) 12-16 v) 16-20

b) What could be the length of this plant on the 14th day? Guess.

i) 8.7 cm ii) 9.9 cm iii) 10.2 cm iv) 10.5 cm

c) Will the plant keep growing all the time? What will be its length on the 100th day? Make a guess!

There should be some discussion on the last question. Children should be encouraged to observe growth patterns of many other plants and animals.

(c)

Figure 3.18: Example of a line graph based activity from Mathematics Class 5 textbook. (a) A line graph based activity in Class 5 Mathematics textbook. Figure from Class 5, Chapter 12, page number 169 in Mathematics textbook. (b) Data for the line graph activity. Observations recorded for growth of *moong dal* used to plot graph shown in Figure 3.18a. (c) Questions associated with the graph (Figure 3.18a) in the activity.

Data

The data in a tabular form and the way of obtaining it is provided.

Growth Chart of a Plant Amit sowed a few seeds of moong dal in the ground. The height of the plant grew to 1.4 cm in the first four days. After that it started growing faster.

Amit measured the height of the plant after every four days and put a dot on the chart. For example, if you look at the dot marked on the fourth day, you can see on the left side scale that it is 1.4 cm high.

Now look at the height of each dot in cm and check from the table if he has marked the dots correctly.

Legend, Axes

The axes are labelled and with units.

Close-to-life

Growth of plants is a close-to-life context for the students to observe and record.

Design Aspects

The graph is well made. Background colour (yellow) to the graph could be avoided.

The graph above and the associated questions are good starting exercise in reading graphs. The exercise provides a connection between different representations of data, tabular and graphical. However, after this, we see that line graphs do not appear in Class 6 mathematics, except in the case of the number line. So, there is no build up from what is a good starting exercise in graphicacy.

Class 8

Example 5

Activity 10.2

Use the data given in Activity 10.1 to draw a graph. Take age on the X-axis and per cent growth in height on the Y-axis. Highlight the point representing your age on the graph. Find out the percentage of height you have already reached. Calculate the height you might eventually reach. Tally your graph with the one given here (Fig. 10.1).

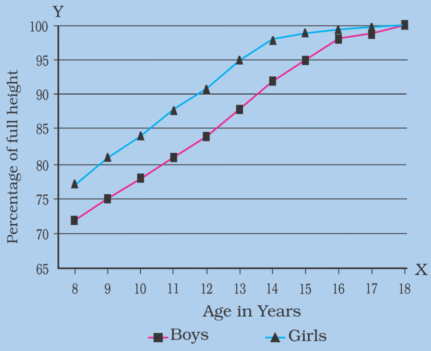


***Fig. 10.1 :** Graph showing percentage of height with age*

Age in Years	% of full height	
	Boys	Girls
8	72%	77%
9	75%	81%
10	78%	84%
11	81%	88%
12	84%	91%
13	88%	95%
14	92%	98%
15	95%	99%
16	98%	99.5%
17	99%	100%
18	100%	100%

Figure 3.19: A graph showing percentage height with age. Figure from Class 8 science textbook, Chapter 10, page number 115. The data for this graph is in the table shown in the Table on the right.

Function

The activity asks the students to plot the data given in previous activity. The activity also asks some questions to be answered by the graph drawn by the students.

Reference

The activity gives a graph drawn from the data for the students to compare their hand-drawn graphs.

Data

An activity earlier provides the data in the form of a table.

Legend, Axes

Axes are labelled with units. The legends for points showing boys and girls are explained.

Close-to-life

The activity asks the students to plot their own height-age on the graph, to see where they are placed on the graph. The activity also asks few interpolatory and exploratory questions to be answered. The graph relates to differential growth in males and females which is close-to-life for the age group in Class 8, as the students are themselves undergoing these changes.

Design Aspects

The legends could have been placed next to the line graphs.

We think this is a good activity as it allows individual learners to find answers for their data. The questions asked requires the learners to use their age as a starting point. The questions span all three levels of graph use described by Bertin. For example, **Level 1:** Point your age on the graph, **Level 2:** Find out the % height you have achieved, and, **Level 3:** Calculate the height you might eventually reach. This activity can be a good collaborative activity when the students exchange their data with others in the class.

Class 9

Example 6

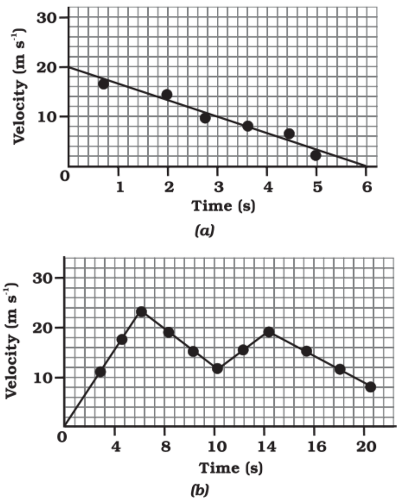


Fig. 8.7: *Velocity-time graphs of an object in non-uniformly accelerated motion.*

Figure 3.20: Velocity time graphs from text showing non-uniformly accelerated motion. Figure from Class 9, Chapter 8, page number 106 in science textbook.

Function

The graphs are presented as examples of non-uniformly accelerated motion.

Reference

Main text refers to both the graphs

> Fig. 8.7(a) shows a velocity-time graph that represents the motion of an object whose velocity is decreasing with time while Fig. 8.7 (b) shows the velocity-time graph representing the non-uniform variation of the velocity of the object with time. Try to interpret these graphs.

Data

The textbook does not give the data for the graphs.

Legend, Axes

Both the graphs have units on both the axes.

Close-to-life

Currently, the examples are not close-to-life. Experiences of non-uniform motion in real-life could be used to contextualise the examples. For example, see the monograph by Gerhart & Nussbaum (1966).

Design Aspects

The background grid is too dense. It can be reduced for a cleaner graph. A redrawn version is shown in Figure 3.21.

Though the caption of the graphs says that they represent graphs in "non-uniformly accelerated motion", in the first graph this is not the case. The first graph is a graph of uniformly decelerating motion. This example is a clear case of an **incorrect** graph in the textbook. Comparing this graph to the one of uniform acceleration (Figure A.31), the students might get a misconception that uniform acceleration can result only from *increasing* speeds and not from *decreasing* speeds.

The text asks the students to "interpret the graphs", some pointed questions, which can be answered by analysing the graph, should have been added. For example, "how can you describe the motion as depicted in the second graph?"

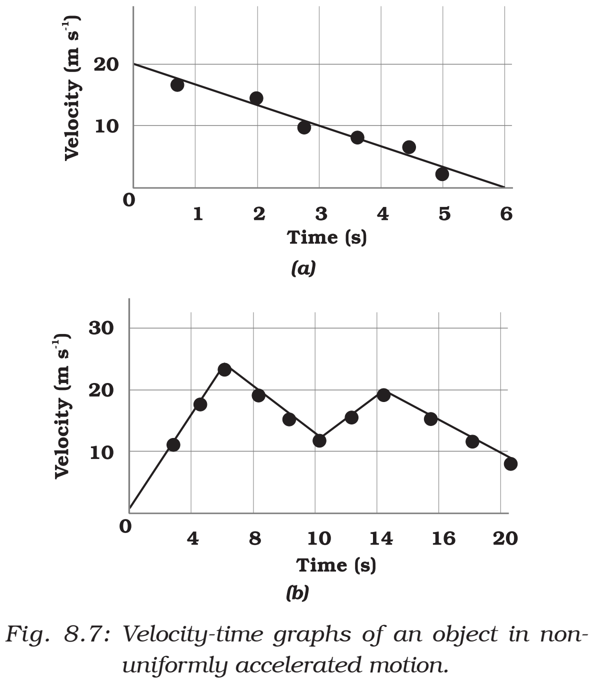


Figure 3.21: Redrawn: Velocity time graphs from text showing non-uniformly accelerated motion. Figure from Class 9, Chapter 8, page number 106 in science textbook.

Example 7

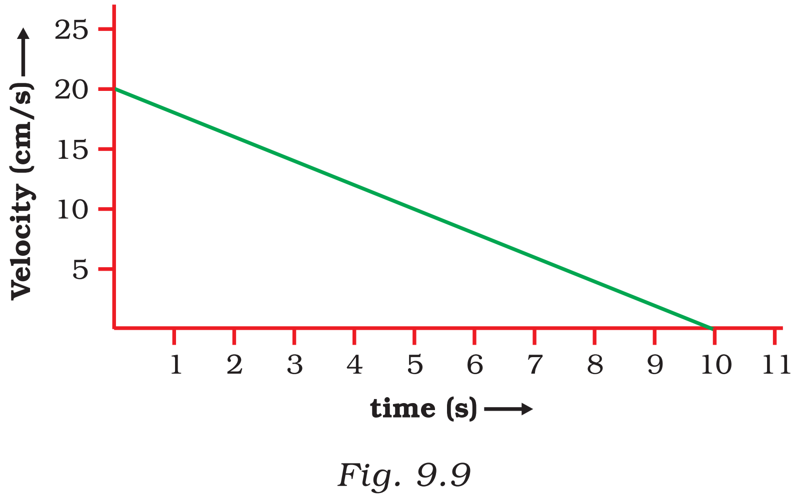


Figure 3.22: A velocity time graph from a problem in textbook. Figure from Class 9, Chapter 9, page number 121 in science textbook.

Function

The graph is a part of a problem in which the students have to read values from the graph to solve the problem. Figure 3.22 is a problem from the Chapter 9, Class 9 science textbook, Figure 9.9 in the textbook. In this problem, the students are presented with a Time vs Velocity graph for a ball moving along a straight line on a table and are supposed to find out the force exerted on the ball by the table.

> ``**Example 9.5** The velocity-time graph of a ball of mass 20 g moving along a straight line on a long table is given in Fig. 9.9 (Figure 3.22 here). How much force does the table exert on the ball to bring it to rest?''

Reference

The main text refers to the graph. The main text gives a physical interpretation of negative sign in the answer.

Data

The textbook does not give the data.

Legend, Axes

Axes are labelled with units.

Close-to-life

The ideas of acceleration and deceleration are experienced every day.

A real-life situation like a movement of a ball and providing the different measurements to make the graph can be the setting for this problem.

Design Aspects

There is no chart-junk, though the graph can be made more readable by adding grid lines. Grid lines should be for aiding the reading of graph, and there should be a balance between the readability and non-readability.

The solution to this problem is given in the textbook. The required force F is given by the equation

$$F = m \times a$$

where m and a are mass and acceleration respectively. The graph allows us to find the acceleration a, as the initial and the final velocity at start and end time are known. Thus initial velocity $u = 20\,\text{cm/s}$ and final velocity is $v = 0\,\text{cm/s}$, and this change happens in 10 s. So the acceleration is

$$\begin{aligned} a &= \frac{v-u}{t} \\ &= \frac{(0-20)\ \text{cm/s}}{10\ \text{s}} \\ &= -2\ \text{cm/s}^2 \end{aligned}$$

So the required force is $F = 20\text{g} \times -2\ \text{cm/s}^2 = -0.0004\text{N}$. In the solution to the exercise in the textbook it is noted that:

> The negative sign implies that the frictional force exerted by the table is opposite to the direction of motion of the ball.

Apart from this, there is no interpretation of the problem regarding the graph given or the physical situation involved. Many leading and exploring questions about the motion of the ball can be asked and answered from this graph. For example, can we just by looking at the graph tell that the acceleration will be *negative*? It turns out we can. The acceleration of a body is positive if it is increasing its speed over time, whereas it is negative if the speed is decreasing. This is the physical interpretation of the sign of acceleration. How is this related to a *Time vs Velocity* graph? The slope of the line in this graph is the required acceleration.

If the line has a negative slope, then the acceleration is negative, while if the line has a positive slope the acceleration is positive. This simple rule enables one to understand many problems associated with motion in the physical context.

The next set of graphs in Class 9 appears in the context of sound.

Example 8

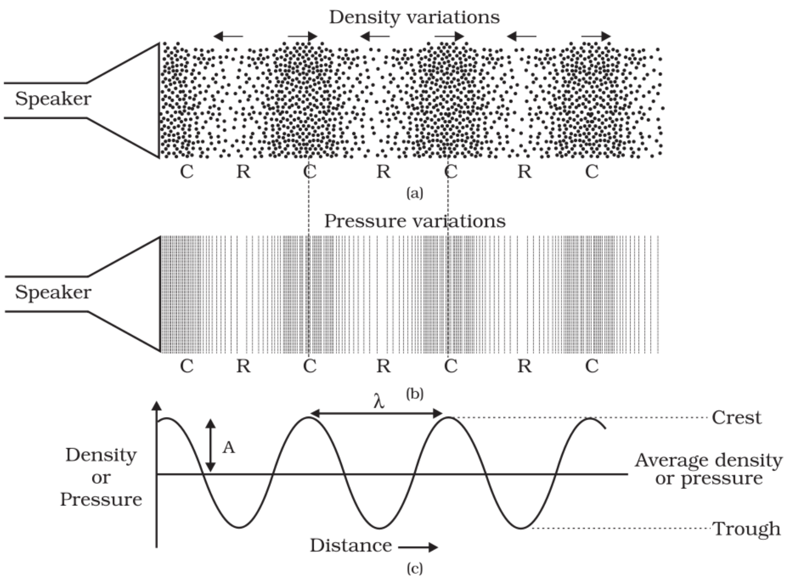


Fig. 12.8: *Sound propagates as density or pressure variations as shown in (a) and (b), (c) represents graphically the density and pressure variations.*

Figure 3.23: A graph showing variation of density during propagation of a sound wave. Figure from science Class 9, Chapter 12, pg. 164.

Function

The graphs shown in Figures 3.23, A.43 and 3.25 appear in Chapter 12 *Sound* in grade 9 textbook. This set of figures are graphical representations of the sound waves.

Reference

The main text refers to the graphs, but a full integration of different parts of the graph with the textual content is absent.

Data

The textbook does not give the data.

Legend, Axes

The axes are labelled, but without units.

Close-to-life

We experience different types of sounds daily. This example is a close-to-life context.

Design Aspects

There is no chart-junk, but the readability of the graph can be improved by certain modifications as shown in Figure 3.24.

In Figure A.41 the physical variation of density of air or pressure in air associated with a sound wave is represented graphically. There are three parts to this graph. Parts *12.8(a)* and *12.8(b)* show the density and pressure variation in the medium as a sound wave travels through it. Though the figure shows a speaker as the source of the sound wave, there is no reference to the source of sound (the speaker in this case) in the main text. Instead, the text directly refers to part *12.8(c)* which is the actual graphical representation. Instead of building the graphical sinusoidal graph with a physical basis of variation of density and pressure, it starts with the graphical form as a given.

> A sound wave in graphic form is shown in Fig. 12.8(c), which represents how density and pressure change when the sound wave moves in the medium. (*p. 163, Class 9, science*)

Thus, the textbook chooses an approach in which the abstracted result, the graphical form of a sound wave, in this case, is presented *before* considering the actual physical observations. Observations about what happens in the medium with regards to density and pressure when a sound wave travels should have come first in the explanations. Moreover, this could be used logically to form the graphical form shown in part *(c)* of the figure. However, we see no such effort. Parts *(a)* and *(b)* though have a wealth of information are not directly referred to in the text concerning their content. The reference in the main text is as below:

> Fig. 12.8(a) and Fig. 12.8(b) represent the density and pressure variations, respectively, as a sound wave propagates in the medium. (Class 9 science, p. 163)

Since the two parts (*a*) and (*b*) do contain the actual physical observations, they should be used logically to form part *c*. Another thing to note is that there is a line which connects parts *a* and *b* where there are compressions in the medium. A better way to represent this entire figure would be to show vertical lines connecting all three parts. A graph in this manner would make clear that the three graphs are referring to the same physical space on the *X-axis*. Right now, there seems to be an ambiguity in this as a line connects only the first two parts. We think, all the compressions and rarefactions should be mapped on the graphical form to show its physical basis (Figure 3.24). There is a label called *A* in part *(c)*, the immediate explanation is not in the main text. The explanation that *A* is the amplitude appears on next page (*p. 165*) with discussions about the pitch, frequency in between. The explanation for this could be a continuity to the discussion about frequency λ, defined in the context of the compressions and rarefactions. This approach would make the referencing in the later figures easier. None of the physical quantities discussed has any units. Figure 3.24 shows a redrawn version of this figure, with the caption explaining how to read the figure.

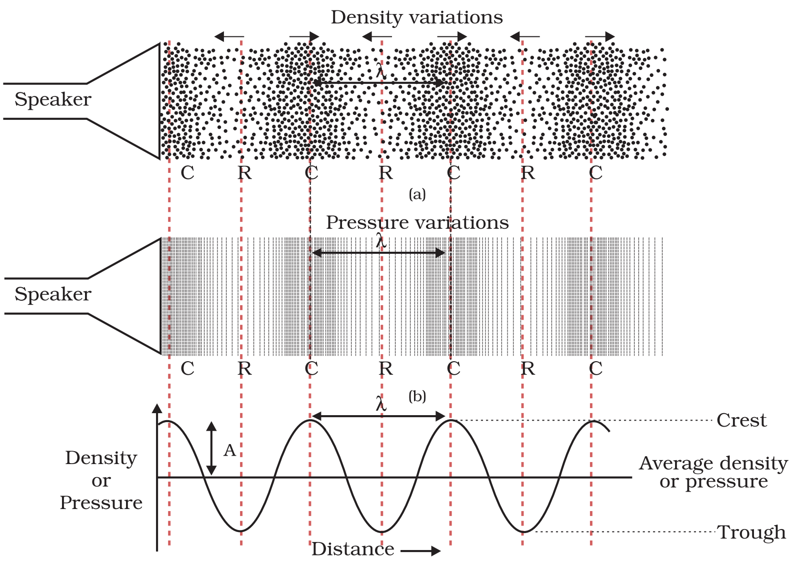


Fig. 12.8 *(redrawn)* Sound propagates as density or pressure variations as shown in (a) and (b), (c) represents graphically the density and pressure variations. The troughs and crests in (c) correspond to compressions (C) and rarefactions (R) in (a) and (b). The wavelength λ is the distance between two consecutive crests (or compressions) or troughs (or rarefactions). The amplitude A is the magnitude of maximum disturbance on either side of the mean value.

Figure 3.24: Figure redrawn from Class 9, Chapter 12, page number 70 in science textbook.

Example 9

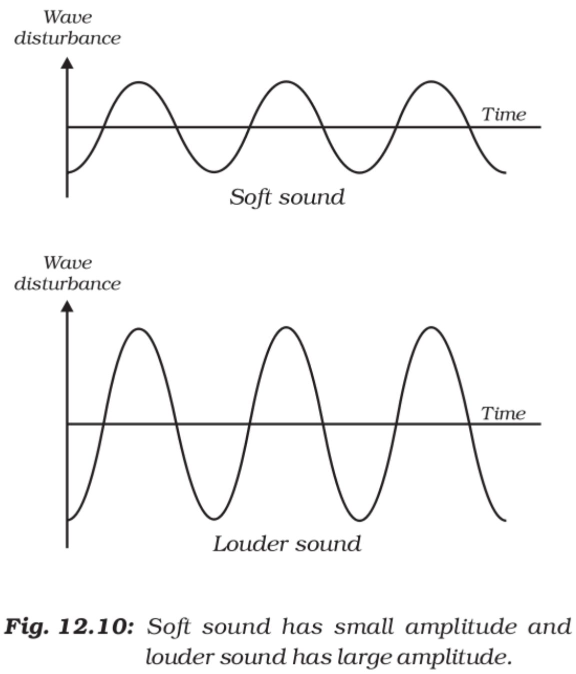


Figure 3.25: A graph showing the difference in amplitudes of soft and loud sounds. Figure from Class 9, Chapter 12, page number 165 in science textbook.

Function

Figure A.45 shows the third graph in Chapter 12 of class 9 science textbook. This graph is very similar to the graph in Figure A.43, but in this case, the emphasis is on the amplitude of the sound wave. The *X-axis* represents time in this case, while the *Y-axis* represents "Wave disturbance". Either axes do not have any units.

Reference

The figure is referred to in the main text as under:

> Fig. 12.10 shows the wave shapes of a loud and a soft sound of the same frequency. (*p. 165, Class 9, science*)

Data

The textbook does not give the data.

Legend, Axes

The axes have labels, but no units.

Close-to-life

We experience different types of sounds daily. This example is a close-to-life context.

Design Aspects

The graphs do not have chart-junk. However, a combination of the two graphs to be compared can be helpful in emphasising the points made in the text.

The meaning of amplitude *A* from part *(c)* of Figure 3.23 is explained in the textbook. However, interestingly, the figure itself has no label for amplitude *A*. Superimposing the two graphs using the same *X-axis*, can improve this graph vastly. Providing a grid will help in easy comparison of the amplitude and time values. The presence of a grid will make the comparison along the *Y-axis* much easier. In the current version of the figure, the reader needs to make the comparison of the graphical values situated at different positions on the page. The studies on the graphical perception imply that certain graphical designs are perceived more easily than others. Cleveland & McGill (1984) claim that position along a common scale is the most accurate perceptual feature while reading graphs. By putting the two waveforms together, we can easily see that sound waves with different amplitudes can have same frequencies. Figure 3.26 shows the redrawn version of the graph with the modifications discussed above.

The discussion around the graphs should have some real-life examples and activities related to wave motion and sound that bring out the points made in graphs The inclusion of a figure showing the different parameters with different waves will be beneficial. For example, a graph showing two waves with different frequency and amplitude. Currently, the graphs two graphs seem to be of the same kind: with equal frequency but different amplitude, and with equal amplitude but different frequency. Figure 3.27 shows one such graph.

Class 10

The final textbook analysed in this part is the Class 10 textbook. The textbook has just four graphs in total. The graphs appear in the context of Ohm's Law, pH Values, distribution of energy sources and pollution levels in the river Ganga.

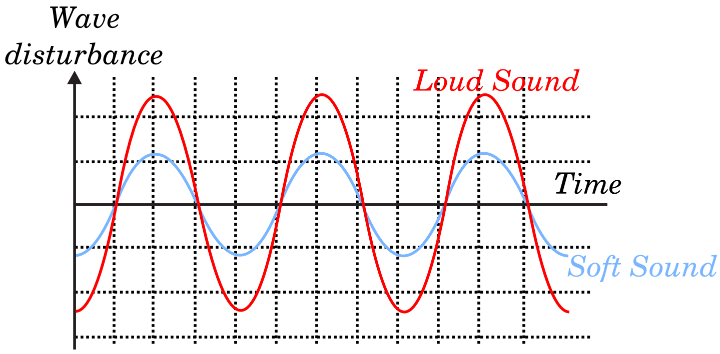


Fig. 12.10 *(redrawn)* Soft sound has small amplitude and louder sound has large amplitude. In this figure the sound wave shown in red has higher amplitude frequency, while the sound wave shown in blue has smaller amplitude. Note that both the waves have same amplitude so their troughs and crests coincide. For each trough and crest note that the maximum amplitude for red wave is larger than the blue wave. For example, during crests the blue wave is little over one unit on the grid, while the red wave is well over two units. We can also have sound waves which are different in frequncy and amplitude.

Figure 3.26: A graph showing the difference in amplitudes of soft and loud sounds. The figure is redrawn version of Figure 3.25.

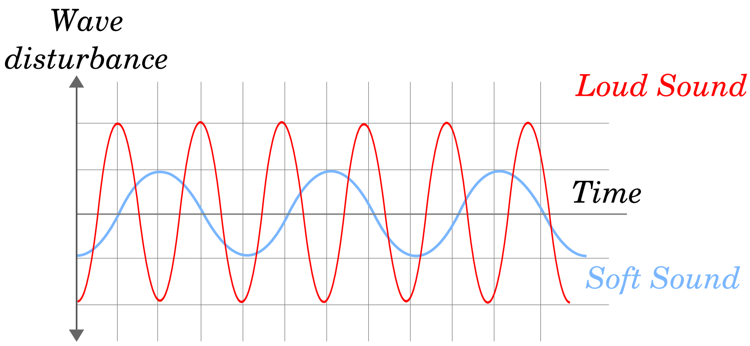


Figure 3.27: A graph showing the comparison of the amplitude and freuqency in two different waves.

Example 10

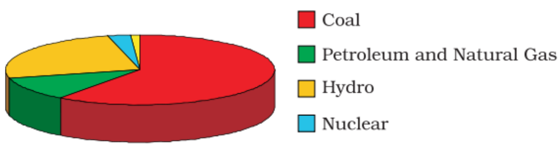


Figure 14.1
Pie-chart showing the major sources of energy for our requirements in India

Figure 3.28: A pie chart showing major sources of energy in India. Figure from science Class 10, pg. 244.

Function

The pie chart shown in Figure 3.28 appears in the section on conventional sources of energy.

Reference

The main text refers to the pie chart in passing:

> The fossil fuels are non-renewable sources of energy, so we need to conserve them. If we were to continue consuming these sources at such alarming rates, we would soon run out of energy! In order to avoid this, alternate sources of energy were explored. But we continue to be largely dependent on fossil fuels for most of our energy requirements (Fig. 14.1).

Data

The textbook does not give the data.

Legend, Axes

The data is colour coded in different sectors of the pie-chart.

Close-to-life

Energy usage is a close-to-life context.

Design Aspects

The 3-D representation of 2-D data presents its own problems. In this case, we detail out the false perceptions due to the design of this graph.

The pie chart is used to make a case for alternative fuels. The main text does not give the source of the data used in the pie chart. The text does not provide the year for the data. Providing the year for the data sets it in a historical perspective. A time series of resources showing the trends over the years would have been much better. Showing such trend over the years would also show how newer (non-conventional) sources of energy have made an impact in the recent years. Also, such a graph will show the trends of whether the annual production of the resources has increased or decreased. The pie chart here clearly shows that coal and petroleum and natural gas as the principal sources for the energy requirements. However, the exact numbers are absent. An interesting exercise would be to investigate actual fuel usage in their lives by the students. When done in the classroom collaboratively, such exercise can lead to interesting questions being asked and answered. Giving the students access to data for making conclusions based on the analysis can be the base for such an exercise.

The chart is in *3-D* which is not the needed for the essentially 2-D data. This is aptly described by Tufte in his works as *chartjunk* or *ducks* (Tufte, 2001). Robbins (2012) elaborates on the same theme very clearly:

> These examples demonstrate that the way to read three-dimensional bar charts depends on the software used to create them. But the reader rarely knows what software was used so has little hope of reading them correctly without the values printed. Even PowerPoint and Excel, two programs that come packaged together in the same suite, use different algorithms to plot their graphs. Therefore, you should never use a three-dimensional bar chart for two variables. A properly drawn two-dimensional chart shows the same information more effectively and avoids misinterpretation. (p. 27)

To find out the angular measure in this 3-D chart is difficult due to the perspective. To find the angles and hence the percentage of the sources, we need to make the chart 2-D. The reconstructed chart, along with the approximate values of angles and percentages are shown in Figure 3.29.

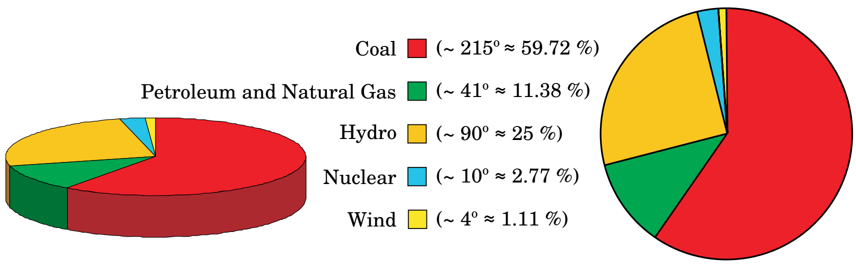


Figure 3.29: The 3-D pie chart for energy sources (Figure 3.28) converted to 2-D and the approximate values for percentages shown.

The reconstruction of 3-D to 2-D reveals interesting statistics. The percentage values of the sources approximately are:

Coal	59.72 %
Petroleum & Natural Gas	11.38 %
Hydro	25 %
Nuclear	2.77 %
Wind	1.11 %

Since the text does not give the source of this data, a direct verification is not possible. The second volume of *Twelfth Five Year Plan* (Planning Commission, Government of India, 2013) gives the official statistic for this information. Chapter 14 in the *Twelfth Five Year Plan* on the theme of energy has the relevant statistics. Particularly in Tables 14.4 and 14.5 give the data for the different sources of energy. In Table 14.4 the trends in the supply of primary commercial energy are given in the measure of *million tons of oil equivalent*. This quantity is the energy equivalent of burning million ton of oil. In Table 14.5 (Table 3.6 here) the percentage share (actual, provisional and projected) of different fuels in energy supply is given. Using this data, we can see the trends in the different sectors over last decade. The percentage of the fuels are shown in Figure 3.30. Though in recent years due to a major push towards solar and wind energy projects, the projected numbers have changed towards greater percentage of renewable energy.

Fuel Type	2000-01 Actual	2006-07 Actual	2011-12 Provisional	2016-17 Projected	2021-22 Projected
Coal and Lignite	50.36	53.22	53.45	55.41	56.90
Crude Oil	37.45	33.41	31.51	26.04	23.29
Natural Gas	8.49	6.99	10.32	13.46	13.17
Hydro Power	2.17	2.53	2.17	1.79	1.73
Nuclear Power	1.49	1.24	1.57	2.26	2.95
Renewable Energy	0.04	0.22	0.98	1.43	1.97

Table 3.6: The percentage share of each fuel in the total commercial energy supply, source p. 134, *Twelfth Five Year Plan Vol 2*.

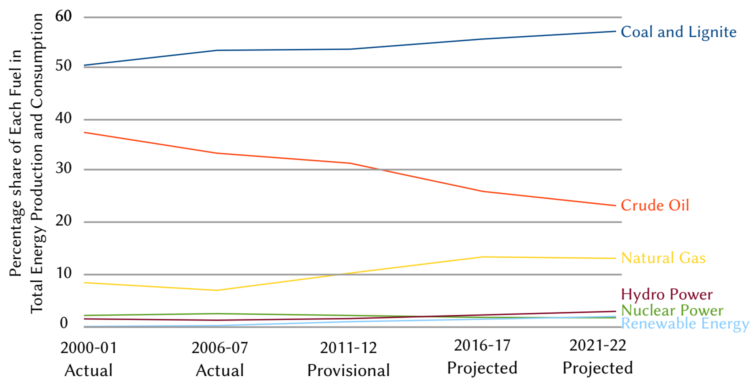


Figure 3.30: The share of each fuel in the total Energy Production and Consumption, Data from Table 14.5 of *Twelfth Five Year Plan Vol 2*.

If we try to match values calculated from the 3-D pie chart in the textbook to the values from the *Twelfth Five Year Plan*, they do not match up. Though the values that we have calculated from the 3-D pie chart are approximate, they represent what is shown in the diagram (Figure 3.28), and its major trends.

When we plot a graph of above data the discrepancies are much clearer. In Figure 3.31 the average value percentage of the three years shown in Table 3.7

Fuel Type	Textbook	12^{th} FYP (*Avg.*)	Ratio
Coal	59.72	52.34	1.14
Petroleum & Natural Gas	11.38	42.72	0.26
Hydro	25.00	2.29	10.91
Nuclear	2.77	1.43	1.93
Renewable	1.11	0.41	2.68

Table 3.7: Comparison of percentage values from the textbook (extracted) and the data from *Twelfth Five Year Plan* (FYP). The FYP has separate data on Crude Oil and Natural Gas. Here the two are added to be compatible with textbook data. Similarly, the textbook only mentions wind energy, whereas the FYP mentions a broader category of Renewable energy. Values for 2000-01, 2006-07 are actual values, while that for 2011-12 are the projected values.

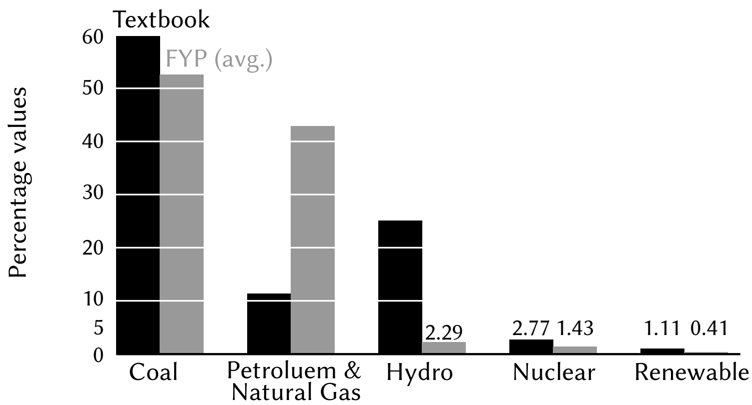


Figure 3.31: Share of Each Fuel in Total Energy Production and Consumption, Data from Table 14.5 of *Twelfth Five Year Plan Vol 2.*

are compared against the extracted values from the textbook. In case of coal, the textbook shows ∼ 60 % while the FYP has 52 %. For Crude Oil and Natural Gas the numbers are off by a multiple of 3.5 (∼ 12 % in textbook instead of ∼ 42 % in FYP). The case for hydroelectric is even worse. In the textbook, it appears as a major player with 25 %, whereas in actual production hydroelectric is just a tenth of that at ∼ 2.3 %. In the case of nuclear energy, it is 2.77 % in the textbook, while it is ∼ 1.4 % in FYP. In case of renewable energy for the year 2000-01 the numbers are off by order of 100 (1.11 % and 0.04 %)! Though they are closer to the average values, 1.11 % in the textbook and 0.98 % in FYP average. Looking at the table above we see that there is a gross misrepresentation of data in the pie chart. Only for coal and nuclear power the order of magnitude presentation is correct. Rest of them are off the mark.

In Figure 3.30 we can see the trends for the different sources of energy for the coming years. Though the dependence on crude oil is projected to decrease in the coming few years from about 37% to 23 % the dependence on coal is increasing. The total output of hydro, nuclear and renewable sources is small (all below 5%) and remains so for the projected years. With renewable energy sources being debated and discussed in a major way with implications for the environment, we feel an immersive, data-rich activity can take place in this regard. This example and its analysis makes strong cases for (a) making the primary sources of the data available to the students, to make sense of the debates and their implications, and, (b) avoiding the use of 3-D graphics when they are displaying 2-D information

The Ti*k*Z and PGF Manual (Tantau & Wibrow, 2007) has a chapter *Guidelines on Graphics.* Among several other things, the manual provides certain do's and don'ts regarding various aspects of graphics. For a 3D pie-chart, they give the following commandments in Section 4.6 *Plots and Charts*:

- Do not use 3D pie charts. They are *evil.*
- Consider using a table instead of a pie chart.
- Do not apply colours randomly; use them to direct the readers focus and to group things.
- Do not use background patterns, like a crosshatch or diagonal lines, instead of colours. They distract.
- Background patterns in information graphics are *evil.*

(emphasis in original, p. 51)

Example 11

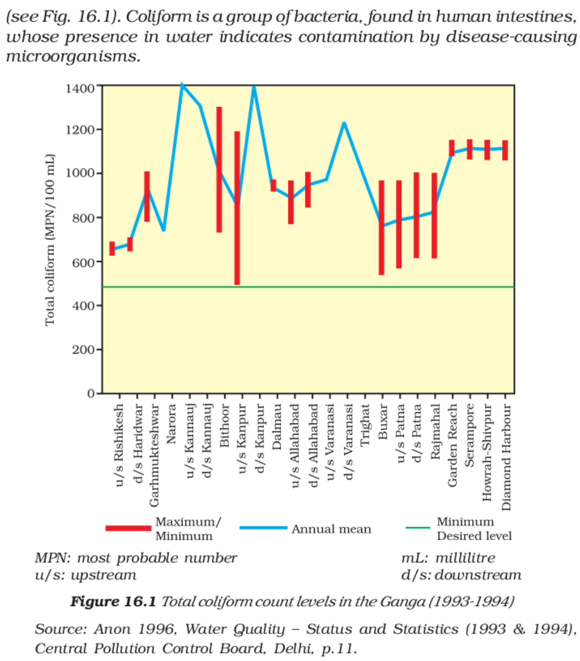


Figure 3.32: A graph showing total coliform levels in Ganga river. Figure from Class 10 science textbook, pg. 267.

Function

The graph displays the pollution levels in Ganga to make a rationale for the GAP.

Reference

The main text refers to the graph.

Data

The main text gives the data source, but not the data in a tabular form.

Legend, Axes

The graph has legend keys. The Y axis has the count of Total coliform in MPN/100 ml. The places for the data collection are on the X axis. The places are shown to be equidistant when they are not.

Close-to-life

With water purification being one of the pressing issues in everyday life this graph presents a close-to-life context.

Design Aspects

The graph could have been redrawn to fit in scales.

This graph is perhaps the most complex graph in the entire sample of textbooks. The textbook gives the source of the graph as *Water Quality - Status and Statistics (1993 & 1994), Central Pollution Control Board, Delhi.* The graph finds a mention in the main text with these lines:

> Awareness about the problems caused by unthinkingly exploiting our resources has been a fairly recent phenomenon in our society. And once this awareness rises, some action is usually taken. You must have heard about the Ganga Action Plan. This multi-crore project came about in 1985 because the quality of the water in the Ganga was very poor see Fig. 16.1). Coliform is a group of bacteria, found in human intestines, whose presence in water indicates contamination by disease-causing microorganisms.

However, unfortunately, there is almost no discussion *about* the graph itself in the textbook. The graph has Total coliform (MPN/100 ml) on the *Y-axis*. The textbook does not explain the process or method to arrive the numbers. There is no explanation of the unit of the of the count MPN. MPN is the *Most Probable Number Index* and relates to the number of bacteria in the water. A brief explanation of this would have been helpful to situate the graph in the context of the discussion in the textbook.

The *X-axis* of the graph presents towns and cities in the course of Ganga river. The graph in textbook gives both upstream and downstream data for some of the places. The upstream and downstream data itself could have been a good source of discussion in the main text. For example, determining whether by passing through a city or town how much does the river gets polluted? Is this related to any peculiar industries being present in those towns? Why there are peaks in certain towns (for example, Narora)and not in others (for example, Dalmau)?

Also, if there is any variation in the counts during the different seasons is not clear from the graph. The variation due to seasons can be an excellent question to be asked. Apart from this, there are several 'peaks' in the graph, which need an explanation. For example, why does the NPR suddenly peaks at Narora is not clear. Another thing that is not clear from the graph is the geographical distances involved. Currently, from the graph, it seems that the places on the *X-axis* are all equidistant from each other when they are not. The cities in the graph span almost the entire extent of Ganga in India from Rishikesh (~ 10 km) to Diamond Harbour (~ 2500 km). The graph can create a false impression that the data is from equal distances across the length of Ganga. A better way to create the graph would be to use the actual distances from the origin of the locations on the *X-axis*. The MPN on the *Y-axis* should have been related to other variables near the observation sites. Also, the textbook should have given the data in tabular form. Providing data over the years, since the start of the GAP, would have added another dimension to the discussions around the graph enriching it further.

Another point of note is the rationale for showing this graph. The main text says the *Ganga Action Plan* (GAP) was started in 1984, but the text reads as if the data in the graph was the rationale for this project. However, the data is from the years 1993-94. Perhaps, showing the data from years before 1984 would have provided a better justification for the formation of the Ganga Action Plan (GAP).

Many position papers are providing extensive data on the *Ganga Action Plan*, for example, (Alternate Hydro Energy Centre, 2009; Central Pollution Control Board, 2013). The data from these position papers can be the points of discussion. In the data provided in these reports give, along with the *Coliform*, two other significant indicators for analysing the water quality. First one is the *Biological Oxygen Demand* (BOD) and the second is the *Dissolved Oxygen* (DO). There two parameters along with the *Coliform* activity usually are indicators for the biological health of the river.

An entire narrative can be built using these data sets. (a) How bad was the quality of water before the *GAP* and what measures have been taken to improve it? and, (b) how successful it has been. The first graph of the narrative can be represented by some parameter which has seen change before and after the *GAP*.

Pre-GAP The data from the early 1980s show that the pollution in river Ganga was at alarming levels. Various indicators of pollution were beyond their prescribed limits. The Ganga Action Plan was formed with the objective of preventing the pollution of Ganga and to improve its water quality to acceptable standards.

For example, the designated best use standards according to Central Pollution Control Board *(CPCB)* are as under:

As an example, the data from pollution indicators from pre-GAP period (1982) is shown in Figure 3.33.The data is from *Status Paper On River Ganga* (Alternate Hydro Energy Centre, 2009).

The GAP over the years has shown improvement in the various parameters

Class	Designated Best Use DBU	pH	DO mg/l	Indicator BOD mg/l	Total Coliform MPN /100 ml
A	Drinking Water Source without conventional treatment but after disinfection	6.5-8.5	≥ 6	≤ 2	50
B	Outdoor bathing (Organised)	6.5-8.5	≥ 5	≤ 3	500
C	Drinking Water Source with Conventional treatment followed by disinfection	6.5-8.5	≥ 4	≤ 3 *Free Ammonia*	5000
D	Propagation of wild life and fisheries	6.5-8.5	≥ 4 *Electrical Conductivity*	1.2 mg/l *Sodium Absorption Ratio*	*Boron*
E	Irrigation, industrial cooling and controlled waste disposal	6.5-8.5	2250 ℧1/cm	26	2 mg/l

Table 3.8: The Designated Best Use Classification of Inland Surface Water.

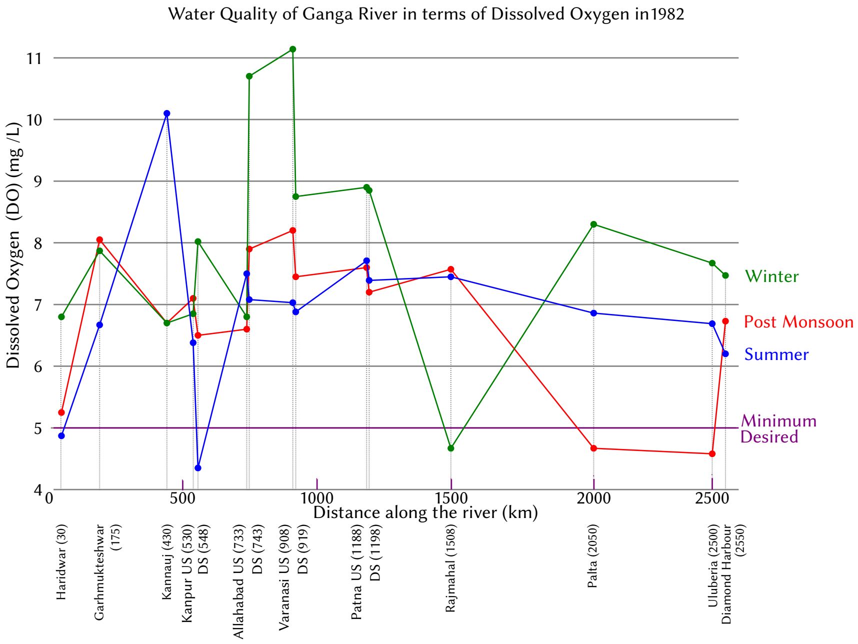


Figure 3.33: Variation of Dissolved Oxygen (*DO*) with seasons in Ganga, 1982. The numbers in bracket next to city names indicate distance in kilometres along the river.

that measure the pollution levels. Given the importance of Ganga Action Plan the graph should have revealed this aspect. The graphs and the data that they come from should challenge the students into asking questions from the data, making relevant conclusions about the effectiveness of the project, significant roadblocks etcetera. However, this is entirely missing in the glancing treatment that this single complex graph receives in the textbook.

3.6 Visualising the qualitative data

In this series of graphs that follow we have visualised the qualitative parameters of the graphs in Science textbooks. For each of the graph (indicated by the title of the graph), we are visualising how the parameters of that graph fare in our analysis. Table 3.9 gives the three scale rubric used to judge the parameters in the graphs in the Science textbooks. A blue coloured • in the comb plots indicate a positive trait for a given category in the graph. Similarly, a red coloured • dot indicates a negative trait for a given category in the graph. Categories which are not applicable or have neutral traits are shown on the X-axis in these graphs with a white circle ◦. For example, in the sample graph shown below (Figure 3.34)

1. the categories Function (F), Reference (R), Data Used (D), Legend/ Axes (L), have been rated with a negative trait,
2. the categories Integration (I), Design aspects (DA) are neutral, and,
3. the category Close-to-life (C) is rated with a positive trait.

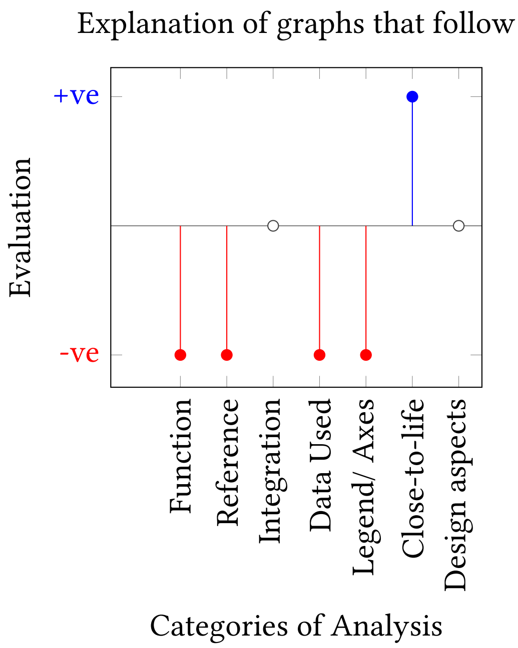


Figure 3.34: A sample graph explaining the summary graphs of qualitative analysis from the textbook analysis. In the graphs that follow we have used abbreviations of the parameters: Function (F), Reference (R), Integration (I), Data Used (D), Legend/ Axes (L), Close-to-life (C), Design aspects (DA).

The categories of analysis are indicated by their starting letters, indicated on the X-axis of these graphs as discussed in Table 3.4. According to this scheme, the graph is better contextualised and designed if it has more traits on the positive side.

Parameter / Trait	● Positive +1	○ Neutral 0	● Negative -1
Function (F)	if the reasons for introducing the graph are clear and well elaborated.	if the reason for introducing the graph is clear, but it is not well elaborated.	if the reasons for introducing the graph is not clear.
Reference (R)	if there are multiple references to the graph accompanying the text.	if the graph is referred only once in the accompanying text.	if the graph is not referred in the accompanying text.
Integration (I)	if there is an effort to include the graph and its features in the main textual narrative.	if the graph is mentioned only once in passing in the accompanying text.	if the graph is not integrated with the accompanying text.
Data Used (D)	if the data is provided in the form of a table with source and/or how it was obtained, bonus if the students collect/use data themselves.	if the data is provided in the form of a table, but the source or how it was obtained is not provided.	if the data is not provided.
Legend/ Axes (L)	if the graph is labelled and the units are given on the axes.	if either the labels or the units are not given.	if both labels and units are not given.
Close-to-life (C)	if the graphs are set in the context of real-world experience and are elaborated.	if the graph is set in the context of real-world experience, but is not elaborated.	if the graphs are not set in the context of real-world experience.
Design aspects (DA)	if there are no extraneous features on the graph.	if the graph has features which can be modified easily to make it better.	if the graph has to be redrawn completely.

Table 3.9: The rubric explaining how values of parameters for the qualitative analysis of graphs were graded.

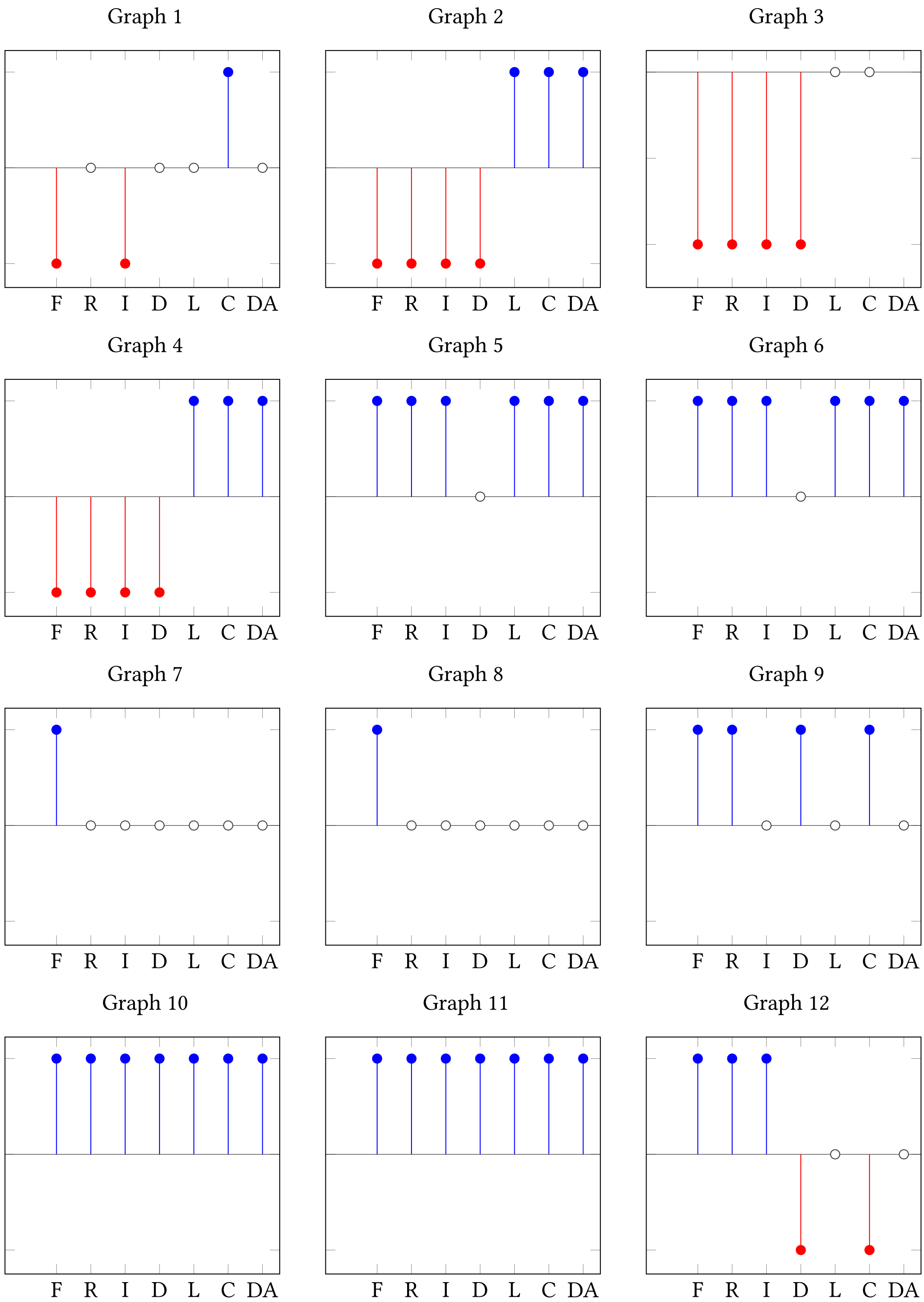


Figure 3.35: A visualisation of data from qualitative analysis of graphs in NCERT Science textbooks. Graphs 1 to 12. *(continued on the next page)*

This visualisation of the qualitative data in the form of the Figure 3.35 helps us in understanding the issues with the graphs at a glance. Though the richness of the

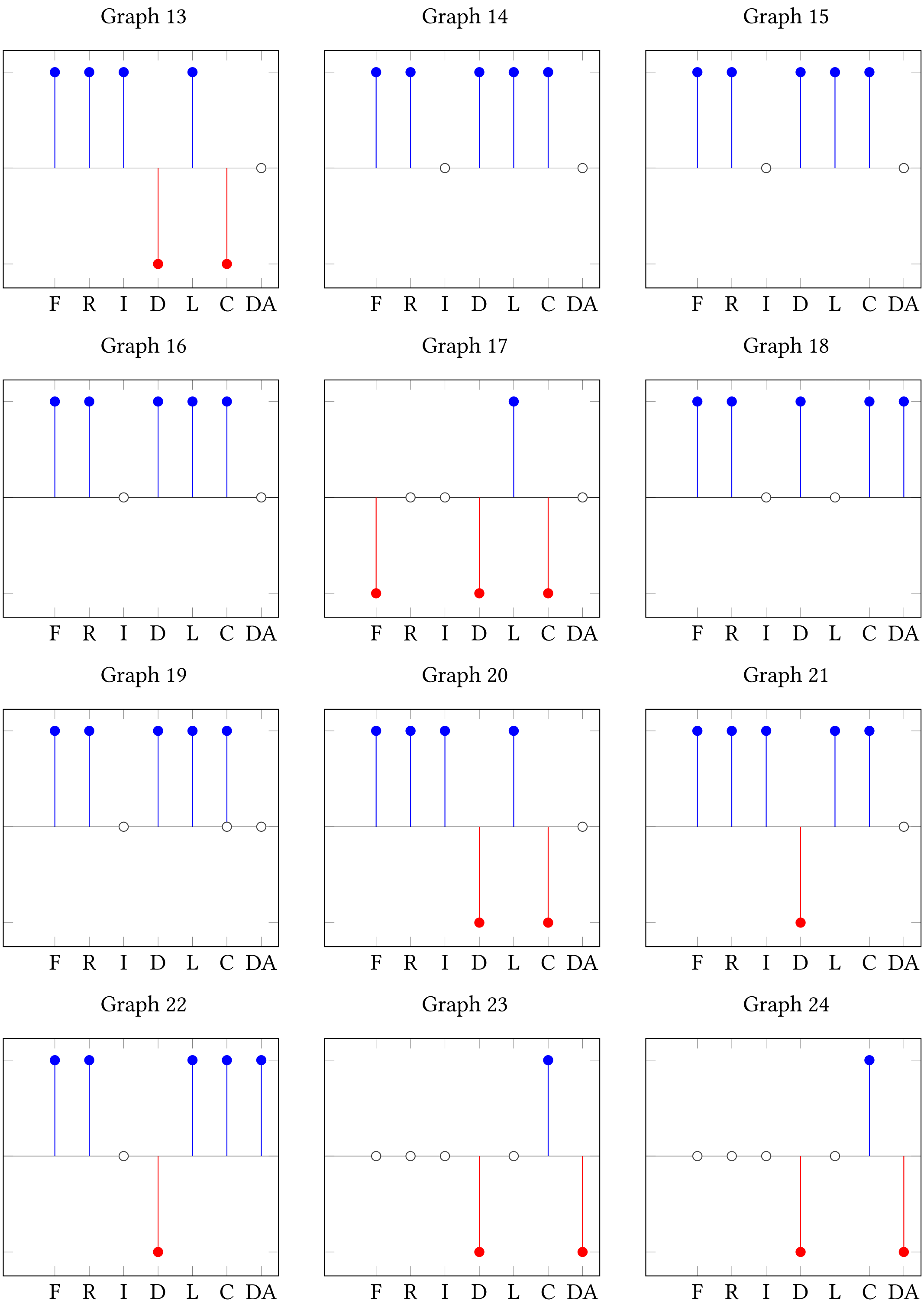


Figure 3.35: A visualisation of data from qualitative analysis of graphs in NCERT Science textbooks. Graphs 13 to 24. *(continued on the next page)*

descriptive analysis is missing, it allows the viewers to understand the fundamental issues we observed. The three-point scale used in the rubric described in Table 3.9

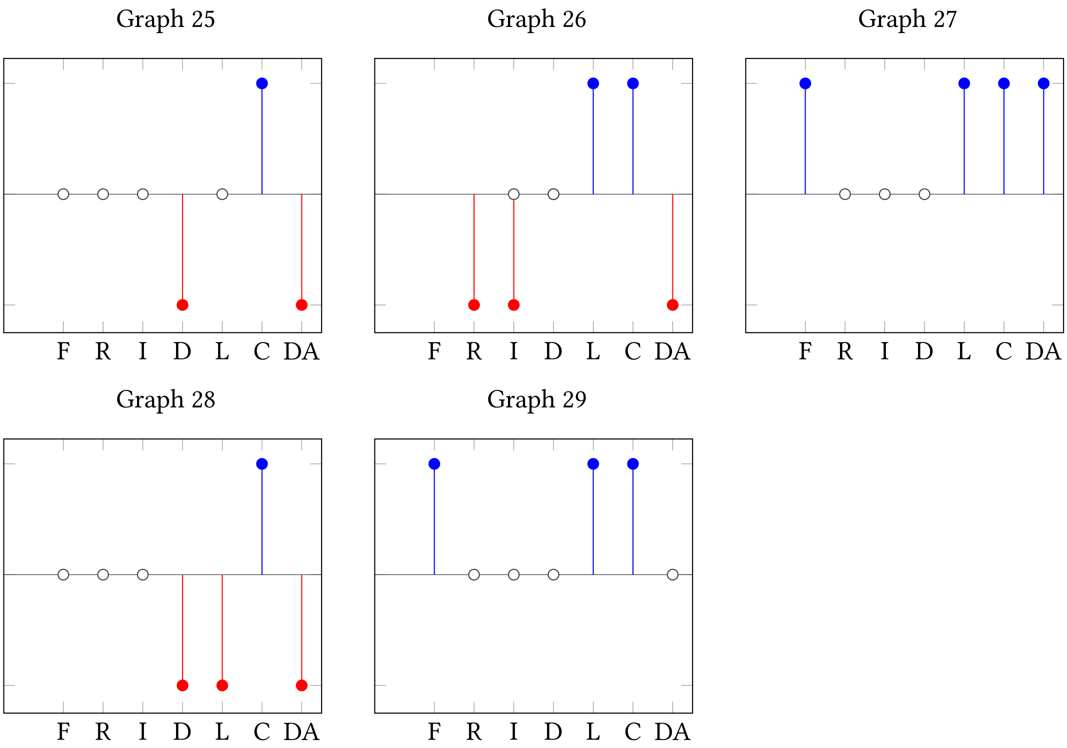


Figure 3.35: A visualisation of data from qualitative analysis of graphs in NCERT Science textbooks. Graphs 25 to 29. The categories are: Function (F), Reference (R), Integration (I), Data Used (D), Legend/ Axes (L), Close-to-life (C), Design aspects (DA).

provides us with a rough scale for rating the graphs according to the parameters used in the analysis. A finer analysis of this type would be possible if a five-point scale is used instead. In such a five-point scale we would be able to point to the issues in much more detail.

3.7 Reflections

Textbooks in the Indian context often serve as the only source of knowledge for a majority of the students. Our analysis of graphs contained in these textbooks demonstrates a lack of coherent design and intent both at the level of production and implementation. This part addresses research question (2). We summarise the findings as follows:

1. In general, the presence of graphs in the textbooks were peripheral. By this, we mean the integration of graphs with the central narrative of the subject matter was either loose or absent. Some exceptions include graphs on motion, where the students were told how the graph was constructed from a data table.

2. Only a few probing questions were asked about graphs which could lead the students to analyse the graph critically.

3. The graphs with real-world data which the students could collect were mostly absent, except in two cases. However, there is immense potential in some of the activities for bringing in real-world data and its interpretation.

4. For most of the graphs, neither the source nor the method of data acquisition was provided. This is in contrast to the scientific practice of doing so.

5. The link to topics in the Mathematics textbooks is entirely missing.

6. Axes in almost all graphs were labelled. In some graphs, the units on axes were not given. However, there is no discussion on how to choose scales and the effect on the graph due to changing of scales.

7. Redesigning of some graphs can improve their readability. We have shown how simple redesigning made the graphs more readable and integrated with the text.

8. Most of the graphs appear in the context of physics, graphs in the domains of biology and chemistry are highly under-represented.

9. Graphs 10 and 11 are good examples of how students can use real-world data to ask and answer investigative and exploratory questions from graphs.

10. We could find only *one* exemplary graph among the 29 graphs we examined from the science textbooks (Figure 3.32). This graph has a potential for critical analysis.

11. Regarding comprehension categorisation, most of the tasks fall in the elementary category as per Bertin's classification.

Problems about textbooks are known, for example, NCF 2005 (NCERT, 2005) notes, "Textbooks are written in an ad-hoc fashion, with no attempt to follow a coherent strategy of reading instruction." (p. 41). We see a similar pattern about the reading of graphs in the textbooks. The graphs are seen in isolation between chapters, between textbooks and between classes. The unifying thread in different classes and textbooks concerning the graphs is missing. The narrative around graphs, exemplifying the usage for analytical purpose, for drawing out inconsistencies in the data, is not developed. We not only found that the graphs are sparse, particularly in science, but the nature of most of the graphical activities present does not engage the students with the explicit aim of graphical literacy. It appears that the writers of the study material ignore the central relevance of graphs in teaching scientific topics and treat it as a mere visualisation of data at best or as a static image. This point is particularly concerning as Clement (1985) found that the students rarely question graphs in textbooks although they often misinterpret them.

There is no emphasis on the idea that the graphical practices are universal and not limited to the problem at hand. Instead, in the textbooks, the graphs are presented as separate units required only to deal with the problem at hand. This approach leaves much to be desired to show the students the use of the enormous power of graphical visualisation of data. The opportunities for the students to collect their *own data* in real-world contexts and analyse it are lacking. The norm seems to be to present the students with data, with very limited objectives, mostly for the display of data. Just the presentation of the data relates to the elementary level of Bertin's classification of graph comprehension. The other two higher levels, intermediate and comprehensive relate to the possible interpolatory, and extrapolatory questions that can be answered, possible inferences that can be drawn from the data, are mostly missing. Also, the textbook will become more graphicacy oriented if the design of the graphs is improved.

For graphs on weather, we found that many activities could be done in which the students can collect real-world data. This data can have many resources like newspapers, websites and finally, the students can collect their own data through simple devices. We feel this is not emphasised enough in the textbooks. Though the earlier mathematics textbook (particularly Class 5) has a lot of emphasis on collection and graphical analysis of data by the students, this trend is not continued.

For graphs on weather, the students can perform many activities in which they can collect real-world data, for example, see (Chia, 1998; Lee, 2014; Walker & Wood, 2010). This data can have many resources like newspapers, websites and finally, the students can collect their own data through simple devices. We feel that the science textbooks do not emphasise this enough. Though the earlier mathematics textbook (particularly Class 5) has much emphasis on collection and graphical analysis of data by the students, this trend does not continue.

The most of the graphs in the science textbooks are the graphs related to motion. A close-to-life context was found lacking in the graphs related to motion. The presentation of the entire topic of motion is done in a very abstract way. Close-to-life experiences from the students can be used to enrich this topic. The studies on graphs in the context of motion point to providing concrete learning experiences for the students which enable the movement from concrete physical situations to abstract graphs and back (for example, (Mokros & Tinker, 1987)). Such approaches allow the students to make sense of features on the graphs concerning the physical situations they represent. The graphs should provide for more activities in which the students can collect the data themselves and correlate it to their physical experiences of motion. Also worth mentioning are a few books which deal with the topic of motion experientially in this regard. The GPS (Global Positioning System) devices provide an ideal platform for activities related to motion. We explored this idea of using GPS devices as a tool for learning motion as a position paper, but not tested in the field (Dhakulkar & Nagarjuna, 2011b). This activity, hence, is not reported in the thesis. With regards to the topic of sound, the treatment in the textbook is not set in a close-to-life context. The graphs which are present are missing crucial points they are supposed to highlight for the students. The near absence of any activities regarding the sound and wave motion is striking. Numerous activities could be planned which can provide real-world data to the students for graphical interpretation of the activities.

A summary of our findings addressing the research question (2) *What kind of opportunities do Indian school science textbooks offer to the learners to engage with* *graphs meaningfully?* are as under:

We not only found that the graphs are sparse, particularly in science, but the nature of most graphical activities do not engage or serve the purpose of achieving the aims of graphicacy. There is no effort to build on graphicacy across grades in science. For example, there is no reference to graphs appearing in earlier textbooks. Graphs are presented as isolated entities with elementary objectives, for example, only to display the data. The graphical activities sparingly ask the intermediate and advanced level questions. For example, questions about extrapolating and interpolating data, predicting, inferring are rare. An exemplary usage of graphs is rare. Such an exemplary usage would include forming hypotheses, designing and constructing experiments, collecting data to solve problems and generate answers. For the construction of graphs also, the opportunities are limited.

Our approach to tackling this issue was to develop activities which would be interspersed with the different topics that the students encounter in different textbooks. To address some of these issues we developed a framework to develop activities which presented in the next part of the work.

For the reading of graphs critically one can ask questions which pertain to the production of the graphs (how the graph was produced?), the reasons for creating a graph (why the graph was produced?), the information in the graph (what questions can and cannot be answered on the basis of a graph?). Questions of these kinds are succinctly put forth by in work by Pechenik & Tashiro (1992). In this work, the authors provide a step-by-step guide to using graph interpretation as a rhetorical device to explore and understand the experimental parameters and conclusions in the context of biology. They frame questions for which the students need to explore the graph critically. The questions include: " 1. Looking only at the axes and data, what do you know about how the study was done? 2. Looking now at the entire graph, including the figure caption, what else do you know about how the study was done? 3. What *can't* you tell about how this study was done? 4. What specific question is being addressed in the portion of the study represented by this graph? 5. Why would anyone want to ask the question referred to in Step 4? 6. What is the most important result shown by the graph?" (p. 433) Answering or attempting to answer these set of questions, allows the learners to engage with graphs at a deeper level. Such questions, which promote a deeper understanding of the context of study as well as that of the graphs, should accompany the presentation of graphs in the school textbooks.

We see that there exists a tremendous opportunity to explore and utilise graphs as means of teaching concepts in the sample of textbooks that we have studied. The mere presence of graphs in the textbook is not justified unless it is appropriately related to the subject matter. Graphs are best understood in a context when the learners collect "real-world" data and use graphs to analyse this data. In this way, one can introduce some aspects of critical graphicacy in the classroom. The apparent lack of opportunities for the learners to become *graphicate* in textbooks and curricula, and ways to address them is the central theme of our work.

As for the other graphs which are present in isolation on other topics we feel that the data related to water purity and pollution should be made available to the students. These are close-to-life contexts, which are crucial in many aspects of human welfare.

Another topic that is missing in the graphs is the idea of *indirect* measurement using the graphs. The indirect measurement of quantities can be done by using graphs. For this, we designed activities which involve the same mathematical principle of finding the required quantities using indirect measurement but on two entirely different scales. The first activity was measuring the average diameter of the mustard seeds. This activity can be the first exercise in mathematical modelling of real-world data. Chapter 5 described the field study for this activity. The other scale which we dealt with in another activity was a mega scale, which was measuring the *distance-to-diameter* ratio of the Sun. The Sun measurement activity together with the mustard seed activity provides two different scales, which use similar principles of finding required values graphically. Chapter 6 describes the field study of this activity. The above two activities provide links between the subjects of science and mathematics and are interdisciplinary in nature.

One area where we felt using graphs could open up new avenues for learning the concept was that of electromagnetic induction. The textbook does not give any graphical treatment for this topic. We found that introduction of data collection and the graphs in this topic provides an entirely new aspect of learning. Chapter 7 presents the field study done in this area.

As a proposed remedy to the perceived lack of relevant graphical experiences in the textbook, we address the research questions ③ and ④. These issues form the theme of our next chapter in which we propose a design framework for developing activities which will make connections with various concepts through graphs.

Part II

Learning Contexts

"Constructionism - the N word as opposed to the V word - shares constructivism's connotation of learning as "building knowledge structures" irrespective of the circumstances of the learning. It then adds the idea that this happens especially felicitously in a context where the learner is consciously engaged in constructing a public entity, whether it's a sand castle on the beach or a theory of the universe."

Seymour Papert, *Situating Constructionism*

4

Design and development of the learning contexts

There is a considerable amount of literature on what conditions are best suited for learning to be effective. Some of these ideas have also been crystallised and given names such as project-based learning, immersive learning, activity-based learning. Most of these approaches attempt integrating content and context to create a unified whole that would enable the student to participate in an engaged fashion versus being in a passive reception mode. Within such innovative approaches, content design and contextual placement of the study materials (both conceptual and physical) is given great importance. The idea and demands of graphicacy are perfectly poised at the intersection of design and context and provide us with a fertile environment to explore and pursue these ideas further. In this chapter, we formulate a comprehensive framework within which activities were developed towards the goal of bringing science and mathematics together. We believe that graphs are an essential link in mediating the interaction between the two and we attempt at making this essential role graphs play as explicit and impactful as possible in these activities.

4.1 Introduction

In Chapter 3, we saw that textbooks do not provide the students with enough opportunities to engage with data. The NCF 2005 (NCERT, 2005) looks at the idea of developing inventiveness and creativity via co-curricular activities. The document particularly notes the lack of opportunities to promote these ideas:

> Science education in India, even at its best, develops competence but does not encourage inventiveness and creativity . . . inquiry skills should be supported by language, design and qualitative skills. Schools should place much greater emphasis on co-curricular activities aimed at stimulating investigative ability, inventiveness and creativity, even if these are not part of the graduating exam. (p. 49)

The above quote summarises the need for activities that have elements of creativity, design and investigative ability along with qualitative skills. We feel that the design of activity should inherently use these elements set in a context. Chapters 2 and 3 brought to focus many issues pertaining to comprehending and constructing graphs.

The *NCF 2005 Position Paper on Mathematics* further points:

> This also brings us to the need for **making connections**, within mathematics, and between mathematics and other subjects of study. Children learn to draw graphs of functional relationships between data, but fail to think of such a graph when encountering equations in physics or chemistry. (emphasis in original) (2006a, p. 10)

The above quote can be seen in the context of making meaning from graphs representing physical situations. This quote also highlights some of the problems that we discussed in Chapter 2. Literature from the field of graphicacy studies conducted in the West, strongly support the quintessential role data collection, handling, and analysis provide an additional advantage to the students in learning. NCF 2005 recognises this importance in these terms:

> Data handling, representation and visualization are important mathematical skills which can be taught at this stage. They can be of immense use as ``life skills''. Students can learn to appreciate how railway time tables, directories and calendars organize information compactly. Data handling should be suitably introduced as tools to understand process, represent and interpret day-to-day data. Use of graphical

> representations of data can be encouraged. Formal techniques for drawing linear graphs can be taught. (2006a, p. 17)

Similarly, the *Syllabus* based on NCF 2005 has the following recommendation regarding activities related to processes of science:

> This [elementary classes till 8th] is the stage where children can and should be provided plentiful opportunities to engage with the processes of science: observing things closely, recording observations, tabulation, drawing, plotting graphs - and, of course, drawing inferences from what they observe. Sufficient time and opportunities have to be provided for this. (2006c, p. 137)

Taken together, the NCF 2005 is underlining the need for the student's direct involvement in observation, data generation and analysis. Furthermore, they are arguing for the need to situate these activities within contexts that are relatable and familiar to the lived experiences of a student. Implicit in the NCF 2005 arguments is also it is recognition of the lack of material culture in the teaching-learning practices in India. When the document recommends that observation and recording of data be done in close contact with real-life materials, they are asking that content design must enable the creative movement between the conceptual and the material world of objects that both science and mathematics deal with. We fully agree with their recommendations and attempt here to build activities that would do justice to the requirements placed by the committee on the conceptual and the contextual realms.

Though the proposed activities appear in varied contexts, we have applied a common framework for designing and developing them. The sequence of activities goes from very concrete as in the case of the mustard seed measurement to the highly abstract representation of voltages across coils. Each of the activity presents a different challenge regarding the concepts involved and the abstraction. They increase in complexity, abstract nature, and reasoning skills using graphs. One of the aims is to provide the learners with familiar contexts and purposeful tasks to inculcate skills useful in doing both science and mathematics, for example, see Boohan (2016), Gallagher (1979). In the next section, we look at the pedagogical considerations for the design framework of these activities. These include various factors which influence the comprehension and constructions of graphs as seen in Chapter 2.

4.2 Pedagogical design considerations for the activities

In this section, we present the pedagogic considerations for the proposed framework for the design and the development of the activities. This part addresses Development Objective 1:

Part 2: Development Objective 1

(1) What design principles of a learning activity could comprehensively address the issues of comprehending and constructing graphs informed by the literature situated within a constructionist framework?

Graphs, as seen from this framework, are not seen in isolation. They are organically related to the various concepts and skills that are part of this framework. The meaning-making process from graphs inherently depends on the *context* in which the graphs appear. Moreover, the learning with graphs is enhanced when the context is *familiar* to the learners and is the setting is that of *problem-solving*. So, the situation and the contextual setting of the graphs is an important consideration. As we have seen in both the cognitive and sociological frameworks (Section 2.5), the *prior knowledge* of the learners in the form of both content and practice plays a significant role in learning with graphs. Hence, the framework should make use of the prior knowledge of the learners. The problem-solving context leads to thinking about ways to solve the problems concretely. In this process, the learners can generate *hypothesis*, *design* and *construct* experimental setup. The construction seen in our framework is a crucial aspect and brings in the element of concrete. The *measurements* made with the experimental setup leads to the generation of *data*. However, it is essential that the students themselves collect, handle and process the data. The *multiple representations* (tabular, algebraic, graphical and verbal) of this data allows the learners to engage with the data in different ways. The *analysis* of this data can lead to forming of *mathematical models*, *testing* of the proposed *hypothesis*, *inferring* patterns in the data and *solving* of the stated problem. The *public display* of this entire activity in the form of *classroom discussions* with the datasets and *experiences* of the learners, *presentations* before peers, *written reports* can add multiple dimensions to the learning.

Next, we discuss the various concepts noted above and their significance in learning with graphs. Then, in the subsequent sections, we discuss the setting of the field work and introduce the three representative activities based on this framework and its implementation.

Context: We use the notion of the context as explicated by Janvier & Bednarz (1989). A context here is seen as equivalent to a *situation*. It is a concrete basis from which many abstract mathematical ideas are derived from the real world. Contextualisation lies in the intersection between the phenomena (real-world, concrete situations) and equations (abstracted mathematical formalism), as shown in Figure 4.1. The idea of contextualisation cannot be separated from problem-solving and is distinct from the application of mathematics. Contextualisation in this sense provides an operational space. This operational space allows learners to make the connections between the abstract and concrete, embedded in a problem-solving situation.

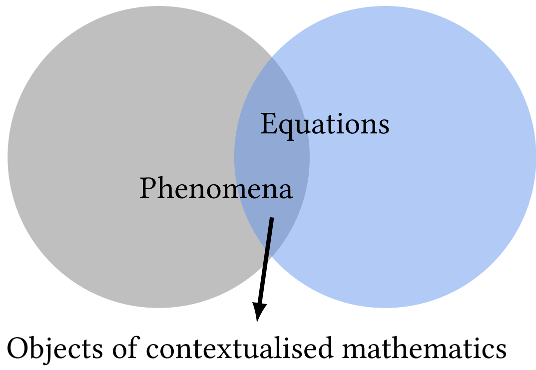


Figure 4.1: A schematic diagram showing the objects of contextualised mathematics. Contextualisation is the space positioned between the real-world. phenomena and abstracted mathematical ideas, from Janvier & Bednarz (1989).

Prior-knowledge: The problems that we give to the learners will be successful in achieving their potential if they are in a familiar context, and have concrete goals which build on their prior knowledge (both *practice* and *content*). For example, the learners might know how to do things, like measuring with a scale or a measuring tape, or they might know about particular facts or concepts. Making use of the prior knowledge during teaching could be very useful. The use of prior knowledge of the learners makes the learners contribute to solving a problem, thus making it a personal endeavour for them. The mathematical transfer of learning in case of a real-world problem is different than in a contextual word problem depicting a realistic situation (Roth, 1996). In case of the Construction-Integration model of understanding graphs by Shah & Hoeffner (2002), out of the three components, two components focus on the background knowledge of the learners about graphs and domain.

Thus we need to design learning situations whose setting is *familiar*, that is *close-to-life contexts* of the learners, and tasks which have *understandable problems*, that is, it should be clear as to *what is to be done.* Once the problem is recognised, the discussions on how to solve the problem, that is *brainstorming* gives the learners a chance to design and construct hypotheses and experiments. Janvier (1981) notes that:

> If situations are meant to help pupils of all academic abilities, their role should be mainly *the development of their capacity to abstract*. Such development *does not depend on the use of an abstract vocabulary* but on a slow and gradual 'drawing-out' (*abs-trahere*) of the main features guided by *some precise intentions*. (emphasis in original, p. 120)

Instead of just telling the students about graphs, Bowen & Roth (1998) claim that, they should be engaged in "investigations in which they use graphical representations to construct relationships based on collected data and which they then support and construct claims from by reading and relating a variety of related written resources (which includes graphs)." (p. 87)

The design of the activities includes this gradual development towards abstract, which should always be grounded in concrete observations and situations. During the discussion about the problem, inputs from the learners are a crucial component of the discourse. Hypotheses about solving the problem should be made, refuted, modified and accepted. This process includes setting a dialogue with the learners, taking inputs from prior knowledge and reasoning, and finally using these to solve the problem.

Multiple Representations: The experimental construction leads to numeric *data generation* which can be represented in multiple ways (*graphical, algebraic, tabular, verbal*). Another crucial point here is the fact that a graphical construction follows from a set of data - often presented in a tabular form. The data itself results from measurements of observations. Given the chain of events presented above and keeping with the approach towards design, we have applied a systematic move from data generation to graph plotting. The organisation of data into tabular representation is an intermediate step in creating graphs. To be able to move between different representations has advantages (Even, 1998; Friel et al., 2001; Mosenthal

& Kirsch, 1990a; 1990b). Even (1998) notes:

> The ability to identify and represent the same thing in different representations, and flexibility in moving from one representation to another, allows one to see rich relationships, develop a better conceptual understanding, broaden and deepen one's understanding, and strengthen one's ability to solve problems. (p. 105)

In the design of the activities, we have emphasised multiple representations of data to enable the students to develop the ability to move between them. All the representations should be grounded in the concrete observations done by the students.

The graphs allow making inferences and predictions, thus testing hypotheses in the process. The activities should be designed so that each experiment or observation is grounded in something concrete and tangible (for example, the number of turns in a coil) and thus the resulting graph could be linked to the phenomenon. The algebraic manipulations can be seen as developing *symbol sense* in the context of using algebraic variables for a given situation. By symbols here we mean algebraic variables. For teaching symbol sense, symbolic manipulations should be taught in rich contexts which provide opportunities to learn when and how to use these manipulations (Arcavi, 1994).

The concrete grounding of observations makes the connection between the abstract features on the graph and the concrete things they represent strong. In many cases, the flow of the process can be both ways: to and fro between the concepts and skills. The double-sided arrows indicate this in Figure 4.2. The graphical analysis based on data in a given situation can lead to inferences, conclusion and solutions to the hypotheses and problems. A *public display* in the form of classroom presentations and discussions of these results can lead to further discussion about the data, models, hypotheses and the problem itself. In some cases, the public display leads to a revision of the earlier ideas and hypotheses and the process can become iterative. The core ideas of the designed activities and their work-flow are shown in Figure 4.2. The flow of the activity can be uni-directional (as noted by ⟶) or bi-directional (as noted by ⟷). These concepts (including skills), when they are relevant in the activities are shown in different typeface as: (`concept`).

Construction and public display: Within this framework, collaboration and public (peer group) display and discussion of the artefacts and inscriptions provided the social space for the unfolding of the activities. The tangible form of the products (like constructing objects, reports, graphs) made public display and discussion pos-

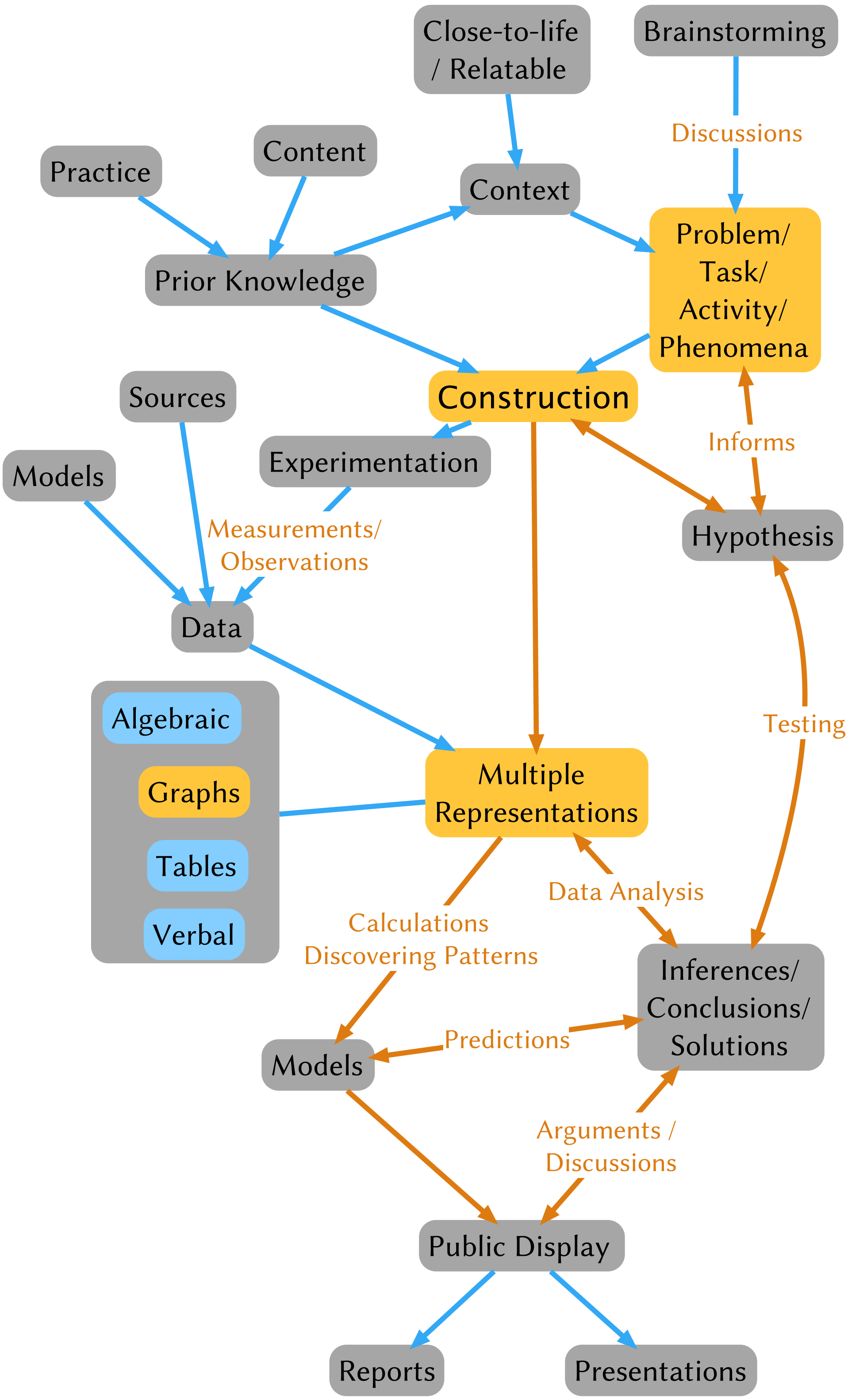


Figure 4.2: A schematic diagram showing core ideas and the work-flow of the activities. We exemplify the work-flow with specific activities when describing them.

sible. This idea is derived from *constructionism* as set forth by Seymour Papert (Papert, 1980; Papert & Harel, 1991) and has parallels in the *studio based* approach to education (Hetland, Winner, Veenema, Sheridan & Perkins, 2007). Because of its greater focus on learning through making, Papert's approach helps us understand how ideas get formed and transformed when expressed through different media, when actualised in a particular context, and when worked out by an individual mind (Ackermann, 2001). As Papert himself notes in *The Children's Machine*:

> *Constructionism* also has the connotation of ``construction set,'' starting with sets in the literal sense, such as Lego, and extending to include programming languages considered as ``sets'' from which programs can be made, and kitchens as ``sets'' with which not only cakes but recipes and forms of mathematics-in-use are constructed. One of my central mathetic tenets is that the construction that takes place ``in the head'' often happens especially felicitously when it is supported by construction of a more public ``sort in the world' ' -- a sand castle or a cake, a Lego house or a corporation, a computer program, a poem, or a theory of the universe. Part of what I mean by ``in the world'' is that the product can be shown, discussed, examined, probed, and admired. It is out there. (emphasis in original) (Papert & Harel, 1991, p. 142)

The construction context is one of the essential aspects of the activities. It allows the learners to navigate between the abstract and concrete levels of the situation in focus. It provides the learners with "object-to-think-with" as Papert puts it. In the case of scientists interpreting graphs, the movement between abstract and concrete is not just one after the other but appears to be simultaneously from concrete to abstract and from abstract to concrete (Roth & Hwang, 2006). Also, the activities are designed in such a way that they can be done by the learners of different grades with different goals and learning outcomes. The same activity can be repeated with a new set of skills, knowledge and a new learning objective.

Here we are following the suggestion of Monk (2003), that the students must repeatedly encounter graphing as a means of communication and of generating understanding, as the students move across the grades. It is not a skill which is to be imparted once and for all, but it should be gradually developed across the grades. Just as literacy, which needs regular practice, graphicacy too needs a similar practice.

Measurement, Data Collection and Handling: Measurement is seen as one of the six fundamental activities for all mathematical cultures (Bishop, 1988). Without measurement, the connections between the abstract mathematical ideas and the real-world cannot be made. The topic of measurement provides a rich opportunity for making the connections between mathematics and science in the classroom. In the case of mathematics, measurement is seen as a part of geometry, thus making no emphasis on data collection and handling. On the other hand, in the case of science, the students are expected to make measurements emphasising concepts like precision, accuracy, relative error, significant numbers (Thompson & Preston, 2004). The Position Paper on Mathematics Education in NCF 2005 (NCERT, 2006a) notes: "Data handling should be suitably introduced as tools to understand process, represent and interpret day-to-day data." (p.17).

Real-world data collection and handling even for simple tasks can be a rich experience for the learners (Curcio, 1987; Wavering, 1989). Classifying data and representing it in various forms can lead to a deeper meaning of data (Hutchison, Ellsworth & Yovich, 2000; Pereira-Mendoza, 1995). In the sociological framework, the background for an interpretative task involving understanding and meaning associated with graphs is derived from the experience with the phenomenon to be modelled mathematically and also from the way in which data is collected (Roth, 2004).

The ability to move between different representations of the same data is not easy (Moschkovich et al., 1993). When the same data is used for further analysis, to find mathematical patterns, to build a model, to predict, to answer questions, to verbalise, the graphs could be used for display and as rhetorical devices. Also, the students experience learning difficulties in variously linked function environments. This indicates the deficiency of linked representations, and it is suggested strongly to provide experiential anchors for function representations (Kaput, 1998, 2). The concrete basis for the activities provide the experiential anchors for multiple representations.

During such an activity many associated concepts from mathematics and science like probability, the statistical measure, experimental error, could also be introduced in a contextualised manner (Lehrer & Romberg, 1996). Also, the familiarity with the data which is collected by the students could lead to successful learning of many fundamental features of graphing (Åberg-Bengtsson, 2006). Such contextualised experiences could help the students to build and expand their repertoiré of skills to understand the mathematical relations expressed in graphs. Hence in the proposed activities, we have the context for calculations and measurements with the collected data.

Graphs: Graphs are central to the activities by design. They form the link between various concepts. The centrality of graphs and its relation to other concepts in the framework is shown in Figure 4.3. We analyse the graphs in the activities with the categories listed. The graphs played a crucial role in the completion of the activities as they do in science.

In his work on graphs, Monk (2003) considers the ways in which graphs can help in meaning-making process. He summarises his points as under:

> 1. By using graphs, the students can explore aspects of a context that are not otherwise apparent.
> 2. The process of representing a context can lead to questions about the context itself.
> 3. Using a graph to analyze a well-understood context can deepen a student's understanding of graph and graphing.
> 4. Students can construct new entities and concepts in a context by beginning with important features of a graph.
> 5. Students can elaborate their understanding of both a graph and its context through an iterative and interactive process of exploring both.
> 6. A group can build shared understanding through joint reference to the graph of the phenomena in a context. (p. 256)

These points are reflected in our own design and observations of the activities. For example, for points 1, 2 and 5 in this list, the exploration of electromagnetic-induction by the students described later (in Chapter 7), provides several such examples. For points 3, 4 and 6, both the mustard seed measurement and the Sun measurement activities have several instances for these.

Use of Computers: We had both hand-drawn and computer-drawn graphs as part of our activities. In case of hand-drawn graphs, the data sets were typically small. The hand-drawn graphs were part of the reports that the learners submitted on the activity performed. This allowed the learner to understand these graphs in relation to the other forms of representations in the report. Adams & Shrum (1990) discusses the effects of the computer-based intervention on construction and interpretation of line graphs of grade 10 students in the context of biology. They found plotting line graphs by hand had a better impact on graph construction tasks than plotting graphs by using a computer. In the case of graph interpretation tasks, they found that "microcomputer-based laboratory exercises that collect and present experimental data to students as "real-time" graphs result in educationally significant achievement on graph-interpretation tasks." (p. 785) Though the advantages of computer drawn-graphs are many, their true potential comes when the data sets are large. Using computers to draw graphs gives many advantages to the young learners (Barton, 1998; Mokros & Tinker, 1987; Thornton & Sokoloff, 1990). Thornton & Sokoloff (1990) lists five characteristics of the computer-based laboratory (for collection and display of data) (a) Students focus on the physical world. (b) Immediate feedback is available. (c) Collaboration is encouraged. (d) Powerful tools reduce unnecessary drudgery. (e) Students understand the specific and familiar before moving to the more general and abstract.

Advantage of Computers: The main advantage of the use of computers is freeing up time for tasks on *thinking* about graphs. The graphing on computers allows for multiple modalities of learning, providing a *real-time* and the mathematical link between a concrete experience and its symbolic representation (Pratt, 1995, 3). Particularly the reduction in cognitive load and time saved from "drudgery" of graph drawing can be used for "What if?" type of discussion questions. The immediate feedback that the learners get while changing the parameters of the graph such as scale or order helps immensely in developing the skill of reading a graph and its meaning (Wavering, 1989). This also indicates that traditional approaches to learning need to be reconsidered in a computer-rich learning environment (Arcavi & Hadas, 2000). Activities with *real-time* data collection and display on the computers have resulted in significant improvement in learning (Adams & Shrum, 1990; Thornton & Sokoloff, 1990). This approach allows the learners to participate in the process of learning similar to that of a scientist working in a laboratory, trying to understand complex factors influencing observations (Nachmias & Linn, 1987).

4.3 The tools

We have used following tools in the designing of the activities, namely expEYES for data logging and GeoGebra for dynamic mathematics. In this section, we provide the reasons for using these tools, sometimes as an aid to the primary activity and sometimes as being central to the activity. One thing that is common to both of these tools, and was also a selection criterion for us, the tools and the software need

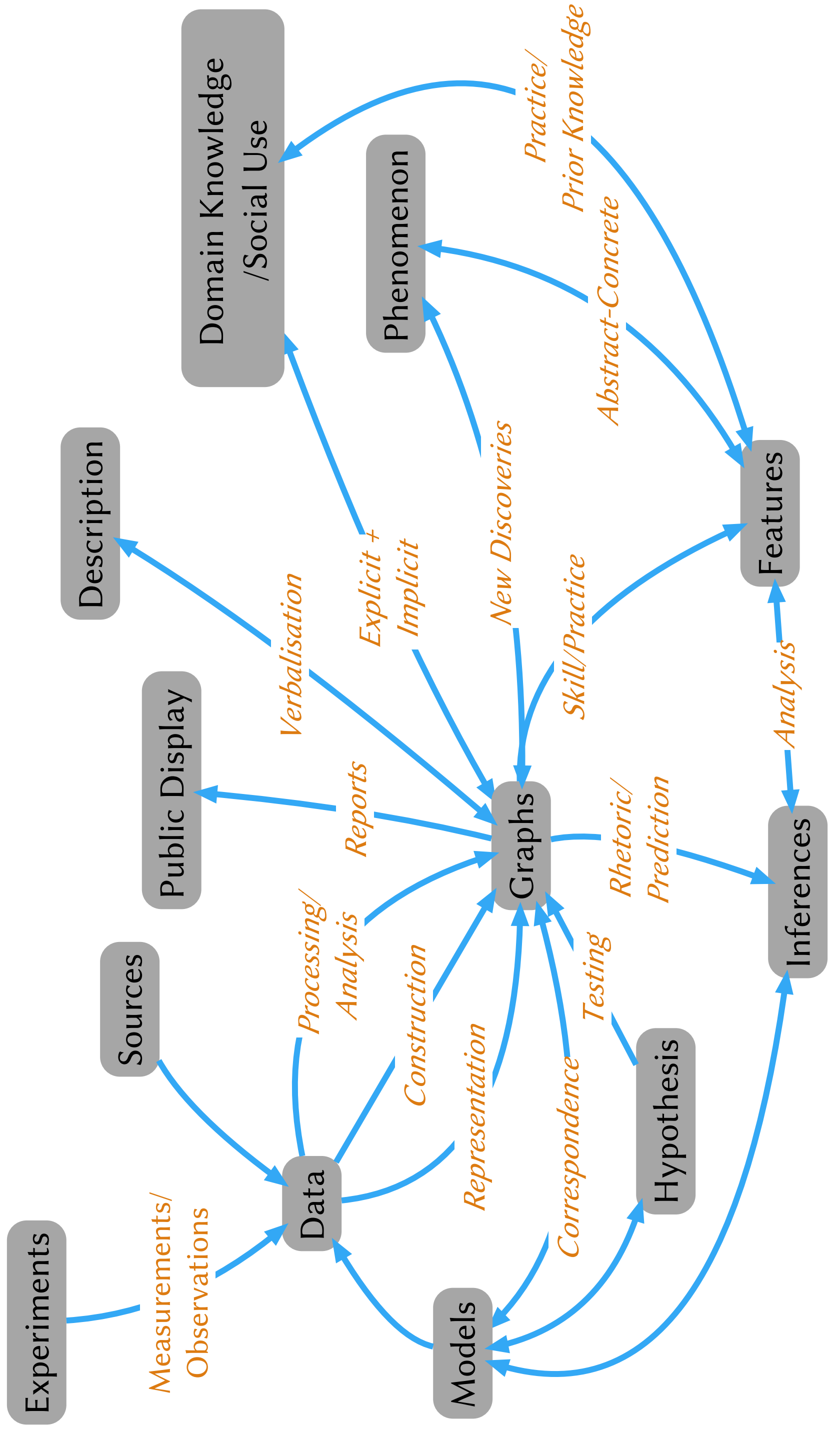


Figure 4.3: A schematic diagram showing major relations of graphs to other components and process in the activities.

to be non-proprietary so that large-scale use will not be expensive or encourage monopoly. There are strong reasons for the software that are used in education to be free, for example, see the page of Free Software Foundation on education (FSF, 2013).

In the current educational system, there is a delayed, or at times no feedback for the students (Hattie & Timperley, 2007). The students are often left on their own to explore topics, and many topics do not easily yield to exploration by traditional methods. The disconnect between theory and practice, especially in case of science, as the students do not have access to laboratory equipment. The position paper on Teaching of Science from NCF 2005 (NCERT, 2006b), makes these points:

> Even well-endowed schools have tended to give only cosmetic importance to laboratory work in the prevailing scheme of things. (p. 20)
>
> Ultimately, there is no alternative but to invest heavily in improving school laboratories and workshops while reducing the importance of external examinations and promoting experimental culture in our schools. We should also have computer-interfaced experiments and projects, besides projects utilizing database from the public domain. (p. 22)

Moreover, in the case where the access is present, often due to proprietary nature of the tools they are also not cost-effective.

Also, another important criteria used for selecting the software to be used in teaching-learning is due to Seymour Papert. In his works, Papert talks about providing the learners with *tools to think with* and *contexts to work with* rather than providing the content (Papert, 1980; 1994). In case of both the tools that we have chosen, we applied these criteria. The tools are platforms for creating knowledge by their users, as opposed to the Computer Based Tutorials (CBT) which provide content to users for consuming knowledge passively. These we think are an essential requirement for any software to be used in education.

With the above criteria in mind, we chose these tools. These tools do not provide content, but help in generating content. In an exploration of the graphical display of data, the reader is looking at two distinct worlds. One is the set of all that is possible, that is the mathematical world, and the other is what is actual, that is the real world. For developing competency in graphicacy, both these domains need to be handled and explored by the students. We used the dynamic mathematics software GeoGebra for this. GeoGebra version (latest at the time of field studies)

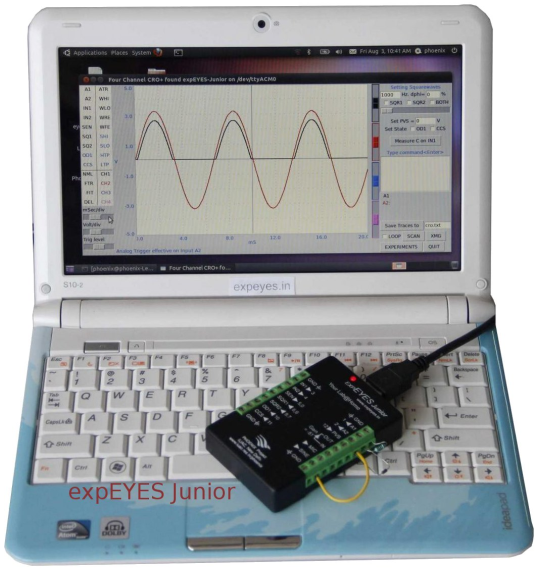


Figure 4.4: The expEYES Junior kit with a netbook laptop.

used in the activities was 4.2, this version provides support for two-dimensional geometry, algebra, and statistics. As of version 5.0 and above, GeoGebra is not an entirely Free Software, as defined by Free Software Foundation. The license used in the release of the software for version 5.0 and above does not allow software to be used freely for commercial purpose, though the source code is publicly available.

The other tool that we have used in our studies is the expEYES (experiments for young engineers and scientists) tool-kit, which is part of the Phoenix (Physics with Home-made Equipment and Innovative Experiments) project of Inter-University Accelerator Centre, New Delhi, India. It is a hardware and software framework for developing science experiments, demonstrations and projects without getting into the details of electronics or computer programming. Design of expEYES combines the real-time measurement capability of micro-controllers with the ease and flexibility of Python programming language for data analysis and visualisation. It also functions as a test equipment for electronics hobbyists and engineering students. The hardware designs are open and royalty-free commercial production is allowed.

One of the other major drawback in the current educational system, particularly in India, is the over-emphasis on theory, with little connection being made to the real-world (NCERT, 2006b). Though the students may understand the concept theoretically, and can solve problems numerically when it comes to the practical part they are found struggling. In such a state of affairs, a device like expEYES comes into the picture. The expEYES toolkit was developed with the purpose of providing a bridge between the theory and practice. It is a low cost, accessible device to the students to provide "hands-on" experience. expEYES makes it possible to attach different sensors to the device and collect data automatically at a micro-second resolution.

Thus we have zeroed upon the tools: one which lets users explore the possible mathematical world, other which empowers them to collect data from real-world in a fast and convenient way.

Before going ahead with the presentation of these field studies, it is important to point out a fundamental reality of scientific experimentation that is pertinent to this chapter. The fantastic work of Peter Galison on the material culture of microphysics (Galison, 1997) points to the role of engineering and fabrication of materials in the construction of scientific theories and narratives about them. Most importantly, we realise the quintessential role engineering plays in the development of scientific thought. Given the embedded nature of engineered artefacts (like sensors, etc.) that play a fundamental role in scientific experimentation and form the bedrock for theorisation, we designed an experimental system that brings together the material culture of experimentation alongside graphicacy. An important point to note here is that although on the surface, the role played by graphicacy seems marginal regarding the larger activity, it constitutes the quintessential element in the learners' experience that we believe provides the *Aha!* moment. This framework is an attempt to bring engineering and physics together while exploring the role graphicacy can play in this union.

4.4 Introducing the field studies

In this dissertation, we are presenting three field studies that were done around the activities[1] that we had designed. The three activities that we are reporting are: Measuring the Mustard Seeds (MS) Chapter 5, Measuring the distance to the Sun (SM) Chapter 6 and Exploring Electromagnetic Induction (EMI) Chapter 7.

The MS and SM tasks involved the collection of small sets of data that were recorded using rudimentary measurement tools such as rulers. Raw data gener-

[1]We use the words *activity* and *task* interchangeably in this report.

ated using such methods were amenable to manipulation using simple formats like graph paper. The experiment reported in this chapter was conducted to further the students' skills regarding handling a significant amount of data points and also to introduce automated systems that hold potential to enrich graphicacy. Furthermore, specific subject matter, such as electromagnetic induction, demand sophisticated data generation abilities and analysis. Towards this end, we exploited instruments and accessible materials that are already available in India and have been specially developed for educational purposes.

The three tasks described below address development objective (2):

Part 2: Development Objective 2

What are the required experiences and technological tools that support the comprehension and construction of graphs informed by the design principles and whether they can be implemented?

The tasks themselves involve construction and data collection in a problem-solving context. Each of the tasks establishes a real-world connect to mathematical concepts in familiar real-world contexts. The first two tasks have technology use at crucial points during the task (use of dynamic mathematics to explain the meaning of variation in slope), while the third task is wholly dependent on the technology used (for collection, display and analysis of data). In each of the task, the students gave varied responses, and mainly, for the first two tasks not all of the students were able to complete the tasks successfully.

4.5 Research design

In the three activities reported here, we have adopted a mixed approach in design and analysis. The Mustard Seed (MS) Task and the Sun Measurement (SM) Task have a similar methodology and were implemented with the same set of students. The third task of Electromagnetic Induction (EMI) was done in a case study mode with a set of two students. We now describe the approaches used in the three studies reported in this work.

Sample

All the three studies reported here were conducted at *Muktangan Vidnyan Shodhika* (MVS), at Inter-University Centre for Astronomy and Astrophysics (IUCAA) in Pune. MVS is also popularly known as the *Science Centre* and was also the workplace of the toymaker and educator Arvind Gupta during the time of our the field studies. MVS organises various programmes in science and astronomy outreach and also conducts night sky watching sessions. The development and field testing of the first two activities (MS and SM tasks) were iteratively carried out over three years (2012-2014) at MVS. An exception to this is the electromagnetic induction activity, which was a case study conducted with two students. The students in all the three studies were from urban and mostly English medium schools. Table 4.1 provides the information about the students in the three tasks reported here. For the MS and SM tasks, we have used the data from the year 2012 in this dissertation.

Information	**MS and SM Tasks, 2012 Data**	**EMI Task**
Number of students	137 (79 Male, 58 Female) Four batches of about 30 students B1: 33, B2: 34, B3: 30, B4: 36	2 (2 Male)
Education Board	Maharashtra: 125 CBSE: 8 IB: 2	Maharashtra: 2
Medium of schooling	English: 99 Semi English: 29 Marathi: 7 Hindi: 2	English: 2
Class	Class 8: 6 Class 9: 123 Class 10: 4	Class 10: 2
Data sources for research	Student reports on the activities, researchers' observations, photographs, pre- and post test data.	Video recorded student interviews, digital artefacts, researchers' observations, photographs.
Use of computers	For demonstrations by the researcher.	By the students for collecting and plotting data.
Duration of the task	MS Task: Spread over two days. SM Task: Spread over three days.	Spread over five days.

Table 4.1: The information about the student sample in the three activities.

Mustard Seed (MS) and the Sun Measurement (SM) Tasks

The MS and SM tasks were done in the context of a five day long *Astronomy Summer Camp* at the MVS. Each year the camp is attended by about 120 students, divided in four batches. Each batch has about 30 students. This programme had an emphasis on Astronomy as a subject. The students were taught basics of observational astronomy in the programme, and it is in the setting of this summer camp that the two activities were embedded. A printed version of *Handbook of Activities* (Appendix C) was given to all the students. The two activities (MS and SM) are similar regarding (a) the use of indirect measurement and construction of a mathematical model, (b) the mode in which they were implemented and, (c) the sample of the students who did these activities.

In both these activities, we have kept a physical parameter which could be varied, and the result of this variation could be seen in concrete terms. The data collected in case of both the activities are similar. We have collected three main types of data. The first data set is in the form of a written Pre-Test that the students were administered before and after we began the 5-day camp. The questionnaire for the Pre-Test can be seen in the Appendix C. Some of the questions in the Pre-Test were objective type, and for some other, the students had to write elaborate responses. The second data source for these two activities are the detailed reports which the students submitted after performing these activities. These reports were rich and contextualised presentations of each students' approach to the activities and gave us a glimpse of the way in which activities were performed. The reports submitted by the students typically had Aim, Apparatus/Materials needed, Procedure, Assumptions/Precautions, Observations, Graphs, Equations, Diagram Conclusions, Predictions as major sections. Typically each of these reports was of 2-3 pages. The third source was the researchers' observations while conducting the classes and activities, including photographic evidence collected during the classroom discussions and performing the activities. A combined analysis of these three data sources is presented in the respective chapters on MS and SM Tasks. Along with this, we have also looked at Science and Mathematics textbooks for the prior concepts which are required for the performance of these activities. By looking at the textbooks, we are confirming our assumption that most of the conceptual background needed for performing these activities is within the grasp of the students. This was also tested when the activities were actually performed, and the students submitted the reports. Overall the two activities were observed for three years during 2012-2014. The activities still continue to be performed during the summer camp which is an annual event at the MVS. We have presented the data analysis based mainly on the first year (2012) of the study. The focus in the presentation of these two activities is on describing the *process* by which the students performed the activities along the line of the design framework that we have described.

Electromagnetic Induction (EMI) Task

The EMI task was carried out in a different setting. In this task, only two students worked for five days, intensely for a single project. The students were explained the task and the tools on the first day and carried out the task over the next four days. In case of EMI Task, due to the highly contextual nature of the task in constructing and designing experiments, we used an explanatory case study approach with two students. According to Yin (2003) case studies are particularly useful when one is trying to explain the phenomena situated in a context.

For the EMI Task, we have three primary data sources. The first source is the semi-structured interviews that the researcher did with the students on a daily basis. These interviews were transcribed and the text analysed for the episodes of meaning-making and transitions from abstract to concrete. The footage from the video along with the transcribed text is used as a descriptive report for the activity. The second source was the researchers' notes about the students and their interactions. Finally, the digital and physical artefacts created by the students formed the third source of data. The physical artefacts included the coils with various parameters, while the digital artefacts included the data files and the graphs they generated.

In the next three chapters, Chapter 5 MS Task, Chapter 6 SM Task, and Chapter 7 EMI Task, we look at each of these field studies in detail.

5

Measuring the mustard seeds

In this chapter, we describe the activity for measuring the average diameter of the mustard seeds using indirect measurement. We call this the Mustard Seed (MS) Activity. We present a descriptive analysis of the responses of the students and the problems that they faced in the context of how it is a step towards mathematical modelling using a real-world setting.

We first discuss the essential mathematical, statistical and science background required for this activity as covered in the textbooks until class 8 and 9. After this, we discuss the same and setting of the where the intervention. Next, we discuss the workflow of the activity and detail various steps as seen in the light of the design framework described in the Chapter 4. Finally, we discuss the results, implications and limitations of the activity.

5.1 Introduction

Measurement in the real-world forms the much of the basis of knowledge about the world. The concept of measurement is essential for both science and mathematics learning. (cite NCF project2021). Though there is a difference in the way, science education and mathematics education communities approach the topic (Thompson & Preston, 2004). In mathematics, measurement is typically seen as a subset of geometry and not necessarily involving the collection of data or its analysis. The last two aspects (data collection and analysis) are essential (at least theoretically/normatively) in science along with the associated statistical concepts like errors, accuracy etc. In mathematics, the idea of *distance* is typically seen as a number between two points which is unique (definition of distance), while in science it is related to real-world measures (2004). While arguing for the use of measurement as a bridge between science and mathematics curricula Thompson & Preston (2004) see it as an ideal opportunity:

> It is an ideal opportunity in measurement education to bridge this gap between measurement instruction in mathematics classrooms and the need for measurement in science. For example, mathematics teachers can use problems from science classes as a basis for measurement instruction in their own classrooms. Similarly, mathematics and science teachers can coordinate labs, with each sharing responsibility for solving the mathematical aspects of the lab activities. In so doing, both the mathematics and science teachers take responsibility for measurement education, particularly areas such as estimation, relative error, precision and accuracy, significant digits, indirect measurements, and so on. (p. 517)

In our work, we look at how the context of measurement can be used to help the students construct and comprehend graphs. Echoing the idea put forth in the above quote, the design of the MS activity considers the measurement as a starting point to combine many concepts from science and mathematics. The subsequent analysis and finally the graphs obtained at the end provide the consolidation. Measurement, in this case, is set in concrete real-world context and provides meaningful learning opportunities for meaning-making from graphs made from the data collected in these measurements.

After the students learn about measuring dimensions in real life using direct methods, it is crucial that they learn how to measure very small and very large dimensions or dimensions that are not directly accessible. For example, large trees, the distance between the Earth and the Sun, or very small objects like microbes, pollen, or the thickness of a hair. Under these situations the use of indirect measurement techniques becomes imperative. Indirect measurement often requires the use of alternative physical models and the corresponding mathematical models. For example, in order to measure the height of a building, the alternative physical model may be an analogous triangle, and the mathematical model would be the equations describing the proportionality using either similarity of triangles or basic trigonometry. In some cases, even the physical model may not be amenable to direct experience or direct measurement. In such cases, mathematical models provide a way to measure the required dimensions that often depend on some assumptions and approximations about the physical world.

We see the activity described in this Chapter as the first exercise in middle or high school students creating mathematical models from observations. The activity involves finding the average diameter of the mustard seeds. The observational data resulting from this activity leads to a simple linear mathematical model with an accessible concrete physical basis from the real-world. This simple task provides rich opportunity and a context to learn several linked topics in measurement, mathematical modelling, graphicacy and statistics. We present this as a template to be used for developing a series of activities for learning indirect measurements, physical and mathematical models. The use of graphs in this activity acts as a bridge between the symbolic mathematical model and the physical basis which it represents. We next review how the mathematics and science textbooks approach the topics of measurement.

5.2 Review of the textbook topics

In this section, we review the content of the textbooks for the topics that are requisite to the activity described in this Chapter. We find that most of the mathematical background is covered in the preceding years. We analyse both the NCERT textbooks and Maharashtra board textbooks for the topics.

Mathematical Background

The essential background knowledge and skills required for the MS activity are data collecting and handling, statistics (averages and their meaning) and the concept of dependent and independent variables, linear equations in two variables. We look at each of these topics and how they appear in the textbooks next in this section. Associated with the concept of measurement are statistical quantities which involve notions of average, accuracy, assumption and errors. We look at where these topics appear in the textbooks.

Class 5 mathematics: In Class 5 mathematics *Chapter 10 Tenth and Hundreds* discusses aspects of measuring with a scale. This textbook uses many real-world situations for activities which can be done with a scale. The textbook also discusses various units like the centimetre, millimetre and others. Many everyday objects like pencils, currency notes, vegetables etc. are used in the measurement activities in this chapter. For example, in Figure 5.1 the students are supposed to measure the length of a vegetable in both centimetres and millimetres. These kinds of activities provide the students with an estimate of the dimension of real-world objects. It would have been an interesting variation to use actual vegetables brought by the learners as a part of this activity. Such a variation would lead to natural variations in the lengths measured opening avenues for many stimulating discussions.

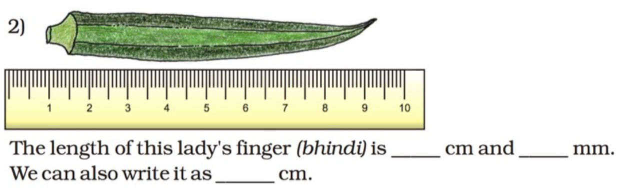


Figure 5.1: An activity to measure the length of okra or *bhindi* using a scale in Chapter 10 of Class 5 mathematics book of NCERT, p. 136. This Chapter has various exercises that deal with measurement of lengths set in everyday contexts.

We notice a shift in the approach toward the topic of measurement after Class 5 in mathematics textbooks of NCERT. We see a change in the informal and close-to-life approach seen in the earlier texts to a more formal approach from Class 6 onwards. Even the design and illustrations of the textbooks display the shift in the approach.

Class 6 mathematics: The mathematics textbooks after Class 5, continue to explore the idea of measurement in the context of mensuration. Perimeter and area of geometrical figures, both regular and irregular, feature significantly in these chapters. In Class 6 mathematics *Chapter 1 Mensuration* explores the basic ideas of area and perimeter of various geometrical figures like rectangles, squares and triangles. They also explore the topics of area and perimeter with some non-regular shapes. In the same book *Chapter 9 Data Handling* introduces the students to the ideas of collecting and tabulating data. The chapter has exercises in which the students create and explore pictographs and bar graphs. This section on *Bar Graphs* (Section 9.7) explores them various examples.

In statistics, bar graphs are used to compare discreet categorical variables, while histograms are used to show the distribution of quantitative variables. However, the content of the chapter does not maintain this distinction. Instead, both categorical, as well as quantitative data, are visualised using bar graphs. This class introduces the concept of *variables* in the context of Algebra (Chapter 11). The concept of variables and their manipulation is at the core of algebraic thinking.

Class 7 mathematics: In case of Class 7 mathematics *Chapter 3 Data Handling* further develops the theme in Class 6. The students are introduced to collection of data and statistical quantities of *mean*, *median* and *mode* using various examples. Bar graphs and tables are used to illustrate and discuss examples. Chapter 4 discusses simple equations in one variable. It sets up many equations with one variable in varied contexts. In Class 7 mathematics textbook of NCERT (NCERT, 2007a) the concept of variables is developed further. A variable is defined as under:

> *The word variable means something that can vary, i.e. change. A variable takes on different numerical values; its value is not fixed.* Variables are denoted usually by letters of the alphabet, such as x, y, z, l, m, n, p etc. From variables, we form expressions. The expressions are formed by performing operations like addition, subtraction, multiplication and division on the variables. (emphasis in original, p. 78)

The concept of variables is eventually developed in the context of simple equations with one variable (Chapter 3) and algebraic expressions (Chapter 12).

Class 8 mathematics: In Class 8 mathematics Chapter 2 deals with linear equations with one variable. Chapter 5 *Data Handling* introduces the students to the ideas of collecting, organising and displaying data using bar graphs, histograms and pie charts. The chapter also deals with the frequency distribution of values using tables and histograms.

Chapter 15 on Graphs is perhaps the most important unit on graphs in the range of textbooks that we considered. It starts with an introduction of various types of graphs like a bar graph, pie chart, histogram and line graphs with examples for each.

The MS Task and the Sun Measurement (SM) Task have measurements as essential, core component in them. The mathematical apparatus in the form of multiple representations used to describe, understand and analyse the data makes it possible to make sense for the students. These two tasks are both complementary and supplementary to each other. On the one hand, they have structural similarity in their approach to the task involving data collection, analysis. Both the activities are based on the idea of indirect measurement, while the lengths involved are differing by a large order of magnitude. The linear equations in two variables represent the simplest and a first form of mathematical functions that the students encounter. Linear equations can also be used to represent a variety of real-world situations.

Science background

The idea of measurement lies at the core of the doing science. The concept measurement and the subsequent analysis of the data obtained are essential for testing of theories, and in some cases helps in creating them. Graphs play a vital role in this process: to visualise data and to find patterns from the data.

In the case of science textbooks, as we have seen in the Chapter on textbook analysis, the linear functions in science textbooks appear in the context of uniform motion (Class 8) and Ohm's law (Class 10). Though as we have seen in the analysis of science textbooks, in both these the real-world data collection is not used. Linear equations in one variable appear in mathematics textbooks starting in Class 8. However, there is no systemic effort to build on the concepts learned in the mathematics classes.

In case of Maharashtra Board, Class 6 science textbook covers the relevant topics of indirect measurement, accuracy and error. *Chapter 4 Measurement* introduces the students to the ideas pertaining to measurements of various physical quantities like length, area, volume and temperature. The chapter also discusses a standard method of calculating the volume of an irregular object by measuring the displacement of water (Figure 5.2).

In the next chapter, *Chapter 5 Estimates of Measurement* two method of indirect measurement are discussed (Figure 5.3). The first method is about finding the thickness of pages of a book, and the second method is for finding the diameter of

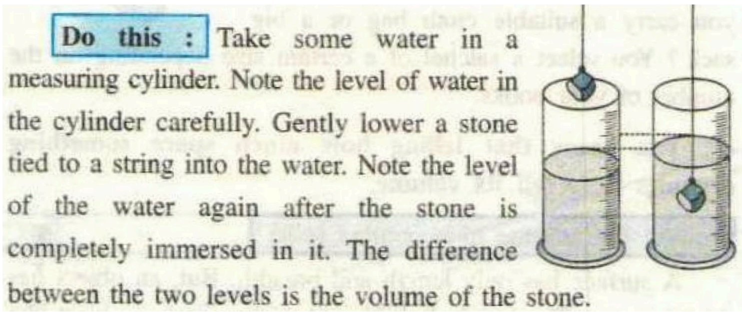


Figure 5.2: An activity describing the measurement of the volume of an irregular object by measuring the volume of the displaced water by the object. From Maharashtra State Board Class 6 science textbook, Chapter 4, p. 40.

the pencils. Both these methods use a similar technique to the MS task to find the required length.

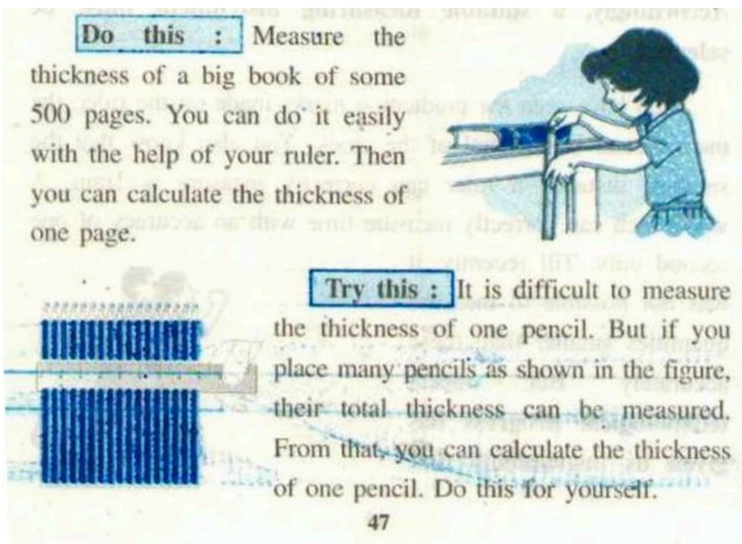


Figure 5.3: Two activities describing the use of indirect measurements to find the thickness of pages of a book and diameter of a pencil. Both these activities are structurally similar to the MS task. From Maharashtra State Board Class 6 science textbook, Chapter 5, p. 47.

Thus we see that the mathematical, statistical and graphical background needed for this activity is covered in the syllabus of the students in the previous years. How the students could recall and use these concepts in the MS activity will be discussed with examples from the students' work in the following sections.

5.3 Field study

The activity was designed and developed over three years along with the Sun Measurement (SM) task during the years 2012-14. Each year we had a different set of students from urban Indian schools. For each year we had about 120 students mostly from class 9. Most of the students had English as the medium of instruction. Each year around 4-5 students, who had Marathi or Hindi as a medium of instruction submitted reports in Marathi and Hindi respectively. For these students, the instructions were repeated in the Marathi and Hindi. The students were part of a special summer camp, with a variety of activities. The MS task was the first activity in the set, giving them an opportunity to develop the idea of scale and measurement. The subsequent activities built on this idea, of using indirect measurements and using mathematical models to solve the problem at hand.

The data collected during the development and field testing consisted of researchers notes during the classroom discussions, photographs and reports submitted by the students. All the students in the sample did not submit the written reports for various reasons. We discuss in a descriptive form the analysis of the reports submitted by the students when they completed this activity.

5.4 Setting the context: Mustard seeds

An Indian kitchen is a versatile place where many spices and ingredients mingle to produce a variety of cuisines. Though each part of India has a unique style of cooking, there are many things that one will find in all kitchens. One of them is the mustard seed, of the family *Brassica*. In many of the cuisines, the mustard seed is a requisite, to give a *tadka* or flavour to the food. In many Indian languages, the mustard seed is metaphorically used to denote something of a very small size. There are three common varieties which, and all of them are small so that a direct measure of a single seed with scale is difficult. Also, the seeds are approximately *spherical* in shape this helps us in setting the problem to find an *average diameter* for the seeds. Two varieties (*B. juncea* and *B. alba*) have seeds almost double size of the other one (*B. nigra*), as shown in Figure 5.4. The figure also shows the variation in the size and shape of the seeds of the same species. These two characteristics of the seeds provide opportunities for classroom discussion as we will see later. In the next section, we describe the workflow of the MS activity.

Figure 5.4: Commonly found mustard seeds, and their sizes. From left to right *Brassica juncea*, *Brassica alba* and *Brassica nigra*. The relative size difference makes the mustard seeds well suited for the activity. Also, note the variation within each type of seeds and their approximately spherical shape.

5.5 Workflow of the activity

In this section, we describe and discuss the overall workflow of the MS activity. We also highlight the applicable concepts and skills from the design framework Figure 4.2 from the Chapter 4 are shown in brackets with a different formatting and font as `(concept)`. The design framework as seen in the context of the MS task is as shown in Figure 5.5.

The MS task was carried out over two days. On the first day, a few different possible approaches to measure the diameter of the mustard seed were discussed with the students. The students were asked to do actual measurements of seeds in their homes. Also, how and why a mathematical model (both algebraic and graphical) may fit the observations was discussed. The students were asked to write detailed reports on the task.

Timeline of the events in the MS Task

Day 1

Session 1: Estimating the size of everyday objects, introduction to the problem, finalising the method for measurements.

Homework: Measurements on the mustard seeds at home, data collection and report writing. Printed instructions in the Student Handbook.

Day 2
Students submitted the written reports from the homework exercise done on the previous day.
Session 2: Discussions on reports from the homework exercises. Collaborative plotting using GeoGebra, mathematical modelling.

On the next day, the data was collected from all the students and plotted using GeoGebra on a projector. An engaging pattern emerged from this collaborative plot. The mathematical model was again discussed with the data from all the students which helped in understanding the physical meaning of terms in the mathematical model. In the following part, we narrate the significant events when learning took place to highlight how a simple task can be so rich in its learning outcomes. The length of each discussion session with the students was typically about an hour.

The principal steps in the MS task can be seen below. In each step, we have indicated the essential concepts pertaining to that step. This, however, does not exclude other concepts from being present in these steps. The tagging of the activity in this way provides a way to look at the MS task in the light of the design framework of the activity.

Steps in the Mustard Seed Task

1. Discussion on estimating the size of everyday objects. `(prior knowledge)` `(discussion)`
2. Discussion on the direct and indirect methods of measuring `(prior knowledge)` `(context)` the mustard seed diameter `(problem)`. `(discussion)`
3. Discussing possible sources of error while making the measurements. `(discussion)` `(experiments)`
4. Measuring the length of aligned mustard seeds in varying numbers and recording it. `(data)` `(hypotheses)` `(experimentation)`
5. Collecting data in the form of a table and plotting the data on a graph. `(graphs)` `(multiple representations)`
6. Performing simple calculations and statistical analysis on the data collected. `(calculations)`

   `(construction)` `(multiple representations)`
7. Making a mathematical model from the data that was collected, making connections of the mathematical model to observations. `(models)` `(inferences)` `(analysis)`

(8) Reporting the work done, with graphs, tables and diagrams `(public display)` `(construction)`.

(9) Collecting data from all the students and plotting the graph using GeoGebra. `(public display)` `(multiple representations)` `(graphs)`

(10) Classroom discussion about the reports, and the combined graph. `(inferences)` `(conclusions)` `(discussions)`

In the following section, we illustrate each of these steps in the workflow with examples from the students work.

1. Discussion on estimating the size of everyday objects.

Concepts/Skills: `(prior knowledge)` `(discussion)`

All the students were familiar with mustard seeds. We start the discussion with examples which are similar to the mustard seed task involving indirect measurement. One of the tasks was the indirect measurement of the width of a thread. A method of doing this is usually by winding the thread on an object (for example, a pencil) and finding the width for a given number of turns. We would get the average width of the thread by dividing this length by the number of turns. Another task that was discussed was how to find the thickness of one page of a book. This task involves measuring the thickness for a given number of pages, and then the average thickness is found out by dividing the length by the number of pages.

After either of these two warm-up discussions, the students were asked to *guess* the approximate diameter of one mustard seed. For this purpose, some mustard seeds (*B. juncea*) were shown to them. During the discussions that ensued the students usually came up with guesses from a few millimetres to a few centimetres.

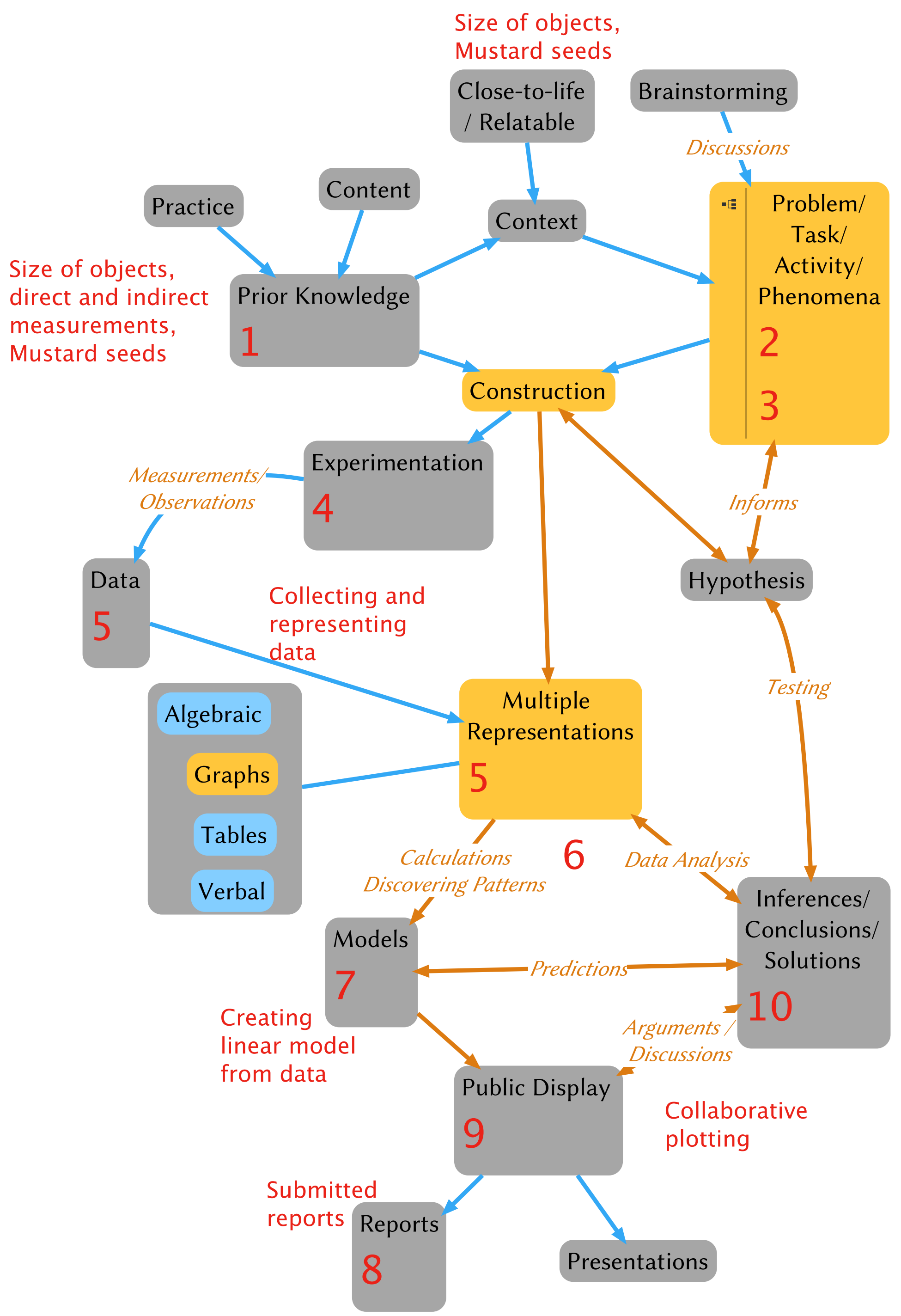


Figure 5.5: A description of the Mustard Seed task in the design framework described in Chapter 4. The red numbers in the figure indicate the steps given in the workflow.

2. Discussion on the direct and indirect methods of measuring the mustard seed diameter.

Concepts/Skills: (prior knowledge) (context) (problem).

In the next part of the discussion, the students were encouraged to come up with ideas for measuring the diameter of a seed. The students were asked the question "How can we measure the diameter of the mustard seed?" During the discussion that followed some of the students came up with some ingenious methods of measuring the diameter. One of the students suggested that a thread should be wound on the seed, and then the length of the thread can be measured easily with the help of a scale.

Another student suggested an even more elaborate method: we can find out the volume of displacement of water due to one seed and then from the volume of the water displaced we can find out the volume of the mustard seed, and from this volume, we can find out the radius and hence the diameter. The students were already familiar with the properties of a circle (area and circumference) and a sphere (volume), which are to be used in the above two methods. Both the methods described above are derived from the method described in the textbooks used to find the volume of irregular objects (Figure 5.2).

One of the students said that a divider from a geometry box could be used to hold the seed, and then the distance between the points of the divider could be measured on the scale (Figure 5.6). Further, another student rolled a seed in a piece of paper and measured the diameter of the roll (Figure 5.7).

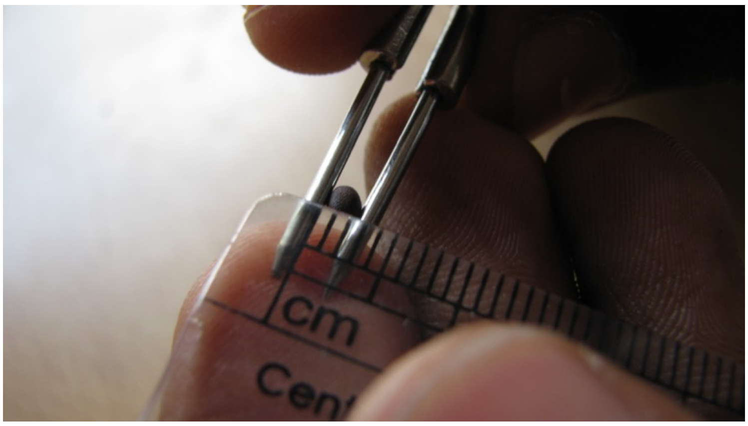


Figure 5.6: A student measuring the mustard seed diameter by using a divider.

Despite the discussions in the warm-up on similar tasks it was interesting to note that it never occurred to the students to use more than one seed in the measurement. We observed this consistently across all the three years of the activity, though the students were aware of the concepts of average quantities, as evidenced by the written reports. This inability of the students to apply concepts known to

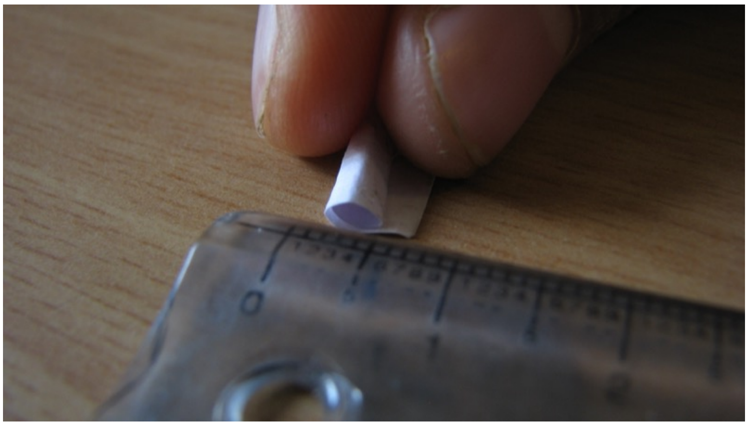


Figure 5.7: A student measuring mustard seed diameter by rolling a paper around it and later measuring the length of the roll.

them may indicate the transfer of mathematical knowledge from one context to another is often not easy.

To drive home the point regarding the need for taking an average of the readings we asked the students to look at the mustard seeds and to tell "Are all of them were of precisely the same size and shape?" As we can see from Figure 5.4, there is a variation in size and shape even in the same species, and the students noted this. The students answered that all the seeds were not of the same size. They indicated that some of the seeds were larger, while some of the seeds were smaller. This answer was a result of a concrete observation done by the students. Some of the students responded to this trigger of variation in size by saying that we need to take an average of many values. The discussion included rhetorical questions like "Will the value of the measurement be correct for all seeds if one happens to measure a single smaller or a larger seed?" In this process, we brought in the rationale of doing multiple measurements on different seeds and taking averages of the readings. This discussion helped the students to realise that variation in the size of the seeds implied that we need to measure the averages.

We then reminded the students about the discussions in the warm-up tasks. An agreement was reached by deciding to use a scale (smallest scale division is count 1 mm) to measure the average diameter. In the discussion that followed the procedural details of the task were worked out. The method involved aligning a number of mustard seeds along a scale and measuring the length covered by them. Then the average diameter for each one of the sets of seeds was found. The precautions to be taken were discussed.

Figure 5.8 which shows a photograph and a drawing made by one of the students to explain the procedure. The students were guided to take measurements of sets of 5, 10, 15, 20, 25 and 30 mustard seeds, though some the students went up to 40 seeds. After this, possible ways of making a mathematical model from these observations were discussed. Also, the *assumptions* required for such models were discussed. This discussion on modelling was further elaborated on the next day when the data from the students were collected and plotted collaboratively.

3. Discussing possible sources of error while making the measurements.

Concepts/Skills: `(discussion)` `(experiments)`

In the discussion following the finalisation of the method for measurement, the students were asked about possible errors that might arise while performing the experiment. Some of the students said aligning the seeds correctly was necessary as if there is a gap between the seeds it will introduce an error in the measurement. Some of the students suggested that the divisions on the scale should be carefully counted. The students were also asked to write down the problems and possible errors that they may create while performing the measurements.

4. Measuring the length aligned mustard seeds in varying numbers and recording it.

Concepts/Skills: `(data)` `(hypotheses)` `(experiments)` `(observations)`

The students did the experiments at their home. Asking the students to perform the task at home was a part of the design of the MS task. Performing the task at home brought variation in the data collected by the students owing to different types of mustard seeds used in different homes. The students aligned the seeds in multiples of 5 and measured the resulting lengths and recorded the observations in the form of tables.

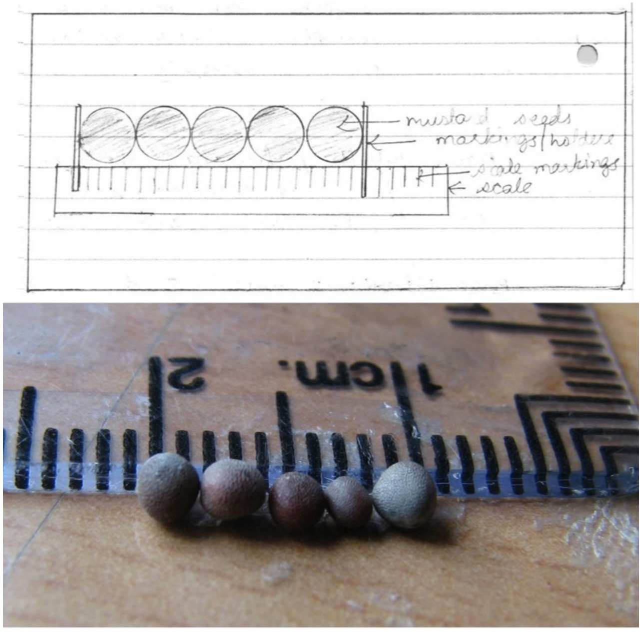


Figure 5.8: The aligning of mustard seeds for measuring the resulting length. The top photograph shows the placement of 5 seeds along a scale. A schematic drawing of the setup by one of the students. Placeholders which hold the seeds together can be seen in the drawing.

5. Collecting data in the form of a table and plotting the data on a graph.

Concepts/Skills:`(graphs) (multiple representations)`

The students submitted written reports which were supposed to include the procedure, precautions that were taken, possible errors, observations, tables, pictures and conclusions. In their reports, the students were also told to plot a line graph for the number of seeds versus the total length that they measured for these seeds.

The observations on the different sets of the seeds were recorded in the form of a table. The table had data in the following format: Number of seeds - Length in mm/cm - Average diameter for 1 seed Figure 5.9. Some of the students used calculators to get answers up to 4 decimals. This provided an opportunity to discuss significant numbers, the concept of least count, and rounding off in the class on the next day. The most commonly reported error was of the alignment and placement of the seeds with the scale. This error was the most irritating for some of the students although it was fun for others. Although the students did not realise it, Observations :-

Sr	No. of seeds	dist.(mm)	$\bar{x}$ (mm)
1	5	10	2
2	10	20	2
3	15	29	1·93
4	20	38	1·9
5	25	49	1·96
6	30	56	1·86

Figure 5.9: A typical observation table from the report of the students. The columns in the table typically included Sr. No., Number of seeds (n), Length of Seeds/Distance (L), Average (L/n).

the problem was aggravated at times by the presence of static electric charge on the seeds. One of the students actually glued the seeds on the paper to overcome this problem!

Almost all of the students who drew the graph could plot the data points correctly. Not all of them drew best-fit lines through the points that they had plotted. Some of the students drew the graph on a plain paper rather than using the graph paper they had been given. Only one student drew both a bar graph as well as a line graph. The students were not given specific instructions in choosing the scales, but they were asked to write the scales on the graphs that they drew. While most of the students chose a scale of 5 seeds per unit for the X-axis, various scales were used for the Y-axis. Many of the students chose the same scale as the actual readings, with 1 mm on the graph paper being equal to 1 mm of the actual measurements (Fig. 5.10). One of the students plotted the values of the average diameters that were obtained from the measurements against the number of seeds in each set.

6. Performing simple calculations statistical analysis on the data collected.

Concepts/Skills: (multiple representations) (calculations)

The students found out the average diameter for each set of observations by di-

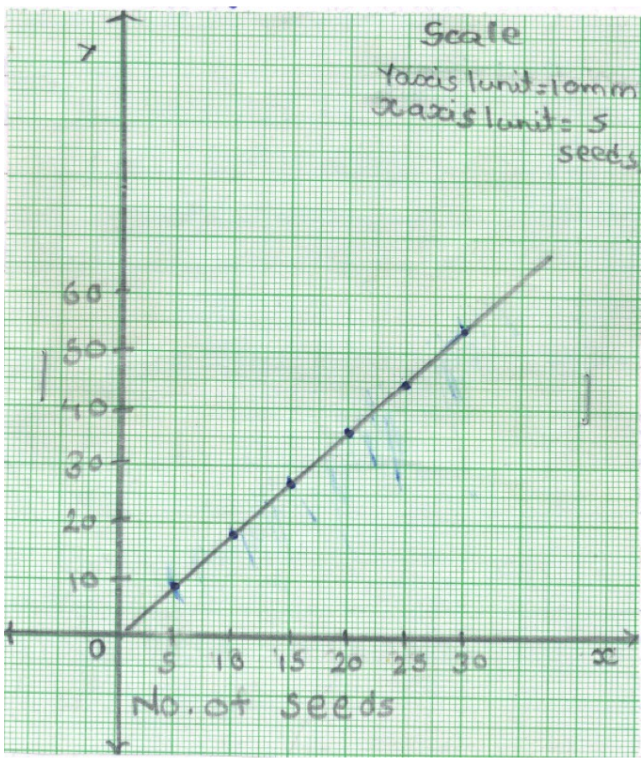


Figure 5.10: Graph one student made of the number of seeds vs. the length. Here the Y-axis has scale 1 mm = 1 mm and X-axis has scale of 5 seeds = 1 cm.

viding the length obtained for each set of seeds with the number of seeds. Some of the students also found the average of averages. Some of the students included average calculation as part of table (Figure 5.9), while others did separate calculations as shown in Figure 5.9.

7. Making a mathematical model from the data that was collected, making connections of the mathematical model to observations.

Concepts/Skills: `(models) (inferences) (analysis)`

On the next day, the students came with their written reports, and they were asked "What is the average diameter you have found?" The answers varied, sometimes almost double than some answers. Through some probing on why the answers varied, it emerged that there were, in fact, two different varieties of mustard seeds. This fact, the students did not realise initially, because each of them had only one of the varieties at their home, and so had the data only for one type. This natural variation in the size of the seeds provided us with an opportunity to discuss what the existence of two sizes would mean for the mathematical model. During the discussion, the students were asked if there are any mathematical relationships between the number of seeds and the lengths that they had measured for them. First of all, it was noted that as the number of seeds increases, the total measured length also increases. So, it was agreed upon that the measured length of the sets of seeds L is directly proportional to the number of seeds n:

$$L \propto n \tag{5.1}$$

After agreement that these two quantities are in direct proportion, the discussion was taken further by introducing a proportionality constant d. Hence the mathematical relationship between the two quantities L, and n can be written as

$$L = d \times n \tag{5.2}$$

At this point, the students were reminded of the straight line equation in the form

$$Y = m \times X \tag{5.3}$$

where m is the slope of the line. We then compared the two equations for similar terms. The total length L and the number of seeds n in equation (5.2) is analogous to the Y and the X values respectively in equation (5.3). The proportionality constant d in equation (5.2) can be seen analogous to the slope m in equation (5.3). All this leads to the fact that equation (5.2) is indeed an equation for a straight line. After showing that the mathematical relationship that is expressed by equation (5.2) is a straight line, it explains why we can draw a reasonably straight line passing through all the points.

8. Reporting the work done, with graphs, tables and diagrams.

Concepts/Skills: (public display) (construction) (reports)

The students were asked to submit written reports of their work. The report writing was a common instruction for both the MS task and Sun Measurement task discussed in the next chapter. The instruction to the students can be seen in Figure 5.11.

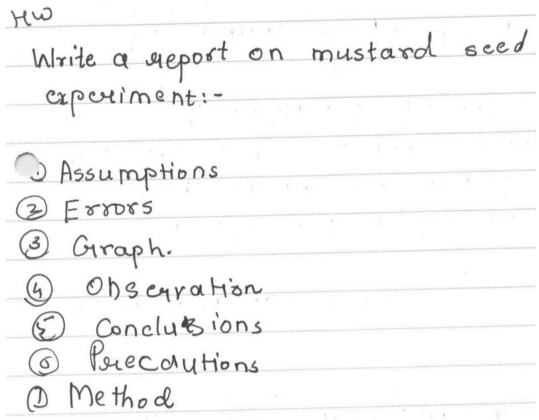


Figure 5.11: The "Homework" given to the students for performing the MS task, the broad sections of the report to be submitted can be seen.

In some cases, the order was changed, and some sections were subsumed under others or were absent. The actual reports included the following sections for most of the students. (a) Aim / Experiment (b) Apparatus (c) Procedure (d) Assumptions (e) Errors (f) Precautions (g) Diagrams (h) Observations: Tables (i) Observations: Graphs (j) Predictions (k) Calculations (l) Conclusions

While there were some commonalities in the reports, each report was also a testimony to the personal styles and approaches of the students. Some reports were impersonal and formal, while others included an experiential component of performing the experiment.

9. Collecting data from all the students and plotting the graph using dynamic mathematics software.

CONCEPTS/SKILLS: (public display) (graphs) (multiple representations)

In this activity, an explicit connection between the mathematical knowledge about averages, direct proportion and linear equations was made and was used to solve a problem at hand. As suggested by the textbook analysis these connections are usually not made. The ideas of independent and dependent variables, errors were embedded in the activity in an organic way. This activity could be seen as the first step in mathematical modelling and making use of graphs to understand and explore the model in the context of solving a problem. As evident from their reports, many of the students could think concretely about the slope of the graph as the average diameter of the mustard seed. This linking happened with the help of discussions around the two graphs in the classrooms.

A collaborative exercise was done on the second day to emphasise the understanding of the slope of the line and its physical meaning in the mathematical model. The length of each set of seeds (5 seeds, 10 seeds, 15 seeds, ...) was collected and displayed on a spreadsheet in GeoGebra a dynamic mathematics software. After this, the average lengths of each of these collected values were plotted against the corresponding number of seeds in each set (Figure 5.13). When the points were connected with the help of straight lines, two *distinct* lines emerged, corresponding to each type of seed. It led to a discussion on the question “Why did we get *two distinct* lines?”, leading to physical meaning of the slope in the mathematical model.

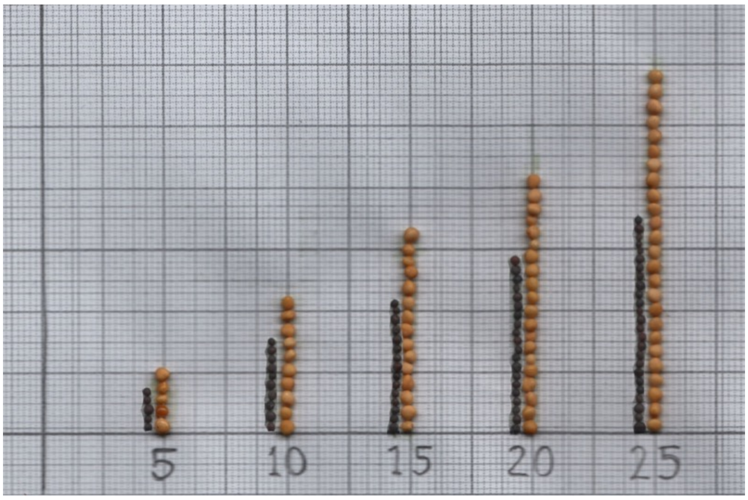


Figure 5.12: A ‘bar graph’ with the two types of mustard seeds. A difference in the heights of the seed columns is clearly visible, this difference relates to the different *slope* of the lines. The students were shown a similar photo during the classroom discussions.

10. Classroom discussion about the reports, and the combined graph.

Concepts/Skills: (inferences) (conclusions)

Discussion followed on what is the physical meaning of the slope of the line. Figure 5.12 where both the sizes of the seeds are aligned next to each other shows this clearly. The difference in the size of the seeds is manifested as difference in the "height" of the corresponding columns seeds. A line passing through each of these columns will have a different slope. The line corresponding to larger seeds will have a larger slope, Figure 5.13 shows this.

In this case, we have a concrete physical observation that the sizes (diameters) of the two types of seeds are different. On the other hand in the mathematical model, the slopes of the lines are different. Thus we can relate the abstract change in the slope of the mathematical model to a concrete observation regarding the size of the mustard seed. This point was discussed at length in the classroom. It was helpful to use GeoGebra to visualise how the lines would have looked if the slope was different, meaning if the size of seeds was different. For example, "if the seeds had an average diameter of 0.5 mm or 3 mm, where would these lines be drawn with respect to the existing lines?" This way the use of graphs for understanding the meaning of the slope in terms of associated lengths was made clear. The students were also asked to plot the collaborative average on their own graphs. Drawing this 'average plot' shows how much deviation the students' readings have from the average values. This average plot led to interesting discussions about different aspects of statistics like averages, standard deviations and the need to take multiple measurements.

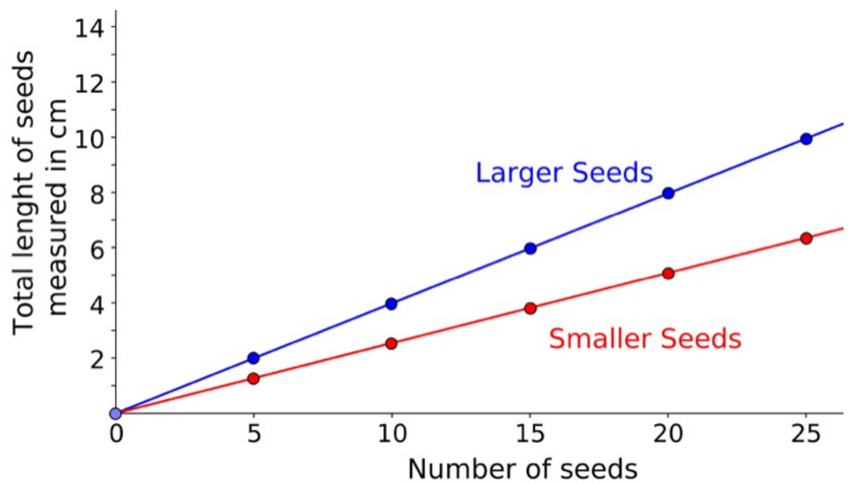


Figure 5.13: Typical graphs of lines for two different types of seeds drawn in GeoGebra from the average readings taken from the students. The difference in their slopes is linked to the difference in the size of the seeds.

Another use of graphs in the context of modelling is their use as calculating and predicting devices. The students were shown with an example in GeoGebra, how to find the length for a given number of mustard seeds. For example, to find the length for a set of 50 mustard seeds in a row, we need to take a line which is parallel to the length axis (Y) and find the point of intersection of this line and the line made from observations. Similarly, we can also find the number of seeds, if we know the length of the set of seeds, by using a line parallel to the number of seeds axis (X). In one of the batches, the students were asked some practice questions for these types of predictions (Figure 5.14).

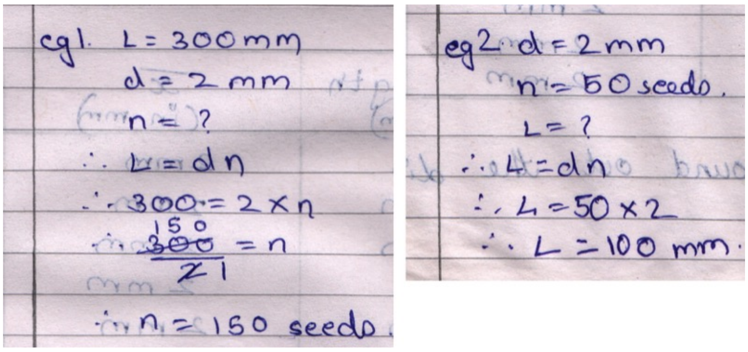


Figure 5.14: A response from a student on the *predicting* question. The students were asked to (1) find out the number of seeds if the length was 300, and (2) the length of 50 seeds.

The students who solved these questions used the algebraic method as shown in Figure 5.14, after forming the *mathematical model*. The results were in agreement between the methods and the actual observations, within the error margins. Though, not all of the students in that batch could answer the predicting question. The use of algebraic method was despite the fact that they could find required values from a graph. In another activity, we had given them the task to find squares/cubes and square-roots/cube-roots by plotting graphs of x^2 (during the classroom discussions) and finding the required values using intercepts. This activity was performed successfully (Figure 5.15) by most of the students.

5.6 Analysis of the reports

In this section, we discuss in detail the Section wise peculiarities of the reports submitted by the students. Not all of the students submitted the completed reports. Typically the size of the reports was 2-3 pages including a graph.

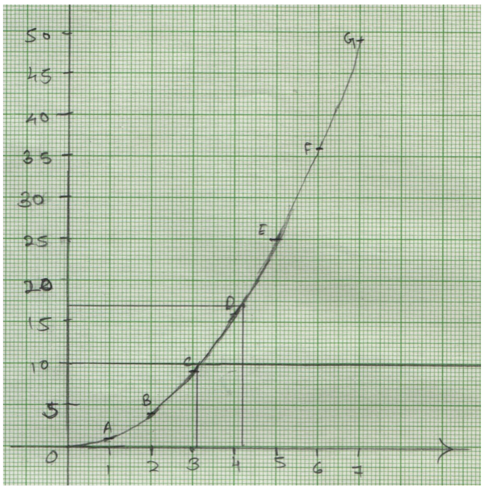
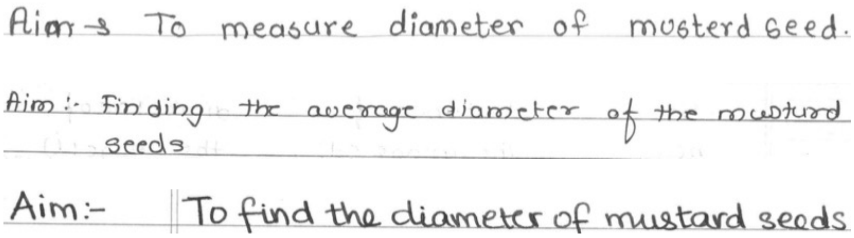


Figure 5.15: Finding of squares and square-roots by using the graphical method.

> **Please Note:** The images included from the student's reports in this section should be rather seen as running *text quotes* from the reports. Hence we have not numbered the images appearing in this section as Figures.

Aim: Most of the students had the aim of the activity as "Aim: To measure diameter of mustard seed" or "Aim: Finding the average diameter of the mustard seeds".

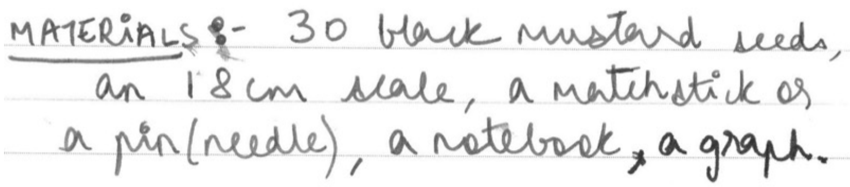


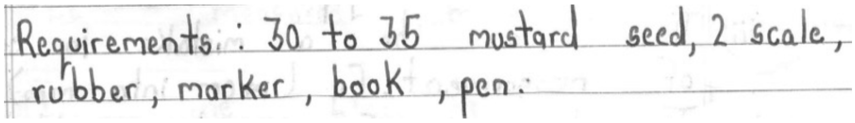
Aim :- To measure diameter of musterd seed.

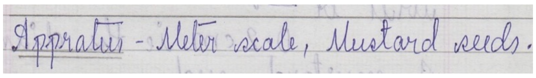
Aim :- Finding the average diameter of the musturd seeds

Aim :- To find the diameter of mustard seeds

Apparatus: In case of the "Apparatus " the students typically listed mustard seeds, (two) scale(s), white paper, Fevicol (an adhesive), pins, notebook, graph. Some of the students used, "Requirements" or "Materials Used" instead of Apparatus. A "notebook" was listed in the requirements as during the classroom discussions we had shown them use the folds of a book to align the seeds in a line without leaving gaps between them. Some of the students also listed the number of seeds that they used in the activity. Some of the students listed only mustard seeds and a scale.

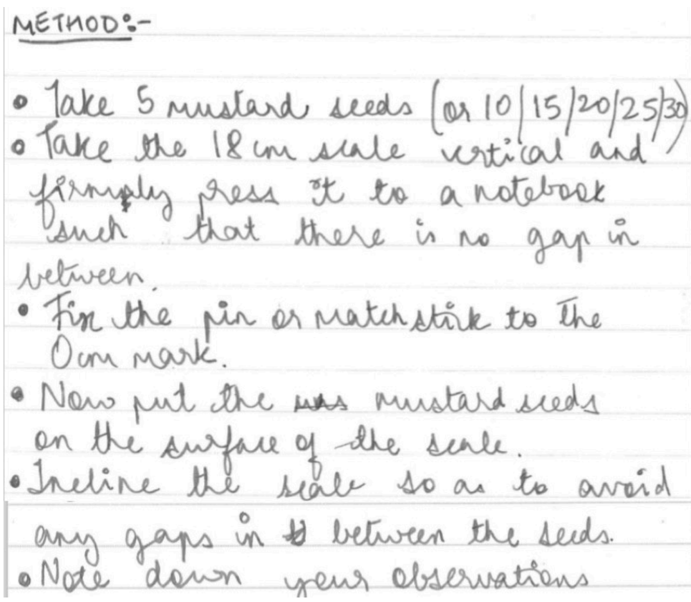

MATERIALS :- 30 black mustard seeds, an 18 cm scale, a matchstick or a pin (needle), a notebook, a graph.


Requirements : 30 to 35 mustard seed, 2 scale, rubber, marker, book, pen.


Apparatus - Meter scale, Mustard seeds.

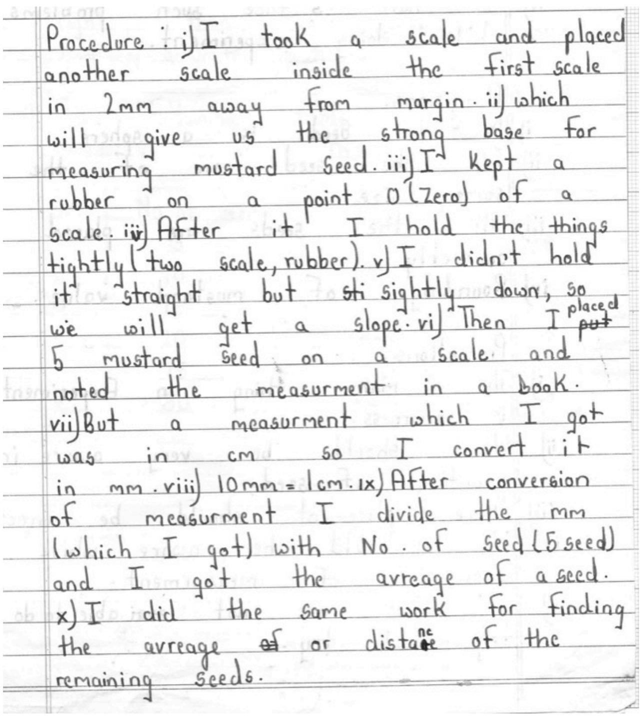
The difference in the list of materials used by the students was also indicative of the detail of the rest of the report. Typically, the students who had detailed list of materials also had detailed reports in other sections too.

Procedure: The students typically wrote how they did perform the activity. The students were told that "By reading your report, another person should be able to repeat the experiment." As a result of this, some of the students reported detailed and step-by-step procedures for performing the activity. For example, the report of student shown below also gives the rationale for each of the steps:


METHOD :-

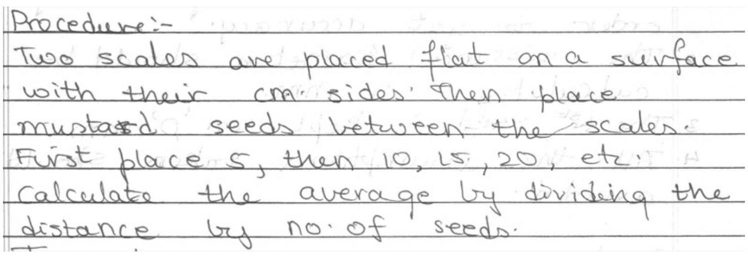
- Take 5 mustard seeds (or 10/15/20/25/30)
- Take the 18 cm scale vertical and firmly press it to a notebook such that there is no gap in between.
- Fix the pin or matchstick to the 0 cm mark.
- Now put the ~~was~~ mustard seeds on the surface of the scale.
- Incline the scale so as to avoid any gaps in ~~b~~ between the seeds.
- Note down your observations

Another student gives these steps for the procedure which are detailed:



Procedure : i) I took a scale and placed another scale inside the first scale in 2mm away from margin. ii) which will give us the strong base for measuring mustard seed. iii) I kept a rubber on a point 0 (Zero) of a scale. iv) After it I hold the things tightly (two scale, rubber). v) I didn't hold it straight but ~~sti~~ sightly down, so we will get a slope. vi) Then I ~~put~~ placed 5 mustard seed on a scale and noted the measurment in a book. vii) But a measurment which I got was in cm so I convert it in mm. viii) 10mm = 1cm. ix) After conversion of measurment I divide the mm (which I got) with No. of seed (5 seed) and I got the avreage of a seed. x) I did the same work for finding the avreage ~~of~~ or distance of the remaining seeds.

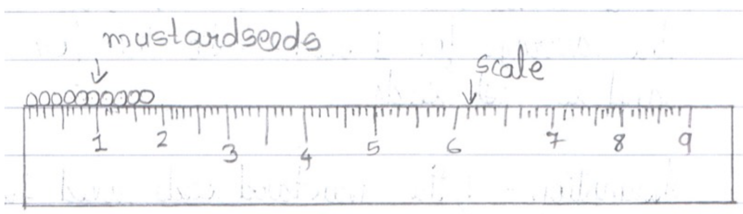
A few of the students gave very spartan descriptions of the procedures followed like the one quoted below:



Procedure:-
Two scales are placed flat on a surface with their cm sides. Then place mustard seeds between the scales. First place 5, then 10, 15, 20, etc. Calculate the average by dividing the distance by no. of seeds.

In the reports describing the "Procedure" the algebraic variables are being defined operationally: *counting* the *number of seeds* and *measuring* the *length of the aligned seeds.*

Diagrams: Most of the students did not draw a schematic or any other type of illustration in the report. The students who drew diagrams typically showed the positioning of the aligned seeds with respect to a scale like the one shown below:



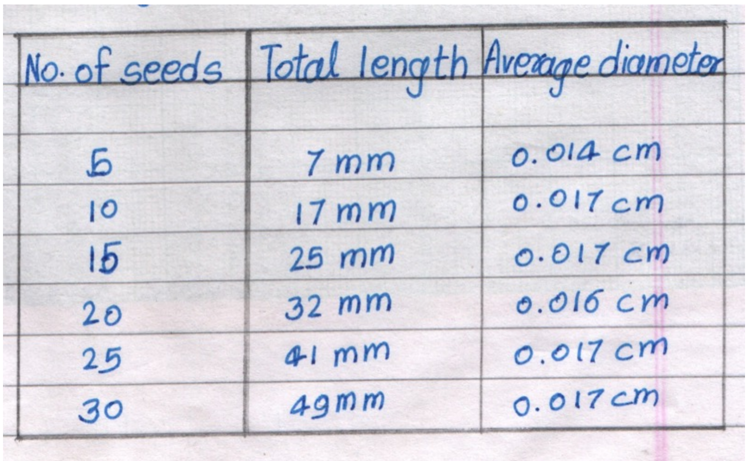
Another typical diagram drawn by the students is shown in Figure 5.8, where it is compared with a photo of the same setup. It was interesting to note that many of the students who drew schematic diagrams for the SM Task, did not draw one for the MS task. However, all of the students who drew schematic diagrams for the MS Task did draw one for SM task too.

Observations - Tables: All the students presented their observations in the form of tables. As described earlier the tables typically had columns containing the number of seeds, measured length of the number of seeds, and average diameter as calculated from these two. Almost all of the students used units (cm or mm) correctly and consistently. The table column head also has the numerical values of the algebraic variables used. The data table in this sense was a representation showing the manifestation of the algebraic variables in this activity. The tables are the first category of multiple representations that the students are using in the MS task. In the subsequent steps, the data from the table is used for creating the algebraic and the graphical representations.

Sr. No.	No. of mustard seeds	Distance (mm)	$\bar{x}$ (cm)
1	5	6	1.2
2	10	12.5	1.25
3	15	18	1.2
4	20	25	1.25
5	25	31.5	1.26
6	30	38.5	1.28
7	35	45	1.29
8	40	51	1.28
9	45	58.5	1.3
10	50	66	1.32

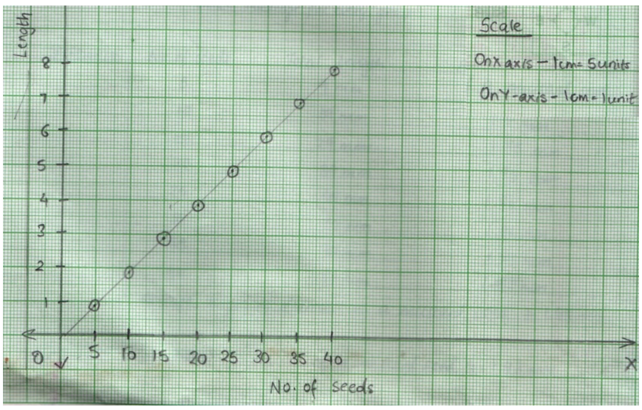
	No. of seeds	Distance (mm)	Diameter (in mm
1.	5	7	1.4
2.	10	9	9.
3.	15	28	2.8
4.	20	35	3.5
5.	25	47	4.7
6.	30	55	5.5

Only in very few cases (like the one shown below) we found confusion in use and conversion of units between (cm and mm).

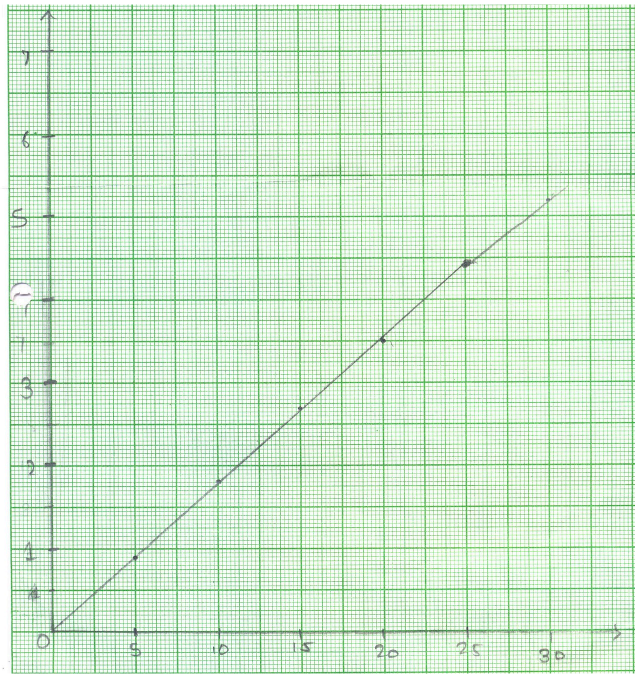


No. of seeds	Total length	Average diameter
5	7 mm	0.014 cm
10	17 mm	0.017 cm
15	25 mm	0.017 cm
20	32 mm	0.016 cm
25	41 mm	0.017 cm
30	49 mm	0.017 cm

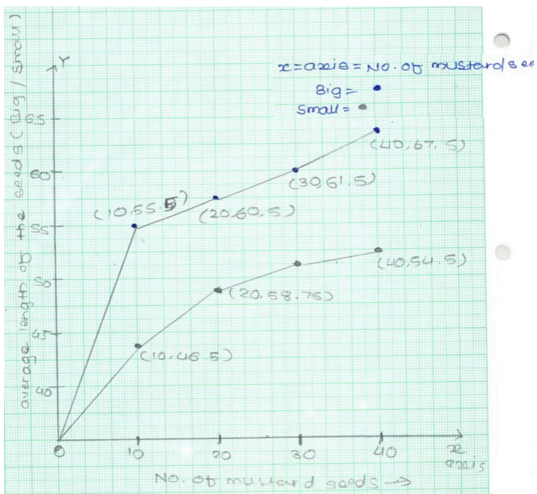
Observations - Graphs: Most of the students who made tables also created graphs. In general most of the students were able to plot the data points correctly on the graph. The students correctly chose the scales of the graph by themselves. Most of the students used graphs papers with 1 mm^2 squares to plot the graphs. A typical graph made by the students is shown below. Most of the students included the labels on their axes and provided the scale that they had used in plotting the graph.



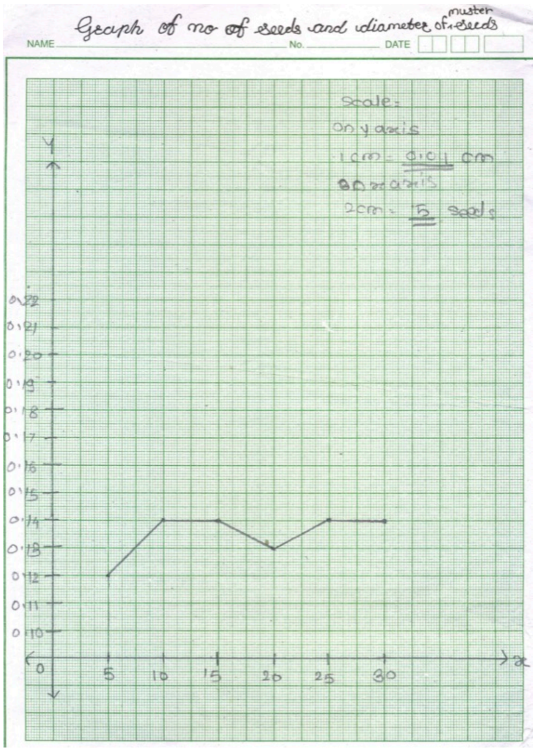
A few of the students plotted graphs on the ruled answering sheets by using the rules of the pages as a "grid" for the Y-axis. Some of the students did not give any label to the axes or any legend and just plotted the data as numbers.



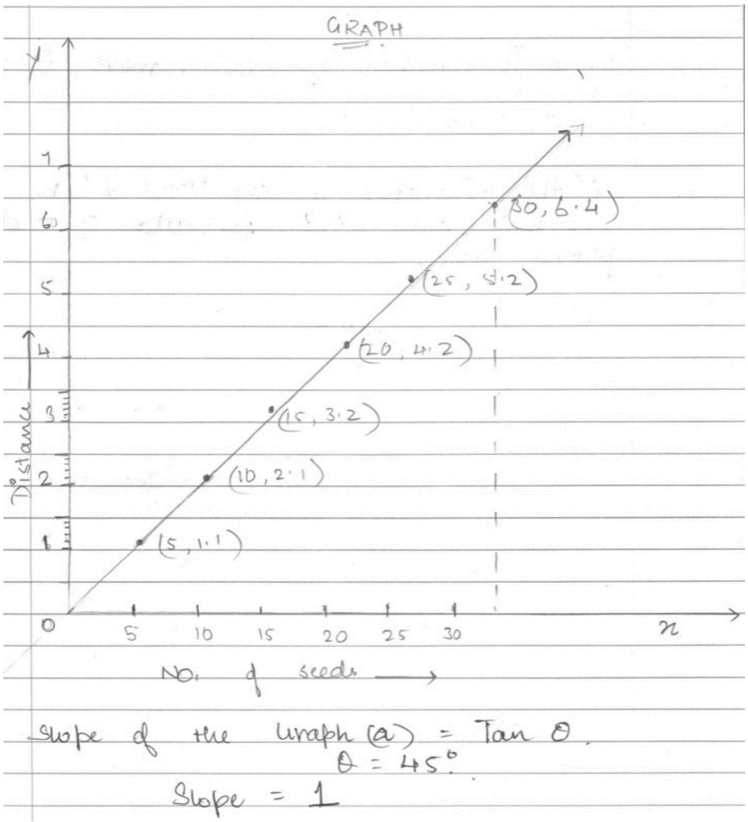
One student had access to both types of seeds and plotted the graph of both the seeds as shown below. The resulting graphs are not straight lines as expected, but show a rather peculiar "plateau". Though the data is provided on the graph as coordinates of points, we cannot be sure if these were the correct values that she obtained.



A few of the students plotted the *average* diameters that they obtained for different set of seed values.

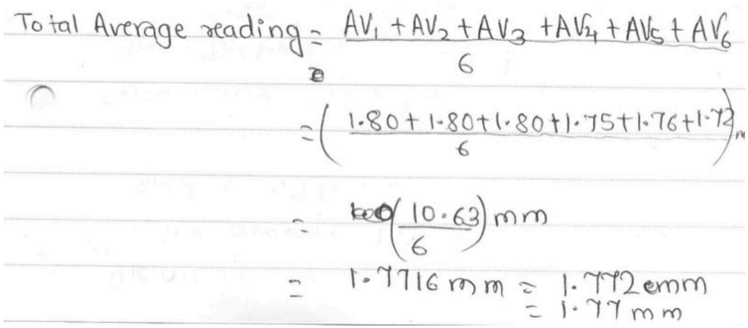


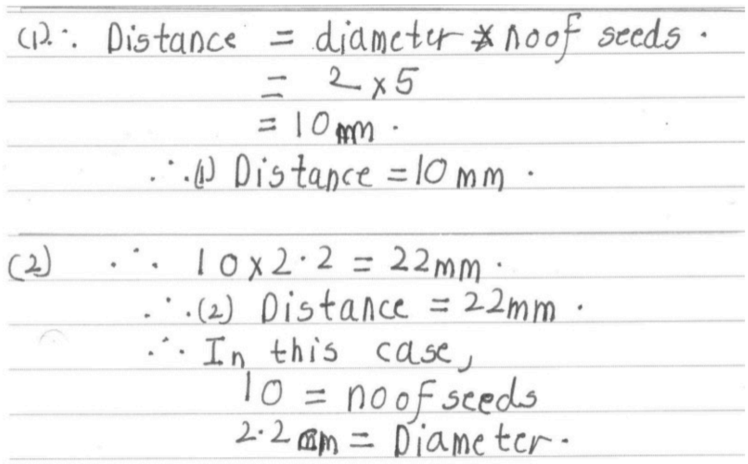
Only two exceptional students in the first batch actually could understand the implications of the plotting the graph and its relation to the mathematical and physical model, *before* the discussions in the classroom on the next day. Though more students understood the algebraic linear equation model, this perhaps points that linking of the graphical representation and the physical situation it is depicting does not come without effort. The classroom discussions that followed from the collaborative plotting helped the students to understand the physical concept of the slope of the graphs.



The use of graphs in understanding the physical situation and mathematical model based on it became the central theme in the classroom discussions on the next day. The graphs students made and their data was collected using a spreadsheet. The graphs resulting from the data thus collected was plotted using the dynamic mathematics software GeoGebra. The use of GeoGebra allowed exploration of the slope as an indicator of the physical size of the seeds measured.

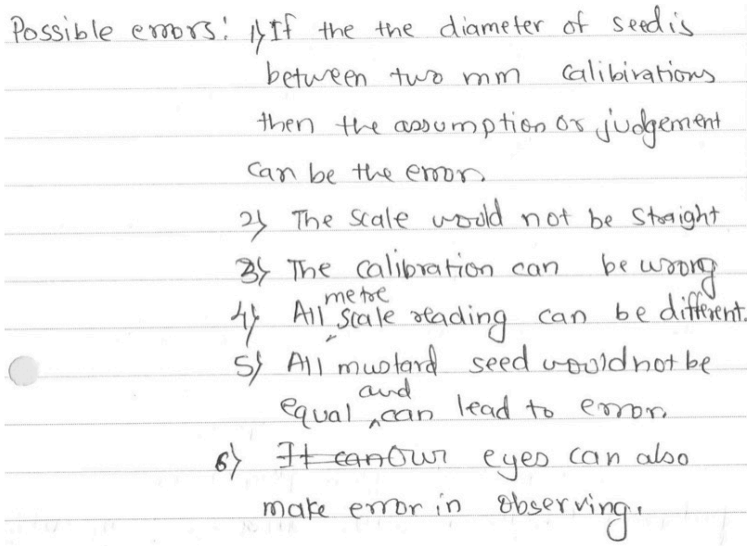
Calculations: Most of the students did the calculations beforehand, and only added the calculated values in their respective tables. In some cases, the students did separate calculations to calculate the average diameters.



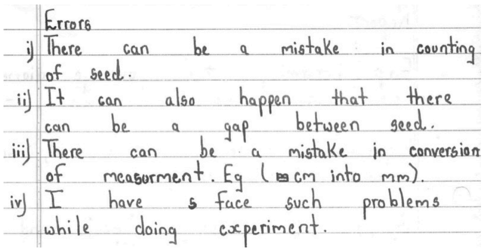

(1) ∴ Distance = diameter × no of seeds.
= 2 × 5
= 10 mm.
∴ (1) Distance = 10 mm.

(2) ∴ 10 × 2.2 = 22 mm.
∴ (2) Distance = 22 mm.
∴ In this case,
10 = no of seeds
2.2 mm = Diameter.

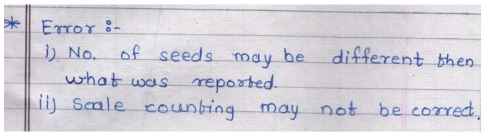
Errors The students reported various errors that they thought were relevant to the activity. Some of these were actual errors, while others were wrongly classified as errors. For example, look at the student response below:


Possible errors: 1) If the the diameter of seed is between two mm calibrations then the assumption or judgement can be the error.
2) The scale would not be straight
3) The calibration can be wrong
4) All metre scale reading can be different.
5) All mustard seed would not be equal and can lead to error.
6) ~~It can~~ Our eyes can also make error in observing.

Some of the students combined assumptions and the errors into one section, often confusing the two. The errors typically included (a) problems with the counting of seeds (this becomes especially relevant when the number of seeds is large, and one can easily lose the count of seeds), (b) the problem with gaps left between the seeds while aligning them, as this would lead to incorrect length being measured for given number of seeds, (various strategies were discussed to overcome this, some of them mentioned in the *Procedure* and the *Precautions* parts) (c) incorrect measurement resulting from incorrect alignment of the seeds with the measuring scale, (d) incorrect reading of the scale, (e) rounding off the measurement to the nearest whole number, and (f) incorrect measuring scale markings.

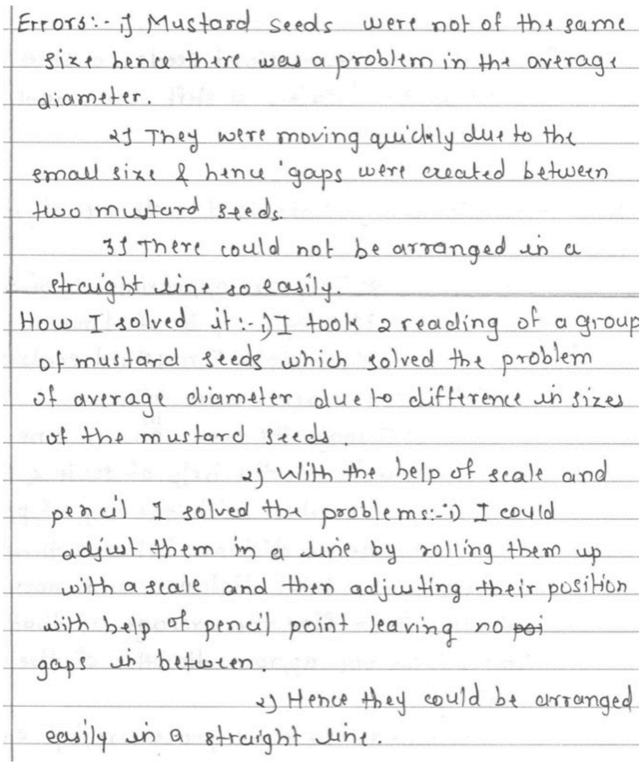


Errors
i) There can be a mistake in counting of seed.
ii) It can also happen that there can be a gap between seed.
iii) There can be a mistake in conversion of measurment. Eg (cm into mm).
iv) I have s face such problems while doing experiment.



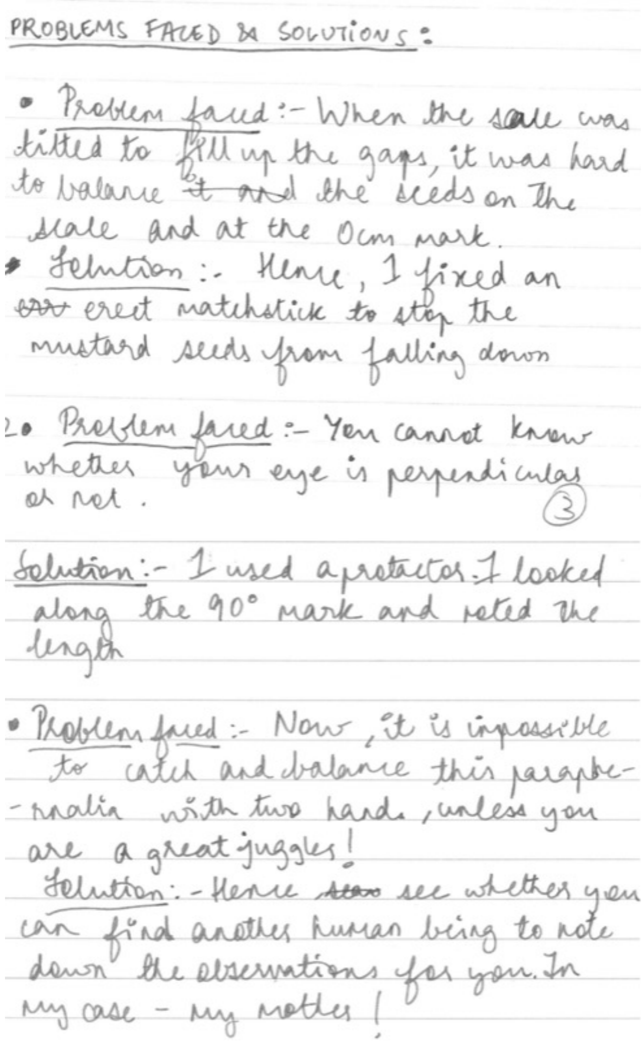
* Error :-
I) No. of seeds may be different then what was reported.
II) Scale counting may not be correct.

Very few of the students actually wrote in their reports about the problems that they faced and the ways in which they solved them:



Errors:- 1) Mustard seeds were not of the same size hence there was a problem in the average diameter.
2) They were moving quickly due to the small size & hence 'gaps were created between two mustard seeds.
3) There could not be arranged in a straight line so easily.
How I solved it:- 1) I took a reading of a group of mustard seeds which solved the problem of average diameter due to difference in sizes of the mustard seeds.
2) With the help of scale and pencil I solved the problems:- 1) I could adjust them in a line by rolling them up with a scale and then adjusting their position with help of pencil point leaving no ~~poi~~ gaps in between.
2) Hence they could be arranged easily in a straight line.

Another student provided a first-person account of how she dealt with the problems faced while performing the activity.


PROBLEMS FACED ~~&~~ SOLUTIONS :

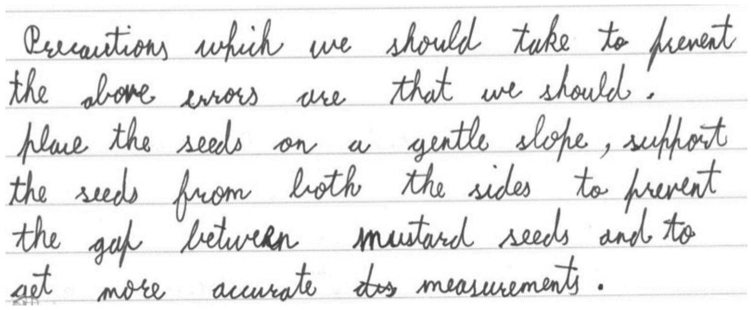
• Problem faced :- When the scale was tilted to fill up the gaps, it was hard to balance ~~it and~~ the seeds on the scale and at the 0cm mark.
• Solution :- Hence, I fixed an ~~...~~ erect matchstick to stop the mustard seeds from falling down

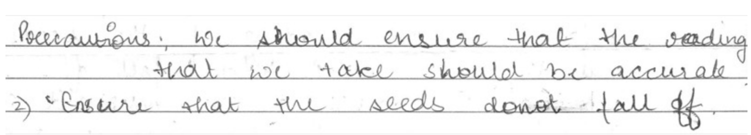
2• Problem faced :- You cannot know whether your eye is perpendicular or not. (3)

Solution :- I used a protactor. I looked along the 90° mark and noted the length

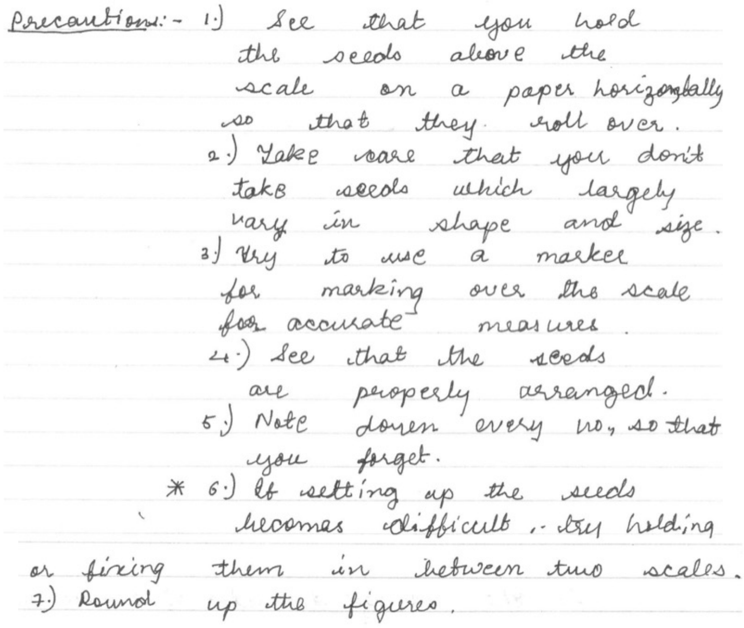
• Problem faced :- Now, it is impossible to catch and balance this paraphe-rnalia with two hands, unless you are a great juggler!
Solution :- Hence ~~...~~ see whether you can find another human being to note down the observations for you. In my case - my mother!

Precautions: The "Precautions" listed in the reports were ways to overcome the "Errors" reported above. Many times the students combined Errors and Precautions into a single section. Most of the precautions that the students reported related to the proper alignment of the seeds, without any gaps.

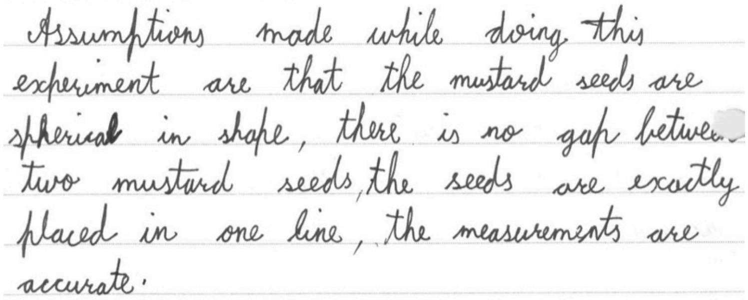

Precautions which we should take to prevent the above errors are that we should: place the seeds on a gentle slope, support the seeds from both the sides to prevent the gap between mustard seeds and to get more accurate ~~...~~ measurements.

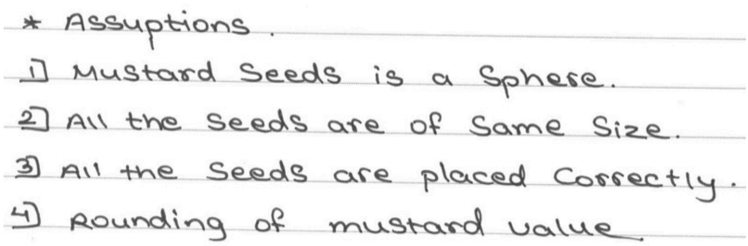

Precautions: We should ensure that the reading that we take should be accurate
2) Ensure that the seeds donot fall off.

Some of the students gave detailed precautions like the one below, which also deal with finer details of experimentation.

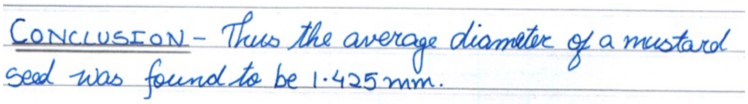

Precautions:- 1.) See that you hold the seeds above the scale on a paper horizontally so that they roll over.
2.) Take care that you don't take seeds which largely vary in shape and size.
3.) Try to use a marker for marking over the scale for accurate measures.
4.) See that the seeds are properly arranged.
5.) Note down every no. so that you forget.
* 6.) If setting up the seeds becomes difficult, try holding or fixing them in between two scales.
7.) Round up the figures.

Assumptions: During the classroom discussions we touched upon the idea of assumptions that we were making while creating the mathematical model. Typically the assumption that most of the students reported were: (a) the seeds are spherical in shape, (b) the seeds are all of the same size, (c) the scales that were used are accurate, (d) seeds were arranged (aligned) accurately, (e) the measurements are correct and (f) rounding to the closest marking on the scale is done correctly.

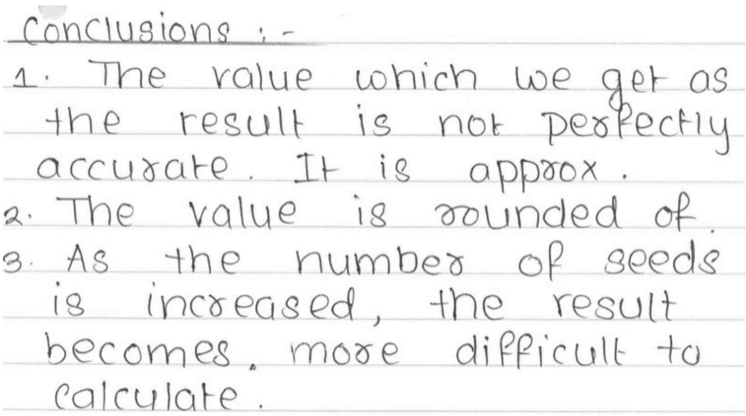

Assumptions made while doing this experiment are that the mustard seeds are spherical in shape, there is no gap between two mustard seeds, the seeds are exactly placed in one line, the measurements are accurate.

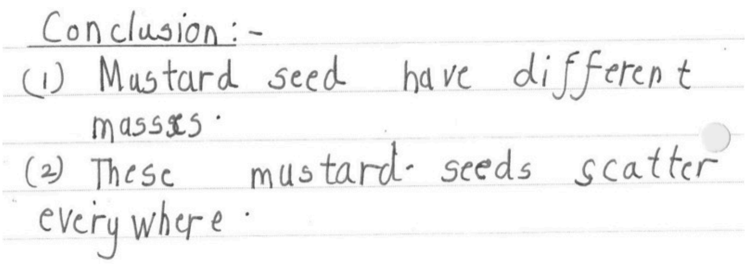

* Assuptions.
1] Mustard Seeds is a Sphere.
2] All the Seeds are of Same Size.
3] All the Seeds are placed Correctly.
4] Rounding of mustard value.

Conclusions: Most of the students who submitted the reports got the numerical results correctly. But many of them wrote different things in the "Conclusions" section. While some of the students wrote straight forward conclusions like the one below:

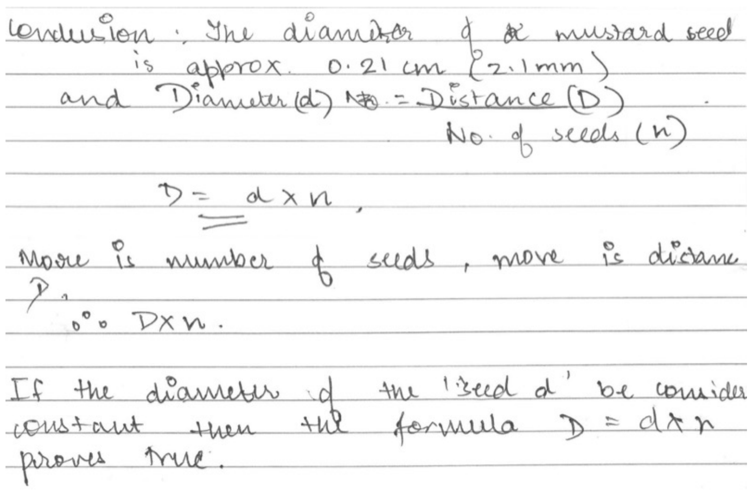

CONCLUSION – Thus the average diameter of a mustard seed was found to be 1·425 mm.

Some of the students wrote different types of conclusions, which did not include a numerical answer (though they had calculated the answer).


Conclusions :-
1. The value which we get as the result is not perfectly accurate. It is approx.
2. The value is rounded of.
3. As the number of seeds is increased, the result becomes more difficult to calculate.


Conclusion :-
(1) Mustard seed have different massses.
(2) These mustard seeds scatter every where.

Only very few of the students did the mathematical modelling part using the proportionality arguments in their conclusion.


Conclusion : The diameter of a mustard seed is approx. 0.21 cm (2.1 mm)
and Diameter (d) = Distance (D) / No. of seeds (n)

D = d × n

More is number of seeds, more is distance D.
∴ D ∝ n.

If the diameter of the 'seed d' be consider constant then the formula D = d × n proves true.

The writing about an experience can be seen as a *third-order* experience for the writer, the first two orders being thinking and talking to another person respectively (Dix, 2006). The writing of the reports, especially the descriptive parts of the hands-on components made the experiences of the students richer. The student reports discussed here show the variety of responses the students had for the task. Though seemingly simple, not all of the students completed the task successfully. However, the task was immersive, and the students could understand what was to be done "Measuring the average diameter of the mustard seeds". The hands-on component which included "performing" an "experiment" provided an added impetus to the task.

5.7 Discussions

As we have seen earlier, *graphicacy* is defined as the "ability to understand and present information in the form of sketches, photographs, diagrams, maps, plans, charts, graphs and other non-textual, two-dimensional formats" (Aldrich & Sheppard, 2000). The MS task described here can be seen as the first step towards larger work on developing critical graphicacy skills. During this activity, the students (a) first engaged with the problem to be solved, (b) formed a mathematical model to describe the situation, and, (c) finally collected and analysed data to solve it.

In *Critical Graphicacy* Roth et al. (2005) take a stance that "our aim as critical educators is not just the provision of opportunities for the students to become graphically literate; rather, we want the students to develop critical graphicacy, that is, we want them to become literate in constructing and deconstructing inscriptions, the deployment of which is always inherently political ” (2005, p. xxviii). In our study on Indian textbooks, reported in Chapter 3, we found that the presence of graphs is limited and opportunities to use them critically are almost non-existent. In such a scenario it is important that such opportunities be provided to the students. In the MS task, the emphasis was on using and understanding graphs in the context of mathematical modelling of real-world data and measurements. The MS task can be seen as the first step in the direction of making the students graphically critical and literate. The MS task also provides a concrete context for the students to use multiple symbolic systems (tabular, graphical and algebraic in this case) and to understand the relationships between them.

Context of the activity: The context of situating the activity is significant in developing and deriving abstract mathematical ideas (Janvier & Bednarz, 1989). In the MS Task, the context was that of estimating the size of an everyday object (the mustard seeds). The simplicity of the task, set in a familiar context, made the task tangible to all. In this case, the context allowed the learners to make connections between abstract and concrete, as also pointed by Janvier & Bednarz (1989). The success MS task was also dependent on the choice of the mustard seeds, and the natural variation among them (Figure 5.4). Due to the natural difference in the sizes, the graph obtained by collating the values of all the students gave two *distinct* lines. This fact helped in further establishing the connection between the concrete (size of the seed) and the abstract (slope of the line) in this context. The MS task can be further expanded by measuring other seeds (like different grains) or other objects by the same method and plotting the resulting values on the same graph. By doing this, the resultant graph will clearly show the variation in the size of the objects as lines with *different* slopes. Such a graph would further help establish the applicability of indirect measurement in the real-world.

Prior knowledge: As seen from the cognitive framework of understanding graphs, the prior knowledge of the students about graphs, as well as about the subject of study plays an important role (Shah & Hoeffner, 2002). The students already had prior ideas about direct measurements, plotting graphs, calculating averages, direct proportion, linear equations, and some methods of indirect measurement. As evidenced by the classroom discussions and the reports, the students used these concepts in a concrete, real-world problem-solving context. Without the students having these skills and concepts, the activity would not be a success.

Mathematical modelling: The physical situation, in this case, was modelled using a linear equation in the form of $y = m \times x$. Though linear equations are one of the first and the simplest mathematical functions that the students learn, studies have shown that the students have difficulties in understanding the interpretation of the slope and the intercept (Chiu et al., 2001; Schoenfeld et al., 1993). The mathematical model was established in this case using an intuitive and physical approach, with an appeal to direct observations of the mustard seeds. In the MS task, through the use of a contextualised problem, we have given the students an opportunity to attach physical meaning to the variables in a linear equation. The simplicity of the model, as well as that of the context, helped to make these connections.

Measurement, real-world data and data handling: The MS task involved relatively simple measurement of real-world data using a scale and an everyday object (the mustard seed). However, even a simple collection of real-world data and its handling can be a rich learning experience (Curcio, 1987; Wavering, 1989). As we have witnessed the MS task indeed was a vibrant learning experience for the students. The idea that they can perform "experiments" to collect and analyse data was substantiated by the MS task and the SM task. In the sociological framework of graph comprehension, the knowledge about the way in which data is collected plays an important role in meaning making with graphs plotted with that data (Roth, 2004).

Multiple representations: The MS task is a rich context for the students to appreciate and develop the idea of multiple representations. The ability to move between different representations of the same data is not easy (Moschkovich et al., 1993). However, the movement between the representations of the same data "allows one to see rich relationships, develop a better conceptual understanding, broaden and deepen one's understanding, and strengthen one's ability to solve problems." (Even, 1998). This is also essential for understanding the concrete (physical) and the abstract (mathematical) relationship. In the MS task, the physical measurement on the mustard seeds led to data in the form of observation tables. From the data in these tables, the student's calculated required quantities, formed equations and constructed graphs. Bell & Janvier (1981) argue that: " . . . the use of tables proved a powerful tool to study "how variables change". The results conclusively show that the table approach certainly spelt out many ideas to the extent of making possible transfers from tables to graphs. Consequently, results suggest that the use of tables should be included in our graph teaching scheme." (p. 41) Most of the students could make this connection of taking data from tabular representation and convert it into a graphical representation. The data was also presented in an algebraic representation in the form of a linear equation. In the MS task, the students always had the concrete physical situation while collecting and handling the data. The writing of reports gave the students space to reflect on what they had done, how had they done it. As writing is seen as a third-order experience (Dix, 2006), verbalising the experience helped to establish the concreteness of the context in yet another representation.

Graphs: Graphs in the MS Task played several roles and were connected to many processes as components. We found that most of the students did have the ability to construct graphs from tabular data. The interpretation of the features of the graph was brought in through the collaborative plotting of graphs, and classroom discussions. We use the points made by (Monk, 2003), seen earlier in the discussion in (the list can be seen on page 131 here). Particularly three points from this list are relevant to the MS task.

(a) *Using a graph to analyze a well-understood context can deepen a student's understanding of graph and graphing.* In the context of MS task, the concrete, physical nature of the context and of data collection, leads to an understanding of graphs and graphing.

(b) *Students can construct new entities and concepts in a context by beginning with important features of a graph.* In the context of the MS task, the discussions on the formation of the mathematical model led to establishing the physical meaning of the slope of the graph as the average diameter of the seeds.

(c) *A group can build shared understanding through joint reference to the graph of the phenomena in a context.* The collaborative plotting of the combined data led to a shared understanding of the physical aspect of the situation and its manifestation in the form of the slope of the graph. The collaborative plotting also allowed the students to access the larger and varied data sets of the peers. It was this variation which was crucial in establishing the concrete-abstract relationship.

Limitations and further work: One of the limitations of this study was the limited use of dynamic mathematics software only by the researcher for demonstrations and plotting the data collaboratively. It would be interesting to see how the students perform when they are given access to computers for plotting the data. In the future, we would like to repeat the MS task when all the students are provided with access to computers. Another limitation was the task not carried out in a real classroom setting, so it would be interesting to see how a teacher in a real classroom transacts this task. The MS task could be extended by using measurements from a variety of everyday objects with varied sizes and collating data sets from a large number of students.

Though the task and the model were simple, not all the students could come close to the expected result. Some of the students could not go on to make the mathematical model on the first day. Only after the classroom discussions on the second day they could do so. This occurrence perhaps points to the fact that bridging the gap between abstract mathematical knowledge and the real-world is not trivial. By making the students aware of the fact that the same mathematical model can describe different objects, one can perhaps hope to overcome this problem.

It is vital to bring to the classroom tasks, which are simple, but rich enough to raise discussions of several interrelated concepts in a close-to-life context. Other tasks such as measuring the thickness of paper or the diameter of a thread can be done in continuation to this task. This would emphasise the power of mathematical modelling to the students: that using the *same* general linear model, we can model systems which are *not* similar to each other. The next chapter (SM task) focuses on a similar task, in which the same linear model is used to solve a problem in a completely different context and scale, approached by a different pathway. A few such experiments can act as a springboard to scientific modelling and would help the students find the links between the models and the real-world.

This part of the dissertation was first presented as a paper titled "**Measuring the mustard seed: A first exercise in mathematical modelling**" in epiSTEME 5 Conference in Mumbai (Dhakulkar, Dhurde & Nagarjuna, 2013) conference and subsequently published as a journal article titled "**Measuring the mustard seed: an exercise in indirect measurement and mathematical modelling**" in *School Science Review* (Dhakulkar, Dhurde & Nagarjuna, 2015).

6

Measuring the distance to the Sun

In the activity reported in this chapter, the students estimate the ratio of the diameter of the Sun to its distance from Earth, the Sun Measurement (SM) activity. The SM activity revolves around indirect measurement on an astronomical scale with the use of geometry. The setting of the class, the sample and the context for the SM activity is the same as Mustard Seed (MS) activity. The SM activity is complex than the MS activity, with higher mathematical concepts and constructions required for completing the task. As in the MS activity, we shall first discuss the essential mathematical, astronomical and science process background required for this activity as covered in the textbooks until grade 8 and 9. We then discuss the workflow of the activity and detail various steps with the background of the design framework described in the Chapter 4. After this, the responses of the students and the problems they faced are discussed in the context of how it is a step towards mathematical modelling using a real-world setting. Finally, we discuss the results, implications and limitations of the activity.

6.1 Introduction

The Sun measurement task is more involved than the mustard seed task. In the Pre-Test we asked students the question: "How can we measure the distance to the Sun?" The classroom discussions based on the answers of the students led to questions pertaining to understanding how we can do another type of indirect measurement.

For example, some students used the idea of **speed** = **distance** × **time**, knowing the speed of light and time light takes to reach us from the Sun, the distance can be found out. However, when such responses were probed a bit deeper, by asking "How do we know that it takes about 8 minutes for the light to reach us from the Sun?" the students could not answer. Some students, used a "rocket ship" to find out the distance, just like we can find the terrestrial distance using a car. The answers themselves gave insights into the way students think. The role of prior knowledge in the student's attempts at problem-solving became clearer. The students did not associate the geometry they had learned in the context of triangles for real-world measurement. This observation perhaps implies that the knowledge transfer across domains does not happen without effort, but may happen when an explicit effort is made towards achieving it. Such a core aspect of science should become an explicit learning objective of science teaching.

The reports of the students contain a detailed account of the activity. These reports contain the method, procedure, observations and results from the activity. The data was represented in the form of a table, in the form of an equation and in the form of a graph. The connection between various representations that the students are making can be seen in these reports. The slope of the graph in this case also is concretely connected to the solution problem. However, the interpretation of the slope is different in this case. This result is verified by algebraic means also. This activity establishes a rich learning experience for the students, addressing in part, development objective ②. The use of technology is essential but is limited in this case.

In SM task, we asked the students to measure a fundamental quantity regarding the Sun: *the distance to diameter ratio*. This ratio was known since ancient times, for example, see the discussion in (Rogers, 1960). This ratio is manifested to us in the form of the angular diameter of the Sun in the sky which was known to us historically. Knowing the current estimates of distance and the diameter, we can calculate backwards, the angular size of the Sun. The mean distance to the Sun according to the present calculations is approximately 1.496×10^8 km (D), and the diameter of the Sun is 1.39×10^6 km (d). This makes the ratio of distance to diameter equal to

$$\frac{D}{d} = \frac{1.496 \times 10^8 \, \text{km}}{1.391 \times 10^6 \, \text{km}} = 107.54 \approx 110$$

The angular diameter in radians (δ) then can be calculated using the formula

$$\begin{aligned} \delta &= 2 \arctan \left(\frac{d}{2D} \right) = 2 \arctan \frac{1}{2 \times 107.54} \\ &= 2 \arctan 0.0046 = 0.0092 \, \text{radians} \\ &\approx 0.52^\circ \end{aligned}$$

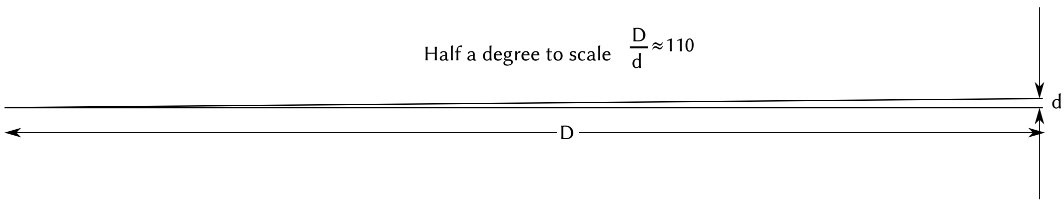


Figure 6.1: An illustration showing the measure of half-a-degree. The longer sides are about 110 times the base where the arrows are depicted.

In this drawing, we have kept the ratio of 110 between the sides (length D), moreover, the base (length d), giving the resulting angle of approximately 0.5°. Measuring the ratio $\left(\frac{d}{2D} \right)$ is relatively easy, but measuring the actual distance and diameter is not. We had to wait till 17th century to find the accurate values of the quantities in this ratio. Though there were methods, for example, that of Aristarchus (Hirshfeld, 2004), from antiquity to find the absolute value of the distance. The observations based on the transit of Venus gave us the first definitive values of the distance to the Sun, see for example series of books which deal with this topic in detail (Anderson, 2012; Lomb, 2012; Wulf, 2012).

6.2 Review of the textbook topics

The ideas about astronomy are introduced to the students in various classes using the school textbooks. Each of the chapters in textbooks essentially contains a large amount of factual information. The Solar System and its constituents form a substantial portion of the syllabus. We discuss the content of both NCERT and Maharashtra Board (MHB) textbooks in this section which are pre-requisites for the SM activity.

NCERT Textbooks

We review the content from three NCERT textbooks in the context of the astronomical background.

Class 3 Geography: The Class 3 Geography textbook introduces the Sun and the stars in *Chapter 1 Let Us Observe The Sky* and in *Chapter 2, This is How the Earth Moves!* it discusses the movement of the Earth around the Sun.

Class 6 Science: Basic knowledge about the solar system its members can be found in the first Chapter *The Earth In The Solar System* in Class 6 textbook of Geography and Class 8 textbook of Science. In the first case, the information is part of an infographic which shows the *Solar System* (Figure 6.2.)

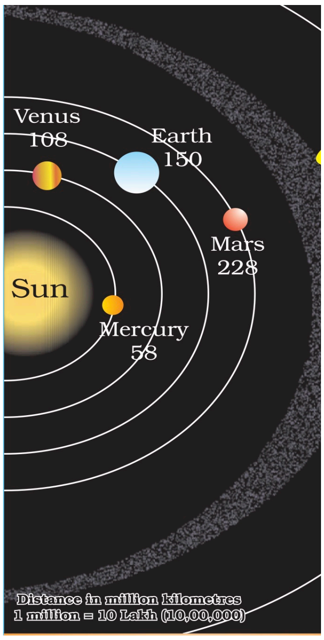


Figure 6.2: Factual information about the distance to the Sun in the textbook. The information is not given directly, but students have to multiply '150' by '1 million' to find the actual distance. This is a cropped image from an infographic depicting distance and other information about members of the Solar System. (Class 6 Geography textbook, Chapter 1, p. 3).

Class 8 Science: Further in Class 8, in the Chapter on *Stars and the Solar System* there is similar factual information about the Solar System. We noticed in one of the tasks during the 5-day long camp, the students usually reproduced such factual information very well. For example, when asked about what do they know about the *Solar System*, the students usually produced a list of factually correct points. Figure 6.4 shows a typical response from the students.

> The Sun is nearly 150,000,000 kilometres (150 million km) away from the Earth.
>
> The next nearest star is Alpha Centauri. It is at a distance of about 40,000,000,000,000 km from the Earth. Can you read this distance in kilometres conveniently? Some stars are even further away.
>
> Such large distances are expressed in another unit known as **light year**. It is the distance travelled by light in one year. Remember that the speed of light is about 300,000 km per second. Thus, the distance of the Sun from the Earth may be said to be about 8 light minutes. The distance of Alpha Centauri is about 4.3 light years.

Figure 6.3: The distance to the Sun is given as information in a box. From NCERT Class 8, Science, Chapter 17, p. 222)

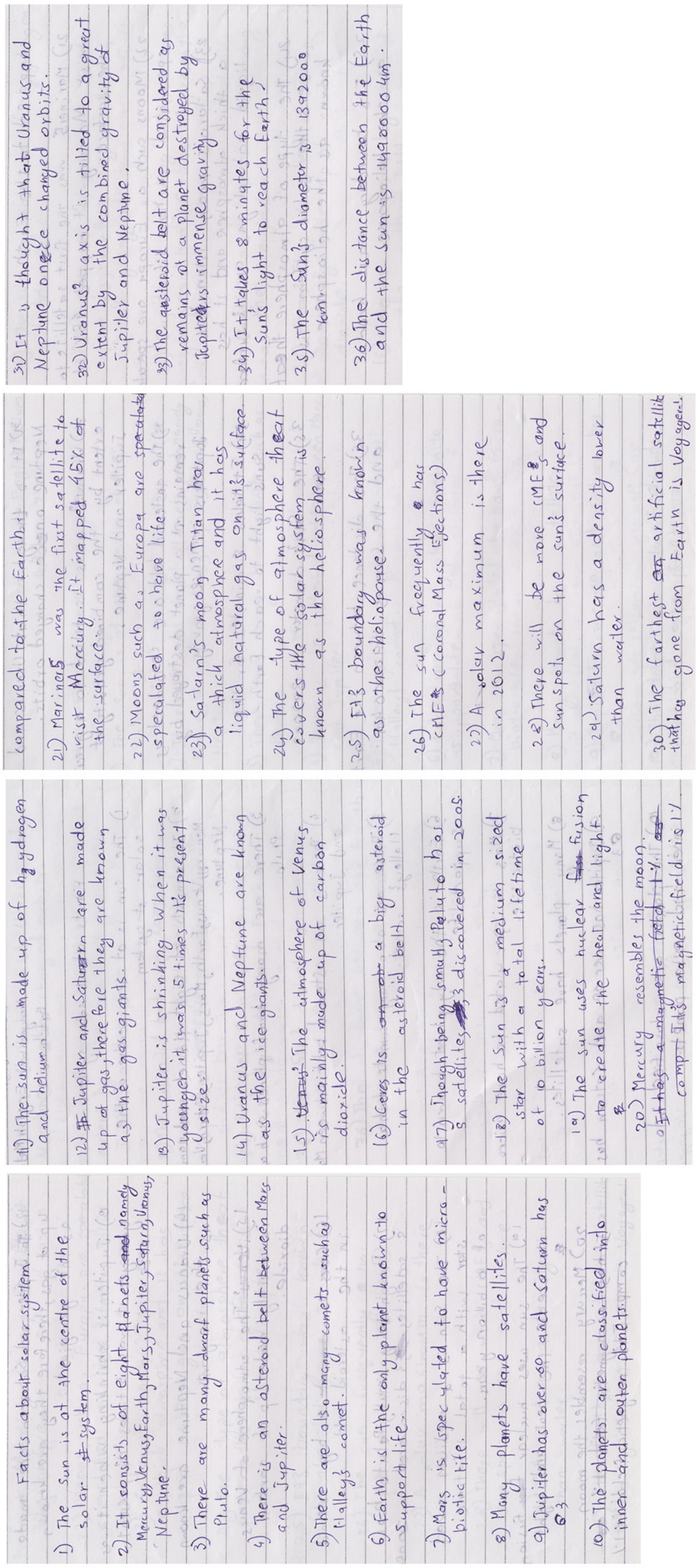
Maharashtra Board Textbooks

In case of Maharashtra Board textbooks, the topic of astronomy occurs in both Geography and Science textbooks.

Class 6 Geography: In Class 6 Geography textbook, a lot of factual information about the solar system can be found in Chapter 1 *The Solar System*. While in Chapter 2 *Motions of the Earth and their Effects* the distance to the Sun is introduced with the concepts of *aphelion* and *perihelion* with the aid of a diagram (Figure 6.5).

Class 8 Science: In Class 8 the textbook on Science has Chapter 1 *Stars and Our Solar System* which has some factual information about the Sun as shown in Figure 6.6.

Though there are some activities based on gestures and bodily movements depicting the solar system, there are no activities with *quantitative* measurements. Another example in this case which might have influenced the students answers



Facts about solar system.

1) The sun is at the centre of the solar system.
2) It consists of eight planets namely Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune.
3) There are many dwarf planets such as Pluto.
4) There is an asteroid belt between Mars and Jupiter.
5) There are also many comets such as Halley's comet.
6) Earth is the only planet known to support life.
7) Mars is speculated to have micro-biotic life.
8) Many planets have satellites.
9) Jupiter has over 60 and Saturn has 63
10) The planets are classified into inner and outer planets.
11) The sun is made up of hydrogen and helium.
12) Jupiter and Saturn are made up of gas, therefore they are known as the gas giants.
13) Jupiter is shrinking. When it was younger it was 5 times its present size.
14) Uranus and Neptune are known as the ice giants.
15) The atmosphere of Venus is mainly made up of carbon dioxide.
16) Ceres is a big asteroid in the asteroid belt.
17) Though being small, Pluto has 5 satellites, 3 discovered in 2005.
18) The Sun is a medium sized star with a total lifetime of 10 billion years.
19) The sun uses nuclear fusion to create the heat and light.
20) Mercury resembles the moon. It's magnetic field is 1% compared to the Earth's.
21) Mariner 5 was the first satellite to visit Mercury. It mapped 45% of the surface.
22) Moons such as Europa are speculated to have life.
23) Saturn's moon Titan has a thick atmosphere and it has liquid natural gas on its surface.
24) The type of atmosphere that covers the solar system is known as the heliosphere.
25) Its boundary was known as the heliopause.
26) The sun frequently has CMEs (Coronal Mass Ejections)
27) A solar maximum is there in 2012.
28) There will be more CMEs and sunspots on the sun's surface.
29) Saturn has a density lower than water.
30) The farthest artificial satellite that has gone from Earth is Voyager.
31) It is thought that Uranus and Neptune once changed orbits.
32) Uranus' axis is tilted to a great extent by the combined gravity of Jupiter and Neptune.
33) The asteroid belt are considered as remains of a planet destroyed by Jupiter's immense gravity.
34) It takes 8 minutes for the sun's light to reach Earth.
35) The Sun's diameter is 1392000 km.
36) The distance between the Earth and the Sun is 149000000 km.

Figure 6.4: Factual information about the Solar System given by one of the student, other students' responses were in similar to this.

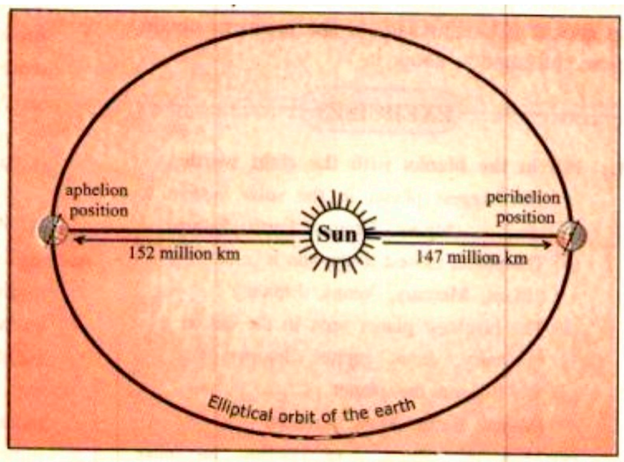
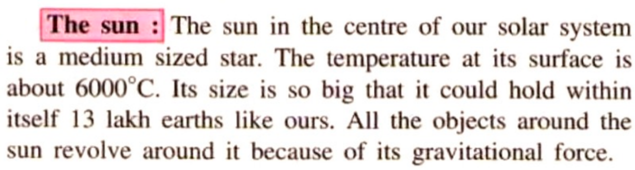


Figure 2.1 : Perihelion and aphelion positions

Figure 6.5: A figure depicting the distance of the Sun from the Earth during aphelion and perihelion. The figure is highly distorted and not to scale, but this is not mentioned in the text. From Maharastra Board Class 6, Geography p. 4.



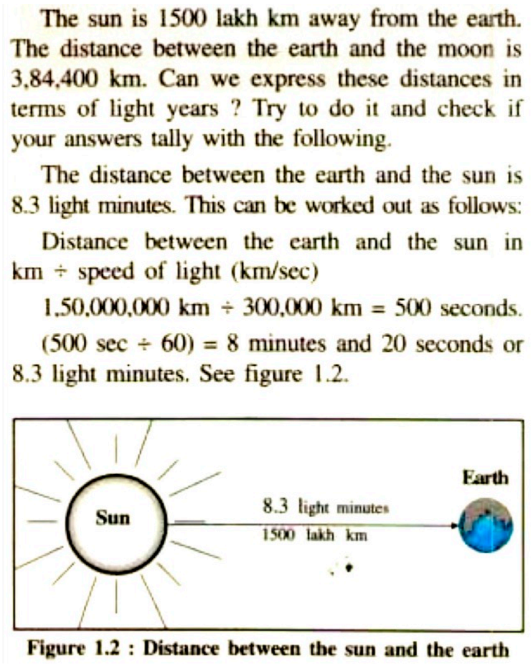
The sun : The sun in the centre of our solar system is a medium sized star. The temperature at its surface is about 6000°C. Its size is so big that it could hold within itself 13 lakh earths like ours. All the objects around the sun revolve around it because of its gravitational force.

Figure 6.6: Factual information about the Sun in Science Textbook. From Maharastra Board Class 8, Science p. 6.

in this regard (analysed in the next section) is the illustration shown in Figure 6.7. This illustration and the accompanying text shows the calculation for the distance between the Earth and the Sun in terms of light minutes.

Finally, the other aspect of the task a pinhole camera makes its appearance in Class 7 Science textbook with an accompanied activity (Figure 6.8). In the SM task, the students have to construct a pinhole camera in a cardboard box and observe the resulting image inside the box.

In case of the NCERT textbooks, the pinhole camera has much more detailed information (Figure 6.9). The activity describes a construction of the pinhole camera and to note observations using it. Another activity described is very close to the Sun Measurement Task that we are describing (shown with red rectangle in Figure 6.9. This activity involves observing the Sun through a cardboard but does not require any quantitative measurements.



The sun is 1500 lakh km away from the earth. The distance between the earth and the moon is 3,84,400 km. Can we express these distances in terms of light years ? Try to do it and check if your answers tally with the following.

The distance between the earth and the sun is 8.3 light minutes. This can be worked out as follows:

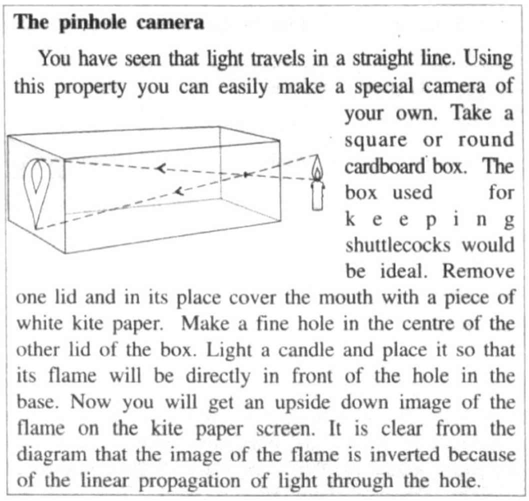
Distance between the earth and the sun in km ÷ speed of light (km/sec)

1,50,000,000 km ÷ 300,000 km = 500 seconds.

(500 sec ÷ 60) = 8 minutes and 20 seconds or 8.3 light minutes. See figure 1.2.

Figure 1.2 : Distance between the sun and the earth

Figure 6.7: Factual information about the Sun in Science Textbook. The text starts from fact that the Sun being 1500 lakh km away, and then proceeds to calculate the distance in terms of light minutes. From Maharastra Board Class 7, Science p. 1.



The pinhole camera

You have seen that light travels in a straight line. Using this property you can easily make a special camera of your own. Take a square or round cardboard box. The box used for keeping shuttlecocks would be ideal. Remove one lid and in its place cover the mouth with a piece of white kite paper. Make a fine hole in the centre of the other lid of the box. Light a candle and place it so that its flame will be directly in front of the hole in the base. Now you will get an upside down image of the flame on the kite paper screen. It is clear from the diagram that the image of the flame is inverted because of the linear propagation of light through the hole.

Figure 6.8: Information about pinhole camera and associated activity in the Maharashtra Board Class 7 Science textbook. The text starts from From Maharastra Board Class 7, Science p. 1.

Mathematical Background

The mathematics textbook of class 8 introduces the students to the similarity of triangles. The SM activity uses the knowledge about the similarity of triangles in a practical way to measure an unknown quantity. Chapter 6 of Class 10 Mathemat-

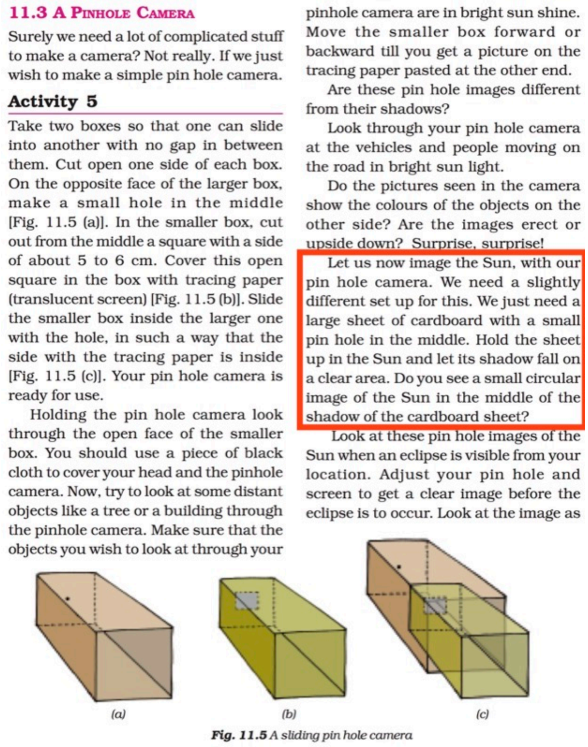


Figure 6.9: Information and activity related to the pinhole camera from NCERT textbook. The text has an activity very similar to the activity of Sun Measurement described in this section. Figure From NCERT Class 6, Science p. 110.

ics textbook of NCERT is on Triangles which provides the mathematical concepts required in this task.

Figure 6.10 shows some of the conceptually equivalent examples from the NCERT textbooks for the SM Task.

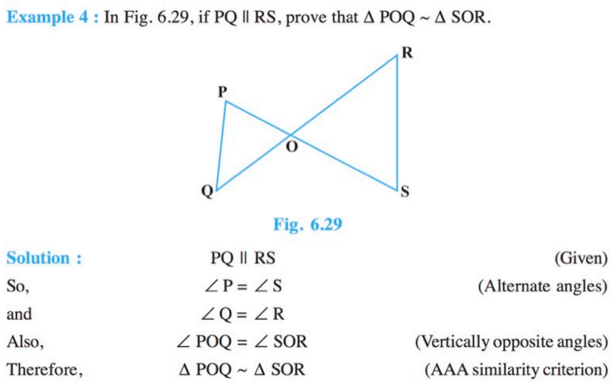
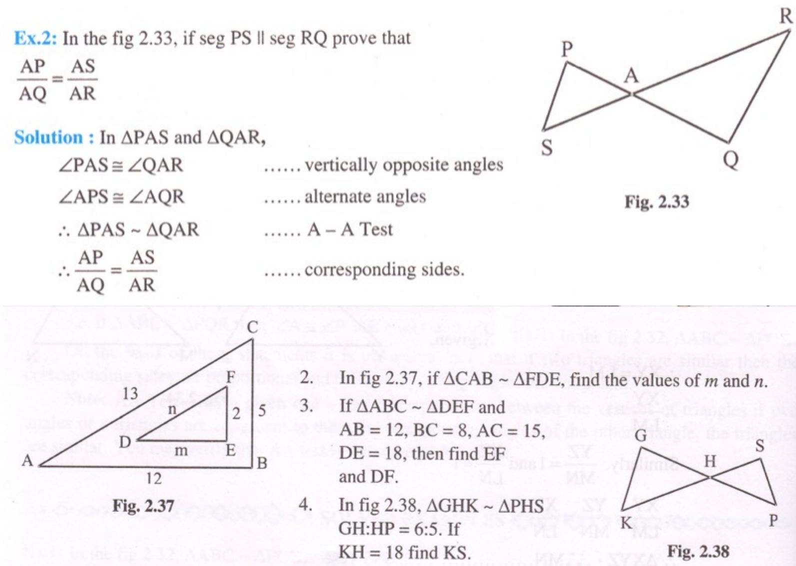


Figure 6.10: Conceptually similar problems to the task using properties of similar triangles. From NCERT Class 10, Mathematics p. 135.

In the case of Maharashtra board *Chapter 3 Triangles*, in Class 9 and *Chapter 1* in

Class 10 are on the topic of similarity of triangles. The examples in these chapters typically involve proving theorems, about the properties of similar triangles. The textbook also emphasises on variety of tests like *Side-Side-Side* (SSS), *Angle-Angle-Angle* (AAA) and *Side-Angle-Side* (ASA) and their converse. Problems typically involve invoking the tests to prove that two or more triangles are similar. Other types of problems involve finding out lengths or angles in one of the similar triangles when the data about one of the triangles is given. Figure 6.11 shows typical problems in this category. We use these properties of the triangles that students have learned in such problems.



Ex.2: In the fig 2.33, if seg PS ∥ seg RQ prove that $\frac{AP}{AQ} = \frac{AS}{AR}$

Solution : In ΔPAS and ΔQAR,

∠PAS ≅ ∠QAR …… vertically opposite angles

∠APS ≅ ∠AQR …… alternate angles

∴ ΔPAS ~ ΔQAR …… A – A Test

∴ $\frac{AP}{AQ} = \frac{AS}{AR}$ …… corresponding sides.

Fig. 2.33

2. In fig 2.37, if ΔCAB ~ ΔFDE, find the values of *m* and *n*.
3. If ΔABC ~ ΔDEF and AB = 12, BC = 8, AC = 15, DE = 18, then find EF and DF.
4. In fig 2.38, ΔGHK ~ ΔPHS GH:HP = 6:5. If KH = 18 find KS.

Fig. 2.37

Fig. 2.38

Figure 6.11: Conceptually similar problems to the task using properties of similar triangles. From Maharastra Board Class 9, Mathematics p. 39, 40.

We see that the syllabus covers most of the concepts required for this task. In this activity we organise these scattered concepts into a coherent whole, integrating elements and concepts from both mathematics and science. In the design of the activity, we have kept connections with concrete objects, processes and events while making these connections.

6.3 Field Study

The sample of the study was the same as that of the MS activity. The same set of students participated in this activity, as described in Section 5.3 in the last chapter. The activity was designed and developed over three years along with the Mustard

Seed (MS) task during the years 2012-14. Each year we had a different set of students from urban Indian schools. For each year we had about 120 students mostly from 9th grade. Most of the students had English as the medium of instruction. Each year around 4-5 students, who had Marathi or Hindi as the medium of instruction submitted reports in Marathi and Hindi respectively. For these students, the instructions were repeated in the Marathi and Hindi. The students were part of a special summer camp, with a variety of activities. The MS task was the first activity in the set, while SM task was done subsequently. The crucial difference between the two tasks was that the SM task was a collaborative group activity, which involved concrete constructions. The experimental observations in the case of the SM task were performed in groups (Figure 6.30.).

The data collected during the development and field testing consisted of researchers notes during the classroom discussions, photographs and reports submitted by the students. Though, all of the students did not submit the written reports for various reasons. We discuss in a descriptive form the analysis of the reports submitted by the students when they completed this activity. The focus in the part that follows is more on describing the *process* by which the students performed the activity along the lines of the design framework.

6.4 Workflow of the activity

In this section, we describe the workflow for the SM Task. We also highlight the applicable concepts and skills from the design framework Figure 4.2 from the Chapter 4 are shown in brackets with a different formatting and font as `(concept)`. The design framework as seen in the context of the SM task is as shown in Figure 6.12.

Timeline of the events in the SM Task

Day 1

Session 1: Discussion of answers from the pre-test, discussions on similar triangles, pinhole camera and its construction.

Session 2: Construction of the Solar pinhole camera.

Homework: Proof of similarity.

Day 2

Session 3: Taking observations from the constructed Solar pinhole camera.

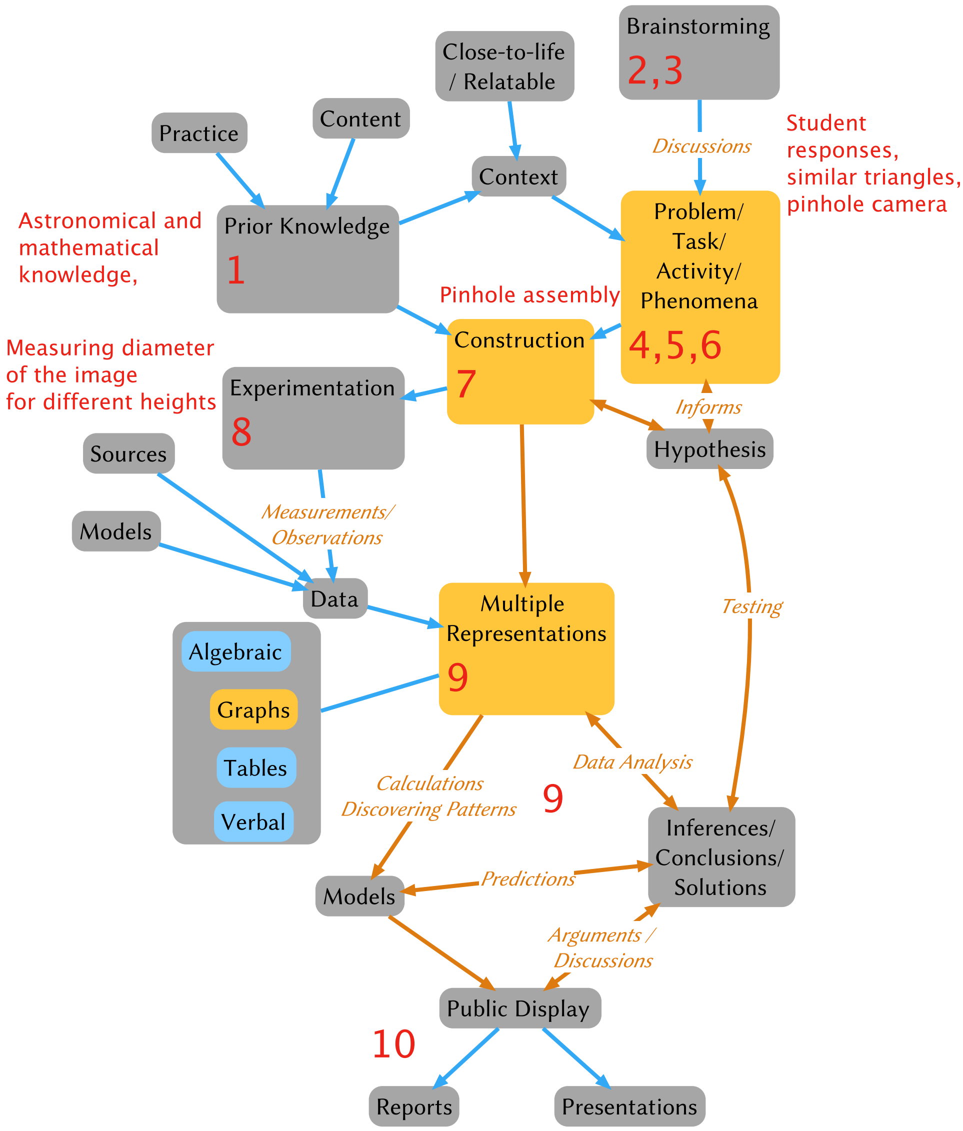


Figure 6.12: Description of the Sun Measurement task in the design framework described in Chapter 4. The red numbers in the figure indicate the steps given in the workflow.

> *Homework:* Report writing on the basis of observations taken.
>
> **Day 3**
>
> *Session 4:* Discussions on reports from the homework exercises. Collaborative plotting using GeoGebra, mathematical modelling, order-of-magnitude answers.

The activity was completed in three sessions, spread over three days. On the first day we had classroom discussions around (a) the answers that were given by the students in the pre-test on the questions relating to this activity, and (b) the working of a pinhole camera and understanding it from a perspective of similar triangles. The pinhole camera was constructed in the next session following this. On the next day in the morning, the students made the observations using the pinhole camera that they had constructed. The students then wrote a report based on the observations, and final classroom discussions were done on the next day with the results of the reports. The length of each discussion session with the students was typically about an hour.

The work-flow for the activity was as follows:

Steps in the Sun Measurement Task

1. A pre-test question about how to find the distance from the Earth to the Sun. `(prior knowledge)`
2. Classroom discussions on the student responses to the question above. `(discussions)` `(prior knowledge)`
3. Classroom discussion using GeoGebra on similar triangles and their properties and how they can be used to solve the current problem. `(construction)` `(models)` `(multiple representations)`
4. Creating a 'mathematical model' to depict the situation and identifying the main components. `(hypothesis)` `(model)`
5. Demonstration and working principle of a pinhole camera. `(construction)` `(experimentation)`
6. Discussions on possible errors, precautions to be taken while performing the measurements. `(experimentation)` `(brainstorming)`
7. Construction of the pinhole camera and assembly for the experiment. `(construction)`
8. Measuring the values of the diameter of the image for different heights on the scale. `(data)`
9. Analysing the observations and estimating the distance to the Sun or its diameter by using numerical calculations and by using the slope of the graph. `(analysis)` `(models)` `(inferences)`
10. Writing a report including the required estimate, graphs, tables, and a description of the experiment. `(public display)` `( multiple representations)`

In the next part of this section, we look at each of the steps in detail.

1. A pre-test question about how to find the distance from the Earth to the Sun.

Concepts/Skills: (collaboration) (prior knowledge) (discussions)

In our Pre-Test (Appendix C) for the students, one of the questions related to this task was "How can you measure the approximate distance between the Sun and the Earth, the Moon and the Earth?" As a response to this answer, some students quoted the distance to the Sun and the moon as a known quantity. This can be related to the factual information that the textbooks are giving to the students. Figure 6.13 shows a typical response from the students.

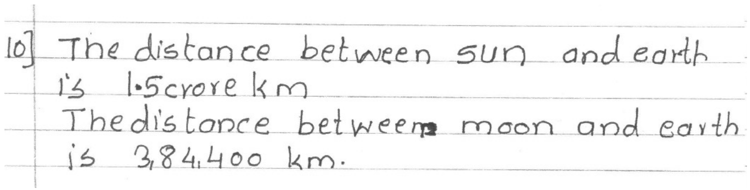
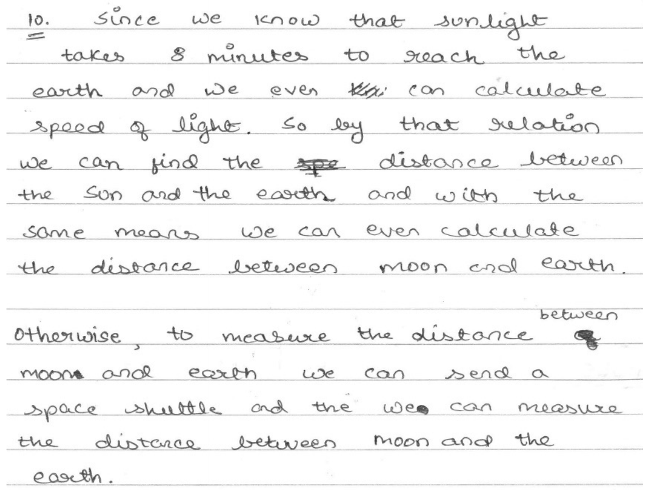


Figure 6.13: A typical response of a student giving values of distance to the Sun and the Moon as a response to the pre-test questionnaire. A *crore* is equal to 10^7.

Many students could not answer the question in the pre-test. This was despite the fact that the textbook has many instances of this factual information. Quite a few students gave very curious responses to this question. These students combined facts known to them to arrive at the required answer. Typically, the relation between speed, time and distance was the core idea, distance = speed × time. In most cases, students used the speed of light to find the distance. Typical responses from students using this method are shown in Figures 6.14 and 6.15.

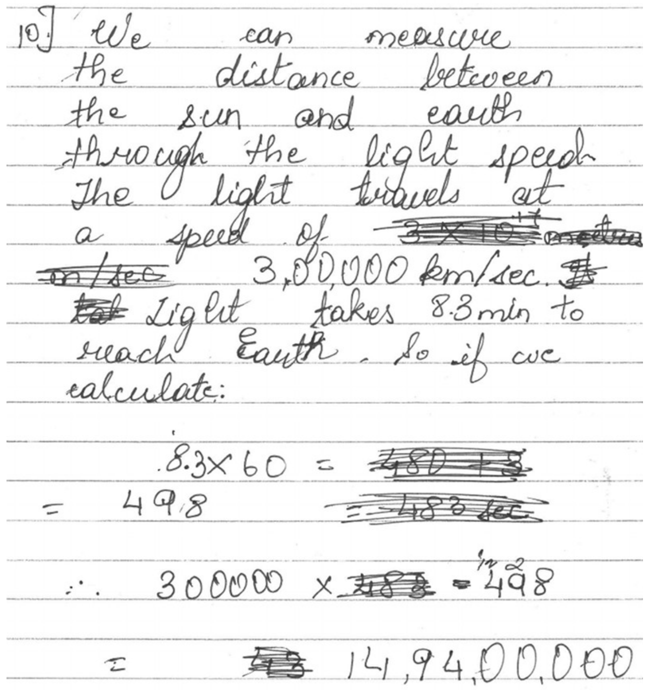
Some students added the actual numerical values in their responses and calculated the distance to the Sun from these values. These answers were close to the actual value of the distance to the Sun.

We observed quite a few variations on this theme. In some cases, the students made errors on the values of the time taken by light to travel (for example 8 years or 8 seconds instead of 8 minutes). Some students invoked the use of spaceships or rockets to measure the distance. Figure 6.16 gives a typical response of the students in this category. Perhaps this was an extrapolation of the students from the terrestrial experience of finding distance by driving vehicles to that of the space.


10. Since we know that sunlight takes 8 minutes to reach the earth and we even can calculate speed of light. So by that relation we can find the distance between the Sun and the earth and with the same means we can even calculate the distance between moon and earth.

Otherwise, to measure the distance between moon and earth we can send a space shuttle and the we can measure the distance between moon and the earth.

Figure 6.14: A typical response of a student knowing that (a) it takes about 8 minutes for the sunlight to reach the Earth, and, (b) speed of light is about 3×10^5 km/s. The student then combines these two facts to find the distance to the Sun by using the relation speed = distance × time.


10] We can measure the distance between the sun and earth through the light speed. The light travels at a speed of 3,00,000 km/sec. Light takes 8.3 min to reach Earth. So if we calculate:

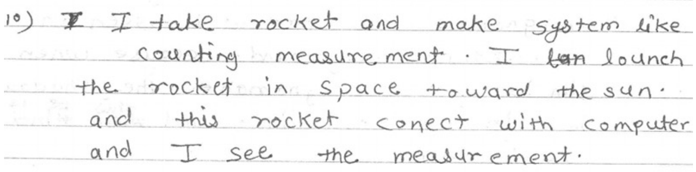
.8.3 × 60 =
= 498

∴ 300000 × 498

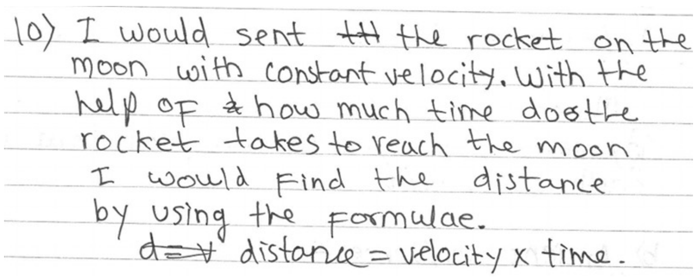
= 14,94,00,000

Figure 6.15: A response of a student using numerical values of the speed of light and time taken by light to reach the Earth to find the distance to the Sun.

When asked during the discussions how do we know that it takes about 8 minutes for the sunlight to reach Earth, students were clueless and only knew this information as a matter of fact (as evidenced in the textbooks). In another response with a spaceship, the student also mentions the changing distance between


10) I take rocket and make system like counting measurement. I launch the rocket in space toward the sun. and this rocket conect with computer and I see the measurement.

(a)


10) I would sent the rocket on the moon with constant velocity. With the help of how much time do the rocket takes to reach the moon I would find the distance by using the formulae.
distance = velocity x time.

(b)

Figure 6.16: Typical responses from students involving a rocket or a spaceship to find the distance to the Sun. (a) In this response, student plans to use "counting measurement" on board a rocket to measure the distance. (b) The use of the speed-distance relation in the context of rocket use.

the Sun and the Earth during seasons. Sometimes the responses were incoherent, except that the students made use of words like "light year", "telescope", "satellite" and "space-ship" etc.

Only a couple of students actually did present some credible method of finding the distance, through the use of geometry. Figure 6.17 shows the credible response using geometrical reasoning to find the distance to the Sun.

Some of the responses and the factual data contained in them formed the basis of introducing the task of measuring the distance to the Sun during the classroom discussions that followed. As we have seen in the examples from the textbooks (Section 6.2), the emphasis is on just presenting *factual data* to the students. There is no attempt even to engage the students with questions of how we can find out, or how the scientists found out about these quantities. If we provide the students only with the factual data, the students will have only superficial factual information about it. This was clearly evident in the responses that we have seen in this section.

At this point, one is reminded of the critique that Kumar (1988) presents on the historical evolution Indian examination system and the associated textbook culture it promoted. According to Kumar the centralised examinations in which a person other than the teacher, who taught the students, examined the students played an essential role in establishing the textbook culture. The examination con-

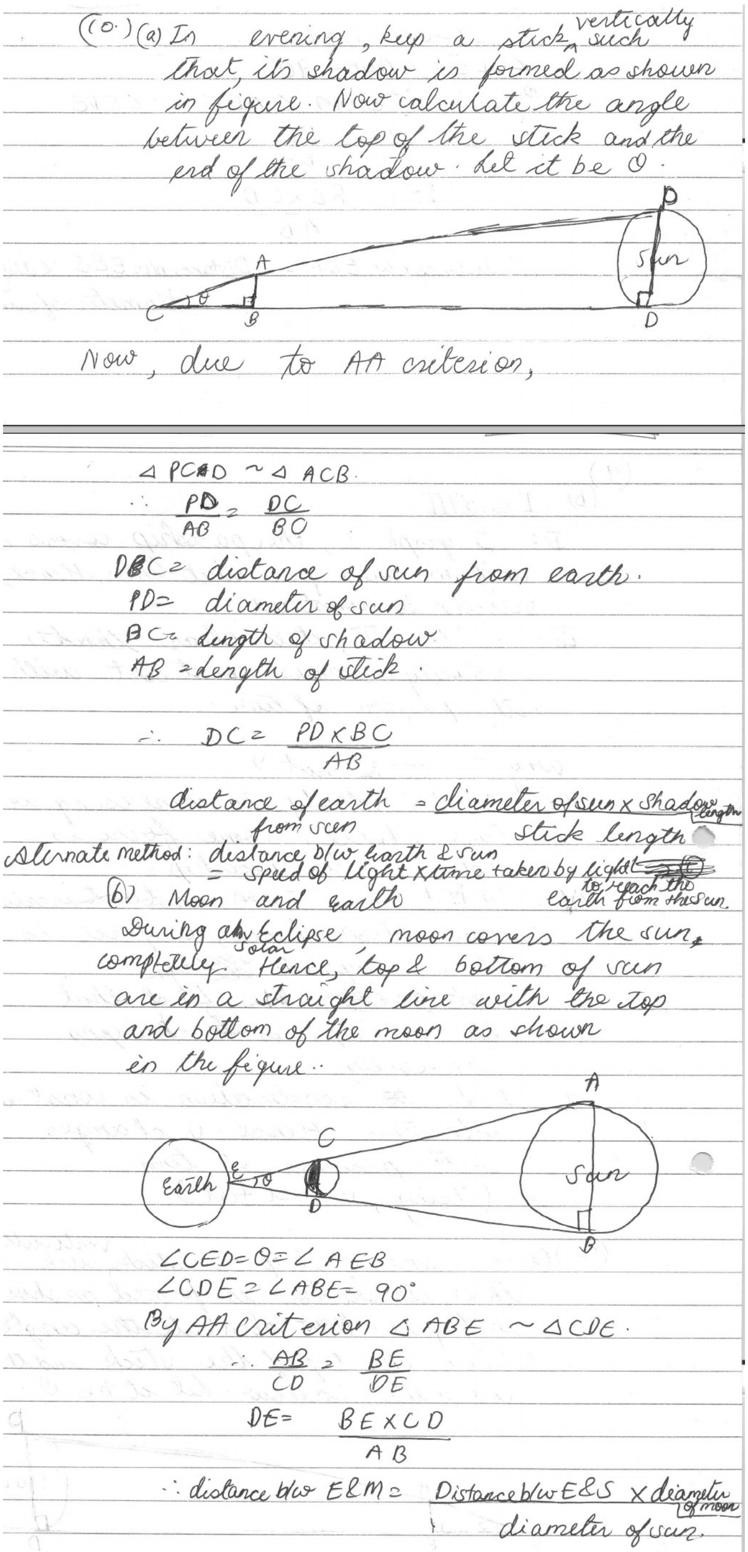


Figure 6.17: A response of a student using geometry to find the distance to the Sun. Note the use of similar triangles in determining the required distance.

ducted thus was intricately linked to the textbooks, and "students were examined on their study of specific texts, not on their understanding of concepts or problems." (p. 458). This mode of examining students gave very high prominence to the *reproducing* the text from memory, rather than *understanding* the content of the text. The standard examination included essay type written answers, and all other modes which could be used for examination were left out of the system. In this regard, practical and vocational skills suffered, and subjects like science which depended on these suffered too.

2. Classroom discussions on the student responses to the question above.

CONCEPTS/SKILLS: `(discussions) (prior knowledge)`

We asked the students some overarching questions on this topic during the classroom discussions:

- How do we know the distance to the Sun or to the planets?
- How do you think we can measure this distance?
- Can we use a measuring tape or a measuring scale that they have to find this distance?

Most of the students, except for a few of them, were unaware of possible answers to the three questions posed above. Some of the students suggested that this information must be true (for example, the distance and size of the planets and the Sun) *because* it was mentioned in the textbook. This observation is perhaps is an indication that students consider the information in the textbooks as unquestionable. However, when asked where this information has come from, they had no idea. But someone must have found this information. How did they do it? Can we also do it? Through the classroom discussions around questions like these, we attempted to build a case for how to make observations which will measure the needed distance.

The Pinhole Camera

We discussed two more themes in this regard. The first was that of a *pinhole* camera and the second was situating similar triangles in the setup. We gave the students a demonstration of the pinhole camera in the classroom. To conceptually clarify the working of the pinhole camera, we made use of a drawing ray-diagram depicting the working of the pinhole. Figure 6.18 shows the typical drawing made for this discussion.

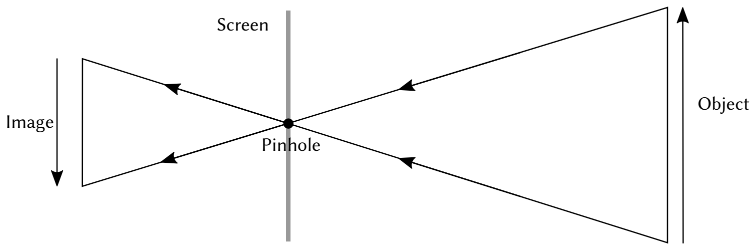


Figure 6.18: A schematic diagram of the pinhole drawn in the classroom.

The ray diagram was used to explain why the image of the object in case of a pinhole camera is *inverted.* To carry the discussion forward, the students were also shown various images of the *bokehs* that are formed due to the sunlight seeping through the branches (this is mentioned in the NCERT textbook (Figure 6.9), but not in the Maharashtra Board textbooks). Then they were told that this happens due to the fact that the small openings between the leaves through which the sunlight comes out act like a *pinhole* camera. So the round bokehs that we see on the ground are actually the images of the Sun. This fact becomes very clear during Solar eclipses. The bokehs which are usually round in shape actually show the current phase of the Solar eclipse. Figure 6.19 is a vivid demonstration of the fact that the bokehs are actually the images of the Sun due to the *pinhole* camera effect.

The size of the image of the Sun in the case of this case is dependent on the height of the pinhole from the ground. If the diameters of the pinholes are different, but they are at the same height, we get images of different sharpness and brightness. Figure 6.19 shows this, the sharp but relatively dim images are from smaller apertures (to the right of the yellow leaf in the centre), while the relatively bright but diffuse images are from larger apertures (at the bottom right of the yellow leaf). An interesting discussion on this phenomena, and how it can be used to deduce the height of the tree foliage is given in first section Minnaert's classic book on optical and meteorological phenomena *The Nature of Light and Color in the Open Air* (Minnaert, 1954).

Figure 6.19: The eclipsed Sun and its pinhole image. The top photograph shows a partially eclipsed Sun, taken with a filter. The lower photograph shows the images of Sun during the eclipse under a large tree. The images of the Sun clearly show the eclipsed Sun.

At this point, the students were again asked: “Can we use any of the information that we have just learned about the pinhole cameras and the bokehs of the Sun to answer the questions asked at the start?”

The answer was again the negative, (except for very few students who gave the credible method described in Figure 6.17). Students could not see a way to connect the images with distances that we were seeking. This observation is perhaps an indication that students cannot transfer mathematical knowledge that they have learnt in one context, in this case, that of similar triangles in geometry, to another context, in this case, real-world measurement of distance based on similar triangles. Most of the students even with the presentation of the ray diagrams from the pinhole camera, in which the similar triangles are readily identifiable (as in Figure 6.18) could not make the connection to the properties of the similar triangles that they knew. The fact that they had the requisite geometrical understanding, but only in the context of geometrical problems as described in the textbooks was apparent in this case. The practical, real-world usage of these problems is not found in the textbooks, the knowledge is highly *situated* in the context of textbook-based problem-solving. We made these connections by concretising the measurements using similar triangles in case of the pinhole camera. This connection was then further elaborated by *constructing* an *adjustable* pinhole camera and taking observations from it. The geometrical knowledge was brought back to interpret the collected values in the context of the problem. Further representing these values in multiple ways and arriving at the *same* result using multiple means made the linkages stronger. Finally, when seen with the MS Task, this provides a firm basis for looking at the linkages that mathematics has with sciences and real-world applications.

3. Classroom discussion using GeoGebra on similar triangles and their properties and how they can be used to solve the current problem.
4. Creating a mathematical model to depict the situation and identifying the main components.

Concepts/Skills 3: `(hypothesis)` `(model)`

Concepts/Skills 4: `(construction)` `(models)` `(multiple representations)`

We are presenting steps 3 and 4 together as they become intertwined with each other during the classroom discussions.

The next step in the SM task was to establish the geometrical connections to the schematic diagram of the pinhole camera (Figure 6.18). The students were already aware of the similarity of triangles, as was evidenced in the written reports. During the classroom discussions it was asked: "When can we say that given two triangles are similar?"

For answering this question the students listed out the various tests that are used for establishing the similarity in triangles, like *AAA*, *ASA*, *SSS* discussed earlier. The basis of the tests is on the fact that the *ratio* of sides and the angles in the case of similar triangles is always *equal*. The discussion was carried forward with the help of establishing some cases in which the similarity of a few triangles was established invoking these tests. Through these discussions, the students also knew that if one side of one of the similar triangles is known, then from the ratio we can find out the other side. This idea was established again through some examples of similar triangles. We used the dynamic mathematics software GeoGebra for illustrating and aiding the classroom discussions on this topic. The dynamic nature of the software was used to create variations on the theme of similar triangles. The use of dynamic geometry software helped us to portray various types of similar triangles and illustrate the various tests used for proving similarity of the triangles with excellent clarity. Particularly useful was to show the constant ratio of the sides when the absolute values of the sides were changing along with the shape of the triangle.

At this point, we turned back to the pinhole camera and asked students if they can use anything from these discussions on the similar triangles for finding the solution to the problem. Some of the students could reason that the triangles were indeed formed in the case of the pinhole camera were indeed similar triangles. We assumed that the plane of the image and the source are parallel. The task of proving that these two triangles are indeed similar was given to the students as a *homework* exercise.

The next day the students brought to the classroom a *proof* of similar triangles for the pinhole diagram. This proof rests on the assumption that the image and the object are in parallel planes, a short version of the proof in the textbook can be seen in Figure 6.10.

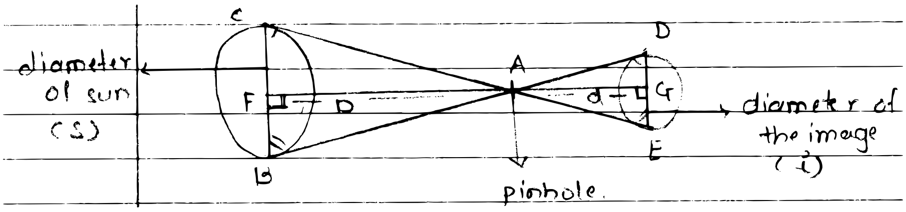


Figure 6.20: An illustration that was drawn by a student, depicting a schematic diagram of the pinhole camera. It shows the geometric constructions used in the calculations.

Proof of similarity

The proof typically follows these steps:

Let us label the image as shown in Figure 6.21. We assume that the image DF and Object AB are parallel. C represents the pinhole.

We also note that the two triangles, $\triangle ABC$ and $\triangle CDE$ are isosceles. Since DF and AB are parallel, we have

1. $\angle EDC = \angle CBA$, alternate angles between parallel lines
2. $\angle DEC = \angle CAB$, alternate angles between parallel lines
3. $\angle ECD = \angle BCA$, opposite angles between intersecting lines

Hence $\triangle ABC \sim \triangle CDE$, by the AAA or the angle-angle-angle test. As a result of this similarity we have:

$$\frac{AB}{DE} = \frac{DC}{CB} = \frac{EC}{CA}$$

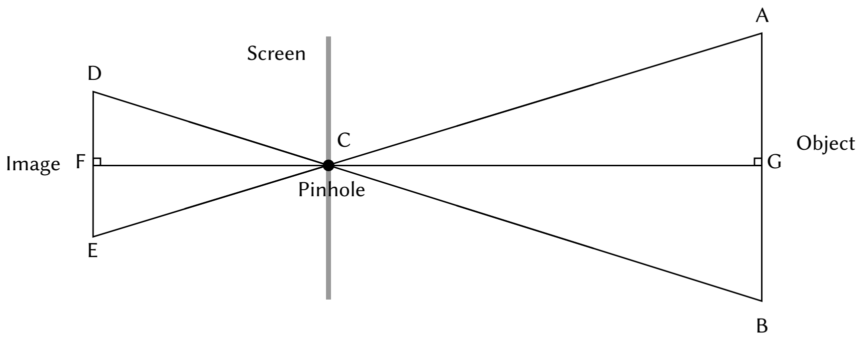


Figure 6.21: Proving the similarity of triangles in the pinhole camera.

A few more constructions are done on this figure. We drop perpendiculars from C to AB and to DE, creating two new points F and G. By construction we have $\angle EFC = \angle CGA = 90°$. Since $\triangle ABC \sim \triangle CDE$, we also have $\triangle GBC \sim \triangle EFC$ and $\triangle GBC \cong \triangle GAC$, $\triangle DFC \cong \triangle EFC$. From this we can say:

$$\frac{FE}{BG} = \frac{FC}{CG} = \frac{EC}{CA}$$

Or rearranging the terms, and considering that $EF = 2ED$, $GB = 2BA$ we get:

$$\frac{ED}{FC} = \frac{AB}{CG}$$

Thus if we can measure lengths of two sides of a triangle, we can know the ratio of the sides. This ratio will be the same for the corresponding sides of all triangles similar to this one. The next step in the discussion was to link this to the distance to the Sun.

This proof of similarity, based on the responses brought by the students, was discussed in the classroom and some of the doubts that the students had regarding this were clarified with the help of other students in the class. Again for this we, used GeoGebra and showed that the *ratio* of the sides remains invariant in case of similar triangles, though the values of the sides may change.

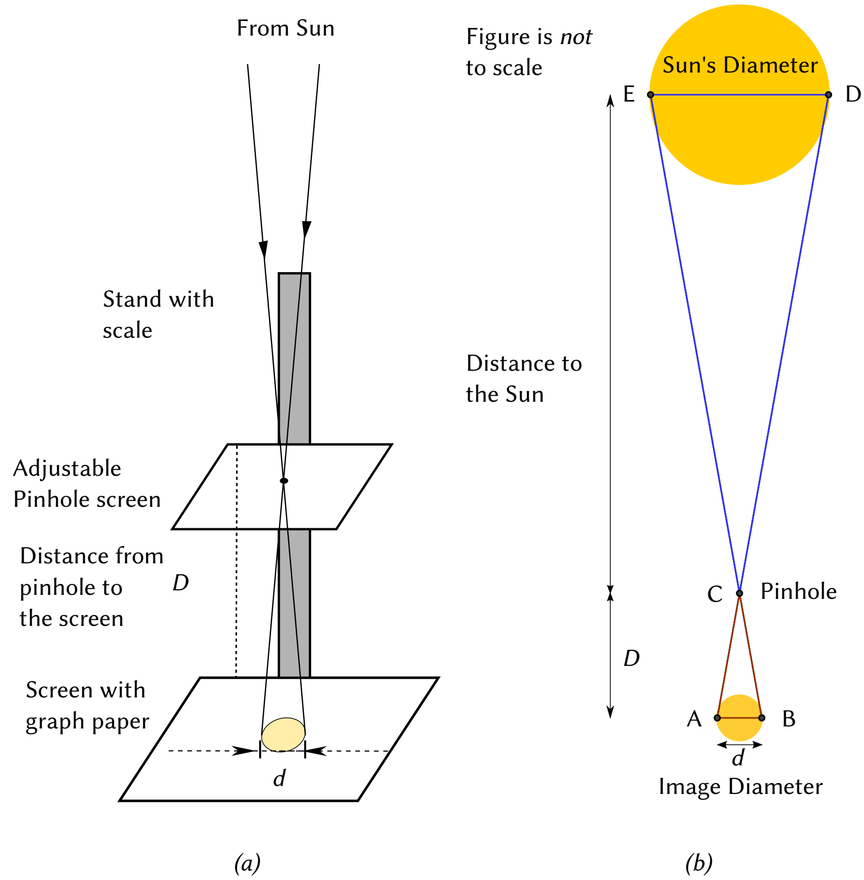


Figure 6.22: (a) Schematic diagram of the pinhole camera showing the Sun's image on the screen below. (b) Geometrical diagram of the pinhole camera showing the Sun and its image on the screen below (The figure is not to scale).

5. Demonstration and working principle of a pinhole camera

Concepts/Skills: (construction) (experimentation)

At this point, the students were taken outdoors and were shown a pinhole camera in which the distance between the image and the object could be changed. This setup is shown schematically in Figure 6.22. The setup consists of an aluminium sheet in which a pinhole is made. This sheet is attached to a stand, and we can vary the height between the pinhole and the base of the stand. The height of the pinhole (distance D in Figure 6.22 (a)) can be measured with the help of a scale attached to the stand. The screen at the base of the stand has a 1 mm graph paper attached to it. The image of the Sun is projected on this screen. We can measure the width of this image (distance d) readily with the help of the markings on the graph paper (with a least count of 1 mm).

If we draw a geometrical diagram for the schematic setup as shown in Figure 6.22 (b), we get the similar triangles which were discussed earlier. The figure is *not* to scale, this fact was emphasised in the classroom discussions. Emphasis was given on this aspect of the scale of the image in the classroom discussions. Though, it is frequently found that textbooks fail to mention this, especially when presenting astronomical illustrations. Figure 6.5 shows an example of this misrepresentation.

The entire setup of the pinhole needs to be aligned with the position of the Sun in the sky. Such alignment can be achieved by resting the stand on a support and inclining it so that the image of the Sun appearing on the screen is nearly circular, and not oval. Figure 6.23 shows this an illustration drawn by one of the students. The non-alignment is a significant source of errors in the subsequent measurements.

In Figure 6.23, which is labelled in Marathi) shows the setup resting on a chair which provides the support for incline. A demonstration was given to the students to form the image of the Sun on the screen. After this, they practised the operation of the setup, varying the height of the pinhole above the screen. This action of changing the distance gives different sizes and sharpness of the resulting image. The larger the distance D larger is the image diameter d, but the brightness of the image is reduced and sharpness increased.

This fact of the increase in the size of the diameter with the increase in the distance is linked to the ratio between the sides of the similar triangles. We are essentially changing the side of one of the triangle, so the other sides must scale. This idea was dynamically illustrated very well by using a simple simulation in dynamic mathematics software GeoGebra. Figure 6.24 shows a typical screenshot of the GeoGebra showing the similar triangles used during the discussions. In this simulation, we can change the distance between the pinhole (point C) and the screen (segment AB), which results in the change in the distance of the image. So the ratio $\left(\frac{d}{D}\right)$ should be constant for different values of D.

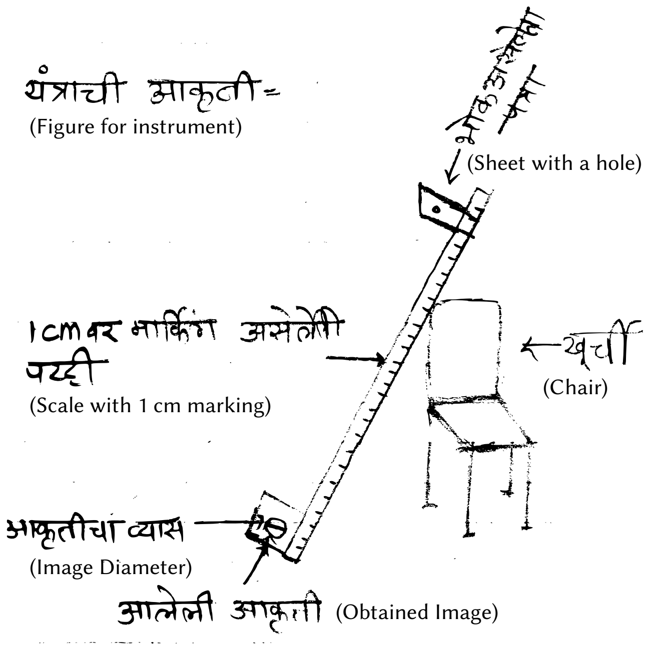


Figure 6.23: The setup for pinhole camera inclined to get the right image of the Sun as depicted by a student. The labels of the drawing are in Marathi, with the translation of the labels given in the bracket below them.

6. Discussions on possible errors, precautions to be taken while performing the measurements.

Concepts/Skills: (experimentation) (brainstorming)

The primary assumption in the mathematical model is that the screen should be parallel to the plane of the Sun's disk. The discussions also included how to observe the readings on the scale. The reports that students wrote had sections on possible errors, and precautions taken while performing the experiment.

7. Construction of the pinhole camera and assembly for the experiment.

Concepts/Skills: (construction)

After these discussions, the students were provided with a thick paper with which they could construct the pinhole and the screen holder. The students were instructed in how to construct the pinhole by cutting and folding the thick paper.

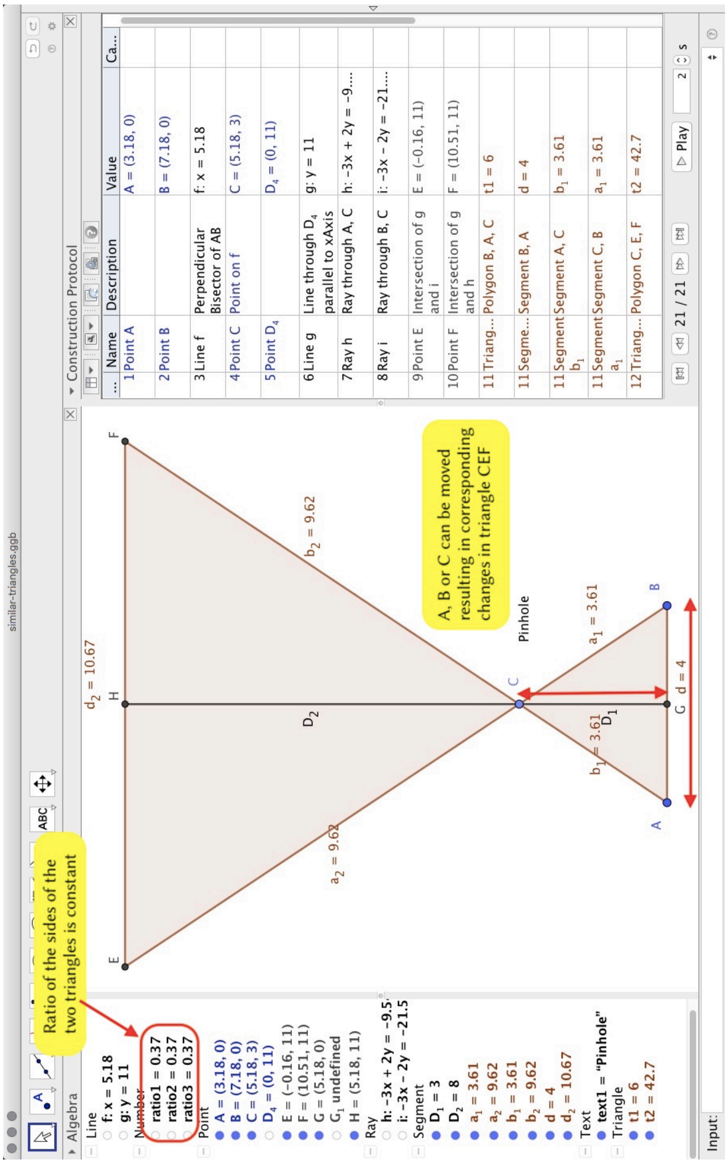
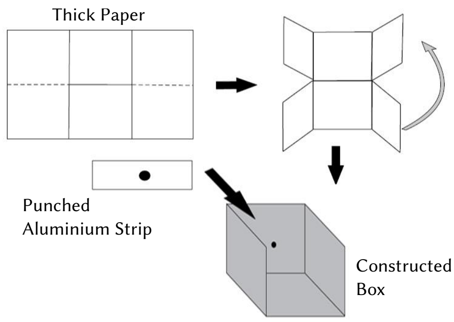


Figure 6.24: A schematic diagram showing the similar triangles in GeoGebra. This diagram was constructed in the class, explaining the student's step-by-step process. When the points A, B, C are moved the dimensions of $\triangle ABC$ change, and correspondingly dimensions in $\triangle CEF$ also change, but the ratio of the sides of the two triangles remains constant.

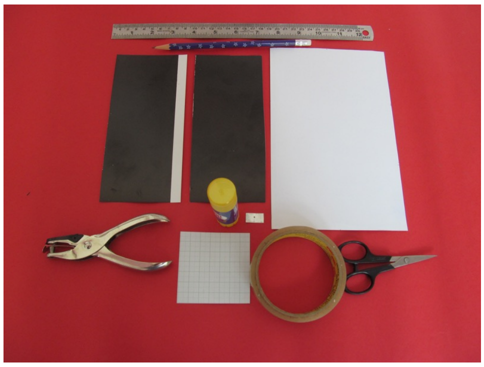
Figure 6.25 shows the steps, materials and tools for the construction of the pinhole.

The process of constructing the pinhole was explained to the students by demonstration. After this, a mentor helped the students to construct the pinhole. Figure 6.26 shows the collaborative construction activity of the students.

The Student Handbook has the following steps for construction of the pinhole.



(a)



(b)

Figure 6.25: Materials and steps for constructing the pinhole required for taking the observations. (a) Steps for folding the box. (b) Materials and tools for constructing the pinhole: a scale, a punch, glue stick, a pair of scissors, adhesive tape, graph paper, thick paper, aluminium foil, and a pencil.

Steps for construction and use of pinhole camera

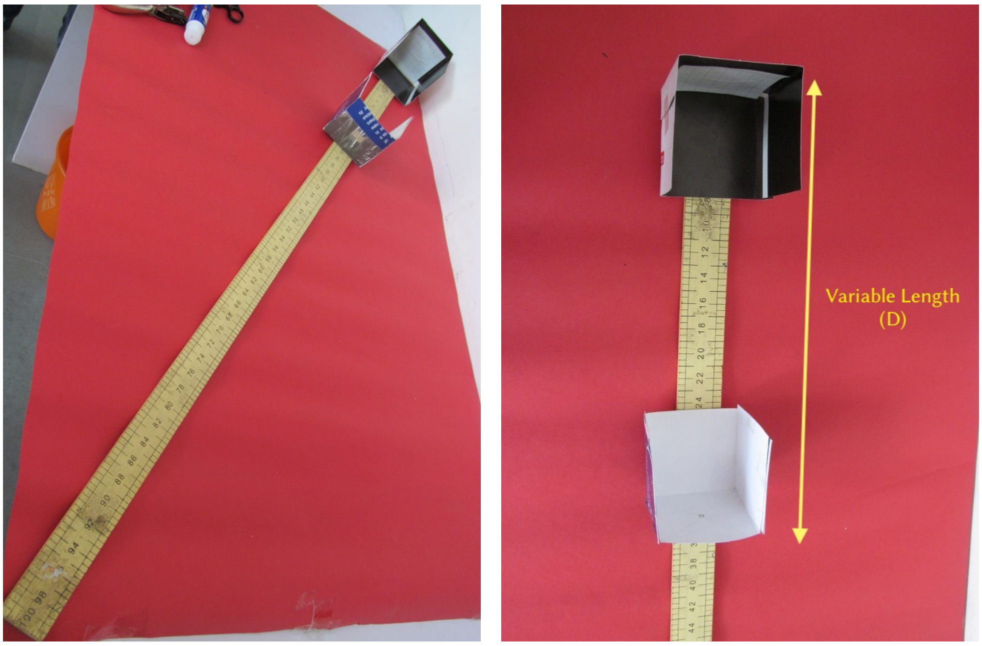
We have to make an instrument for this task which will create an image of the Sun that can be measured. Let us call it the ``Solar Camera''. This is basically a ``pinhole'' mounted on a scale. This pinhole gives us an image of the Sun which can be obtained on a screen at a certain distance from the pinhole.

We will first prepare box like structures using cardboard sheets. Take the rectangular card-sheet and fold it to get six squares on the surface as shown in the figure (Figure 6.25 here). Then cut the sheet in the way shown and fold and paste it to make a partial box. Make two such boxes. On one of the boxes we will punch a hole on top of which we can place the pinhole. The pinhole can be made out of a discarded Aluminium can. We use the other ``box'' for the screen. Attach a graph paper on one side of this box (from the inside) so that we can see the dimensions of the image and read them off it easily.

A strip of an Aluminium can was used to make the pinhole. This strip was punched with a pin to create the pinhole. The strip was then placed over the hole punched in one of the boxes. This box became the moving pinhole holder for the observation. The other box was used as the screen with a graph paper attached to it. To move the pinhole box on the scale, a paper cover was attached to the

Figure 6.26: Students constructing the pinhole cameras required for the observations as a group.

scale which could move freely. The pinhole box was then attached to this cover. Figure 6.27 shows the completed setup of the apparatus constructed.



(a) (b)

Figure 6.27: The completed setup of the pinhole camera with an adjustable mount for the pinhole. The mount containing the pinhole is adjustable along the scale. We can see the image from the pinhole at the bottom of the setup on the graph paper. (a) The complete assembly of the apparatus constructed. (b) Detail of the box mechanism.

For the observations the box with the pinhole was attached to a scale with 1 cm marking, while the box with graph paper was attached to the bottom of the scale. This completed set-up is shown in Figure 6.28.

In Figure 6.28 the movable paper cover can be seen near the hand of the student holding the pinhole box. The readings for the height of the pinhole above the screen were read directly off the scale, using the edge of the box as a pointer.

The students practised adjusting the screen to get a sharp and round image of the Sun on the screen. Typically this was done as a group activity, where a group of two to three students constructed the boxes and took the measurements. Figure 6.30 shows the group of students taking the observations using the setup.

The students performed the activity in the period between 10 to 11 AM, as at high noon, it was too hot outside. The students collected the data by adjusting the distance between the pinhole and the screen, measuring and recording the two

Figure 6.28: Students observing the solar image with the completed setup of the pinhole and the scale. The student on the right adjusts the movable box, while the other student observes the image of the Sun formed in the lower, fixed box.

Figure 6.29: The Solar image obtained by the set-up as seen against the graph paper on the lower screen. We can see the Solar image in the centre of the square-shaped shadow of the upper pinhole box. The diameter of the image was read off the graph paper.

distances d and D. Typically the students took three readings by adjusting the distance of the pinhole.

Figure 6.30: Students observing the solar image and taking readings from the setup in groups.

8. Measuring the values of diameter for different heights on the scale.

Concepts/Skills: `(data)` `(experiments)` `(multiple representations)`

The following task was given to the students (for details see the *Student Handbook* in Appendix C):

Sun Measurement Task

1. Obtain a clear image of the Sun on the screen for a given value of D.
2. Find out the corresponding value of the image diameter d from graph paper on the screen.
3. Repeat the above two steps for at least three different values of D.
4. Write down the values in a tabular format.
5. Find out the ratio $\left(\frac{d}{D}\right)$ for each of the values that were obtained.
6. Find out the average value of this ratio.
7. If the value of $d = 1.392 \times 10^5$ km what is the value of D? (Alternatively, for some groups we have given the distance D and asked to calculate the diameter of the Sun d.)
8. Plot the values of D against d on a graph paper. Choose the units and scales appropriately.
9. If possible, try to draw a straight line which passes through the points that you have drawn.

9. Analysing the observations and estimating the distance to the Sun or its diameter by using numerical calculations and by using the slope of the graph.

Concepts/Skills: (analysis) (models) (inferences)

After the collection of data, the students then analysed the results. The subsequent calculations and predictions from the analysis were contained in the report that they submitted.

10. Writing a report including the required estimate, graphs, tables, and a description of the experiment.

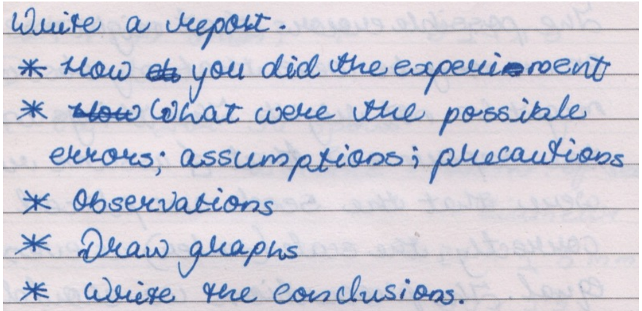
Concepts/Skills: (public display) (reports)

After the submission of the reports, we followed them up with classroom discussions on the report on the third day. The discussions were around the experimental method, the mathematical model, observations and calculations, with examples and data taken from the reports of the students. Particularly the meaning of slope of the line in this case with the calculations that students had done, was discussed explicitly. The discussions also clarified the similarities and differences between the Sun Measurement task and the Mustard Seed Measurement, particularly in reference to the physical meaning of the slope in the two cases.

6.5 Analysis of the reports

As in the mustard task, the students were asked to write a report for the task that they performed. Typically they were asked to address basic questions about the task in the report. Figure 6.31 shows these questions. In the following section, we present an annotated summary of the reports generated by the students.

> **Please Note:** The images included from the student's reports in this section should be rather seen as running *text quotes* from the reports. Hence we have not numbered the images appearing in this section as Figures.



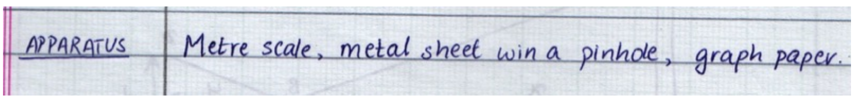
Write a report.
* How you did the experiment
* ~~How~~ What were the possible errors; assumptions; precautions
* Observations
* Draw graphs
* Write the conclusions.

Figure 6.31: Basic questions to be addressed while writing the report for the task.

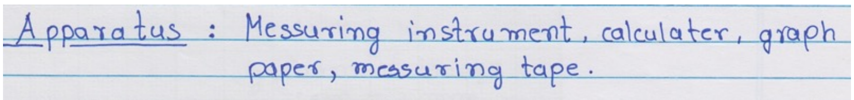
The students did not follow this order of questions. Instead, most of them choose to present the reports in the form that they were all familiar with in the school. Most of the students submitted a report with 2-3 pages, along with graphs. The students in a group, usually from the same school, had very similar reports. The writing of reports, as in the mustard seed task, was a homework exercise. A typical of the report consisted of following sections:

Aim: Most of the students wrote the aim as "To find the approximate distance between Sun and Earth"; "To measure the distance between Sun and Earth"; "To estemate the distance between Earth and Sun (sic)"; "To study diameter and distance of the Sun from the earth"; "To find the distance between the Earth and the Sun"; "To find the distance between Earth and Sun by a simple instrument"; "Calculating distance between Earth and Sun"; "To measure the distance between earth and sun by taking into consideration the diameter of sun".

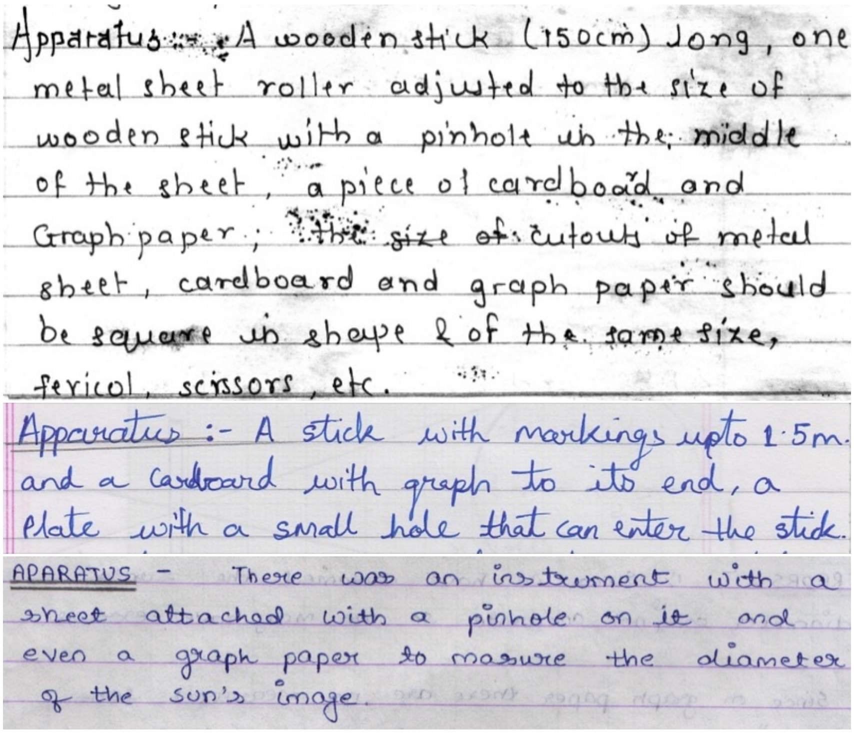
Apparatus: Most common categories in this included, pinhole, graph paper, scale, support. In some cases, the students listed the objects they used while performing the experiments. For example, "Metre scale, metal sheet win a pinhole, graph paper (sic)"; "Wooden scale, small metal plank, Graph paper, Sunlight (sic)".



APPARATUS	Metre scale, metal sheet win a pinhole, graph paper.


Apparatus : Messuring instrument, calculater, graph paper, messuring tape.

Some of the students provided a descriptive list of materials, for example:

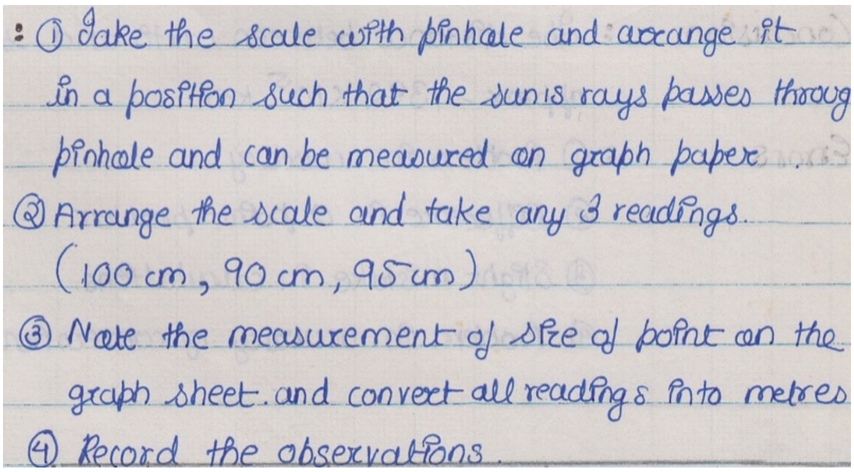

Apparatus :- A wooden stick (150cm) long, one metal sheet roller adjusted to the size of wooden stick with a pinhole in the middle of the sheet, a piece of cardboard and Graph paper; The size of cutouts of metal sheet, cardboard and graph paper should be square in shape & of the same size, fevicol, scissors, etc.

Apparatus :- A stick with markings upto 1·5m. and a Cardboard with graph to its end, a plate with a small hole that can enter the stick.

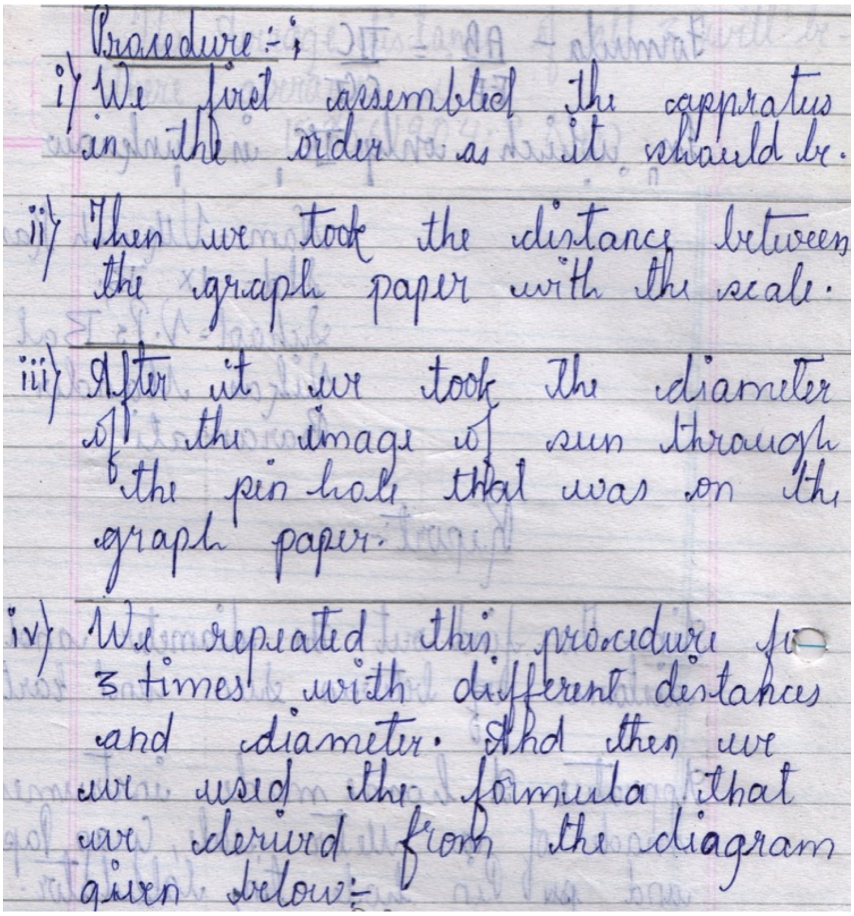
APARATUS - There was an instrument with a sheet attached with a pinhole on it and even a graph paper to masure the diameter of the sun's image.

A few students also added "A sunny day" to the list, indicating that they understood this experiment could not be done on a cloudy day. A few students had "scientific calculator' ' in the list of materials.

Procedure: Typically the procedure that was given in the report varied among the students. The figure below shows a typical description of the procedure:


: ① Take the scale with pinhole and arrange it in a position such that the sun is rays passes throug pinhole and can be measured on graph paper.
② Arrange the scale and take any 3 readings. (100 cm, 90 cm, 95 cm)
③ Note the measurement of size of point on the graph sheet. and convert all readings into metres
④ Record the observations.

The steps as written in the reports varied amongst students. For example, compare the procedure given above to the step given below.



Procedure :-

i) We first assembled the apparatus in the order as it should be.

ii) Then we took the distance between the graph paper with the scale.

iii) After it we took the diameter of the image of sun through the pin hole that was on the graph paper.

iv) We repeated this procedure for 3 times with different distances and diameter. And then we we used the formula that we derived from the diagram given below:-

Some of the students linked the procedure to observation tables and diagrams in the report.

Diagrams: As opposed to the MS Task, most students drew a schematic diagram showing the similar triangles in the setup in the form of the Sun and its image formed via the pinhole. The figure below shows a typical diagram:

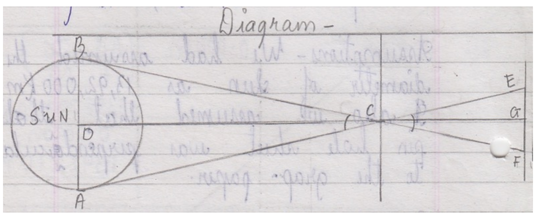


Figure 6.23, seen earlier shows a descriptive diagram of the experimental setup. Figure 6.32 shows another descriptive diagram which has some hints of realism in the drawing.

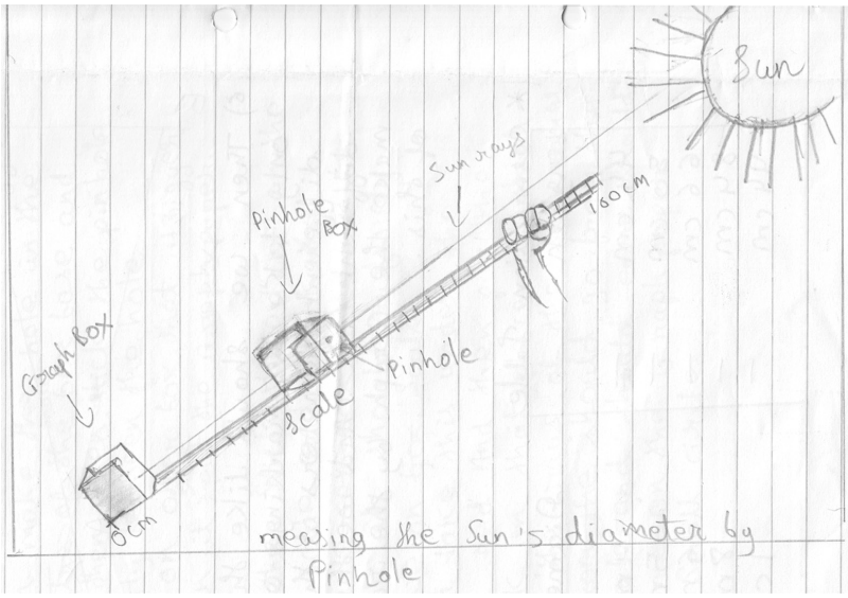


Figure 6.32: A realistic figure of the SM Task setup drawn by one of the students.

In very few cases the diagram was intermediate as shown in the Figure 6.33.

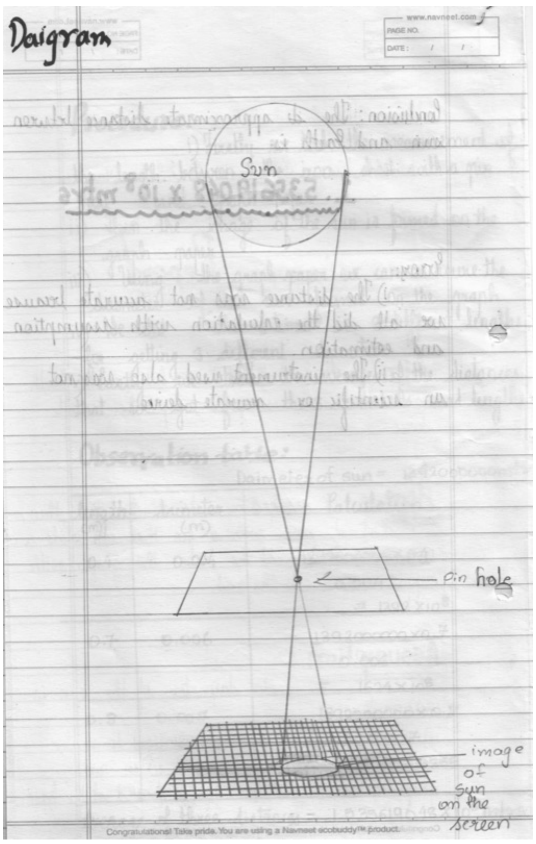


Figure 6.33: A schematic figure of the SM Task setup drawn by one of the students.

Observations - Tables: Observations reported by were mostly in the form of tables. Typically the table consisted of following columns:

Sr. No., Distance on Scale, Image (Diameter), Diameter/Distance of the Sun (Calculated).

Some of the students also converted the units from metre and millimetre to meter and kilometre.

Sr.no	distance on scale	Image	diameter of sun .
(1)	100cm	1 mm	~~13.92~~ 13,92 x 10^3 km
(2)	90cm	0.9 mm	13,92x10^3 km
(3)	95cm	0.9 mm	1392 x 10^3 km.

Convert ⇒

Sr.no	distance on scale	Image	diameter of sun
(1)	1m	0.01 m	1392x10^6
(2)	0.9m	0.009 m	1392x10^6
(3)	0.95m	0.0095m	1392x10^6

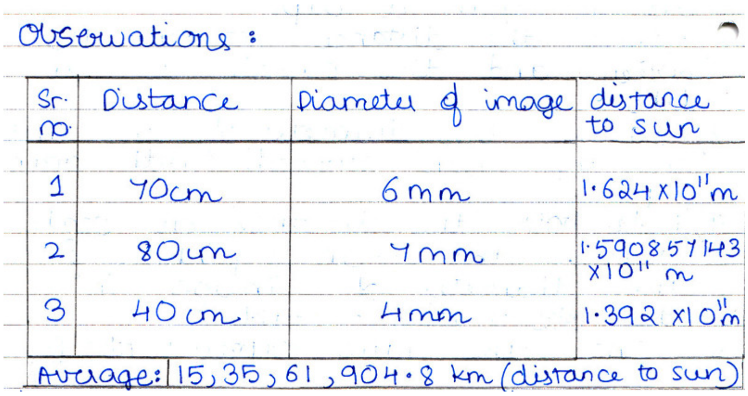
Sr.no	Image dist.	∅ of Image.	Distance of sun.
1	50 cm.	5mm	139.2 million kms.
2	70 cm	7mm	139.2 million kms.
3.	100 cm.	10mm.	139.2 million kms.

The use of index powers while calculating the distances was done correctly by most of the students.

observation Table

distance	diameter of image	dist. between sun and earth
0.30 m.	0.004 m.	104.4 x10^6 km
0.50 m	0.005 m	139.2 x 10^6 km
0.80 m	0.008 m	139.2x10^6 km

Some students calculated the average values from the three readings in the tables.



Observations :

Sr. no.	Distance	Diameter of image	distance to sun
1	70cm	6 mm	1·624 x 10¹¹ m
2	80cm	7 mm	1·590857143 x 10¹¹ m
3	40 cm	4 mm	1·392 x 10¹¹ m
Average: 15,35,61,904·8 km (distance to sun)			

Observations - Graphs: Most students could draw the graphs correctly from the table of observations. Many students also wrote the final answers (the distance/diameter of the Sun) near the observation points on the graph. That most students could plot simple mathematical functions correctly was tested with another activity. This activity involved asking students to plot the x^2 function and to find a square-root or a square from the graph.

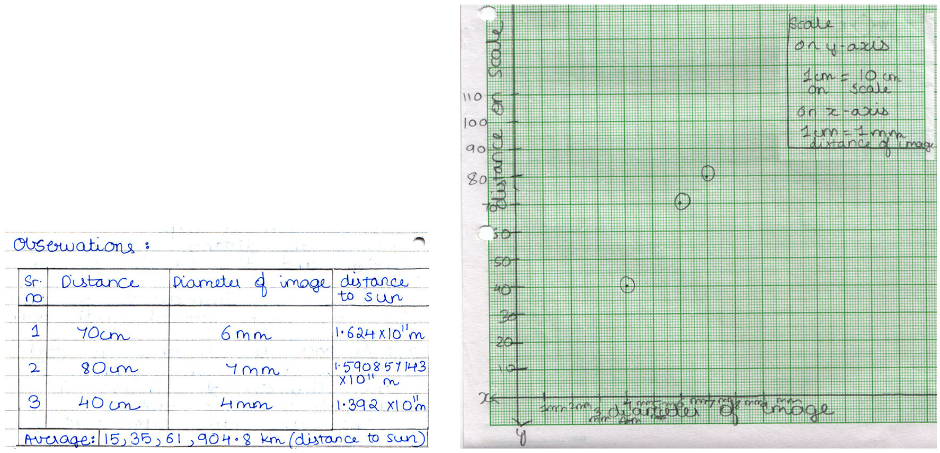
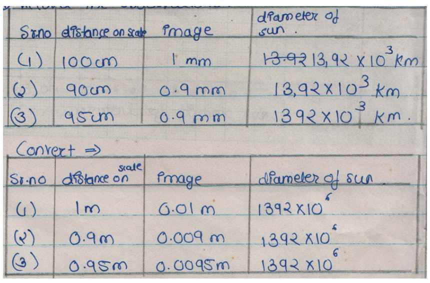


Figure 6.34: Typical set of values for d and D by a student. The figure on the top shows a table with the values obtained from the instrument. It also shows the calculated distance to the Sun. The lower figure represents the graph drawn using data in the table.

While some students did get approximately correct values of the ratio $\left(\frac{d}{D}\right)$, not all were able to get it. A typical set of values obtained by the students of d and D and the graphs resulting from them are shown in Figures 6.34 and 6.35.

Some of the common problems that students had while taking the values and plotting graphs was the choice of readings and the choice of scales. Some of the students like the one in Figure 6.34 had readings too close, so the resulting graph does not adequately cover the graph paper. Whereas, some students who took the readings spread over 100 cm choose scales such that their graphs became clustered in one region. Figure 6.35 shows some examples of this issue. Perhaps this problem will be overcome when plotting the data with a computer.



Sr.no	distance on scale	image	diameter of sun .
(1)	100cm	1 mm	~~13.92~~ 13,92 X 10^3 km
(2)	90cm	0.9 mm	13,92X10^3 km
(3)	95cm	0.9 mm	1392X10^3 km .

Convert ⇒

Sr.no	distance on scale	image	diameter of sun .
(1)	1m	0.01 m	1392X10^6
(2)	0.9m	0.009 m	1392X10^6
(3)	0.95m	0.0095m	1392X10^6

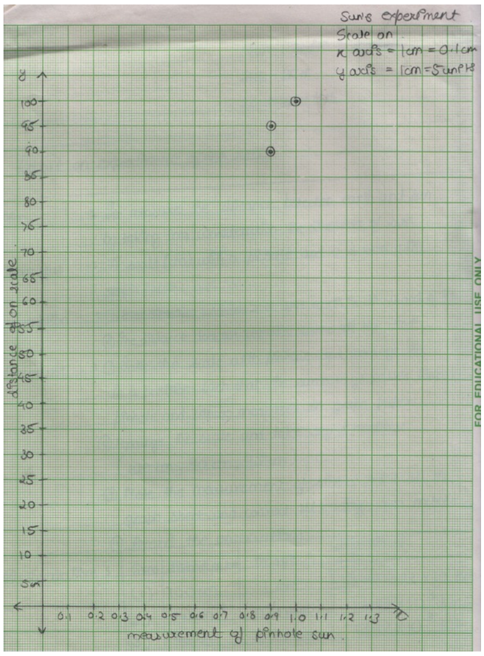
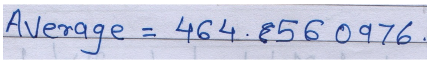
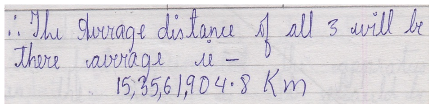


Figure 6.35: A typical set of values for d and D reported by one of the students. Here the middle column is incorrectly labelled as mm, it should be cm. This issue is corrected in the graph by the student where the legend for the X-scale shows 1 cm $=$ 0.1 cm. The student did not cover the entire range of values for D, but restricted to values of 90 cm, 95 cm and 100 cm. This results in the graph which has values in only one section. The same reading of d for the two values of D result from the relative inaccuracy of the instrument. While calculating the student has incorrectly used the conversion for a 1 mm = 0.01 m, hence the order of magnitude in the answers is off by a power of 2.

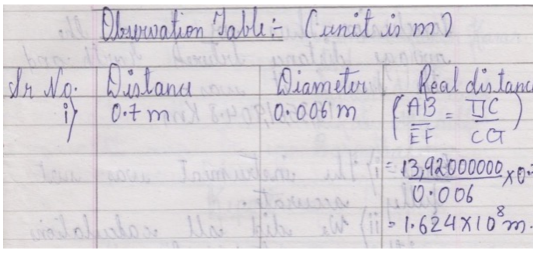
The fact that almost all students could plot the points correctly on the graphs from tables indicates that they could do this without any problems. Though, the interpretation of the graph is another matter. The reports do not contain the aspect of connecting the slope of the graph to the ratio of distance to diameter. This connection was discussed in the classroom on the next day when the students brought their reports.

Calculations: Most students could do the numerical calculations correctly with the observations they had. Since the reports were written at home, they might have done the calculations using a calculator. This is reflected in the significant numbers that many students have in their calculations. While most students showed the calculations explicitly, few students directly gave the results. For example, see the figure below:


Average = 464.856 0976.

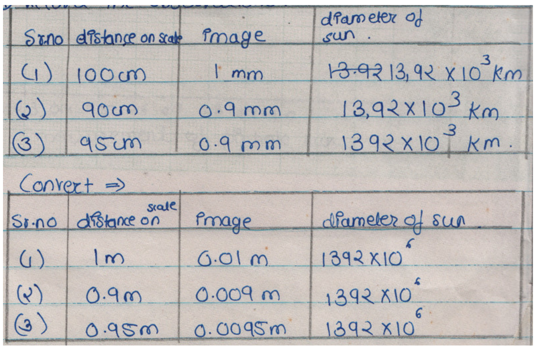

∴ The Average distance of all 3 will be there average i.e –
15,35,61,904·8 Km

Most of the students presented the final results with proper units. Some students provided detailed step-by-step calculations to get to the final answer.


Observation Table :- (unit is m)

Sr No.	Distance	Diameter	Real distance
i)	0·7 m	0·006 m	(AB/EF = ΠC/CG) = 13,92000000×0·7 / 0·006 = 1·624×10⁸ m.

Some students did the calculations inside the observation tables, for example:

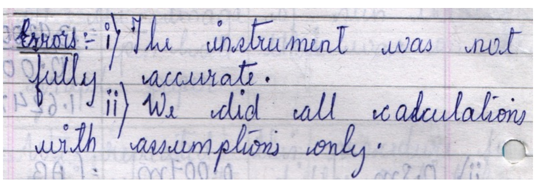


S.no	distance on scale	Image	diameter of sun
(1)	100cm	1 mm	~~13.92~~ 13,92 × 10³ km
(2)	90cm	0.9 mm	13,92×10³ km
(3)	95cm	0.9 mm	1392×10³ km.

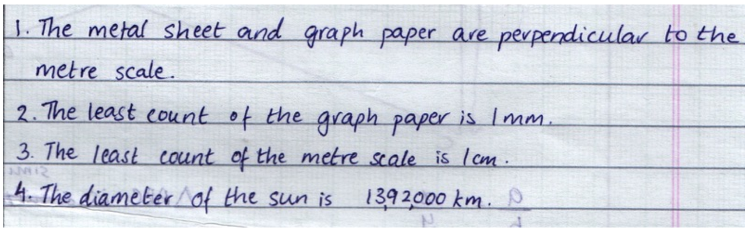
Convert ⇒

Sr.no	distance on scale	Image	diameter of sun
(1)	1m	0.01 m	1392×10⁶
(2)	0.9m	0.009 m	1392×10⁶
(3)	0.95m	0.0095m	1392×10⁶

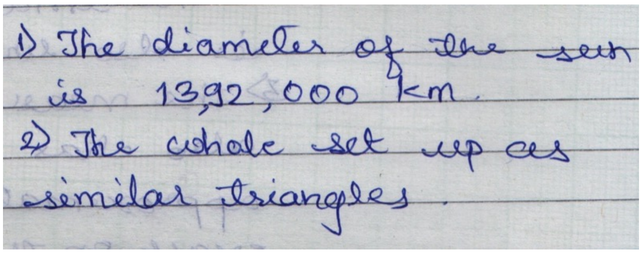
Errors: This category asked students to present the various errors they think might come in performing the experiment. Particularly, we asked what they think are the problems with their observations. Not all students reported the errors. Many students reported errors regarding the accuracy of instrument used. Another major category was the calculations that they did.


Errors :- i) The instrument was not fully accurate.
ii) We did all calculations with assumptions only.

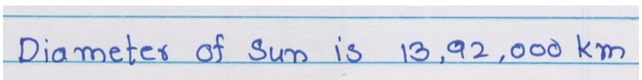
Assumptions: The various assumptions that the students stated in their report included aspects of experimentation and calculations. For example, one of the students listed the assumptions comprehensively in both the categories.


1. The metal sheet and graph paper are perpendicular to the metre scale.
2. The least count of the graph paper is 1mm.
3. The least count of the metre scale is 1cm.
4. The diameter of the sun is 1392000 km.

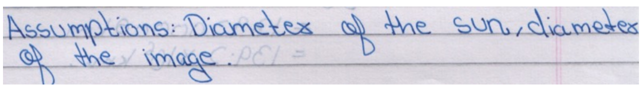
The first assumption here relates to the mathematical model that was used for setting the experiment. The next two assumptions are regarding the accuracy of the measuring instruments that were used: a metre scale and a millimetre graph paper. The final assumption is regarding the calculation and subsequent prediction that they have to do. One student explicitly assumes the similar triangles used in modelling.


1) The diameter of the sun is 13,92,000 km.
2) The whole set up as similar triangles.

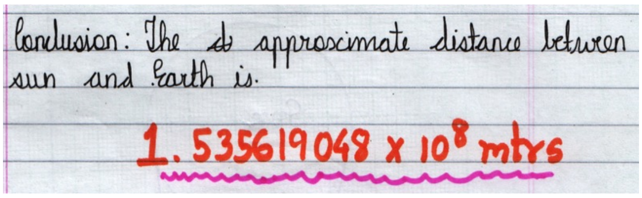
Some of the students gave a subset of the assumptions given in the above example. In other cases, students only gave the assumption regarding the distance/diameter of the Sun. For example,


Diameter of Sun is 13,92,000 km

In one case, the student confused the observation about the diameter of the image on the graph paper to be an assumption.


Assumptions: Diameter of the sun, diameter of the image.

Conclusions: Some of the students presented the conclusion in the form of statements like the one below:



The order of these varied in different reports. The structure of these reports perhaps reflect the cookbook type practicals that students might have done.

Classroom discussions post reports

Overall the students had mixed results in the activity. Not all students got a linear graph when they plotted their observations. This was in contrast to the mustard seed task where almost all the students got the linear graphs. This is perhaps related to the large possibilities of error in the measurements and problem of choosing the scales correctly. This fact led to an interesting discussion on the sources of error in the task, and deviation of the observed values from the mathematical model.

6.6 Discussions

Our textbook analysis suggested that connections between mathematics and other subjects were lacking. There is no coherent approach in designing the mathematics and science textbooks and building on the competencies of each one of them. The textbooks of mathematics and science run parallel narratives, of the same topics. In this activity, we have tried to make one such connection explicit. We used prior knowledge of the learners to create the mathematical model required for the observations and the experiment. The key difference in the SM task and the MS task seen earlier is the use of geometric reasoning as a starting point for designing our experiment and situating the problem. The SM task involved discussions around the idea of similar triangles and pinhole cameras. During the discussions, the similarities between the working of a pinhole camera and similar triangles were brought to notice. We used dynamic mathematics for demonstrating the meaning of similar triangles in the context of the pinhole camera. The discussions continued to finally create a simulation of the experimental setup with the help of the students, using the dynamic mathematics tool (GeoGebra). One of the important outcomes of this part was to enable the students to measure a value (the distance to the Sun) that they knew only as a fact. The textbook only provides this information, as a matter of fact, not giving any sources or methods by which this could be obtained. This is another lacuna that we found in our analysis of the textbooks. Most of the graphs in the textbooks that we analysed either don't give the data used in plotting a graph and or don't give the source of the data or explain how it was obtained. In this activity, such an attempt was made.

The next step involved the students constructing the experimental setup required for the measurements. Students formed groups and were guided by mentors while constructing the pinhole cameras and their mounts (Figure 6.26). The making of the pinhole cameras and assembly of the experimental setup involved many design decisions. The assumptions and precautions to be taken during the activity were discussed with the students. The discussions also involved various types of errors that the students might face during the experiment. The errors were discussed again regarding the data collected by the students when the reports were submitted. The experiment was a group task as it required more than one person to complete. Each student in the group had a chance to perform all aspects of the experiment, by taking turns at specific activities. The change in parameters could be concretely connected to the graph that they plotted. For example, the change in the distance resulted in a change in the size of the image obtained.

The distance to diameter ratio of the Sun is a unique quantity. It makes the Solar eclipses possible as this ratio is approximately same for the Moon. And yet, this ratio can be measured with simple instruments, and the setup can be analysed using results from geometry which are known to the learners from the school textbooks. The activity takes the idea of device construction based on a mathematical model and enables the learners to observe, collect and analyse data required for solving the problem. We can look at the SM task in another way; it also enables the learners to find a way to measure a quantity, which they only know as a matter of fact. In this way, the use of multiple representations is organically embedded in the activity. The discussions in the classroom, based on responses from the Pretest, helped in creating a tangible method to achieve that. The pinhole camera is a fascinating concept for the learners, and ease of its construction makes it possible to be doable, and it is a low-cost/no-cost activity as most of the resources needed are available easily. The act of writing the report and discussions based on them afterwards created a dynamic classroom enriched due to variations in the responses.

The mustard seed activity also generates a similar linear graph, but the *meaning* of the slope, in that case, is *different*. Since the two activities were done together, they illustrate the idea that *same* mathematical model can be applied in a variety of contexts and situations. Exposing the students to a variety of situations which can be depicted through graphs will perhaps allow them to become *graphicate*. Experiences like this will help them understand the nature of mathematical modelling, understanding which they can address all three levels of graphical competence as put forth by Bertin (2011). In both these activities (the MS and SM tasks) the concrete nature of the context and the constructions involved, use of the pre-existing knowledge aided to the formation of graphs. The activities together can be considered as an invitation to *graphicacy*, an introduction to the idea of becoming *graphicate*. Structurally the two activities are similar, though the SM task is complex, both conceptually and operationally. The topic of indirect measurement forms a vital unit in science and mathematics. The SM activity along with the MS activity presented in Chapter 5 covers the *micro* and *mega* scales of modelling and indirect measurement using the same mathematical model.

Context of the activity: The Sun Measurement task is set in the context of estimating a known quantity (distance to the Sun) by first creating a mathematical model for designing the setup for the task. The SM task can be seen in the context of the broader theme of the *Astronomy Summer Camp* at MVS which these studies were conducted. The general fascination about astronomy and knowledge of astronomical facts in the students (for example, see the typical response given in Figure 6.4), helped us to situate this task.

Given that the context is important in situating the activity (Janvier & Bednarz, 1989), we see the SM task as an experiential introduction to the theme of measurement in astronomy. In contrast to the MS task, the distances estimated in this case were several orders of magnitude larger. Most of the learners were aware of the distance to the Sun as factual knowledge, as revealed in the Pre-test responses, but were oblivious to methods that could lead to the measurement. So, the context of the SM task was set in designing a method to estimate this factual knowledge. The problem posed thus was concretely established within the theme of the camp. After performing the MS task, the SM task further establishes the idea of indirect measurement, in a different context, by a different route.

Mathematical modelling: As in the case of the MS task, the physical situation, in this case, was modelled using a linear equation in the form $y = m{\times}x$. However, the route taken to form this mathematical model was different from the direct, physical and intuitive approach used in the MS task (for example, compare Figure 5.12 and Figure 6.21). The creating of the mathematical model involved two steps: the first step was to recognise the similar triangles in the setup of the pinhole camera, the second step was to apply this information in the form of a linear equation in the context of the problem. The simulations created using the dynamic mathematics software, helped the students to visualise the consequences of changing the parameters in the setup. As in the MS task, in the SM task, the contextualised problem was used to attach meaning to variables in two mathematical topics: similar triangles, and linear equations. The model was not as simple the MS task but was based on concrete, hands-on experience. The SM task provided the learners with experience of using the mathematical knowledge set in a real-world problem-solving context. As Roth (1996) point out the mathematical transfer of learning in case of a real-world problem is different than in a contextual word problem depicting a realistic situation.

Measurement, real-world data and data handling: After establishing the mathematical model, and constructing the physical set-up built, the measurements were relatively straightforward. As pointed out by many researchers, relatively simple measurement of real-world data can be rich learning experiences ((Curcio, 1987; Wavering, 1989). The idea that the students can perform "experiments" to "measure the Universe" was established during the activity and the classroom discussions. Handling data is an important skill in both science and mathematics curricula. For example, the *Position Paper on Mathematics Education* in NCF 2005 (NCERT, 2006a) notes: "Data handling should be suitably introduced as tools to understand process, represent and interpret day-to-day data." (p.17). The students collected, tabulated and analysed data to solve the problem at hand. According to Åberg-Bengtsson (2006) the familiarity with the data collected by the students themselves, could lead to successful learning of many fundamental features of graphing. Moreover, as pointed out by Roth (2004), the way in which data is collected plays a crucial role in making meaning with graphs, hence it is important that the students know how the data was obtained. In our analysis of the science textbooks (Chapter 3), this is a lacuna that we have seen in most of the activities related to graphs. In case of both MS task and SM task, we provide the students with opportunities to collect, plot and analyse the data in a problem-solving context set in real-world.

Multiple representations: As in the MS task, the SM task provides similar opportunities for appreciating and developing the idea of multiple representations set in a concrete context. The presence of data in the form of verbal description, tables, equations and graphs, along with the physical setup did help the students make the connection between the multiple representations. The verbal description in the report (Section 6.5), helped to establish the experiential component.

Graphs: Graphs in the SM Task were part of the set of the multiple representations of the data collected by the students. Most of the students could move between the tabular data to graphical. Though due to the difference in the scale between the independent (~ 100 cm) and the dependent variable (~ few mm) led to issues with the plotting of the graphs which were not seen in the SM task, by the same students. For example, the choice of the scale was an issue with many students in the context of the SM task. Also, the linear nature of the graph was not seen in all the graphs, due to significant scope for errors. The interpretation of the slope of the graph is equal to the *ratio* of the distance to diameter, came during the classroom discussions. While the model is a linear one, similar to the MS task, the physical interpretation of the slope of the graph is different. This point was emphasised during the classroom discussions.

In case of the points made by (Monk, 2003), regarding meaning making from graphs (the list of points can be seen on page 131 here). The three points from this list are relevant to the SM task are:

(a) *Using a graph to analyze a well-understood context can deepen a student's understanding of graph and graphing.* In the context of SM task, the concrete, physical nature of the context and of data collection, leads to an understanding of graph and graphing. In addition, the SM task involves the construction of the setup for collecting data and also the realisation that the same mathematical model can be used to describe different situations.

(b) *Students can construct new entities and concepts in a context by beginning with important features of a graph.* In the context of the SM task, the discussions on the slope of the graph led to the idea that the slope was the distance to diameter ratio of the Sun.

(c) *A group can build shared understanding through joint reference to the graph of the phenomena in a context.* The students constructed the pinhole setup collaboratively. The data was also collected collaboratively. During the classroom discussions, the graphical data from various groups was discussed in the context of final results of the task. This sharing of results, helped the students to look at the variations in the data and final results.

Limitations and further work: The SM task could be extended by allowing the students to use another method, like using a reflected pinhole to find the distance to the Sun. In this setup, the advantage is that the image of the Sun is of the order of a few centimetres and the distance from the mirror can be changed as required. One of the limitations of this study, as was pointed out previously in case of the MS task, was the limited use of dynamic mathematics software only by the researcher during demonstrations. It would be interesting to see how the students perform when they are given access to computers for creating the mathematical models of the similar triangles and also for plotting the data. As in the case of the MS task, we would like to repeat the SM task when all the students are provided with access to computers. This is particularly relevant as many students had a problem in choosing scale while plotting the graphs.

A critique of both the MS and SM tasks might be that both these activities can be done without using graphs, for example, using only numerical or algebraic methods. What is then the role or need for graphs in these activities? We argue that, indeed, these activities can be done without graphs, but adding graphs to these aids in learning about multiple representations in a *familiar* context. The study by Preece & Janvier (1992) points that the students need features from familiar contexts to relate to graphs features. In familiar contexts, the students find it easy to understand the features of the graph (Ainley, 1995). The students approach a graph with background knowledge, most of which is irrelevant, and while interpreting the student has to decide the relevant information. The variables in the graph and its shape play a role in the selection of this relevant information. It is one of the facts which adds concreteness to the aspect of using graphs as another tool for understanding and analysing the data. The value that graphs bring, especially allowing the learner to depict a range of scales, and a variety of situations, and graphs can be used to depict quantities which are equal to or are much larger than the scale of the graph itself. Adams & Shrum (1990) discusses the effects of a computer-based intervention on construction and interpretation of line graphs of class 10 students in the context of biology. They found plotting line graphs by hand had a better impact on graph construction tasks than plotting graphs by using a computer. In the case of graph interpretation tasks, they found that "microcomputer-based laboratory exercises that collect and present experimental data to students as "real-time" graphs result in educationally significant achievement on graph-interpretation tasks." (p. 785)

In the next chapter on the EMI task, we look at a context which presents experimental data to students plotted with computers.

The science of electricity is in that state in which every part of it requires experimental investigation; not merely for the discovery of new effects, but, what is just now of far more importance, the development of the means by which the old effects are produced, and the consequent more accurate determination of the first principles of action of the most extraordinary and universal power in nature: and to those philosophers who pursue the inquiry zealously yet cautiously, combining experiment with analogy, suspicious of their preconceived notions, paying more respect to a fact than a theory, not too hasty to generalize, and above all things, willing at every step to cross-examine their own opinions, both by reasoning and experiment, no branch of knowledge can afford so fine and ready a field for discovery as this.

Michael Faraday, *Experimental Researches in Electricity. XI (1838)*

7 Exploring electromagnetic induction

In this chapter, we present the findings of our case study describing design and experimentation to explore and understand the nature of electromagnetic induction. In this task, the students studied the parameters that affect the voltage generated in a coil when a magnet is passed through it. Students made hypotheses about the effect of various parameters that affect the output voltage of the coil. To test these hypotheses, various constructions were made, and corresponding experiments were designed and conducted. The resulting data were analysed to test the hypotheses they had made as part of this task. In this chapter, we first present a review of related topics in the textbooks. Next, we describe the workflow of the task. The student interviews about their explanation of the phenomena are analysed with the focus on meaning-making from the graphs generated. Finally, the possible implications of the task towards learning in a technology-based intervention with graphs as the central focus are discussed.

7.1 Introduction

The opening quote by Faraday sets the tone for this chapter. The students indeed performed the experiments in the spirit that Faraday mentions in the quote. In this task, the students explored the interaction between coils and magnets. The task was done by constructing coils with different parameters and varying other parameters like speed and strength of the magnets. The learners were given a data logger *expEYES* (http://expeyes.in) to collect the data as the output voltage of the coils, when the magnets passed through them. The inspiration for this task came from the many "toys" that can be made to demonstrate the electromagnetic induction. For example, see Arvind Gupta's[1] vast collection of activities based on this topic. The Muktangan Vidnyan Shodhika (MVS, also known popularly as *Science Center*) at IUCAA, Pune where the fieldwork was carried out, was also the place where Arvind Gupta worked at that time (c. 2013).

The passing of the magnet through the coil is a transient phenomenon. It takes the magnet roughly less than $\sim 100\,\text{ms}$ for this event. For capturing the voltage thus generated, a data capturing device *expEYES* was used. The use of the data logging device allowed us to capture the variation of the voltage in the coil, as the magnet passed through the coil. This allowed us to study the phenomena in a *quantitative* manner. In other similar studies, the students usually do not construct the coils to be used in the experiments, for example, see these works (Amrani & Paradis, 2005; Bonanno, Bozzo, Camarca & Sapia, 2011; Kingman, Rowland & Popescu, 2002). Since the number of data points typically ranged from few hundred to few thousand, the plotting of the data with a computer became an imperative. Multiple readings for each set of the parameters created a large amount of data. The parameters that were varied included the coils, magnets and the speed of approach of the magnets. When the graphs for a given parameter are plotted together, they clearly showed the effect of the change in the parameter. For example, in Figure 7.9 we can see the effect of changing polarity and strength of the magnets.

[1]http://arvindguptatoys.com

7.2 Review of the textbook topics

The task is situated in the everyday context of electric motors which run on the principle of electromagnetic induction (*EMI*). The students knew factual information about the phenomenon through their textbooks (described below). They knew factually of various manifestations of the EMI, for example, reversing of deflection of the magnetic needle when the polarity of the magnet is changed. They were also aware of the various parameters that affect the EMI, for example, the number of turns in the coil. The problem that was posed to the students was to test and verify how these parameters affect the induced voltage.

NCERT Textbooks

The phenomenon of electromagnetic induction is usually covered in the secondary school. In the NCERT textbooks that we have surveyed, the topic is covered in *Chapter 13 Magnetic Effects of Electric Current* of Class 10 Science textbook. The NCERT textbook has two suggested activities in this regard, Activity 13.8 and Activity 13.9 (Figures 7.1 and 7.2 here).

The first activity, Activity 13.8 (Figure 7.1 here) in the NCERT science textbook shows a schematic description of a coil attached to a galvanometer and a magnet. The activity then asks to move the magnet towards the coil and then away from the coil. Activity then tells about the deflection in the galvanometer during the process. The activity then informs that the galvanometer did not move when the magnet was stationery. Finally, the activity asks the students to conclude from these observations.

The second activity, Activity 13.9 (Figure 7.2 here), shows induced *EMF* between the two coils. The second set up has two coils attached to the same pipe. One of the coils is attached to a battery, and the other coil is attached to a galvanometer. When a current is passed through the first coil, the galvanometer attached to the second coil shows a deflection. The conclusion that is drawn from these observations is that the current is induced in the second coil only when there is a *changing* current in the first coil. The discussion then proceeds to *Fleming's Right Hand Rule*. We see that this activity is a purely *qualitative* demonstration, with little or no scope for any quantitative measurements.

Activity 13.8

- Take a coil of wire AB having a large number of turns.
- Connect the ends of the coil to a galvanometer as shown in Fig. 13.16.
- Take a strong bar magnet and move its north pole towards the end B of the coil. Do you find any change in the galvanometer needle?

(a)

- There is a momentary deflection in the needle of the galvanometer, say to the right. This indicates the presence of a current in the coil AB. The deflection becomes zero the moment the motion of the magnet stops.
- Now withdraw the north pole of the magnet away from the coil. Now the galvanometer is deflected toward the left, showing that the current is now set up in the direction opposite to the first.
- Place the magnet stationary at a point near to the coil, keeping its north pole towards the end B of the coil. We see that the galvanometer needle deflects toward the right when the coil is moved towards the north pole of the magnet. Similarly the needle moves toward left when the coil is moved away.
- When the coil is kept stationary with respect to the magnet, the deflection of the galvanometer drops to zero. What do you conclude from this activity?

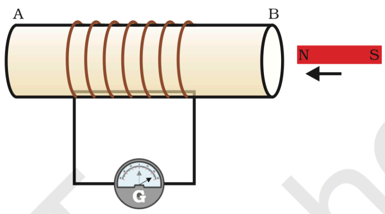


Figure 13.16
Moving a magnet towards a coil sets up a current in the coil circuit, as indicated by deflection in the galvanometer needle.

(b)

Figure 7.1: An activity from the textbook to demonstrate phenomena of electromagnetic induction by relative motion between a coil and a magnet. From NCERT Science textbook Class 10 p. 233-234. (a) An activity for mutual induction in coils. (b) The schematic diagram for mutual induction in coils.

Activity 13.9

- Take two different coils of copper wire having large number of turns (say 50 and 100 turns respectively). Insert them over a non-conducting cylindrical roll, as shown in Fig. 13.17. (You may use a thick paper roll for this purpose.)
- Connect the coil-1, having larger number of turns, in series with a battery and a plug key. Also connect the other coil-2 with a galvanometer as shown.
- Plug in the key. Observe the galvanometer. Is there a deflection in its needle? You will observe that the needle of the galvanometer instantly jumps to one side and just as quickly returns to zero, indicating a momentary current in coil-2.
- Disconnect coil-1 from the battery. You will observe that the needle momentarily moves, but to the opposite side. It means that now the current flows in the opposite direction in coil-2.

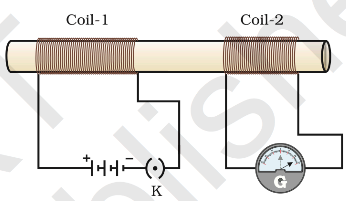


Figure 13.17
Current is induced in coil-2 when current in coil-1 is changed

Figure 7.2: An Activity from the textbook to demonstrate phenomena of electromagnetic induction between two coils. From NCERT Science textbook Class 10 p. 235.

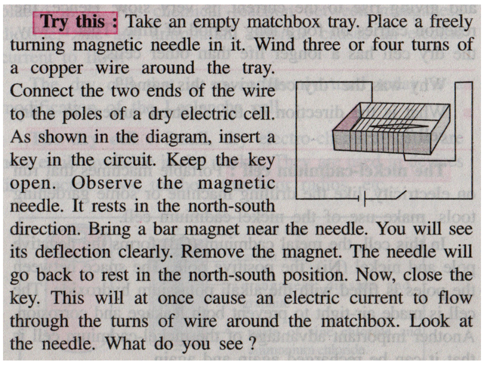
Maharashtra Board Textbooks

In the Maharashtra board (the students had studied these books) the topic of *EMI* occurs in Class 8 and Class 10 textbooks. The activities are along similar lines to the NCERT textbooks.

Class 8 Science: In Class 8, the first activity (Figure 7.3a) asks the students to see the deflection of a magnetic needle when a current carrying wire is wound around it. The second activity (Figure 7.3b) is closer to the core idea EMI task. This activity asks the students to create a coil of 1 cm diameter and as many turns of wire as possible, it is not clear how many are deemed to be sufficient. The activity then asks two questions regarding what will happen when the magnet is moving faster relative to the coil. The second question is regarding if there would be an induced EMF if the magnet is stationary. Finally, the third activity (Figure 7.3c) depicts the interaction of a coil with a permanent magnet, when a current is passed through the coil. The entire set-up is shown in a very abstract and schematic way. It does not give a clear picture of the actual set up, or actual coils or multimeter. We think when a topic is being introduced the abstraction in the visualisation of the set up should be minimal. With this kind of depiction of the setup, the students may not be able to identify real instruments when they see them. We suggest that depictions of set up in elementary introductions should be closer to the actual devices that the students might be using.

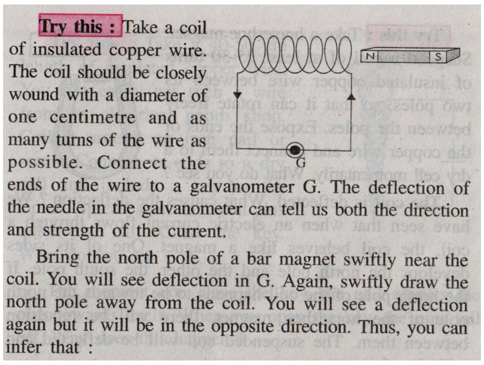
Class 10: The Class 10 textbooks of Maharashtra State have two activities, which are very similar to the NCERT activities. *Activity 5.9* (Figure 7.4) involves moving a magnet near to a coil connected to a galvanometer. The deflections in the coil are noted as the magnet is moved relative to the coil. The second activity, Activity 5.10, is a demonstration of mutual induction between two coils. A current is passed through one of the coils, while deflections are noted in the other coil via a galvanometer.

We make a couple of observations regarding these activities in the textbooks. (1) Typically the activities listed in the textbooks do not provide detailed information on the construction of the coils, for example, the gauge of the wires is not given. Due to this, the demonstration may not work as expected. For example, if we take too thick wires, the induced *EMF* is too small. Specifying the materials and construction steps in the demonstration are very important for the successful execution of the demonstration. (2) All the activities that we have looked at are



Try this : Take an empty matchbox tray. Place a freely turning magnetic needle in it. Wind three or four turns of a copper wire around the tray. Connect the two ends of the wire to the poles of a dry electric cell. As shown in the diagram, insert a key in the circuit. Keep the key open. Observe the magnetic needle. It rests in the north-south direction. Bring a bar magnet near the needle. You will see its deflection clearly. Remove the magnet. The needle will go back to rest in the north-south position. Now, close the key. This will at once cause an electric current to flow through the turns of wire around the matchbox. Look at the needle. What do you see ?

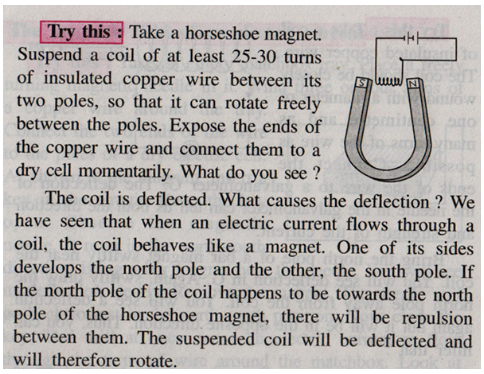
(a)



Try this : Take a coil of insulated copper wire. The coil should be closely wound with a diameter of one centimetre and as many turns of the wire as possible. Connect the ends of the wire to a galvanometer G. The deflection of the needle in the galvanometer can tell us both the direction and strength of the current.

Bring the north pole of a bar magnet swiftly near the coil. You will see deflection in G. Again, swiftly draw the north pole away from the coil. You will see a deflection again but it will be in the opposite direction. Thus, you can infer that :

(b)



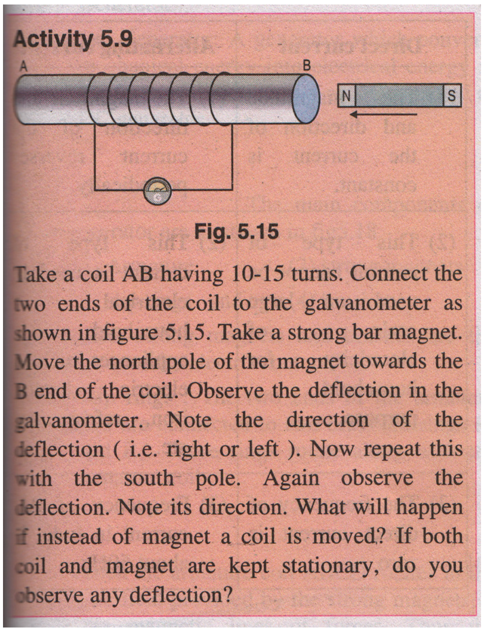
Try this : Take a horseshoe magnet. Suspend a coil of at least 25-30 turns of insulated copper wire between its two poles, so that it can rotate freely between the poles. Expose the ends of the copper wire and connect them to a dry cell momentarily. What do you see ?

The coil is deflected. What causes the deflection ? We have seen that when an electric current flows through a coil, the coil behaves like a magnet. One of its sides develops the north pole and the other, the south pole. If the north pole of the coil happens to be towards the north pole of the horseshoe magnet, there will be repulsion between them. The suspended coil will be deflected and will therefore rotate.

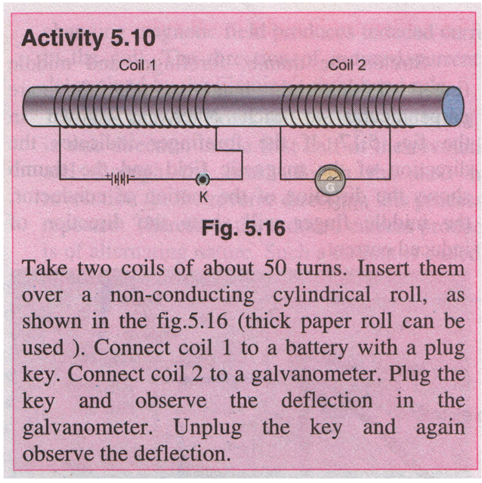
(c)

Figure 7.3: Activities from the textbook to demonstrate phenomena of electromagnetic induction between two coils. From Maharashtra Board Science textbook Class 8 p. 108. (a) Deflection of a magnetic needle by a magnet. (b) Deflection in a galvanometer connected to a coil, when a magnet is passed near the coil. (c) Deflection of a coil near a magnet, when current is passed through it.

essentially *qualitative* in nature. In EMI task we took these essentially *qualitative* demonstrations and converted them to *quantitative, hypothesis-driven, testable experiments.* (3) In the current format of these demonstrations, there is no scope for using graphs in any form. In the activities that we have designed, graphs play a crucial and central role in the process and allow the students to develop a deeper understanding of the phenomena. Graphs in this task make the *visualisation* of the phenomena possible and open up many subtleties in the phenomena which are otherwise "hidden" in the qualitative demonstrations.



(a)



(b)

Figure 7.4: Activities from the Maharashtra Board textbook to demonstrate phenomena of electromagnetic induction. From Maharashtra Science textbook Class 10 p. 55. (a) Interaction between a coil and a magnet, on the lines of the activity presented here. (b) An activity based on mutual induction between two coils.

7.3 Field study

The EMI task was conducted as an open-ended exploratory study with two students who had just given their Class 10 exams. The students were from an urban English medium school. One of the students (designated as AA) in the group had some exposure to hobby electronics and was adept at using computers. The other student (designated as AJ) was unaware of computers and electronics. However, he had good conceptual and theoretical understanding of the subject matter. The students had come to MVS, IUCAA in Pune for a Summer Project in Science. The duration of the project was for five days. During this time the students and the researcher regularly interacted each day. At the end of each day, a video was recorded, summarising the activities of the day. The researcher also took independent notes during the daily interactions and the video recordings. The video recordings were transcribed and analysed in detail to look at the learning discourse that transpired. The screen grabs from the video recordings along with the transcripts give an idea of the learning process during and after the experiments. The physical artefacts and the data generated by the students and their notes were used as another data source. Finally, researchers notes and observations form the third data source. In the following sections, we provide a detailed descriptive analysis of the constructing and learning episodes with the students.

7.4 Workflow of the activity

In this section, we describe the workflow for the EMI Task. We also highlight the applicable concepts from the design framework Figure 4.2 from the Chapter 4 are shown in brackets with a different typesetting as `(concept)`. Figure 7.5 shows the design framework as seen in the context of the EMI task.

Steps in the Electromagnetic Induction Task

1. Discussion about the nature of electromagnetic induction and parameters it depends on. `(prior knowledge) (close-to-life) (discussions)`
2. Observing and explaining the demonstration of the setup using expEYES. `(discussions) (graphs) (data) (multiple representation) (explanation)`
3. Defining the problem and defining experimental parameters for coils, magnets and speed of approach. `(brainstorming) (construction) (hypotheses)`
4. Constructing induction coils from wires with various parameters as decided above. `(construction) (experimentation)`
5. Taking observations by varying different parameters. `(experimentation) (data)`
6. Organising and plotting the data using GNUPLOT. `(graphs) (multiple representations)`
7. Analysing the data collected and testing the hypotheses. `(analysis) (testing) (hypotheses) (multiple representations)`
8. Explaining the features on the graphs in terms of physical phenomena. `(models) (inferences) (rhetoric)`
9. Writing a report explaining the process of the experimental procedure, the hypotheses and inferences. `(discussions) (reports)`
10. Presenting the work in front of the peers. `(public display) (presentations)`

Apart from the video recorded interviews during the project work, the students

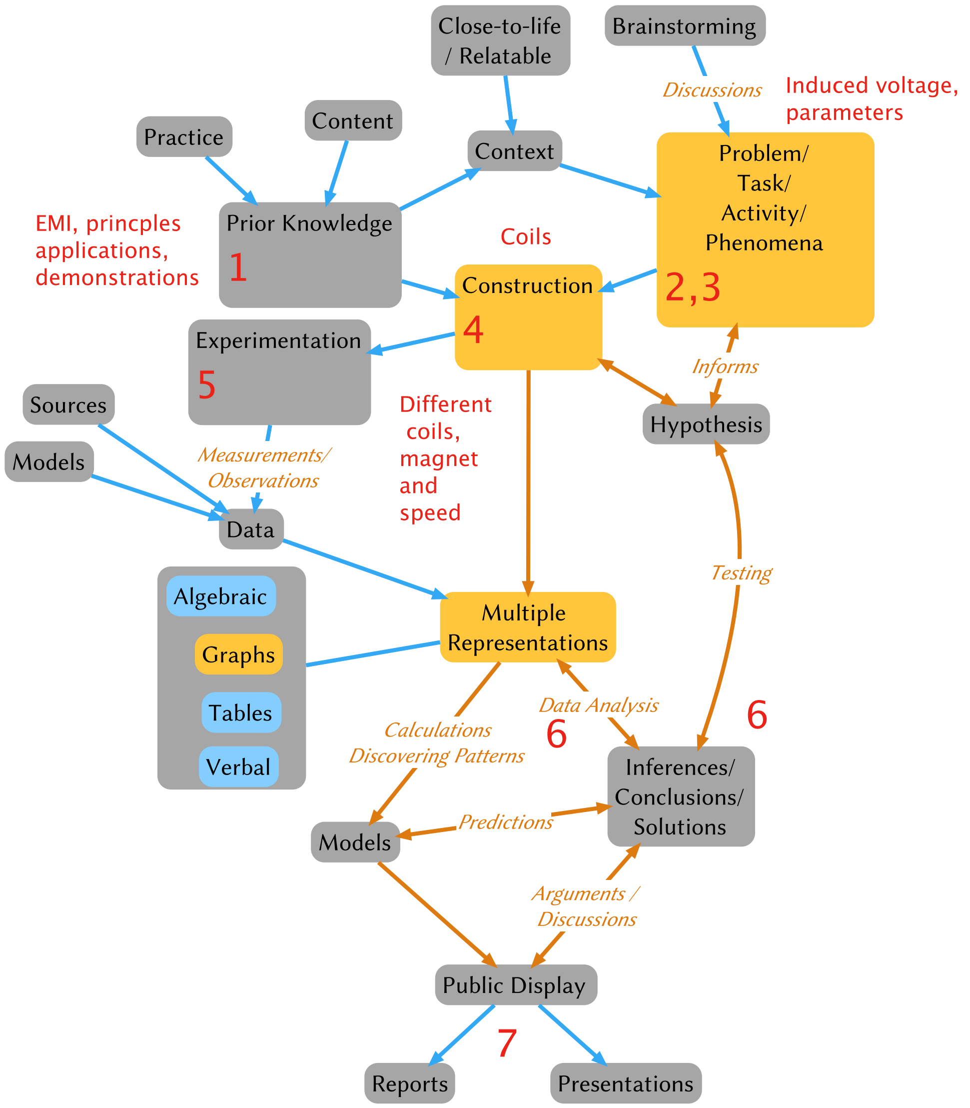


Figure 7.5: Description of the EMI task in the design framework described in Chapter 4. The red numbers in the figure indicate the steps given in the workflow.

interacted with the researcher as and when needed. The students interacted with the researcher, when they needed help or had some doubt. The box below shows the major events that occurred over the five days of the project.

Timeline of the events in the EMI Task

Day 1

Introduction to the problem and demonstration using expEYES, explanations of the students to the demonstration, finalisation of parameters and experimental design

Day 2

Practising working with expEYES, plotting with GNUPLOT, Design and construction of coils, trial experiments.orking
Days 3 and 4
Performing experiments. Recording and organising data. Plotting the data with GNUPLOT. Inferring, hypothesis testing.
Day 5
Report writing. Presentations among peers to explain: What was done? How was it done? and main results of the experiments.

In the remaining section, we describe in detail each of the steps in the workflow.

1. Discussion about the nature of electromagnetic induction and parameters it depends on.

Concepts/Skills: `(prior knowledge)` `(close-to-life)` `(discussions)`

As we have seen in Section 7.2 on textbook review, the manifestations of the electromagnetic phenomena can be demonstrated qualitatively in the classroom in a variety of different ways. However, for most of the demonstrations, the complete analytical description of the phenomena is too demanding for school students. In some cases, the transient nature of the phenomena prevents one from making any simple quantitative study difficult. The task is also seen in a variety of *Hands-On* activities. For example, see Arvind Gupta's[2] vast collection of activities based on this topic.

For example, the *Magnetic torch*[3] involves a strong magnet inside the empty test tube so that the magnet can move freely inside it. An insulated copper wire is then wound around the test tube. The two ends of the copper wire wound on the cylinder are then attached to a Light Emitting Diode (LED). The LED lights up when the magnet is moved inside the test tube. The explanation of this "toy " is readily given in terms of the *EMF* induced in the coil by the action of the moving magnet. This is a really striking demonstration of the phenomena. The construction details also give the specifications for the wire to be used and the

[2]http://arvindguptatoys.com
[3]http://arvindguptatoys.com/toys/magnetictorch.html

Figure 7.6: An activity from the Arvind Gupta's website to demonstrate phenomena of electromagnetic induction by relative motion between a coil and a magnet. In the version of the generator shown here, a test-tube is used for enclosing the magnet. The LED is attached to the coil wound over the test-tube. The LED lights up when the magnet is moved inside the test-tube.

number of turns of the wire required. In these constructions, there is a certain built-in tolerance concerning the construction parameters. Even if we change the number of turns slightly (for example from 500 to 450), or variation in the gauge of the wire by one or two orders, the demonstration will still work.

The students were first presented with a simple setup of a coil as shown in Figure 7.7. A paper tube is passed through a coil to facilitate the movement of the magnet through the coil. The LED lights up when the magnet is passed through the coil. The students readily understood the qualitative explanation of this demonstration. In this particular case when the magnet is passed through the coil, it changes the flux lines causing a current to flow through the coil. As the *EMF* generated in the coil due to this action, if it is above the threshold voltage of the LED, the LED will light up. The relevant portions of the interview which indicate crucial moments in the learning process are presented here in the form of numbered Dialogues boxes. In the Dialogues boxes, AA and AJ are the two students and AD is the researcher. After demonstrating this setup, the students were asked to explain it. The students came up with an explanation as given in Dialogue 1.

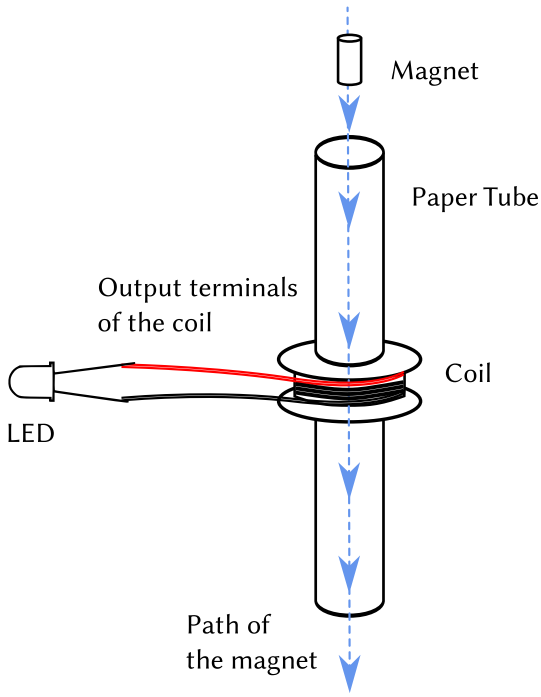


Figure 7.7: The basic setup of the EMI demonstration. We can attach LED to the output terminals of the coil. When the magnet is passed through the coil the LED lights up momentarily. This lighting up of the LED is seen as an evidence of the voltage generated in the coil due to passage of the magnet through the coil.

Dialogue 1

AD: How do you explain the lighting of the LED?

AA: EMF is generated whenever there is a relative motion between the coil and a magnet. In this case we have a coil which is stationary and the magnet is moving. As we release the magnet it falls under gravity. When the magnet goes through the coil, the magnetic lines are cut, and hence current is induced in the coil. Due to this there is a emf generated, which lights up the LED.

Statements in Dialogue 1 shows that the students knew the Faraday's *Law of Electromagnetic Induction*, which was used in the explanation above. They also had the knowledge of Ohm's law in its algebraic form $V = IR$, here V is the voltage, I is the current and R is the resistance. Ohm's law was used by the students to explain many observations subsequently that came out from the experiments.

2. Observing and explaining the demonstration of the setup using expEYES.

Concepts/Skills: (discussions) (graphs) (data) (multiple representation) (explanation)

Observing the phenomena using expEYES

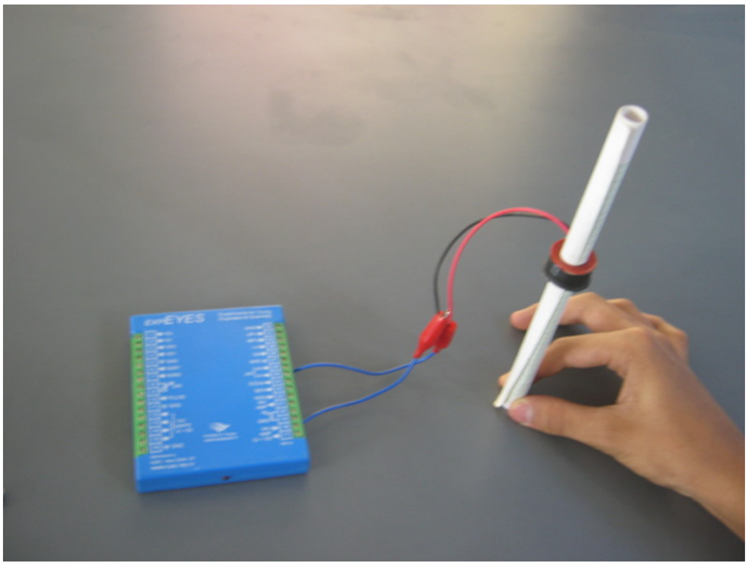


Figure 7.8: Attaching the coil to the *expEYES* device makes the phenomena "visible" by graphs.

When the experimental setup shown in Figure 7.7 is attached to *expEYES* instead of the LED (Figure 7.8), we can measure not only the peak of the voltage but also *observe* the waveform of the phenomena. We use the inbuilt program in the Graphical User Interface (GUI) for *expEYES* for studying electromagnetic induction. The programme for the study of electromagnetic induction requires only two connections to be made (between Terminal A1 and GND of the device). The two ends of the coil were connected using wires to these two terminals. The device is the then connected to the computer via a USB cable. When a coil is connected to the device, the entire event is captured in terms of the voltage at the terminals of the coil during the time of magnet passing through the coil. If we attach a digital multimeter to the coil, we see the peak voltage for a fraction of a second before the reading falls off to zero.

In the interface of the programme (Figure 7.9), pressing the *Start Scanning* button the program looks for possible signals of the experiment. The programme displays the captured signal as a graph as shown in Figure 7.9. The programme also displays the peak values and the time between them. The users can continue to scan for more readings or save the data readings to a text file. The data file has two columns one corresponding to the time and other to the voltage. The observation lasts for 100 milliseconds, and the voltage is recorded every 0.5 ms. Thus, each experimental capture has 200 data points. A typical data file looks like as shown in the Table 7.1.

Time (ms)	**Voltage** (V)
0.000	-0.059
0.500	-0.059
1.000	-0.059
1.500	-0.059
2.000	-0.059
2.500	-0.020
3.000	-0.059
3.500	-0.098
4.000	-0.098
4.500	-0.059
5.000	-0.176
⋮	⋮

Table 7.1: The typical output of voltages from the event of a magnet passing through the coil captured by expEYES.

The data file can be saved with different names, and this feature is useful for plotting the data later. We can also review the graphs from earlier experiments at the same time as shown in Figure 7.9. After the demonstrating this setup, the students were comfortable with attaching the device to the computer, conducting the experiment and saving the data files. The files on the computer were saved with names pertaining to the data that they contain. For example,

```
12cm_std_coil_0.dat
12cm_std_coil_1.dat
12cm_std_coil_2.dat
15cm_std_coil_0.dat
15cm_std_coil_1.dat
```

The above list has files from the speed experiments. The names of the files indicate the distance of the drop (12, 15 cm, a standard coil was used in the experiment (std_coil) and the number of the data set (0, 1, 2). This nomenclature was used for all the data sets that the students collected. The students did multiple observations with each setup. The ease of collecting data with the setup is an immense advantage to perform a large number of variations and repetitions of the experiments in a relatively short time.

These data files then were plotted using the GNUPLOT plotting software.[4] GNUPLOT is a command line driven plotting software which can load data files (like the ones saved by the expEYES programme) and plot them as required. The plotted files can be exported to various graphic formats like *pdf, svg, png* etc. The range, grids and colour in these graphs can be set for different datasets. The students were given and explained the GNUPLOT syntax required for plotting the files. The graphs presented in this section were also created using GNUPLOT. The graphs have been *recreated* by the researcher using GNUPLOT with the same data for clarity in discussions.

When expEYES capture the passage of the magnet through the coil, we see the typical curves as shown in Figure 7.9. There are many features to note in the graphs shown in Figure 7.9. The first striking feature is that there are *two* peaks in the waveform, one positive and one negative. The second feature to note is that for some of the waveforms, the order of the peaks is reversed. The values of both the peak voltages and the time difference between them are displayed.

3. Defining the problem and defining experimental parameters for coils, magnets and speed of approach.

Concepts/Skills: `(brainstorming)` `(construction)` `(hypotheses)`

The task and the parameters

The presence of the graphs for this phenomena opens up an entirely different avenue for the kind of studies that can be done, what kind of observations can be taken. With the graphs now one can think of conducting quantitative experiments, where we can get exact numerical results for the induced voltage. The task that was given to the students was to determine the parameters that affect the induced voltage in the coil. This task was discussed with them after they were shown a basic demonstration of the *Test tube Generator* and after that attaching the coil to *expEYES* and looking at the resulting waveform.

[4] www.gnuplot.info

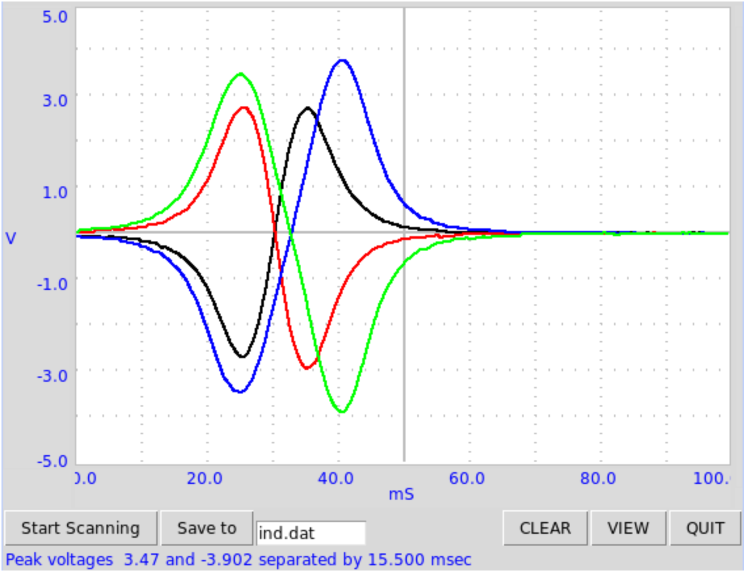


Figure 7.9: A set of typical results from the experiment in Electromagnetic induction (EMI) task as shown in the *expEYES* software. Note the various buttons and the information displayed. The graph shows the voltage recorded across a coil when a magnet is passed through the coil. Each colour in the graph represents a different observation with changes in experimental parameters. Note that (a) there are *two* peaks in the graph, (b) the change in polarity and amplitude for some waveforms, (c) the complete event takes place in about 70 ms, and, (d) the second peak is always larger in magnitude than the first one (e) for each observation the induced voltage is different. This information is completely missing from the qualitative demonstrations.

The students used their previous knowledge about electricity, magnetism and electromagnetic induction to deliberate upon the parameters to be varied. In the discussions three major categories of the parameters emerged: *the coils*, *the magnets* and *the speed of approach*. The categories, the hypotheses about their relation to the induced voltage and the reasons why the students thought they would affect were discussed with the students. In each of these categories, various parameters can be changed. In the discussions, the students provided rationales for choosing the parameters from some of the facts already known to them. Also, a testable hypothesis was made for each of the parameters. The students were given access to various tools and raw materials that they would require for the experiment. This included various tools for cutting, joining, electrical soldering, wires, etc. The students were provided with access to a computer with the *expEYES* setup for collecting data from the experiments. In the next section, we discuss the actual constructions and design of the experimental issues for the three parameters.

A: Experiments with coils The coils are the component which can be the easily changed. According to the students, the main parameter in the coils that would affect the induced voltage was the *resistance* of the material of the coil. The students knew as a matter of fact that the resistance depended on the *material* and the *thickness* of the wire. Hence they reasoned that all the parameters that affected the resistance of the coil would also affect the induced voltage. Dialogue 2 below makes this clear. The students were asked to explain which parameters of the coils they chose and why.

Dialogue 2

AD: Can you explain me what has been told to you so far

AA: So far, main objective of the experiment was to . . . experiment about a coil and a magnet. And we had to . . . Our experiment was pass a magnet through a coil and change various parameters of the coil and magnet and make observations about the experiment.

⋮

AJ: With respect to the coil, the first point is the material of the conductor, length and the cross section area.... Material of the conductor matters for resistivity, and length and the cross sectional area matter in the terms of resistance.

AA: So the resistance of the coil would change, if we change any of these parameters [*both say the same*]

AJ: So changing this would alter the current. [*the reasoning here is deriving from the factors that* AJ *has spoken in the first sentence*]

AA: So the current produced, . . . if all other parameters are constant and if only one of this one is changed and we do the experiment, we would find change in these factors.

In Dialogue 3 the students describe the number of turns in the coil and its implications. This reasoning is based on the notion of the magnetic field and its relation to the number of turns. The students knew about the relation of the number of turns and the amount of current produced as a matter of fact from the textbooks. They ascribed this change to the increase in the magnetic field.

Dialogue 3

AD: . . . So one thing is the materials that you will be using. What is the next thing in your agenda?

AA: The number of turns that is the number of times we have would the conducting material around the core, whatever it is. So if we change that surely we will observe a change in the amount of current produced and it would be possibly because the magnetic field which interacts with the amount of conducting material will be more as there are more turns. So as a result we would have more amount of current when we have more number of turns. Thats why I think, such a thing would happen.

The students also had a notion of "density" of coils as a parameter. This was subsumed under the parameter of the number of turns after a discussion. The next coil parameter that the students consider is the diameter of the coil. They consider the increase in the distance related to a "resistance" to the magnetic field by free space. They did not have a notion about the decrease in magnetic field-strength by the increase in distance as evident in Dialogue 4.

Dialogue 4

AD: What is the next (thing)?

AA: The diameter of the coil.

AD: So what do you expect will happen? When you increase or decrease the diameter?

AJ: If we increase the diameter of the coil then we allow free space between the magnet and the coil. So more the free space more is the resistance of the magnetic field . . . And if the intensity of the magnetic field is reduced the current will surely be reduced. So if we decrease the space, air space between the coil and the magnet we can surely get more current because the resistance of the air for the magnetic field is more than that of the coil.

We summarise the main points of the discussions about the parameters in Table 7.2: We return to the discussion about these hypotheses when we look at the data from the experiments after they were performed.

B: Experiments with magnets The magnets are the second parameter that the students considered for the experiments. They knew from the textbooks that increase in magnet strength would lead to increase in induced voltage. They also considered *size* of the magnet playing an important role in the induced voltage. The reasoning was given in terms of the interaction time of the magnet with the coil. The discussion then centred around how we can change the strengths of the

Parameter	Hypothesis	Reasoning
The material of the wire of the coil.	Different materials have different resistance, hence they will induce different voltage	Known as a matter of fact. The experiment could not be performed due to lack of materials.
The thickness of the wire of the coil.	Thicker wire will produce more induced voltage.	The resistance of the wire depends on the thickness of the wire. A thinner wire will have more resistance, hence will induce a lesser voltage.
The diameter of the coil.	Larger coils will have lesser induced voltage.	The distance between magnet and coil would be more.
The shape of the substrate.	Different shapes will induce different voltages.	No particular reasoning was given.
The number of turns in the coil.	Larger number of turns will induce more voltage.	More magnetic lines of flux were cut as the area was larger.
The material of the substrate.	Different materials will give different induced voltages.	The material will affect the space between magnet and coil and passage of magnetic lines. The experiment could not be performed due to lack of materials.

Table 7.2: The final set of parameters and testable hypotheses for the coils in the experiments.

magnets for the experiment. Dialogue 5 details the strategies on changing the strength of the magnets by attaching them in different configurations.

Dialogue 5

AA: Next thing is about the magnet. We are finished with the coil. The first parameter is the strength. So we can change the strength of a magnet and More the strength more will be the induced current less the strength less would be the induced current. And it is *obvious*, so no . . . I don't feel . . . I don't have any reasoning for that. [*emphasis in voice*]

AJ: After this . . . second factor is the size of the magnet. If we take a short magnet [*Gestures the action with hand*] passing through the coil then the frequency of the current . . . that will be induced, that would be lesser.

AD: Frequency or amplitude?

AA: Yes and I think, smaller the magnet, faster it will pass through the coil so the two poles will interact faster, longer the magnet it would take long time for it to pass through the coil [AJ *nods in agreement*].

Hence the two poles would interact with the long after a long time hence. The North Pole will interact and after sometime the South Pole will interact . . . Some time means it might be very less, but after sometime the South Pole will interact. Keeping all other things constant. Expecting that the speed of the both the magnet is same for the smaller one and the larger one. In that case I think that the . . .

AJ: The direction of the current will be changed after a shorter duration for the shorter magnet. For the longer one, as it has length, the direction of the current will be changed after fixed time.

Thus in case of magnets the experimental design came down to the following two approaches:

§ **Increasing the number of magnets in different configurations:** attaching them pole-to-pole and side-by-side.

§ **Using different magnets:** the students had access to 4 different types (sizes) of cylindrical magnets of varying strength, but we decided against it as there would be no comparable parameter in the magnets, and we did not have access to a Gauss-meter to measure the absolute strengths of the magnets.

In the first strategy, we also discussed whether the way in which we join the magnets would affect the induced voltage. If we have two cylindrical magnets, then they can be joined in two different configurations (shown in Figure 7.11). One is joining them pole by pole, with opposite poles joined together, in this case, the length of the magnet gets doubled. The second method is to join them sideways, in this case, the students reasoned that the strength would be reduced, as lines of force from poles of one magnet would end up on the opposite pole of the second magnet. The students were sceptical whether the magnets would join sideways (as can be seen in the following dialogue). It is indeed the case that the magnets are so strong that it is virtually impossible for them to join sideways (or pole-to-pole for that matter) with similar poles. However, this was a proposition that was to be tested in the experiments, and they were not sure about this line of reasoning as is evident in Dialogue 6.

Dialogue 6

AD: . . . One way of characterising the magnets, we have mostly cylindrical magnets, like this something like this [*AD shows them the cylindrical magnet, both nod in agreement*]. So that is what we tried yesterday. But maybe you can attach another magnet just like this [*AD draws the magnets sideways and shows to them*] instead of attaching pole-by-pole you can just attach the cylindrical part of it.

AA: Cylindrical part of it? [*nods in agreement*]

AJ: But will they attach [*gestures sideways attachment with hands*]

AA: As both of them are of the same shape . . .

AD: See one configuration is like this [*shows them the drawing*] you attach them pole-to-pole . . . something like this [*both nod in agreement*] . . . another configuration is to attach them sideways

AJ: But they are [*inaudible*]

AD: Oh, Thats ok, they are cylindrical.

AJ: But will they attach [*gestures with hand about aligned magnets, Figure 7.10*] to each other only along a single line or . . .

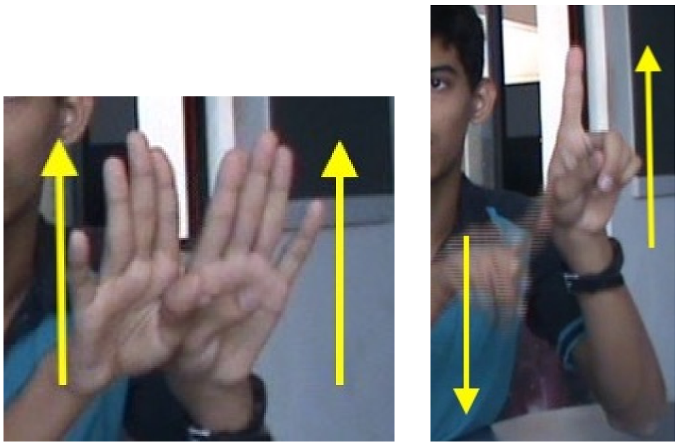


Figure 7.10: AJ explaining the configuration of the magnets with hand gestures.

AD Try it!

AA They would would attach, because they are strong enough to attach themselves to each other but the shape formed would be difficult for us to pass through the tube and

AD Why not? You have tube which is bigger than the size [*than that of the magnet*].

AA While passing through the coil . . .

AD Why, what will happen?

AA Will there be a change because . . .

AD That you can test it. So we cannot predict everything na . . . [*both of them laugh*] So you need to have some . . . Something which you should know . . . don?t know. If you know everything what is the point of doing the experiments. So magnets you can try these two configurations separately, maybe you can try four of them or three of them at one point. See because . . . Will the strength increase linearly with the number of magnets and the induced voltage?

AA That we can predict by seeing the readings.

Thus from a set of two similar magnets, we could produce three different configurations. This design was finalised for experimentation. Table 7.3 shows the final parameters and hypothesis with the reasoning for the experiments with magnets.

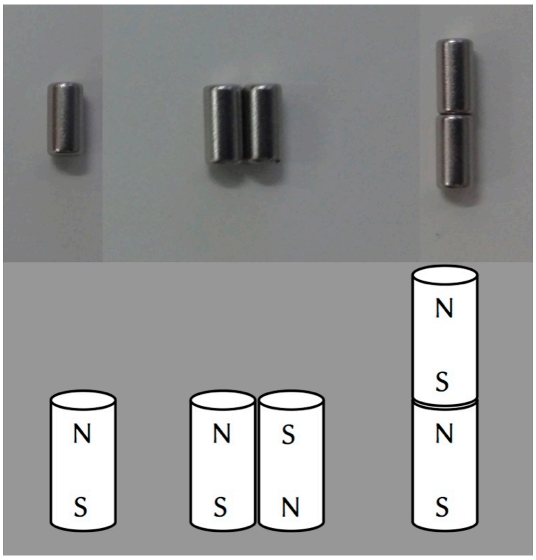


Figure 7.11: The different configuration of magnets for use in the experiments. The pole labels N and S are for an explanatory purpose.

C: Experiments with the speed of approach The students knew as a matter of fact that the induced voltage is directly proportional to the *relative* speed between the coil and the magnet. They also said that the induced voltage would be the same if the coil approached the magnet with the same speed. In the demonstration, the magnet is dropped in the paper tube (no initial velocity) and falls under gravity. So, in this setup "How can one vary the speed of the magnet? " This question was posed to the students. Dialogue 7 shows their deliberation on this matter.

Parameter	**Hypothesis**	**Reasoning**
Magnet strength.	Increasing magnet strength will increase the induced voltage.	Known as a matter of fact and performed with three different configurations of magnets.
Magnet polarity.	Reversing the polarity will reverse the induced voltage.	Known as a matter of fact and reasoning with the direction of the relative velocity between the coil and the magnets.
Magnet configurations.	Configuration with magnets attached side-by-side would give the minimum induced voltage, while magnets attached pole-to-pole will give maximum induced voltage.	In the first configuration, the opposite poles of the two magnets will "cancel" the lines of force.

Table 7.3: The final set of parameters and testable hypotheses for the magnets in the experiments.

Dialogue 7

AJ: So more the relative motion between coil and magnet, more the emf is produced.

AD: Ok. So this . . . I mean . . . you know as a matter of fact or

AJ: Yes we know this as a matter of fact [*smiling*].

AD: How do you plan to make this relative motion? In the experiment?

AA: By using the property of gravity, that objects are accelerating when they fall. [*Note here the precise knowledge about free-fall, this was supported further by use of equations of motion*]

AJ: By changing the distance at which the coil is fixed on the tube for the passing of the magnet and fixing the length of the tube for which the magnet falls, we can alter the . . .

AA: Slope.

AD: Slope? [AA *nods in agreement*]

AJ: . . . velocity of the magnet. Because as the . . . [*gestures the fall with hand*]

AA: Free falling bodies, free falling bodies keep on accelerating, so lower the coil more would be the speed and when the coil shifts upwards then the speed would be less.

AD: Ok. So, I mean you are expecting a difference in the two methods? Using the slope and changing the position of the coil?

AA: Yes. I think that changing position would be easier, because . . . And I also think that the readings would be more accurate because . . .

AJ: We can calculate the length of the tube or channel which we are used for falling the magnet and we know our gravity, the constant of gravity (g) so we can calculate using that the time, sorry the speed of the magnet for the different observations. And then we can calculate for how much, by how much it has changed.

AA: Also it is much easier, as in the slope we would have to measure angle and all so it's not so easy.

In the discussions that followed the students were suggested to do the slanting experiment nonetheless. As they explained later, the slanting setup "dilutes" the gravity, and hence the magnet would fall slower than in the straight setup. Another approach that they suggested was to impart some initial speed to the magnet when it is dropped in the paper tube. However, when asked about the way in which this could be actualised, they gave up on this idea, saying that it would be too complicated to implement.

We discussed on the line of initial speed further. In the discussions, a reference was made to Newton's laws of motion. In the expression for the final speed of the object there are three terms: the initial speed (u), the acceleration g (in our case due to gravity) and the time of fall (t).

$$v = ut + \frac{1}{2}\, g\, t^2$$

During the discussions, it emerged that when an object falls for a longer time, it gains more speed in case of constant acceleration. Another way to put it is: if an object falling under gravity covers larger distance, its speed will be more. So from this, another way of varying the speed was found. If we change the distance between the coil and the magnet, the speed of the magnet when it passes through the coil will also change. Thus varying this distance will also give us different speeds of the magnet. We see this clearly in the last two lines of Dialogue 7.

In both the cases, of the magnets and the speeds, we do not have the quantitative numerical data regarding the strength of the magnets or the actual speed with which magnet is moving. However, for our purpose, this data is not required. Though, one can always add more instruments to get this data in a quantitative format its absence does not affect the primary focus and results of our experiments.

In our case, we can make the inferences regarding the factors on which the induced voltage in the coil depends.

Parameter	Hypothesis	Reasoning
The relative speed between magnets and the coil.	Increasing speed will increase the induced voltage.	Known as a matter of fact and performed with two configurations (straight and slant) for varying speeds.

Table 7.4: The final parameter and testable hypothesis for the speed in the experiments.

Overall the three parameters used in the experiments allowed the students to concretely vary the parameters and see variable results in the dependent parameter, the induced voltage. Table 7.5 shows the parameters that were finalised for the experiments to be performed. Once these were finalised, the students proceeded to construct coils with different parameters as decided.

Parameter	Variation	Other Parameters
Diameter of Coil	0.5 inches	35 gauge wire, with 450 turns
	0.75 inches	
	1.5 inches	
The Number of Turns	250	gauge 30 wire, 1 inch diameter
	500	
	650	
Diameter of the Wire	30 gauge	250 turns and 1 inch diameter
	35 gauge	
	41 gauge	
Magnet Configuration	Single magnet Two magnets pole-to-pole Two magnets side-to-side	Standard coil
Speed of appraoch	**Straight drop:** varying the distance of the coil in the paper tube **Slant drop:** slanting the paper tube at different angles	Standard coil

Table 7.5: The final set of parameters used in the EMI experiments.

4. Constructing induction coils from wires with various parameters as decided above.

Concepts/Skills: (construction) (experimentation)

The Constructions The students were provided with some readymade coils of about 500 to 550 turns with 35 gauge wire. These coils they called "standard" coils. The experiments with variations of magnets and speeds were performed with the standard coils. In cases where the parameters of the coils were to be changed the students *constructed* the coils with required parameters. For this purpose, the students were provided with raw materials and tools for making the coils. The tools included insulated copper wires of different gauges, cutting tools, PVC pipes of different diameters for the substrate, adhesive tapes, glue, etc.

(a) (b)

Figure 7.12: Students constructing coils with required parameters. (a) Winding wires on the coil, one student is holding the bundle of wire, while the other student is winding and counting the number of turns on the coil. (b) Some of the coils constructed by the students.

The construction of the coils provides a very *concrete* basis for understanding the phenomenon. Due to the context of these constructions, the students always had a concrete object to think with when they analysed the experimental results. The coils, in this case, were the "objects to think with" as Papert would have put it. The graphs resulting from the experiment were always looked through the lens of these constructions. The explanations of the students during the interviews reflect this very well.

The construction of the coil was one of the things that the students enjoyed the most during the project. The students explained in detail how they constructed the coils. A basic demonstration video from Arvind Gupta Toys was shown to them. At many places during the construction, the coils required deliberate design decisions. These design decisions and the rationale behind them are amply reflected in the step-by-step process of coil construction as narrated by the students. The idea of using a paper scaffold to hold the wound wire in place over the coil was one such innovation that the students arrived at due to the problem they faced of wires sliding off the substrate.

The students were told that these experiments should be repeatable by others, and for that how they constructed the coils should be known. The steps that they followed in making the coils are listed below (as transcribed and summarised from the interview):

Steps for constructing the coils

1. Select the substrate and cut the required length of the substrate.

Figure 7.13: Students cutting the substrate.

2. Mark the area in which the wire is to be wound, this needs to be uniform, otherwise it will add another parameter which is changed.

3. To keep the turns inside this area, attach a paper to the substrate such that it forms a hollow in this region (Figure 7.14).

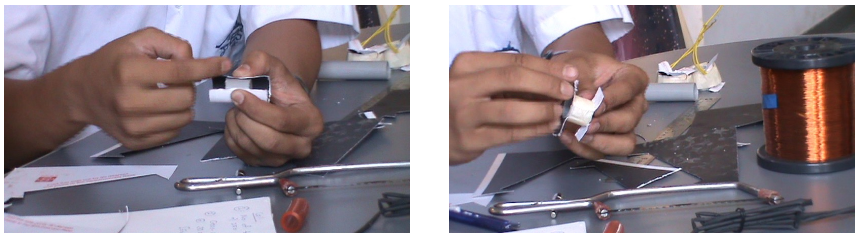


Figure 7.14: Attaching the paper base to the substrate. Folding the paper back so that it forms a hollow.

4. Keeping a sufficient length of wire out (for making the electrical contact), an enamelled copper wire is struck on the substrate.

5. Then the wire is wound on the coil. In this one person was holding the wire roll, while the other was winding the wire on the coil. The counting of the number of turns was done by the person who was winding the wire on the coil (Figures 7.14 and 7.12a).

Figure 7.15: Winding the wire on the coil.

6. Once the sufficient number of turns are made, the wire is cut after keeping a sufficient length for electrical contact.
7. Keeping the start and the end of the wire outside, a tape is wound over the rest of the turns.
8. The electrical insulation is scrapped off the ends of the wires. One way to scrape it is to remove it mechanically using a sharp blade. Another way is to burn the wire so that the insulation burns off leaving the copper exposed.
9. Two regular insulated wires are used for making the electrical contacts, with insulation removed from their ends.
10. A soldering gun is used to make an electrical contact between the copper wire and regular wire.
11. The electric contact thus made is covered with an insulating tape or a *heat-shrink* tube.
12. These wires are wound on the coil, and a tape is put again on them.
13. It is made sure that no mechanical stress is on the electrical contacts.
14. Finally, the coil is tested for electrical continuity. This can be done either by using the continuity tester setting in a multimeter or lighting a LED from a battery using the coil as wire.

As we can see in the detailed step-by-step guide given by the students, they became immersed in the project. In the next step, the students performed the experiments with the constructed coils and variations in magnet and speed configurations.

5. Taking observations by varying different parameters.

Concepts/Skills: (experimentation) (data) (multiple representations)

In this section, we describe the experiments that the students performed with the setup. The students had a clear idea about varying only one parameter during the experiments while keeping other parameters constant. All the data from the experiments was saved on the computer. The graphs that follow are redrawn by the researcher for better clarity for discussions in the thesis using the data generated by the students.

Explaining the phenomena and discovering new facts

After they had done the experiments, the students were asked to explain the typical sinusoidal curve that is generated due to the passing of the magnet through the coil. Figure 7.16 shows one such typical curve. In the experiments that follow, (a) the peak voltage, and, (b) the time at which they occur were found to vary with the parameters. In this case, the students were asked to relate the phenomena that they saw to the graph it was representing (Figure 7.16).

Dialogue 8

AD: Ok . . . So can you explain me the nature of the graph [*that you are holding?*]

AJ: The nature is one wave [*points with fingers along the line of the graph Figure 7.17*] . . . the graph is one complete wave . . .

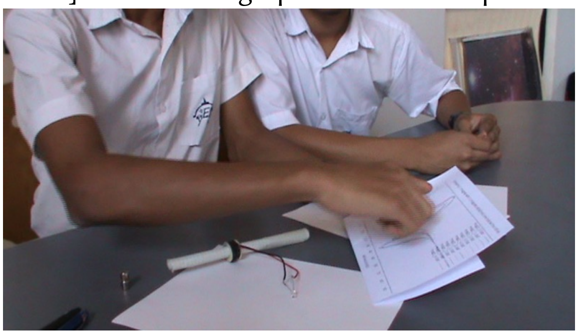


Figure 7.17: Student showing the shape of the graph.

because when we [*takes the magnet and gestures its passage through the coil*] apparently it is *similar* to this . . . [*points to graph, here we can see that the*

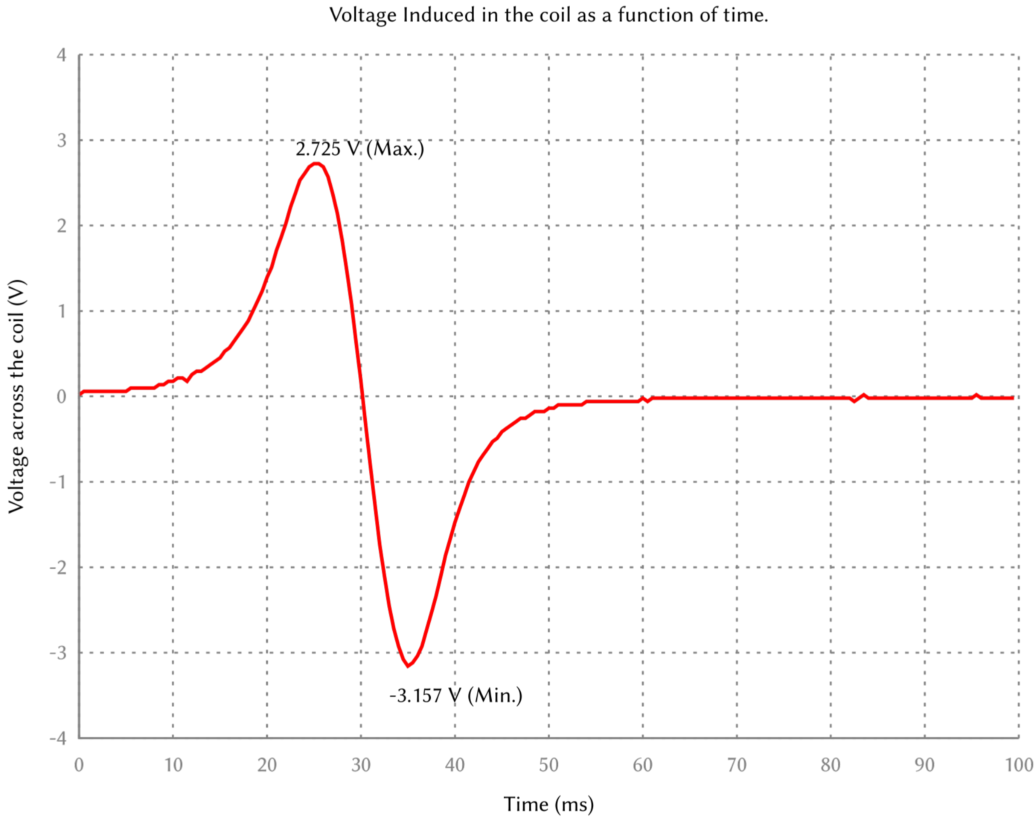


Figure 7.16: A typical curve resulting from the drop of the magnet in the coil. Notice the nature of the waveform, the twin peak voltages and the time taken by the induced voltage to rise from zero to go back to zero.

students have made a direct correspondence between the concrete action of the magnet passing through the coil and the graph representing it] when we pass a magnet through this [*drops the magnet through the coil*] it approaches the coil [*points to the coil*] then . . . moves away from the coil [*gestures with hand the movement away from the coil*] . . .

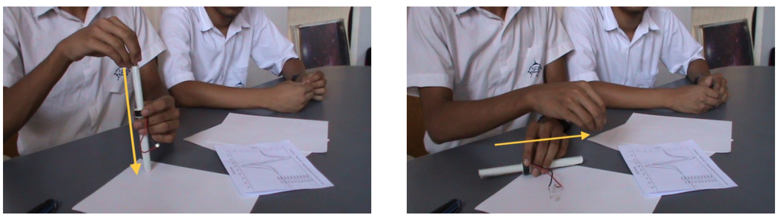


Figure 7.18: Student showing the movement of magnet in the coil.

AD: Ok . . . then . . . So if one has to draw lets say . . . I mean where would be the graph when the magnet is at the centre of the coil? Can you guess that?

AJ: Also to the coil goes . . .

AD: So if you have to place the coil somewhere in this graph, physically, so how would you do that?

AA: So . . . At this point [*points finger to the point on the graph where the line graph intersects with the X-axis*] the magnet would be at the centre of the coil.

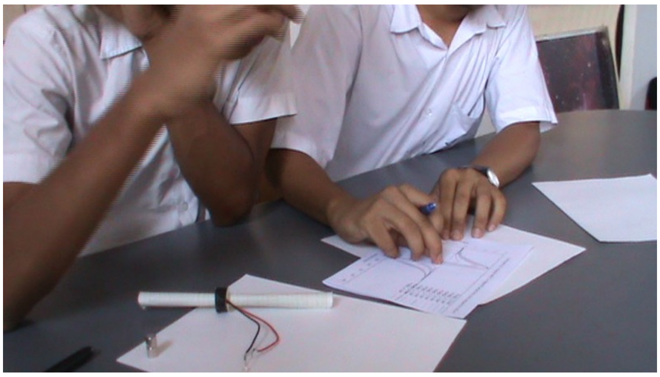


Figure 7.19: Pointing to intersection of the graph with X-axis.

AD: Which point?

AJ: When it is . . . when again it is zero.

AD: It is zero, when it changes the direction . . .

AA: When it is zero, when it changes the direction . . . When it changes the **polarity** from the current . . . at that time the magnet would be at the centre of the coil. [*Note here the students are relating a concrete physical event to a feature on the graph.*]

AJ: When it changes the polarity

AD: Centre of the coil? . . .

AA: And at this peak [*points to the first peak on the graph*] there would be some specific distance from the coil where the magnet is atthat time is at peak in velocity and it is same for this one [*points to the second peak on the graph*] some in this direction [*points to the areas above the below the coil on paper tube, in this case we can see the direct correspondence between the coil and the graph (Figure 7.20).*]

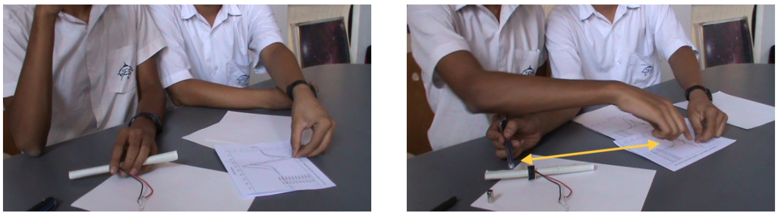


Figure 7.20: Pointing to correspondence between the coil and the graph.

AD: Ok.

AJ: At some specific distance . . .

AA: There must be some specific distance where these two points would be achieved [*points to the two peaks with his fingers Figure 7.21*]

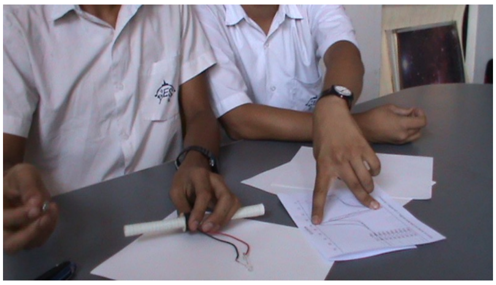


Figure 7.21: Pointing to the two peaks on the graph.

and at the centre it would be zero volts [*points to the intersection with X-axis Figure 7.22.*]

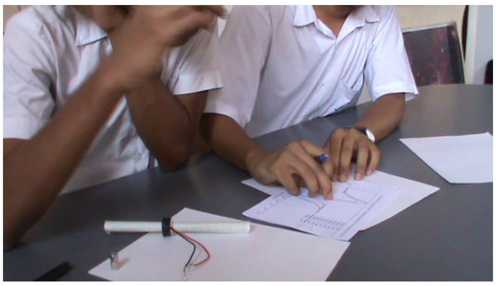


Figure 7.22: Pointing to the intersection of the curve with X-axis.

AD: Ok.

AA: And in between any values will be attained. [*Notice how the movement of the magnet through the tube is being corresponded with the waveform of the graph.*]

AD: Ok. Is it only the velocity of the magnet or also the distance from the coil?

AA: Yes. Also the distance from the coil . . .

AJ: Also the distance from the coil.

AA: Yes, that is also important . . .

AD: Ok, once it is inside the coil . . . I mean what would be the effect . . . See now we now talking about the magnet approaching the coil or going away from the coil .

AA: Once it is inside and if we . . . if we think that it is for there stable . . . then there would be no emf produced . . . so that is why there would be a zero reading [*points to the zero reading in the graph Figure 7.23*] at that, at that one spot.

AJ: At one spontaneous reading

AD: But it is moving at the same time? Right?

AA: So. So when it starts moving away from the centre then some negative voltage would start being produced or when . . . [*Here the Lenz's law is being used in its implicit form.*]

AJ: So it is neither . . . away from the coil . . . but at the centre of the coil.

AA: That would be zero.

AD: That would be the zero mark. I mean . . . Can you draw a X-axis there? With some other . . . may be you can draw it with blue colour so that you know . . . take a scale and draw it . . . [AA*takes the pen to draw*] or use another paper to draw the X-axis . . . from zero [AD *wants them to draw the X-axis scale, they are a bit confused about what exactly is to be done*] . . . X axis I want . . .

AA: X-axis along this [*points to the bottom scale of the graph Figure 7.23.*]

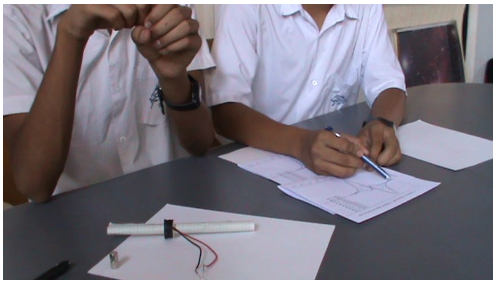


Figure 7.23: Pointing to the bottom scale of the graph.

AD: From zero . . .

AA: From zero [*points to the left corner of the graph, perhaps a habitual thing (Figure 7.24).*]

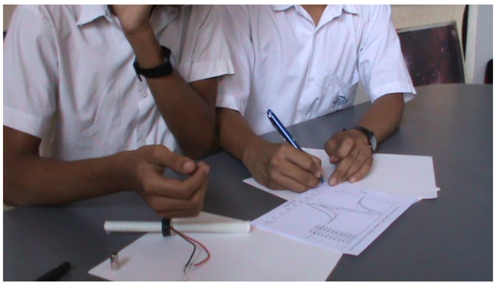


Figure 7.24: Pointing to the bottom scale of the graph.

AD: From zero volts . . .

AA: From zero volts [*points to the horizontal scale showing zero volts*] ok . . . here we have X-axis . . .

AD: You use another paper for that . . . so that you can draw . . . [AA *uses another sheet of paper as a ruler to draw the X-axis line*] by default it doesn't give you the X-axis [*We had used GNUPLOT for plotting the graphs with default options*] is it correct? Ok anyway . . . [AA *draws the line along the X-axis with blue pen*] its ok for our current purpose . . . So where is the point that you . . .

AA: This point [*both point to the intersection point*]

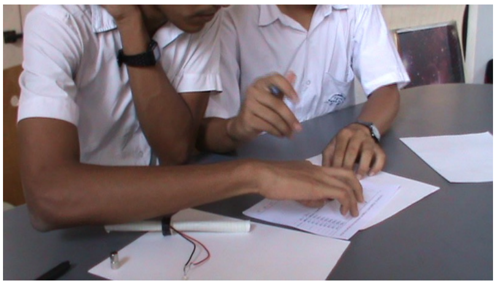


Figure 7.25: Pointing to intersection of the graph with X-axis.

AD: Can you mark it there . . . [AA *marks the point of 0 with origin, both of them points to that point*] Ok so that is the point where . . .

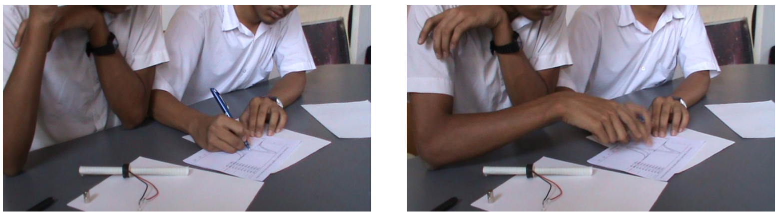


Figure 7.26: Pointing to intersection of the graph with X-axis.

AA: The magnet would be at the centre . . .

The one-to-one correspondence that the students had established from the graph to the physical phenomenon is strongly evident in the Dialogue 8. The coil *became* an "object-to-think-with" for the students. The phenomenon earlier had only a qualitative aspect, but rendering it in a graphical representation allowed a deeper meaning to be established. Students could "see the phenomena" through the graph, and in the process, the phenomenon itself became transparent to them. The movement of the magnet through the paper tube can be schematically understood as shown in Figure 7.27. If we rotate the graph such that the time axis is parallel to the path of the magnet, we can see a one-to-one correspondence between the graph and the phenomenon very clearly (Figure 7.27). This correspondence came out in discussions with the students very well.

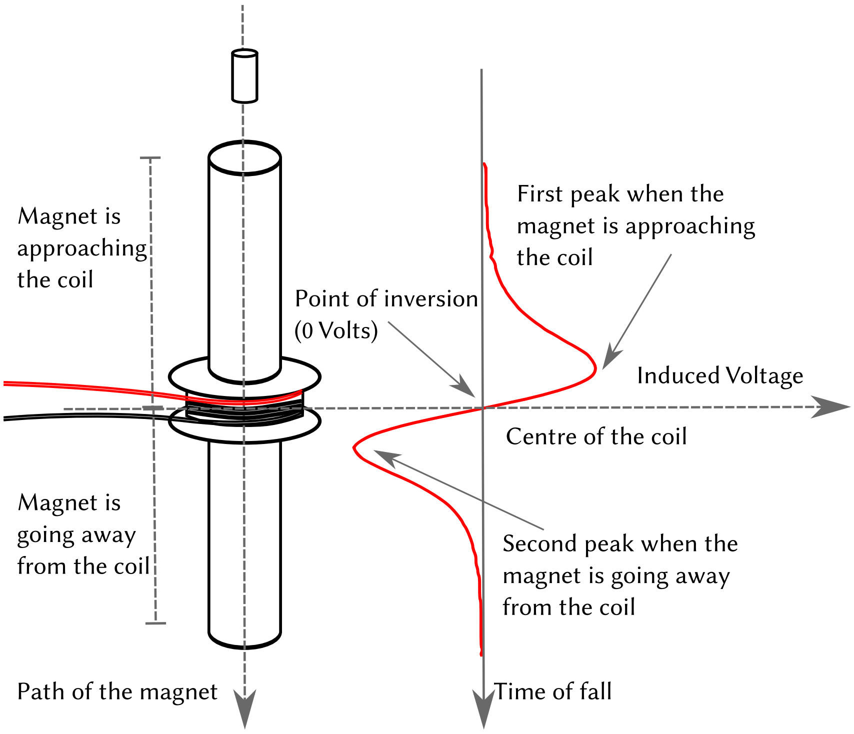


Figure 7.27: The correspondence between the journey of the magnet through the coil and the induced *emf.* The first peak is generated when the magnet is moving towards the coil when the magnet is at the centre of the coil the voltage is zero, the second peak is generated when the coil is moving away from the coil. This is a schematic illustration, the waveform is not scaled to the actual readings.

We think that this was only possible as the students (a) performed the experiments, (b) collected and plotted, (c) analysed the data, and, (d) at each step they had a concrete basis to think. As we see later, some of the peculiar features of the graphs (for example, the slight increase in the voltage of the second peak) allowed the reverse movement from abstract to concrete as well. However, this movement of meaning-making itself was not only from concrete to abstract but also from abstract to concrete.

6. Organising and plotting the data using GNUPLOT.

Concepts/Skills: `(graphs)` `(multiple representations)`

The experiments were performed by varying the parameters given in Table 7.5. For each value of the parameter, the experiment was repeated at least five times. The resulting data was stored in files on the computer, with the nomenclature discussed on page 246. After completing the experiments, the students plotted the data files to make graphs using GNUPLOT software. The students were initially given a short demo of using GNUPLOT and were given the required syntax for plotting multiple data files in the same plot.

7. Analysing the data collected and testing the hypotheses.
8. Explaining the features on the graphs in terms of physical phenomena.

Concepts/Skills 7: `(analysis)` `(testing)` `(hypotheses)` `(multiple representations)`

Concepts/Skills 8: `(models)` `(inferences)` `(rhetoric)`

We are presenting steps 7 and 8 together, as at times the two steps have become very intertwined with each other. These two steps inform us about the process of meaning-making from the graphs by the students. The movement from concrete-to-abstract and abstract-to-concrete is seen from the graphs to the physical phenomenon and vice versa.

Experiments with magnets

In this section, we present the data from the experiments with the magnets. These experiments were done with the standard coil. The students had access to different types (sizes) of strong Neodymium magnets. The students used a longish (~ 5 mm in length) cylindrical magnet for the experiments as shown in Figure 7.11.

A: Inversion of magnet and coil One of the features of the qualitative demonstrations that we have seen earlier is about the polarity of the magnet. If the polarity of the magnet is reversed, then the deflection in the galvanometer is also reversed. We asked the students to repeat the same with the magnet and the coil when *expeEYES* was used. The magnet was dropped in one orientation first, and then the polarity of the magnet was reversed, and it was dropped again. When the LED is attached to the coil, no noticeable difference is seen in the lighting of the LED with the change in polarity. When we connect *expEYES* to the coil we "see" the phenomenon in a completely different light. We not only see the reversal in the waveforms (Figure 7.28) but also note they are symmetric. This graph also takes explains the '−ve' sign in the law (due to Lenz's Law), which says that the induced voltage is always opposing the change in the magnetic field. The graph showing both the waveforms is a direct visual and quantitative demonstration of this fact. At the same time, the peak value of the voltage remains almost same in both the cases (the first and second peaks respectively).

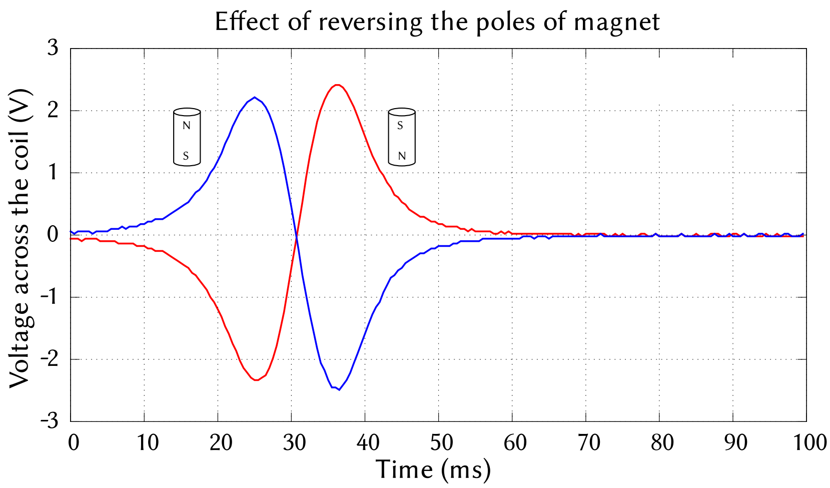


Figure 7.28: Effect of changing the polarity of the magnet on the induced voltage. Notice the complete reversal in phase of the waveform and almost symmetrical waveforms.

The students gave an explanation to the graph in terms of the poles of the magnet. In Dialogue 9 below this is clearly evident.

Dialogue 9

AD: . . . Here is another graph in which there is an inverted peak [AD *shows them a graph with one of the readings reversed*] . . . So can you explain that? [*the second graph is given and the both* AA *and* AJ *look at the graph.*]

AA: Yes. So . . . these are the two places where two magnets are . . .

AD: Let me just explain . . . there are two . . . there is an inverted peak as well. So whereas . . .

AA: So thats our reading error because we had passed it vice-versa [*pointing to the inverted peak*] . . . the magnet the polarity that we had passed through . . . that paper [AA *calls this as a "reading error" as graph for this particular graph had the magnet polarity reversed.*]

AD: Ok, but why does it go like that?

AA: Because . . . the north pole approaches the coil at that time a certain . . . one wire . . . if positive voltage is induced at the same time the south pole approaches at that point negative voltage is induced

After seeing this effect, the students were asked this question: " What effect will changing the orientation of the coil will have on the induced voltage?"

Initially, the students answered that it would not have any effect, only the changing the orientation of the magnets will affect the induced voltage. It turned out that they were not aware of the directional property of coils. They reasoned that the current flowing would not change if it flows in either direction, as the resistance of the coil would not change. However, when they performed the experiment, they found that the induced voltage *did* change in polarity. This change in polarity came as a surprise to the students. What they failed to take into account was the directional nature of the current, which is also related to the polarity of the induced voltage. Another way to understand this is to see the coil itself as a magnet, which has inbuilt directionality. Dialogue 10 gives the explanation given by the students.

Dialogue 10

AD: That's why the inverse peak?

AA: We can consider that with Flemings right hand rule.

AJ: So with that we can find that . . .

AD: Ok, So but at the same time if you also invert coils . . .

AA: Yes then also there would be a difference . . . so if both is interchanged then it wont change and if one of them is inverted then the peak will be inverted.

We thus see that simple qualitative demonstration of deflection of a galvanometer is uncovered in an entirely different light by using the graphical representations. The depth of understanding that the students achieved on the topic of inquiry was very well grounded in concrete experiences.

B: Comparing the magnet strengths As discussed in the previous section, three different orientations of the magnets were used as shown in Figure 7.11. The students knew as a matter of fact that the magnet strength affects the induced voltage. The hypotheses that were put forth by the students in this case were: *the induced voltage would increase as the magnet strength increases.* There was another hypothesis regarding the magnet strength in different configurations: *whether the magnetic field strength would be increased or reduced in the side-by-side configuration*, this is already discussed on page 252 and Table 7.3.

The students performed the experiments with the three configurations and got the graph as shown in Figure 7.29. The difference in the induced voltages due to the three configurations is clearly evident in the graph. Dialogue 11 gives the explanations of the students and their inferences in this case.

Dialogue 11

AA: Yes and I think, smaller the magnet, faster it will pass through the coil so the two poles will interact faster, longer the magnet it would take long time for it to pass through the coil [AJ *nods in agreement*]. Hence the two poles would interact with the long after a long time hence. The North Pole will interact and after sometime the South Pole will interact . . . Some time means it might be very less, but after sometime the South Pole will interact. Keeping all other things constant. Expecting that the speed of the both the magnet is same for the smaller one and the larger one. In that case I think that the . . .

As expected by the students, the configuration which had the two magnets attached pole-to-pole, the longest of the three, gave the *maximum* induced voltage. The students gave the explanation of this based on the *length* of the magnet. In Dialogue 12 they reasoned along the lines of time taken for interaction between the two poles of the magnet and the coil (please note that some of the conversations in the dialogue below are in Marathi, which are with translations).

Dialogue 12

AJ: In this case there are two magnets attached side-by-side . . . in this case this is the graph (marks the graph with a pen) . . . two magnets attached side-by-side . . .

AA: So like this . . .

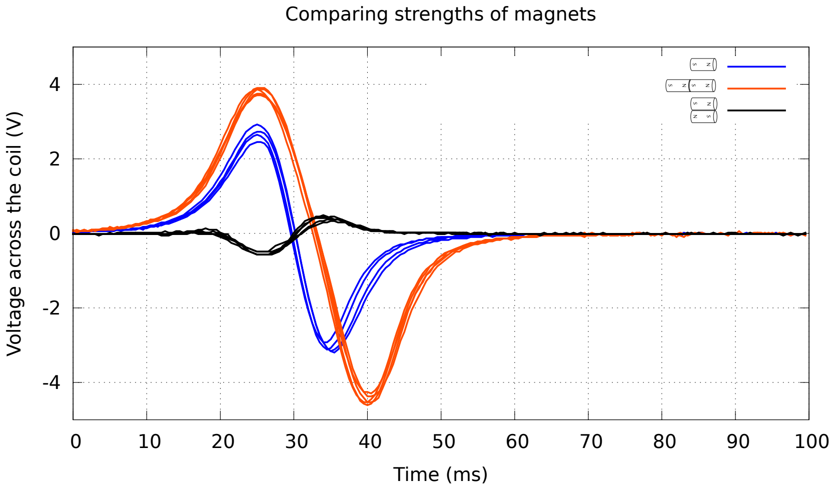


Figure 7.29: The result of comparison of magnet strengths in three different configurations. The orientation of the magnet types are given in the legend.

AJ: And this one is [*pointing to the other line on the graph*] of only one magnet . . . so the timing . . .

AA: So . . . *var-khali* (Up-down.)

AJ: Uh-oh . . . (making a sound in disagreement)

AA: *Hoy hach graph ahe toh* (Yes, this is that graph.)

AJ: *He pan bagh.* (Look at this also.) [*both are contemplating which graph is which*]

AJ: *Teen te minus teen* (3 to -3)

AA: *Ek* (one) [*then nods in agreement*]

AJ: Haa. this is one magnet and this is two magnets attached side by side . . . [*pointing to the two graphs (Figure 7.29 here),*]

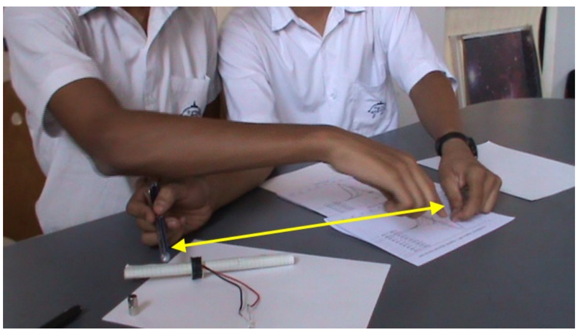


Figure 7.30: Pointing to intersection of the graph with X-axis.

the lengths of the magnets are constants [*showing the length with two hands (Figure 7.30)*] so when we drop them . . . [*shows the dropping action with hands*] so the time remains constant . . . so the time taken by them is constant

. . . but . . .

AD: Ok

AJ: But the . . . magnets are attached inverted [*shows one hand next to another which is inverted (Figure 7.31)*] suppose this is North-South then this is South-North . . . because they have . . . they have to be remain attached to one another . . . the magnetic field produced by one another cancels out.

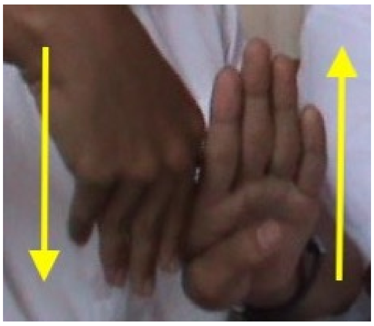


Figure 7.31: Pointing to intersection of the graph with X-axis.

AD: So it is highly reduced actually.

AJ: Yes . . . reduced and very low.

AA: And that is why the emf induced is also less.

AJ: Around 0.5 volts . . .

AD: Ok.

AJ: Very less . . . for one magnet it is around 3 volts . . .

AA: For two magnets [*attached pole-to-pole*] it was 4 volts which were . . . one below the other . . . but those two magnets only when they are side by side they dropped near to 0.5 so thats big dip . . . from 4 to nearly 0.5

AD: You initial guess was what? Two magnets should give you more voltage or less voltage?

AA: No *less* voltage because . . . as they are *opposite* poles are meeting over there . . . emf is *decreasing*.

AJ: Emf is produced in the . . . they will cancel out each other . . .

As we can see in the dialogue above the students reasoned along the action by the two opposite poles of the magnet reduced the induced voltage in the coil. The graphs, in this case, provided a basis on which the students could base their arguments. In this way, the graphs were used as a rhetorical device.

A visual understanding of the orientation of the reason for two magnets being least strong when attached side-by-side can be seen in terms of magnetic lines of force, as shown in Figure 7.32. In the first configuration (left) where the poles are similar, the lines of force are seen to repel. In the second configuration (right), the lines of force can be seen to end on the opposite poles of the magnet, reducing the strength.

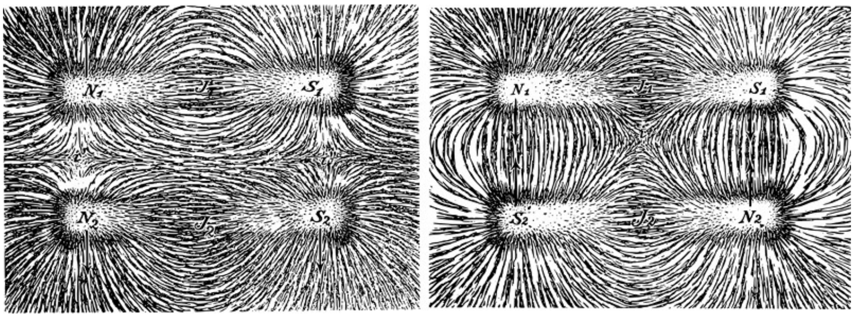


Figure 7.32: Looking at the magnet configuration in side-by-side coils in terms of magnetic lines of force. Notice how the lines of force are different in the two configurations. In the configuration on the left, the lines of force repel each other, while in the configuration on the right, lines of force from the two magnets end on each other, thereby reducing the strength. Both the images from an old 19th-century book on science.

Experiments with coils

In this section, we present the data from experiments by changing the coils. The experiments with coils were the most detailed set of experiments that the students performed. The parameters and the hypotheses about them made by the students can be seen in Table 7.2. For testing these hypotheses, the students constructed the coils with different properties as detailed in Table 7.5. We now present the results and analysis of these experiments as explained by the students.

A: Diameter of the coil We first present the result in which the diameter of the coil mount was changed. The students made three mounts of different sizes (0.5, 0.75, 1.5 inches). The other two parameters were: gauge 30 wire and 250 turns. Figure 7.33 shows the results of this experiment. The hypothesis in this case that the students had proposed was: *induced voltage would reduce as the diameter of the coil was increased.*

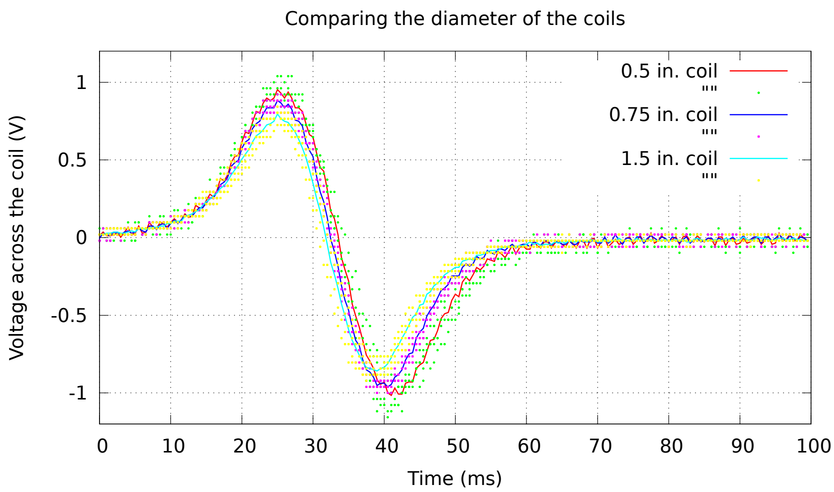


Figure 7.33: Comparing the induced voltage with different diameters of the coil (redrawn by the researcher using student data). Notice that the peak voltage is around 1 V. The solid lines present the average values of the five set of observations for each coil which are represented by coloured dots.

The induced voltage increases with the decrease in diameter. The change is not high, but it is perceptible in the graph. One of the reasons that the induced voltage was not very large is that a comparatively thicker wire (30 gauge) was used. Students had earlier assumed that *thicker* wire would produce *more* induced *emf* due to reduced resistance of the wire. If the experiment were repeated with a thinner wire, the experimental results would have given higher voltages.

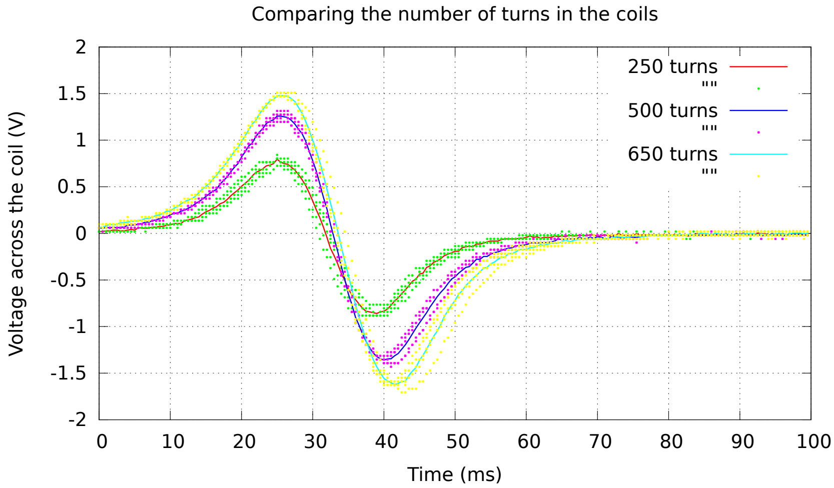


Figure 7.34: Comparing the induced voltage with different number of turns in the coil. The induced voltage is seen to increase with the number of turns. The solid line is the average of 5 readings for each number of turns in coil, the dots represent the actual values of the observations.

Since the students were not aware of this particular fact, this led to the discovery of a "fault" in their reasoning in the experiments concerning the thickness of the wires. This fact came out later in the discussions with them regarding the relatively low voltages obtained for this set.

B: Number of turns The next category of parameters for the coil that we consider is the number of turns. The students had hypothesised: *more number of turns will induce more voltage*. They knew this as a matter of fact. They constructed coils with 250, 500 and 650 turns, with gauge 30 wire. They had aimed to construct the third coil for 750 turns, but could not manage it as construction was not feasible, as wires in the coil spilt out for the higher number of turns.

The experimental graph of the number of turns clearly shows the increase in induced voltage as the number of turns is increased.

C: Diameter of the wire In case of the diameter of the wire the students had hypothesised: *thicker the wire, more voltage it will induce*. The reasoning used in hypothesis was: a thicker wire will have lesser resistance, hence will induce more voltage. Also, the physical handling of the thinner wire was considered difficult by them. Hence they decide to make other coils (for different diameters and number of turns) from the 30 gauge wire (thickest wire made available to them). This is evident from the explanations given by the students when they were asked to describe the coils that they had made for the variation of diameter of the coil and number of turns. The hypothesis, in this case, was found to be untrue. In fact, the hypothesis and experimental facts were precisely opposite. In Dialogue 13 we see how the students made sense of this "fact" in the light of the theoretical ideas that they had.

Dialogue 13

AJ: All for 30 gauge. Varying the number of turns.

AD: Varying the number of turns. This you did not do for 35 or 41 gauge.

AJ: No. 30 gauge is most comfortable.

AD: Why do you say so?

AJ: Because, the wire is thicker and so we can get considerable readings and it also doesn't break while handling.

AA: Because 41 gauge wire is so thin that sometimes we are unable to see it . . . its just hair like and it breaks very easily

AJ: The probability of error is very more . . . very large.

AA: Not error. We may land winding our experiment we will have to again wind the coil.

AD: And for 35 you did not face such problems. It was in between,

AA: For 35 . . .

AJ: On the safer side we . . .

AA: 35 was ok, it was easy to handle . . . not that difficult. [*after a pause*] So the next thing to vary was the gauge [AJ *is writing on paper*] the thickness of the wire.

AJ: Cross-sectional area.

AA: Cross-sectional area. [*repeating*]

AD: Ok. So what is that you know about the gauge of the wires that you are given? So you are given how many wires?

AA: So, we had three gauge of wires, one was 30, 35 and 41 gauge. [AJ *is writing*] . . . So.. as the number increases the thickness decreases . . .

AD: Ok.

AA: So it is inversely proportional. So 30 is the thickest and 41 is the thinnest. So . . . as we had three, the . . . we, we . . . three of them were used and the number of turns were fixed to 250.

Thus we see that the students had a preconceived notion, which was reasoned with the framework of resistance of the wire being the fundamental quantity in determining the induced voltage. Also, the physical aspect of the thinner wire

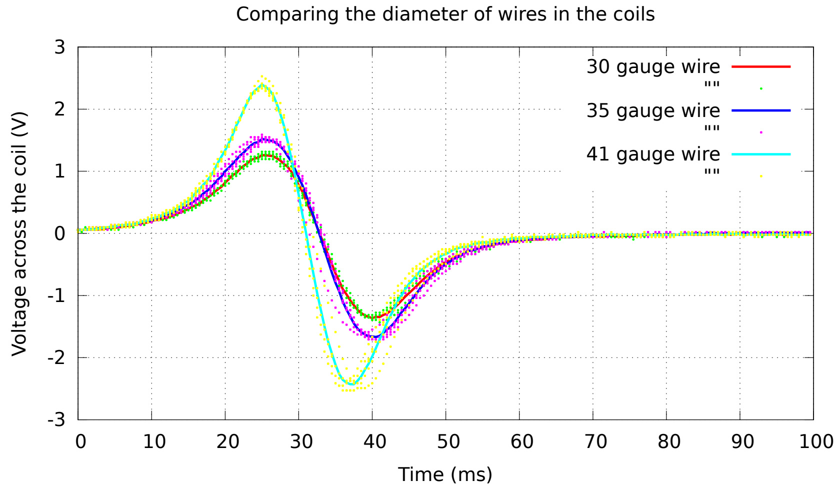


Figure 7.35: Comparing the induced voltage with different diameters (gauges) of the wire of the coil. The solid line is the average of 5 readings for each number of turns in the coil. The dots represent the actual values of the observations.

added to this notion as seen in Dialogue 14.

Dialogue 14

AA: In that case [*gauge of wire*] we had thought that more is the . . .

AA: That the smaller the wire . . . *haan* [*yes*] ok . . .

AJ: Thicker the wire . . .

AA: More the thickness . . .

AJ: More is the current . . . that was true . . .

AD: But we are measuring the voltage here . . . so?

AA: Yes, yes our hypothesis was *more is the thickness more is the voltage.*

AD: Ok. So why don?t you write it down . . . I [*current*] is proportional to t [*thickness of the wire*] or something like that . . . this was the original hypothesis . . . you write it down hypothesis next to that [*AJ is writing it down*].

AJ: But this turned out to be untrue.

AD: And what were your reasons for giving this hypothesis?

AA: No, no. This was not the original hypothesis. Our hypothesis was V [*voltage*] is directly proportional to the . . .

AD: Ok . . . So write down V, not I . . .

AA: The observation was I . . . this was our hypothesis which was untrue

AD: So . . . no, no . . . why did you make such a hypothesis? That you can tell me?

AJ: Because we felt that . . . the thickness was decreased then the resistance would increase . . . and so . . . the . . . hypothesis is not true . . .

AA: The thickness was decreased

AJ: Resistance kami kela ki voltage wadhto [*when resistance is decreased voltage increases (talks to himself and* AA *in Marathi)*]

AA: *Mag to barobar ahe* [*then it is correct*] *Barober ahe to, thickness kami keli tar* [*It is correct, when thickness is reduced*] . . .

AD: No . . . so what was your original hypothesis voltage is proportional to thickness? ok?

AA: Which was wrong and later . . .

AD: More thickness . . . I mean . . . thicker the wire more should be the induced voltage . . . ok so you write it down in that way . . . this is the original hypothesis [AJ *writes down*] Ok . . . and what could be reason for this then . . . I mean so you gave . . . initially you gave a reason that I mean in terms of resistance

AJ: Yes.

AA: Yes. The later was in terms of resistance only but we . . . we had to consider . . . firstly we did not consider the factor of current . . . so when we got the results and had a minor calculation as we not the resistance of that coil . . . after those observe . . . after those calculations we came to know that if we . . . as we have to consider current also . . . the current is directly proportional to the . . .

AJ: The current is directly proportional to the . . . [*resistance*]

We see that the students were not sure about why the thinner wire should induce more voltage. In their conceptual framework, based on the resistance of the coil, this was at odds. So they changed the narrative to consider current instead of the voltage. However, this is not simple. There are multiple factors at play here. One of the factors is that there are many number of turns per unit volume for thinner wire, causing more lines of flux being cut.

Experiments with speed

In the experiments with changing speed, there were two techniques to vary the speed. The students' hypothesis was: *more speed will produce more induced voltage.* This they knew as a matter of fact. We describe the two variations of the speed in this section.

A: Vertical Drop For the vertical drop of the magnet, we needed the drops to be made from specific heights along the paper tube. For this graduated markings were needed on the paper tube. The students achieved this by making the paper tube out of a graph paper. The position of the coil was changed with respect to the markings on the graph paper. Figure 7.36 shows the arrangement schematically. It is with

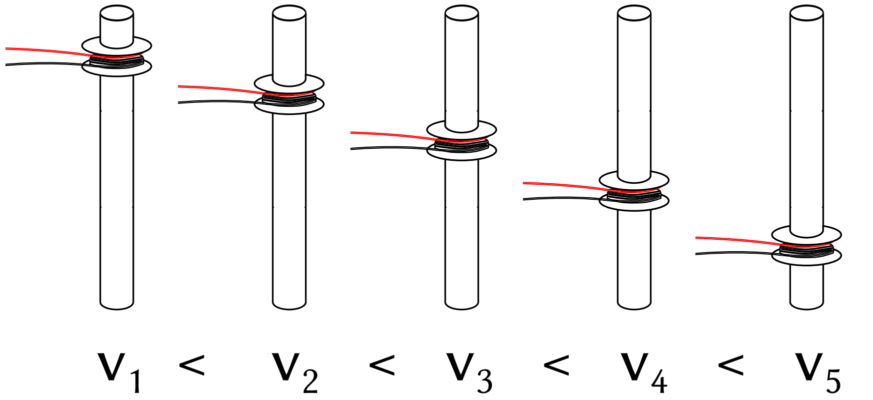


Figure 7.36: A schematic illustration of the setup for varying the speed using the straight drop. Since the magnet is falling freely in the paper tube, the speed of the magnet, when it reaches the coil, increases as the distance of the coil increases from the top.

reference to these markings the readings for this experiment were taken. The labels in the graph, 3 cm, 6 cm etc. in Figure 7.37 are the distance of the coil from the top of the tube. Note the change in the form of the peaks as the speed increases. Not only the peak voltage increases, confirming the hypothesis but peaks are also coming *earlier* with each increase in the speed. By earlier here it is meant that the peak voltage is appearing at earlier time intervals as seen on the graph.

The results of this experiment were according to the hypothesis of the students. Using the equations of motion, they could roughly calculate the speed of the magnet for the different distances. The students discovered a previously unknown peculiarity from the graph. They found that in all the cases, the second peak voltage is always slightly larger than the first peak (for example look at Figures 7.9 and 7.16). When they were asked about this, they explained in terms of the acceleration resulting from gravity. We see how the students invoke Newton's laws of motion to explain this peculiar graphical feature that they discovered in Dialogue 15.

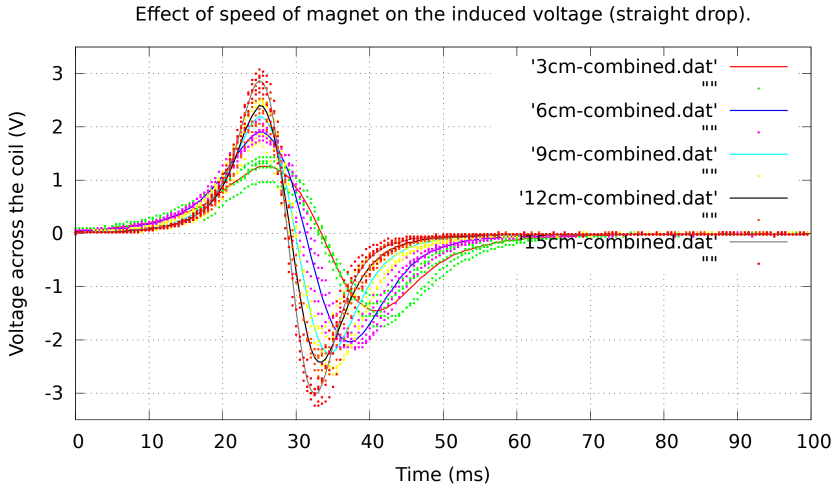


Figure 7.37: Comparing the speed of magnets with induced voltage in the coil by the vertical method. Note the increase in the peak voltage with an increase in the speed, and also the shifting of the first peak and inversion point towards left. The solid line is the average of 5 readings for each number of turns in the coil. The dots represent the actual values of the observations.

Dialogue 15

AD: . . . there is something you told me about the negative peak of the both sides.

AA: Yes. So . . . in each of these observations we found out that the positive peak?

AJ: Was lesser than the negative peak.

AA: So if you take **absolute** value of both of them, then they were not the same, the negative ones [*pointing to the second peak on the graph (Figure 7.38)*] were bigger. So probably we think that the reason was because as it is accelerating

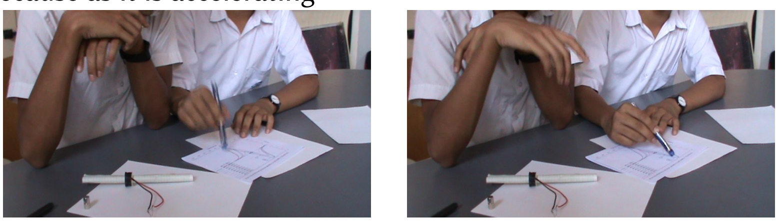


Figure 7.38: Pointing to the two peaks in the graph.

AJ: The approach velocity is greater than the velocity it starts with . . . [*Here we can clearly see the line of reasoning, the correspondence between the motion of the magnet under acceleration due to gravity and its effect on the induced voltage.*]

AD: Yes. Because it takes a **finite** time to pass the coil.

AA: So by the time it accelerates by some minor amount and due to that the lower peaks are more . . . by a margin of nearly 0.1 volts

AD: Ok.

AA: Ok, but then also that coil . . . consistently we are getting . . .

AD: Across all readings . . .

AA: Yes.

This discovery of increase in the peak voltages is a movement from the abstract to the concrete. Though one can theoretically deduce the increase in speed of the falling magnet, the point we want to emphasise is that a peculiar feature in *all* the graphs that they plotted brought this to the notice of the students. In this case, the phenomenon is first "seen" in the graph and after that is "understood" and "explained" in terms of the physical, concrete meaning. This particular aspect of the phenomena is entirely *hidden* in the qualitative demonstrations. It is the power of the graphical representation that brings out this otherwise hidden fact to the notice.

B: Slant Drop The second method that was used by the students to vary the speed was by slanting the tube of the coil. This is shown schematically in Figure 7.39. Figure 7.41 shows the results of these experiments. The coil was kept at a fixed distance from the top of the tube. The tube was held against a vertical scale and observations were taken. Then the tube was moved downwards from the top, and the observations were repeated. This method results in much slower speeds than the vertical drop and the explanation the students gave for this can be seen in Dialogue 16.

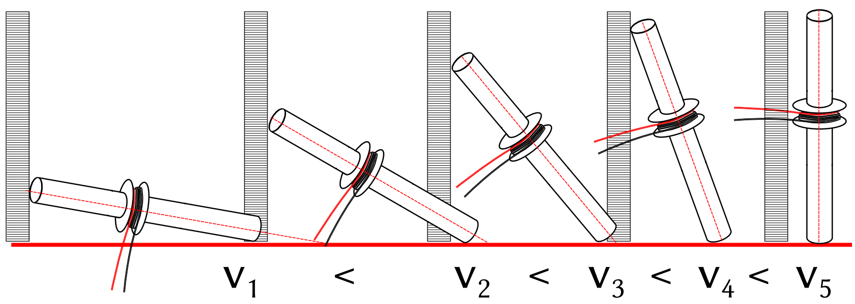


Figure 7.39: A schematic illustration of the setup for varying the speed using the slant drop. The tube is slanted by equal distances along a vertical scale. As the slant the tube increases the acceleration due to gravity (g)is "diluted" and only a component $g \sin \alpha$ is acting on the coil, where α is the angle of the tube from the horizontal.

Dialogue 16

AA: By getting it slanting, in the experiments we will be doing it by slanting, at that time they were approaching very closer values the . . . the absolute values were **closer** [*This is another movement from abstract to concrete.*]

AD: The peaks?

AJ: The difference between the velocities is not much . . . [*Here they are comparing with the speeds with respect to the straight drop method.*]

AD: Ok. Because the . . . ?

AJ: The acceleration is **lesser** . . .

AA: When it is slanting . . .

. . .

AA: Yes about that the relative motion in that case we experimented in two ways by keeping it perpendicular and changing the position of the coil from different heights and in another case keeping it slanting and the coil . . . the position of the coil was fixed but the angle of the tube was changed . . . so in case of changing that angle of the tube . . . when it was . . . the angle was very less . . . we observed a prominent delay in the . . . that wave [*Makes a wave gesture with hand*]

AJ: The duration was increased . . .

AA: The wave was [*Points to the graph along X-axis*] such a long wave . . . so the whole 100 milliseconds were used . . . [*This is the total length of the graph, the device measurements*] so . . . a long wave like this one [*Gestures along the entire width of the graph with his fingers (Figure 7.40), this can be seen as the curve with green dots in Figure 7.41.*]

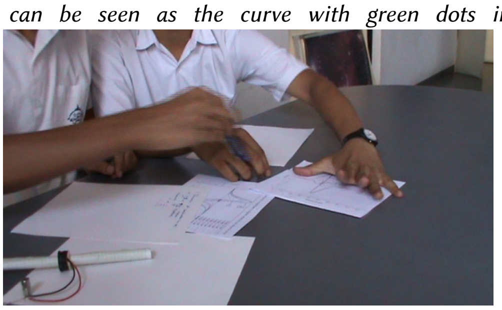


Figure 7.40: Pointing to intersection of the graph with X-axis.

so the complete (shows the width of the hand with extended fingers) . . . is covered . . . [AJ *moves along the graph with his pen*] so as . . . and which . . . which tells us that the velocity of the magnet is very **less** . . . as it is taking a **long** time to pass that coil . . . so that was observed and also . . . [*This is another manifestation of movement from abstract to concrete.*]

AJ: When we changed the heights also . . .

AD: You did not observe any noticeable time delay in that . . .

AA: Yes . . . but in the least one and the most one there was a noticeable delay 3 centimetres and 15 centimetres there was a notable change [AJ *is reiterating the same numbers*] like something around 10 to 20 milliseconds . . .

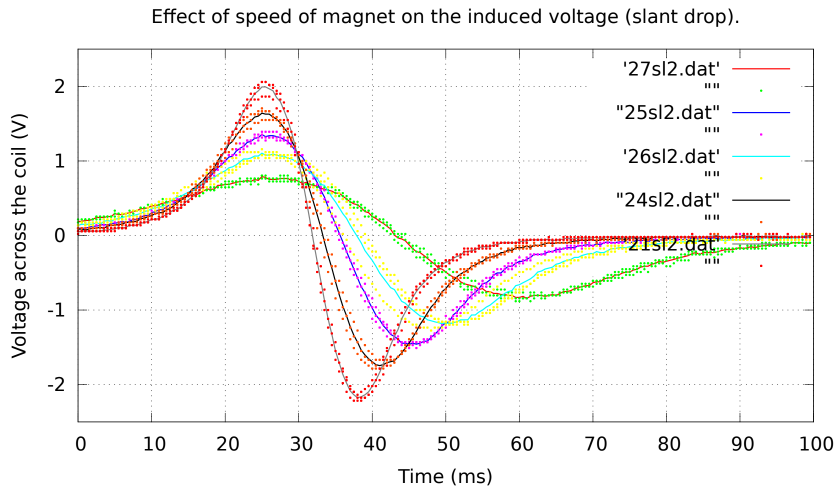


Figure 7.41: Comparing the speed of magnets with induced voltage in the coil by the slant method. Note the reduction in the peak voltage for each of the settings. Comparing this with the graph for the straight drop (Figure 7.36) tells us the effect that the velocity has on the induced voltage. Another aspect to note and compare is the spread of the waveform and area under it, this again comparing this with the straight drop (Figure 7.36) this reveals the relationship between the speed and the induced voltage. The solid line is the average of 5 readings for each number of turns in the coil, the dots represent the actual values of the observations.

Here again, we see another movement from abstract to concrete. Note that the students are reading the values of voltages from the graph and explaining them in terms of the physical parameters and situations in the experiment. When needed, the students made use of theoretical ideas during their explanations. The change in the waveform of the induced voltage is seen as a manifestation of the speed of the magnet. In case of the speed also, we found that the students were able to make the transitions from concrete to abstract and abstract to concrete. This is because they understood each of the parameters in relation to the features of the graphical representation.

9. Writing a report and presenting the work in front of the peers.

Concepts/Skills: (public display) (discussions) (reports)

When the students completed the project, they made a presentation of the work in front of their peers (students who did different projects). During the presentation, the students could combine the demonstration of the phenomena, the change in parameters and the resulting graphs to form a coherent explanation of the overall investigation that they performed.

7.5 Discussions

In the EMI task, we have converted a straightforward and essentially a *qualitative* demonstration of EMI in the textbook to a hypothesis-driven *quantitative* experiment. Due to the transient nature of the phenomenon, and the size of the data, the use of computers was imperative in this task. We now discuss some relevant themes in the task.

Context of the activity: The EMI task was set in a context which was familiar to the students, both via experience and what they had learned in the textbooks. To situate the context concretely, we developed upon the idea of creating a "toy" which works on the principle of EMI. In this task, the context of constructing a toy forms the basis to carry out further investigations. The several parameters which could be varied concretely and ease of recording the results made it very engaging for the students.

Mathematical modelling: The mathematical modelling in the case of the EMI task was different from the other two tasks reported here. The mathematical model, in this case, was a hypothesis about an effect of a particular parameter on the induced voltage in the coil. The students used the ideas of both direct and indirect proportion to construct the hypotheses. The mathematical model, in this case, was approached by use of factual *a priori* knowledge about EMI in conjunction with intuitive reasoning.

Measurement, real-world data and data handling: In the EMI task, the students performed two distinct types of measurements. The first type of the measurement was in the context of creating variations in the parameters. For example, measuring the width of the coil, or the number of turns in the coil. The concept of indirect, qualitative measurement, was implicitly used in the case of the speed based experiments. The second type of measurement was done with the aid of the *expEYES* data logger attached to a computer. The students maintained the record of the physical parameters and their effects meticulously in the form of data files organised on the computer. They could, for example, tell about the experiment by looking at the file name (page 246). Thus, while analysing and explaining the data in the form of graphs, the students were fully aware of the data and method of obtaining it, which is essential for meaning-making from graphs according to the sociological framework for understanding graphs (Roth, 2004).

Use of technology: The transient nature of the phenomenon was captured only because of the availability of a technology which allowed the capturing of this phenomenon. This task addresses the "technological tools" part of development objective (2) in a deeper way.

The technological tool used in this task, lead to easy collection and storage of data in electronic format. The tool also gave immediate feedback in a graphical format, thus enhancing the concrete-abstract connection (Figure 7.9). There are several advantages of using a computer for plotting data (Thornton & Sokoloff, 1990). This provides us with the least criterion that a technological tool should have to help the learners become graphicate.

§ The feedback should be immediate, and this is particularly true for a transient phenomenon in this task.

§ The output should be in a readable format, that is, in a format understandable by the students and be amenable to the graphical representation.

Studies in learning graphs with the help of computers have indicated, activities with *real-time* data collection and display on the computers have resulted in significant improvement in learning (Adams & Shrum, 1990; Thornton & Sokoloff, 1990). During the EMI task, we found that the students could make a conceptual leap, going from textbook-based theoretical knowledge to explaining the phenomenon from the data from experiments they conducted. Also, the movement between abstract-concrete was in both directions. Such a two-way movement was not always the case in the two activities reported previously. This approach allows the learners to participate in the process of learning similar to that of a scientist working in a laboratory, trying to understand complex factors influencing observations, as has been pointed by Nachmias & Linn (1987) in their study.

Multiple representations: In the EMI task, the idea of graphical and tabular representation are embedded at the core. Though the students did not use the tabular data directly, it was the presence of the data in the tabular format that allowed it to be plotted. The verbal description of the graphical features and their correspondence to the physical situations given by the students was meticulous. In this task, the connections between the graphical representation, its verbal description and the physical situation were very well established. Though we did not have, algebraic representation as in the case of the MS and SM tasks, the use of the idea of proportionality (both direct and indirect) was inherent to the EMI task in the form of the different hypotheses about the parameters.

Graphs: The explanations given by the students centred around the graphs. In this task could see the graphs playing multiple roles.

1. The visualisation of large data sets. Without graphs, it would have been almost impossible to "see" the data.
2. Finding patterns in the data. For example, the discovery of the two peaks.
3. The graphs were also the tools for testing the hypothesis that the students constructed. This use was possible as plotting values if several experimental parameters within a single graph allowed a direct comparison of the effect of varying the parameters. For example, the change in the peak voltage. Among the hypotheses made by the students, only one turned out to be incorrect (the thickness of wire).
4. Another aspect was the discovery of new phenomenon which was not expected by the students. This observation represents the flow from abstract to concrete. For example, that there are two peaks in the induced voltage, the second peak in the sinusoidal graph is always higher than the first one
5. At the end of the task the students could *directly argue and explain from the graphs about the phenomenon.* Thus graphs were used as rhetorical devices.
6. The conceptual flow between the abstract graphs and the concrete phenomenon was two-way. The graphs made the phenomenon *transparent* to the students.

> The conceptual flow between the concrete phenomenon and the abstract graph was very well established, and the flow was both ways as is seen in case of scientists (Roth & Hwang, 2006). The reason for this perhaps was the fact that they themselves designed the experiments to test their own hypotheses. This was evident during the discussions and presentation by the students. The students, as they progressed through the task, could "speak" about and "see" the phenomenon in terms of graphs and their features.

Graphical visualisation of the data led to discussions about the effect of changing various parameters which could be seen readily from graphs.

Coil as an exemplary model system: The coils used in this task can be seen as a "model system " for the students to learn various aspects of the scientific process. Model systems in biology are simple systems which can be studied, and the learning from them can be applied to a wider context. The main idea behind the use of the model systems is aptly put by (Kunkel, 2006):

> We are unlikely to ever know everything about every organism. Therefore, we should agree on some convenient organism(s) to study in great depth, so that we can use the experience of the past (in that organism) to build on in the future. This will lead to a body of knowledge in that `model system' that allows us to design appropriate studies of non-model systems to answer important questions about their biology.

Similarly, in case of a physical system, we cannot know everything about the physical system. In our case, the coil is the convenient system which can be studied in detail. The resulting learning can apply to a wider context.

The scientific processes involved in the EMI task included (a) making use of prior knowledge with reasoning to form hypotheses, (b) the construction, the design of experiments to test the hypotheses, (c) collection, visualisation and analysis of data, (d) explanation of the experimental results in terms of physical phenomenon, (e) drawing inferences from graphical data for testing the hypothesis, and, (f) revising hypothesis in case of data not supporting it. The coils provided the student's parameters which they could manipulate and see a tangible result in the form of changing induced voltage in the coil. This correspondence was direct. The ease of constructing the coils and more importantly the ease of conducting several experiments to test the variations was instrumental in establishing the learning that occurred. The setup of the task using data logging device expEYES allowed easy, fast and hassle-free data collection. A set of four to five readings for a given set of parameters could be completed within a span of 2-3 minutes. This allowed the testing the effects of various parameters within a short span of time. The various parameters of the coil, like the diameter, the number of turns and the diameter of the wire provided a concrete basis for creating variations in the “model system”. The variations in these parameters could be easily noticed when the experiments were performed. The *causal* link between the variation and its result for a given parameter was very strongly established due to this. Thus the experiments covered several aspects of the scientific method concretely. Thus the coils as used in this task can be seen as an *exemplary model system* to teach about (a) the phenomenon of electromagnetic induction, (b) the process of science by allowing the students to participate in its core ideas like generating testable hypotheses, designing and constructing experiments, collecting, visualising and analysing data, inferring the results from the experiments, and, (c) graphicacy, which is organically situated in the first two points.

Limitations and future work: The EMI task has several aspects which can be improved. The task was performed with two students, and it would be interesting to see the task being performed collaboratively with a larger group of students. The dynamic collaboration and subsequent dialogues, between peers in a larger group, would result in richer learning contexts. Also, it is not clear, how this task would perform in a real classroom setting, or where there are no computers or data logging devices. The competencies of the teacher required to carry out this investigation in a real classroom are not clear and can be one of the future studies on this theme. The EMI task itself can be extended by many means. The students can be given a much more comprehensive variety of materials to perform the task. Another constraint, in the current task, was the limited timeframe in which the students had to complete the task. A project which runs for a month on the same theme can result in a substantial and sustained increase in the complexity of the learning process. This can perhaps lead to a design of a wind/water generator with required parameters, for example, see Figure 7.42. Finally, the experiments could be performed with multiple coils attached to each other, to investigate mutual induction in coils.

This part of the work was presented as a poster titled “**Exploring the phenomena of electromagnetic induction**” in epiSTEME 6 Conference, in Mumbai (Dhakulkar & Nagarjuna, 2015).

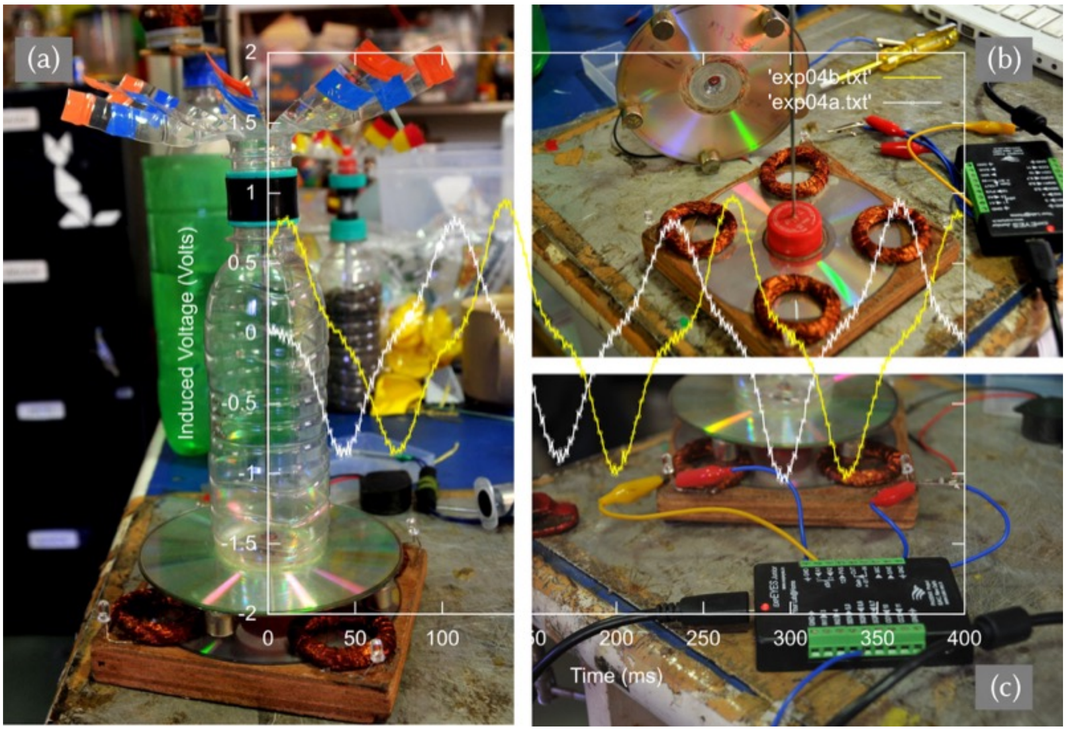


Figure 7.42: A "wind generator" created using four coils. (a) The improvised "fan" at the top of the plastic bottle moves the CD to the base of the bottle. (b) The CD has five magnets attached, and it rotates on top of the four coils attached to the base. (c) The relative motion between the coils and the magnets due to the action of wind on the fan creates induced EMF in the coils. The foreground of the image shows the resultant waveform when the expEYES data logger is attached to two of the coils. This "wind generator" was constructed by Ashok Rupner at MVS.

Graphs should not just be present in the curriculum but should become cognitively available to students. Room must be made somewhere in our already overcrowded curriculum to teach adequate graphing skills.

Brasell & Rowe (1993)

8 Discussions and Reflections

The insights and findings that we have gained from this work are presented here with a focus on implications for the curricular practice. Limitations of the study and directions for further work are also presented.

8.1 Graphicacy as an interdisciplinary skill

The present research work is both exploratory and developmental. It is exploratory in the sense that, we have explored the state of graphicacy in Indian schools as indicated by textbooks. At the same time, it is also developmental in the sense that, we developed a design framework for developing activities situated in a context that is part of an integrated project, which would help the students become graphicate. The testing of these activities in the form of field studies forms another research project with a focus on the process of learning graphicacy along with core practices of science.

What we have attempted through the diverse set of exercises presented here, is to look in them for the common thematic element of graphicacy, but not in isolation. Keeping in mind the concerns from the research in science and mathematics education research, we have explored, designed and developed activities to provide the students with opportunities to handle real-world data in a construction context. One of the overarching goals of these activities was to find and form linkages between mathematics and science, with multiple representations and particularly graphs being the centre in these linkages. Through the experience of actually working concretely, the students should understand that mathematical concepts that they encounter in other subjects are the same. This realisation that mathematics is indeed applicable to other concepts which are *not* part of the mathematical textbook should be incorporated while designing the syllabus. We feel the reason for this is the compartmentalisation of subjects, right from the start of curriculum designing process, culminating in the writing of textbooks of different subjects disconnected from each other. The obliviousness of the science textbooks to the content already covered in mathematics textbooks while dealing with the topic of graphs is indeed worrisome (Chapter 3). The need for the use of mathematics as a means of communication is paraphrased very well in *The Language of Functions and Graphs*:

> Mathematics is a powerful language for describing and analysing many aspects of our economic, physical and social environment. Like any language, it involves learning new symbolic notations, and new `grammatical rules' by which these symbols may be manipulated. Unfortunately, in mathematics, it is possible to learn these rules without understanding the underlying concepts to which they refer, and this often results in mathematics becoming a formal, dull, and virtually unusable subject. When learning any foreign language, pupils are indeed asked to learn a certain amount of grammar, but they are also given opportunities to express themselves using the language, both orally and through `free' writing. In a similar way, it is often helpful to set aside the mechanical, grammatical side of mathematical language and spend a few lessons where the emphasis is on *using* mathematics as a *means of communication*. Using mathematics in this way requires a wider range of skills than have usually been taught or tested in public examinations, and a greater mastery and fluency in some of those techniques that are already included (emphasis in original)(Swan, Malcolm and others, 1985, p. 6).

We need to find ways to integrate different topics which currently are under different subjects organically. Graphicacy being looked at as an essential and critical generic skill, along with literacy, numeracy and articulacy (Balchin & Coleman, 1966), does not find the presence that it deserves in the curriculum. As we have seen in Chapter 1 reading and writing graphs is seen as one of the core skills in today's world. The power of graphs for communicating and analysing data and detecting patterns and making inferences, testing hypotheses, makes them indispensable for any field of academic inquiry. Moreover, the massive number of graphs that are presented in mainstream media and advertisements drive opinions of general readers towards desired conclusions. It is this aspect of graphs and graphicacy, not being limited to any single discipline, we feel can be used to create linkages between different topics covered under different subjects. Though, in our work we have looked in detail mostly the topics related to science, we feel that this will prove to be beneficial for other subjects as well. Monk (2003) suggests that the students need to repeatedly encounter graphs as a means of communication and of generating understanding, as the students move across the grades. However, instead, what one finds in the textbooks that we analysed (Chapter 3) is that the graphical practices are spread far and wide and most often do not make any references or linkages to each other. In case of a given subject, sometimes linkages to a given topic are spread across the grades. Moreover, in case of different subjects, such linkages are almost absent.

While introducing graphs to the students as a first step, they need to be introduced in a context which they are familiar. The already familiar concept and mathematical entities associated with it are now seen in a new light of another representation, namely the graphical representation. Exploring the transition between them leads to a better understanding of the concepts involved. In fact, the studies in mathematics education research indicates that functions and graphs as one of the first places where the students use one form of representation to understand another, graphical and algebraic in this case (Leinhardt et al., 1990). The ability to move between different modes of representation and understanding the meaning between them is one of a desirable quality that we want the students to develop. The emphasis here, of course, is to understand what a thing in one representation means in another representation. For example, what is the physical significance of the steepness of a line in a given context? The National Curriculum Framework of 2005 also indicates that the students must be empowered to collect and analyse their own data. In each of the activities that we have presented here, we have designed linkages to a rich répertoire of concepts, and the graphical activities in each one of them, which make it possible to bridge the gap that we perceived between concepts of mathematical and science topics. The graphical activities set in different contexts provide the mechanism by which the different concepts which are spread across grades and across subjects can be related. In that we see the graphical activities acting like a thread which weaves through the otherwise disconnected set of concepts. Graphicacy is not a skill to be learned in isolation. For that matter, numeracy and literacy also should not be learnt in isolation.

We hope that introducing the students to such diverse set of activities with a thread of graphical representation running through all of them, to help them understand the world around them in a critical way, a world which is full of graphs which are used for both informing and misleading. Just like it is expected of a person who leaves the school to have skills of numeracy and literacy, we strongly recommend that graphicacy be taught, evaluated for all citizens. The design framework presented in this work (Chapter 4) and the activities developed using it (Chapters 5, 6 and 7) do look at the various issues that we have raised.

Science and mathematics education cannot ignore the existence of modern devices and technologies if we have to keep up to date with the changes that surround us. It is perhaps the neglect of the technological developments around in the education that we provide to our children that renders much of the knowledge that they learn without much use. Seymour Papert in his work starting from the 1970s focussed on this aspect of integrating technology with learning. The overarching theme in Papert's vision was the aspect of the children using the computers to solve their problems.

> In many schools today, the phrase ``computer-aided instruction'' means making the computer teach the child. One might say the *computer is being used to program the child*. In my vision, the *child programs the computer* and, in doing so, both acquires a sense of mastery over a piece of the most modern and powerful technology and establishes an intimate contact with some of the deepest ideas from science, from mathematics, and from the art of intellectual model building. (Papert, 1980) (emphasis in original p. 5)

Though now, even after almost 40 years since this quote, we mostly see computer programs being used in the schools to "program the child". However, use of technology can also enable the students to "program the computer " to generate and work with their own real-world data. The presence of ICT tools opens up new avenues of handling graphs by allowing the learners to focus more on conceptual issues related to meaning rather than on mechanical issues of construction of graphs. Using such technologies will enable the students to learn the concepts involved in a much better way from what they will learn passively in the textbook.

We explored this aspect of learning with computers, where the students "program the computer" in Chapter 7 and we look at the learning avenues that open up due to it are not only different in degree but in kind.

8.2 Major outcomes and implications

The broad outcomes of this work can be seen in two parts, matching the organisation of the thesis. The research themed first part of the work was to study the school textbooks for the practices related to graphs and graphicacy. The second part which is developmental in nature led to a framework for designing activities and their field testing. The analysis from the first part gave us an insight of what is present in the curriculum to support the learning of graphs. This gave us an insight into the type of graphs in the textbooks and their usage. This helped us to understand the nature of the problem that we were trying to address. For the majority of the graphs in the textbook, as seen in the analysis in Chapter 3, we found much to be desired. When seen in the light of the parameters used in the qualitative analysis of graphs, some acute problems in terms of design of the graphs and their overall integration and function came out. Some of the pointers from this analysis can be used as guidelines while writing the textbooks in the future. The rubric used for textbook analysis could have been more insightful if a five-point scale had been used instead of a three-point scale. The outcomes are listed as follows:

1. In this work, we identified graphicacy as a neglected item in current scenario of science education in the Indian context.
2. Based on the work we provide recommendations for the curriculum designers:

    § Emphasis on more integrated approach towards the teaching of science and mathematics with graphicacy as a medium of interaction between them.

    § Linkages between different concepts to be explored via construction contexts with graphicacy as a tool for linking.

    § Activities instead of chapters for an organic, integrated approach towards learning. Each of these activities has a special emphasis on connecting to various concepts and in various contexts across subjects and grades. Thus learning happens in the context of construction and we do not have to make extended efforts to connect the concepts.

3. Based on the work we provide recommendations for the textbook designers:

    § The design of the textbooks and presentation of graphs needs an overhaul in terms of the quality of the graphs and their design. This suggestion is valid for other graphic formats like illustrations and photographs too.

    § Students should be exposed to exemplars of excellent design and analysis of graphs as a first step.

    § Many graphs can be redesigned/redrawn to make them more effective.

    § The graphs present in the textbooks should avoid unnecessary decorative design elements.

4. We developed a three-point scale rubric with seven parameters for a qualitative analysis of graphs. For an extensive analysis, when required, this rubric can be extended on a five-point scale.

5. We presented a comprehensive design framework set in the context of the core practices of science, for developing activities to address issues raised in research questions (1) and (2). This addresses development objective (1).

6. We propose and explore the way in which such a skill can be introduced with linkages spanning across the subjects and grades in the form of the learning contexts. These contexts are constructionist in nature, with connections to real-world data.

7. One of the significant outcomes of this work are the model activities which were designed. This addresses development objective (2).

8. Reporting analysis of the field studies of the developed activities. These studies confirm some of the findings of literature and give us an insight into many of the problems that the students face and their possible solutions.

9. Role of ICT in graphicacy: once the skill of graphing is developed, the need is to provide the students with exploring the meaning in the graphs rather than mechanical plotting of the same. Moreover, this is the part where technology helps us at different stages. Exemplars for this kind of technology

are dynamic mathematics software, like GeoGebra and data logging devices, like expEYES and Arduino.

10. Though we have not included the history of graphs in the main text of the thesis, we think this chapter which is included in the appendix (Appendix B) is also an important outcome of the work. Teachers, teacher-educators and science communicators can readily use this.

Limitations

One of the limitations that we are aware of is that the activities were developed in a setting of a summer camp and not in a real classroom. Since the activities were carried out by the researcher, it is not clear what competencies will be required by a teacher to conduct these in a real classroom. However, we have attempted to write the activity chapters in the thesis to be readable by practising teachers and teacher-educators. The focus during the writing of these chapters was to get across the idea of what we are attempting. The use of ICT tools is implicit in almost all the activities, though the dependence and the role of these tools vary according to the requirement of the activities. In Chapter 7, the use of technology is crucial, and some may see this as a limitation in performing the activity, particularly with a large set of learners. However, we attempted to establish how such an activity can lead to a deeper understanding of the phenomenon.

The activities developed in the framework suggested here are by no means a solution to all problems pertaining to graphicacy. The suggested activities can be seen as a start towards developing a more critical and comprehensive approach towards the handling of graphs by the learners. The activities are suggestive and do not cover all the aspects of the syllabus but do address some of the core issues of graphicacy.

Further Work

1. To develop and field test the activities piloted but not reported in the thesis: (a) teaching ideas of motion using GPS data, (b) exploring and understanding the scores of a cricket match with various types of graphs, (c) exploring human senses of hearing and seeing, (d) various signals (AC and DC) in

graphical formats, and exploring mathematical functions using real-world data and dynamic mathematics software

2. Studying the efficacy of re-designing graphs in the textbooks, as suggested in our work (Chapter 3), on student understanding.

3. Studying if there exist any contextual differences in the conceptual understanding of graphs in the contexts of science and mathematics learning.

4. How well do the skills learned in the activities transfer to other domains and contexts. For example, (a) understanding graphs in other school subjects, and, (b) understanding graphs in mainstream media (like newspaper reports and advertisements).

5. Studying how the activities fare in a real classroom setting when the researcher is not the teacher.

6. Studying how the skill of graphical competency maps to required competencies of science and mathematics and to the nature of science.

7. Focussing on the possibility of graphicacy being taken as a concrete and comprehensive measure/outcome for science literacy.

8. Developing graphicacy through data generated by simulations based on mathematical models. For example, data generated with *Agent Based Modelling* system like netLOGO.

Though graphs are considered important by everybody, we have demonstrated sufficiently that Indian school education, does not do justice to such a core "literacy". We have argued that graphicacy as a skill to be developed in the context of learning while doing mathematics and science and not in isolation. Towards this direction, we have contributed a sufficiently rich design framework which introduces graphicacy along with the core practices of science. Using these design principles, we contributed three activities which we hope will be useful for teachers and teacher-educators.

List of Publications

The following articles were published during the course of this study.

1. Dhakulkar, A. & Nagarjuna, G. (2011). An Analysis of Graphs in School Textbooks. In S. Chunawala & M. Kharatmal (Eds.), *Proceedings of epiSTEME 4: International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 127–131). Macmillan

   This article is based on the quantitative part of the textbook analysis presented in Chapter 3 of the thesis.

2. Dhakulkar, A., Dhurde, S. & Nagarjuna, G. (2013). Measuring the mustard seed: A first exercise in mathematical modelling. In A. Jamakhandi, E. M. Sam & G. Nagarjuna (Eds.), *Proceedings of epiSTEME 5: International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 213–219). CinnamonTeal Publishing

   This article is based on the field studies of the MS task.

3. Dhakulkar, A., Dhurde, S. & Nagarjuna, G. (2015). Measuring the mustard seed: an exercise in indirect measurement and mathematical modelling. *School Science Review*, *96*(356), 63–68

   This journal article is based on the field studies of the MS task.

4. Dhakulkar, A. & Nagarjuna, G. (2015). Exploring the phenomena of electromagnetic induction. In S. Chandrasekharan, S. Murthy, G. Banerjee & A. Muralidhar (Eds.), *Proceedings of epiSTEME 6: International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 276–284). CinnamonTeal Publishing

   This article is based on the case study of EMI task.

A student booklet, for the activities, was developed in the context of designing the activities. This booklet has instructions for the two activities presented here (Mustard Seed Task and Sun Measurement Task, along with other activities conducted in the Summer Camp) in Appendix C.

A concept paper for developing activities around GPS data for teaching basic concepts of motion written. This work is not presented in the thesis.

Dhakulkar, A. & Nagarjuna, G. (G.). (2011b). From Geography to physics: How does geography help students learn motion? In S. Hellmann, P. Frischmuth, S. Auer & D. Dietrich (Eds.), *Proceedings of the 6th Open Knowledge Conference, OKCon 2011, Berlin, Germany, June 30 & July 1, 2011.* (Vol. 739). CEUR Workshop Proceedings, CEUR-WS.org

During the duration of the thesis work, we also explored the use of dynamic mathematics in teaching the history of astronomy by forming graphical models for the geocentric and the heliocentric theories. This work is not presented in the thesis.

Dhakulkar, A. & Nagarjuna, G. (G.). (2011a). Epicyclical Astronomy: A Case for GeoGebra. In S. Chunawala & M. Kharatmal (Eds.), *Proceedings of epiSTEME 4: International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 324–328). Macmillan

Part III

Appendix

A

Data from the Survey of the Textbooks

A.1 Quantitative analysis

In this part we present the data from the textbook analysis in a tabulated form. The NCERT textbooks were analysed for presence of graphs, graphical practices and possible activities related to graphs.

The data is presented for each of the books that was analysed in a separate table. The data is presented in following format:

Chapter Number	Page Number	Figure Number	Graph Type	Description of the Figure	Data and Comments
This column has the Chapter number in which the figure occurs.	The page number on which the figure occurs in the textbook.	The index number of the figure, if applicable.	The type of the graph: Line, Bar, Pie or Other.	Details about the figure, its context.	What is the data type and its source, comments about the design of the graph.

Mathematics Textbooks

In this section the data from the NCERT Mathematics textbooks is presented. The books that were analysed were:

- **Class 5** Mathemagic
- **Class 6** Mathematics
- **Class 7** Mathematics
- **Class 8** Mathematics
- **Class 9** Mathematics
- **Class 10** Mathematics

Mathematics Class 5

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
12	159	NA	Bar Graph	Bar graph in a primitive form, tally used for counting and also as a legend. Investigative questions to be answered by looking at the chart.	Number of different favourite pet animals of children in form of a table. This is a good introduction to bar charts, where number count in form of tally marks is used for counting and hence as a measure of height of a particular variable.
12	161	NA	Bar Graph	Bar graph in a primitive form, tally used for counting and also as a legend. Investigative questions to be answered by looking at the chart.	Number of different vehicles passing a street in form of a table. Follow up from the previous task. Here also number count in form of tally marks is used for counting and hence as a measure of height of a particular variable. The follow up task asks students to collect data about trees in their colony and make a tally-bar chart for the data.
12	163	NA	Pie Chart	Pie chart introduced as a Chapati chart and numbers to be matched on pie chart from table. It also asks them investigative questions to be answered using such a chart.	Number of children who help most in house work in form of table. This activity introduces students to pie charts. The follow up task asks students to collect data about what do their friends like to do after school and make a table from this data. Does not tell students how to make a pie chart.

Class 5 Maths: Continued on next page

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
12	165	NA	Bar Graph	Bar graph showing temperature of 4 cities in Summer and Winter. Questions to be answered by studying these graphs. The Y-axis has temperature in degree Centigrade, while X-axis has cities.	Data directly in the bar graph, not shown in form of table. No Source given. A good introduction to bar graphs, but students are not told how to make such a chart. The follow up activity asks students to collect data about temperature of cities from newspapers or television and make a bar chart. The students are to ask questions based on the charts they draw to their peers. Also students are encouraged to relate the temperatures to geographical location of the cities.
12	168	NA	Line Graph	Line / point graph showing length of plant in terms of days. The points are plotted from a table given for the data. The X-axis has number of days, and Y-axis has length of plant in centimetres.	Data of growth of plant as a function of days given in form of a table. The data is from observing of plants of moong seeds. Students are supposed to draw similar graph and check if their graph matches the one in the textbook. Investigative questions to be answered based on the graph.

Table A.1: Survey data from Class 5 Mathematics textbook.

Mathematics Class 6

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
2	30		Number line	Number line introduced	
2	30		Number line	Subtraction on the number line introduced	
2	30		Number line	Multiplication on the number line introduced	
2	34		Number line	Commutativity of addiction and multiplication on the number line	
6	117		Number line	Representation of integers on number line	
6	117		Number line	Representation of integers on number line	
6	117		Number line	Representation of integers on number line	
6	119		Number line	Representation of integers on number line	
6	121		Number line	Representation of temperature on number line	Question is to plot the data on the number line
6	122		Number line	Representation of floors on a vertical number line	Question based on the data
6	126	6.4	Number line	Addition of integers on a number line	
6	126	6.5	Number line	Addition of integers on a number line	
6	126	6.6	Number line	Addition of integers on a number line	
6	126	6.7	Number line	Addition of integers on a number line	
6	127	6.8	Number line	Addition of integers on a number line	
6	127	6.9	Number line	Addition of integers on a number line	
6	128	6.10	Number line	Addition of integers on a number line	
6	129	6.11	Number line	Subtraction of integers on a number line	
6	129	6.12	Number line	Subtraction of integers on a number line	

Class 6 Maths: Continued on next page

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
6	129	6.13	Number line	Subtraction of integers on a number line	
6	130	6.14	Number line	Subtraction of integers on a number line	
6	130	6.15	Number line	Subtraction of integers on a number line	
6	130	6.16	Number line	Subtraction of integers on a number line	
9	189		Bar Graph	Pictograph showing days against the number of absentees	Question based on the data
9	190		Bar Graph	Pictograph showing colours of fridges used by people of different locality	Question based on the data
9	190		Bar Graph	Pictograph showing different modes of transport used by kids in class	Question based on the data
9	191		Bar Graph	Pictograph showing the number of wrist watches manufactured by a company in a week	Question based on the data
9	192		Bar Graph	Pictograph showing the number of tractors that different villages have	Question based on the data
9	193		Bar Graph	Pictograph showing number of girls students in each of the classes	Question based on the data
9	193		Bar Graph	Pictograph showing sale of electric bulbs in a shop on different days in the week	Question based on the data
9	194		Bar Graph	Pictograph showing sale of fruit baskets by different vendors	Question based on the data
9	195		Bar Graph	Pictograph to be drawn with given data of number of students present on each day of the week	
9	196		Bar Graph	Pictograph to be drawn with given data of number of bulbs purchased or a lodging house	

Class 6 Maths: Continued on next page

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
9	197		Bar graph	showing the vehicular traffic over one day at a junction	
9	197		Bar graph	Bar graph showing population of India over last few years	
9	198		Bar graph	Bar graph showing number of students in a school	Question based on the data
9	198		Bar graph	Bar graph showing amount of wheat purchased by the government	Question based on the data
9	199		Bar graph	Bar graph showing the sale of number of shirts from a store in a week	Question based on the data
9	199		Bar graph	Bar graph showing the number of marks obtained by a student in half yearly exam	Question based on the data
9	200		Bar graph	Drawing a bar graph of students in a class and their choice of fruits	
9	201		Bar graph	Drawing a bar graph of expenses in a household	
9	202		Bar graph	Drawing a bar graph of expenses in a household	same as 201 orientation of graph is changed

Table A.2: Survey data from Class 6 Mathematics textbook.

Mathematics Class 7

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
1	1	-	Number line	Number line to show the numbers	
1	4	-	Number line	Number line showing temperature of different cities	
1	7	-	Number line	Associative property using number line	
1	10	-	Number line	Multiplication using number line	
3	69	-	Bar Graph	Bar graph of number of students and their favorite color	Exercise
3	70		Bar Graph	Bar graph depicting marks of students	Exercise
3	71		Bar Graph	Double bar graph showing average hours of sunshine in two cities	Question is to answer which is possible and why
3	71		Bar Graph	Double bar graph showing quarterly and half yearly marks of different students in mathematics	
3	72		Bar Graph	Double bar graph showing watches tested for water proofness	Question is to answer which is possible and why
3	72		Bar Graph	Bar graph showing the pets owned by students of class 7	Question is to answer which is possible and why
3	72		Bar Graph	Bar graph showing the number of books sold by a book store in a year	Question is to answer which is possible and why
9	176		Number Line	Rational numbers on a number line	
9	177		Number Line	Rational numbers on a number line	
9	179		Number Line	Rational numbers on a number line	
9	183		Number Line	Rational numbers on a number line	
9	184		Number Line	Rational numbers on a number line	
9	185		Number Line	Rational numbers on a number line	

Class 7 Maths: Continued on next page

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
9	186		Number Line	Rational numbers on a number line	
9	187		Number Line	Rational numbers on a number line	

Table A.3: Survey data from Class 7 Mathematics textbook.

Mathematics Class 8

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
1	15		Number line	Number line and its extensions	
5	70		Bar graph	Bar graph introduced. Depicts the number of students in academic years in a school	
5	70		Bar graph	Double Bar graph introduced. Depicts the marks of a student in two different academic years in a school	
5	74	5.1	Bar graph	A bar graph depicting the data in the table of number of students and their marks	
5	75	5.2	Bar graph	A bar graph depicting the age of teachers in a school	
5	75	5.3	Bar graph	A bar graph depicting the height of girls in class 7	
5	77		Bar graph	A bar graph depicting the number of hours the children watch the television	
5	77	5.4	Pie chart	Pie charts introduced. Pie charts depicting the hours spent by a child and distribution of age of people.	
5	78	5.5	Pie chart	3 pie charts depicting various information about a class room	asks children to find out the same for their own class
5	78	5.6	Pie chart	Pie chart depicting the information about the different kind of programs watched on a TV	Questions asked about the information provided.
5	79		Pie chart	Example to show how to draw a pie chart	
5	79	5.7	Pie chart	Pie chart showing the expenditure	Questions asked about the information provided
5	80	5.8	Pie chart	Pie chart to be drawn from the given data	

Class 8 Maths: Continued on next page

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
5	82		Pie chart	Pie chart depicting the information about the different kind of music heard by people	Questions asked about the information provided
5	82		Pie chart	Pie chart depicting the information about the marks obtained by a student in different subjects	Questions asked about the information provided
5	83		Pie chart	Display chart data in a pie chart	Exercise
8	119		Pie chart	Pie chart depicting the information about the number of hours the children are helped by the parents	Questions asked about the information provided
15	231	15.1	Bar graph	Bar graph showing marks in mathematics in three years	
15	232	15.2	Bar graph	Double Bar showing comparative sales of fruits over two days	
15	232	15.3	Pie chart	Pie chart showing constituent programs of TV channels in percentage	
15	233	15.4	Bar graph	Histogram showing number of persons and their weight	
15	234	15.5	Line graph	Construction of a line graph	
15	234	15.6	Line graph	Construction of a line graph	
15	235	15.7	Line graph	Line graph showing the total runs scored by two batsman over a year	Questions asked about the information provided
15	235	15.8	Line graph	Line graph showing distance of a car from a city as a function of time	Questions asked about the information provided
15	236	15.9	Line graph	Line graph showing temperature of a patient over a day	Questions asked about the information provided
15	237		Line graph	Line graph showing yearly sales of a company over 5years	Questions asked about the information provided
15	237		Line graph	Line graph showing growth of two different plants in lab conditions	Questions asked about the information provided

Class 8 Maths: Continued on next page

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
15	238		Line graph	Line graph showing actual and predicted temperature for a week	Questions asked about the information provided
15	239		Line graph	Line graph showing distance of a courier boy from town at different times of a day	Questions asked about the information provided
15	239		Line graph	Four line graphs showing temperature time variations	Question is to answer which is possible and why
15	241	15.12	Line graph	Exercises in points and co-ordinate system	
15	241	15.13	Line graph	Exercises in points and co-ordinate system	
15	241	15.14	Line graph	Exercises in points and co-ordinate system	
15	242	15.15	Line graph	4 Exercises in points and co-ordinate system	
15	243		Line graph	Exercises in points and co-ordinate system	
15	244	15.16	Line graph	Exercise to plot price of petrol versus the litre of petrol	
15	245	15.17	Line graph	Exercise to plot simple interest earned and the amount of money deposited	
15	247	15.18	Line graph	Exercise to plot a distance time graph r a constant velocity	

Table A.4: Survey data from Class 8 Mathematics textbook.

Mathematics Class 9

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
1	1	1.1	Number line	The number line	
1	1	1.2	Number line	The number line	No reference in the text
1	7	1.7	Number line	Locating $\sqrt{2}$ on the number line	
1	7	1.8	Number line	Locating $\sqrt{3}$ on the number line	
1	15	1.11	Number line	Representing real numbers on number line	
1	15	1.12	Number line	Representing real numbers on number line	
1	16	1.13	Number line	Representing real numbers on number line	
1	17	1.14	Number line	Representing real numbers on number line	
3	51	3.1	Number line	Street map as a cartesian graph	
3	53	3.3	Number line	Classroom as a cartesian graph	
3	54	3.5	Number line	The number line	
3	55	3.7	Number line	The Axes	
3	55	3.8	Number line	The Cartesian coordinate system	
3	56	3.9	Number line	The Cartesian coordinate system with quadrant	
3	56	3.10	Number line	The cartesian coordinate system with some points	
3	58	3.11	Number line	The cartesian coordinate system with some pots	
3	59	3.12	Number line	Cartesian system in the problem for finding out the coordinates of the points	
3	60	3.13	Number line	The Cartesian coordinate system showing the signs of the points in the different quadrants	

Class 9 Maths: Continued on next page

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
3	61	3.14	Number line	Plotting a point in the system if the coordinates are known	
3	62	3.15	Number line	Plotting a point in the system if the coordinates are known	
3	63	3.16	Number line	Plotting a point in the system if the coordinates are known	
3	64	3.17	Number line	Plotting a point in the system if the coordinates are known	
4	66	4.1	Number line	Plotting a root in 1 D	
4	71	4.2	Line graph	Plotting a root of a line in the system	
4	72	4.3	Line graph	Plotting the graph of an equation of a line	
4	72	4.4	Line graph	Plotting a graph of equation $y = kx$	
4	73	4.5	Line graph	Match the pair of equations with graphs	
4	74	4.6	Line graph	Match the graph with equations	
4	74	4.7	Line graph	Match the graph with equations	
4	76	4.8	Line graph	Equations of lines parallel to the axes	
4	76	4.9	Line graph	Solution of the equation of line parallel to y axis	
4	76	4.10	Line graph	Graph of line parallel to y axis	
9	156	9.9	Line graph	Area of a triangle using a graph sheet.	
14	247	14.1	Bar graph	Bar graph depicting the number of births in each month for students in a class	
14	249	14.2	Bar graph	Bar graph depicting the expenditure per month for different items	

Class 9 Maths: Continued on next page

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
14	250	14.3	Bar graph	Bar graph depicting the weight of students in the class	
14	251	14.4	Bar graph	A bar graph wrongly depicting the information given in a table	
14	253	14.5	Bar graph	Corrected bar graph for 14.4	
14	254	14.6	Bar graph / Line graph	Frequency polygon for distribution of weight of students in a class	
14	255	14.7	Bar graph / Line graph	Frequency polygon for the data given in the table	
14	257	14.8	Bar graph / Line graph	Frequency polygon for the data given in the table	

Table A.5: Survey data from Class 9 Mathematics textbook.

Mathematics Class 10

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
2	22	2.1	Line graph	showing plotting of a linear equation	
2	23	2.2	Line graph	Line graph showing plotting of a quadratic equation	
2	24	2.3	Line graph	Line graph showing plotting of a quadratic equation with comments on zeroes of the equation	
2	24	2.4	Line graph	Line graph showing plotting of a quadratic equation with comments on zeroes of the equation	
2	25	2.5	Line graph	Line graph showing plotting of a quadratic equation with comments on zeroes of the equation	
2	26	2.6	Line graph	Line graph showing plotting of a cubic equation	
2	26	2.7	Line graph	Line graph showing plotting of a cubic equation	
2	26	2.8	Line graph	Line graph showing plotting of a cubic equation	
2	27	2.9	Line graph	Six Line graphs showing plotting various function	Exercise is to find the number of zeros
2	28	2.10	Line graph	Six Line graphs showing plotting various functions	exercise is to find the number of zeros
3	42	3.2	Line graph	Two lines representing two equations are intersecting at a point.	
3	43	3.3	Line graph	Plot of equivalent equations of lines	
3	44	3.4	Line graph	Plot of parallel lines	which do not intersect
3	47	3.5	Line graph	Graphically solving two equations	
3	48	3.6	Line graph	Graphically solving two equations	
7	156	7.1	Line graph	Finding distance between two points in a Cartesian graph.	

Class 10 Maths: Continued on next page

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
7	156	7.2	Line graph	Finding distance between two points in a Cartesian graph.	
7	157	7.3	Line graph	Finding distance between two points in a Cartesian graph.	
7	157	7.4	Line graph	Finding distance between two points in a Cartesian graph.	
7	157	7.5	Line graph	Finding distance between two points in a Cartesian graph.	
7	159	7.6	Line graph	Graph of a problem in class room seating. Asks whether the students are seated in a straight line.	
7	160	7.7	Line graph	Graph of a perpendicular bisector of a line from equation	
7	161	7.8	Line graph	Graph of a problem in class room seating. Asks whether the students are seated along vertices of a square.	
7	162	7.9	Line graph	Graph for section formula	
7	162	7.10	Line graph	Graph for section formula	
7	165	7.11	Line graph	Graph for problem of trisection of a line segment	
7	167	7.12	Line graph	Graph for problem of placing a flag in a race.	
7	168	7.13	Line graph	Graph for finding the area of a triangle.	
7	171	7.14	Line graph	Graph for a problem of finding the area of a triangl in a garden.	
14	290	14.1	Line graph	Less than ogive curve	
14	290	14.2	Line graph	More than ogive curve	

Class 10 Maths: Continued on next page

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
14	291	14.3	Line graph	Median curve	
14	291	14.4	Line graph	Median curve	
14	292	14.5	Line graph	Problem for finding out the profit from the give data	
14	292	14.6	Line graph	Less than ogive curve	

Table A.6: Survey data from Class 10 Mathematics textbook.

Science Textbooks

In this section the data from the NCERT Science textbooks is presented. The books that were analysed were:

- § **Class 5** Environmental Science
- § **Class 6** Science
- § **Class 7** Science
- § **Class 8** Science
- § **Class 9** Science
- § **Class 10** Science

Science Class 7

Chapter	Page	Fig. No.	Graph Type	Description	Data & Comments
7	70	7.2	Line Graph	Graph Showing variation of maximum temperature in August	
13	151	13.5	Bar Graph	Bar graph showing runs scored by team in each over.	
13	152	13.10	Line Graph	Line graph showing change in weight of man with age	
13	153	13.12	Line Graph	Making a line graph	
13	153	13.13	Line Graph	Making a line graph	
13	154	13.14	Line Graph	Distance time graph of a bus	
13	157	13.15	Line Graph	Distance time graph of two vehicles in context of a question	
13	157-158	13.16	Line Graph	Distance time graphs in context of a problem of multiple choice Question regarding constant speed	

Table A.7: Survey data from Class 7 Science textbook.

Science Class 8

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
10	115	10.1	Line Graph	Line graph showing percentage height with age.	
10	126		Line Graph	Line graph and table showing height increase of boys and girls with age.	
5	63		Bar / Line Graph	”Exercise to plot the deficit of energy as A function of time, data values given in a table.”	

Table A.8: Survey data from Class 8 Science textbook.

Science Class 9

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
12	164	12.8	Line Graph	Time line graph depicting wave disturbance and graphical depiction of sound.	
12	165	12.9	Line Graph	Time line graph depicting amplitude of various sounds.	
12	165	12.10	Line Graph	Time line graph depicting of frequenciesvarious sounds.	
9	121		Line Graph	Velocity time graph depicting a problem situation.	Use of multiple colors
8	99	8.1	Line Graph	Line graph of position of an object along a straight line	
8	104	8.3	Line Graph	Time – distance graph of an object moving with an uniform velocity.	
8	105	8.4	Line Graph	Time – distance graph of an object moving with a non-uniform velocity.	
8	105	8.5	Line Graph	Time – Velocity graph of an object moving with an uniform velocity.	

Class 9 Science: Continued on next page

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
8	106	8.6	Line Graph	Time – Velocity graph of an object moving with an uniform acceleration	
8	106	8.7 a	Line Graph	Time – Velocity graph of an object moving with non-uniform acceleration	
8	106	8.7 b	Line Graph	Time – Velocity graph of an object moving with non-uniform acceleration	
8	107	8.8	Line Graph	Time – Velocity graph of an for obtaining equations of motion	
8	112	8.11	Line Graph	Time – Distance graph in context of a problem.	
8	113	8.12	Line Graph	Speed – Time graph in context of a problem.	

Table A.9: Survey data from Class 9 Science textbook.

Science Class 10

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
2	25	2.6	Line Graph	Line graph depicting the variation of pH For concentrations of H^+ and OH^-	The graph curves strangely for OH^- concentration. Makes no sense!
12	204	12.3	Line Graph	Line graph depicting the Ohm's Law, V-I plot for nichrome wire.	Straight line graph, with data points marked x and no data values given.
14	244	14.1	Pie Chart	Pie Chart Depicting major sources of energy For requirements of energy in India	Pie Chart in 3D and Color, No percentage levels [numbers] given.
16	267	16.1	Bar / Line Graph	"Line + Bar Chart Depicting the total coliform Count levels in the Ganga (1993-1994) " Water Quality Status and Statistics	The graph is a bit confusing with no explanations given for MPN, and floating minimum and maximum levels.

Table A.10: Survey data from Class 10 Science textbook.

Social Science Textbooks

In this section the data from the NCERT Social Science textbooks is presented. The books that were analysed were:

- § **Class 5**
  - » Social Science: Environmental Science
- § **Class 6**
  - » Social Science: Social & Politcal Life
  - » Social Science: History - Our Past
  - » Social Science: The Earth - Our Habitat
- § **Class 7**
  - » Social Science: Environment
  - » Social Science: Social Political Life
  - » Social Science: History
- § **Class 8**
  - » Social Science: Political Science
  - » Social Science: History
  - » Social Science: Geography Resources and Development
- § **Class 9**
  - » Geography - Contemporary India
  - » Introducing Sociology
  - » Political Science
  - » History
- § **Class 10**
  - » Geography
  - » Democratic Politics
  - » Understanding Economic Development
  - » History

Environmental Science Class 5

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
1	11		Pie Chart	The diagram uses pie chart in an innovative way, to display the sleeping time of various animals.	

Table A.11: Survey data from Class 5 Environmental Science book.

Earth Our Habitat Class 6

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
5	33		Pie Chart	The 3-D pie chart shows distribution of water in different water bodies.	This figure has no Caption and has no reference in the main text. The presentation of data is ambiguous especially for fresh water. 97.2 % Oceans 2.8 % Stored in Ice Sheets and Ground Water 0.03 % Fresh Water. Pie charts are not introduced in the Mathematics texts in Grades 5 or 4.

Table A.12: Survey data from Class 6 Earth Our Habitat textbook.

Environmental Science Class 7

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
4	20		Pie Chart	The Diagram is used to display the composition Of air in terms of its constituents.	
4	22		Line Graph	A graph depicting the naming of different layers Of atmosphere as a function of height.	The thermopshere supposed to extend Between 80-400 kms is grossly out of scale And exosphere is shown but provides no idea Of its scale and extent. There is a zigzag line which passes through The height column, apparently has no significance.
5	32		Other	Distribution of fresh water on earth.	Uses 2-D shape to depict the different amounts of water, out of scale.

Table A.13: Survey data from Class 7 Environmental Science book.

Social and Political Life Class 7

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
5	63		Bar Graph	Data on boys and girls who leave schools from different social groups	

Table A.14: Survey data from Class 7 Social and Political Life textbook.

Environmental Science Class 8

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
6	67		Bar Graph	Bar graph depicting the distribution of population In various countries.	
6	69		Bar Graph	Bar graph depicting the increase in world population Since historical times.	
6	72		Bar Graph	A population pyramid depicting the age wise and Gender wise distribution of population in a country.	

Table A.15: Survey data from Class 8 Environmental Science textbook.

Political Science Class 9

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
4	70		Line Graph	Voter turn out in the election in UK and India over different years	
4	71		Bar Graph	Voter turn out in the election in US and India over different years as per ethnic background	
4	71		Bar Graph	Those who participated in any election activity in India	
4	71		Pie Chart	People's thought on what their vote has effect	

Table A.16: Survey data from Class 9 Political Science textbook.

Geography Class 9

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
1	2		Bar Graph	Populations seven largest countries of the world.	
4	40		Bar and line graph	Bar and line graph for temperature and rainfall in the cities and students are supposed to plot them	Activity for students
5	44		Bar Graph	Showing the forest cover over in different states in India	
6	54	6.1	Pie Chart	Two pie charts show India's share of the world population and area	
6	54	6.2	Pie Chart	Pie chart showing distribution of population in India	
6	56	6.4	Bar and Line graph	Bar and line graph showing India's growth rate and total population	

Class 9 Geography: Continued on next page

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
6	57	6.5	Pie Chart	Pie chart showing age-wise distribution of population in India	

Table A.17: Survey data from Class 9 Geography textbook.

Understanding Economic Development Class 10

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
2	24	Graph 1	Bar Graph	GDP by different sectors in 1973 and 2003	
2	25	Graph 2	Bar Graph	GDP by different sectors in 1973 and 2003 percentage composition	
2	25	Graph 3	Bar Graph	Sector share of employment	
4	48	Graph 1	Pie Chart	Sources of credit in rural India	
4	49	Graph 2	Pie Chart	Sources of credit as formal and informal in different house holds	

Table A.18: Survey data from Class 10 Understanding Economic Development textbook.

Political Science Class 10

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
7	92		Bar Graph	Shows the people's view about suitability of democracy in their own country	
7	98		Bar Graph	Belief of people in different countries in the efficacy of vote	
7	78		Bar Graph, Line graph, pie chart	People saying they are members of political party	
4	44		Bar Graph	Women in parliaments of the world	
7	50		Pie Chart	Population of different religious groups in India 2001	

Table A.19: Survey data from Class 10 Political Science textbook.

Geography Class 10

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
1	5	1.3	Pie Chart	Land under important relief features	
1	6	1.4	Pie Chart	Two pie charts comparing the general land use in 60-61 and 02-03	
1	7	1.5	Pie Chart	Wastelands in 2000	
4	42	4.14	Pie Chart	Consumption of natural rubber	
5	52	5.2	Pie Chart	Statewise production of iron ore in 03-04	
5	53	5.4	Pie Chart	Statewise production of manganese in 03-04	
5	55	5.6	Pie Chart	Statewise production of copper in 03-04	
5	55	5.7	Pie Chart	Statewise production of bauxite in 03-04	
5	56	5.9	Pie Chart	Statewise production of limestone in 03-04	
6	73	6.5	Double bar graph	Steel production in India and China	

Table A.20: Survey data from Class 10 Geography textbook.

History Class 10

Chapter	Page	Fig. No.	Graph Type	Data Used	Comments
6	129	1	Other	A graph showing growth of London on the banks of Thames	

Table A.21: Survey data from Class 10 History textbook.

A.2 Qualitative Analysis

In this section we present the complete data from the qualitative analysis of graphs in science textbooks. Some of these graphs were presented in Section 3.4 as examples to illustrate the qualitative analysis.

Graphs in science textbooks

Category	Description
Function	What function does the graph serve in the textbook? Whether the function is narrative, organisational, analytical or metaphorical representation.
Reference	Whether the graphs are referred to in the main text? If they are, what is the manner and frequency of reference?
Integration	How well are they integrated into the overall text? How do they go with the flow of the narrative?
Data Used	What is the data used in making the graphs? Is the data provided in a tabular form, is the source of the data provided? Is real data used in making the graphs.
Legend and Axes	Is the graph with key and labels to the axes? Are the variables on the axes with units and labels?
Close-to-life	Does the graph link to any everyday experience of the students?
Design aspects	Is the graph well designed? Does it have unnecessary decorative elements?

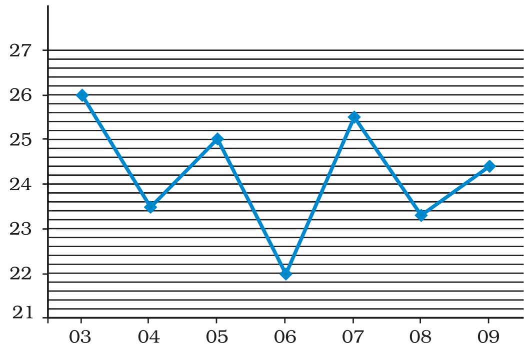
Table A.22: The categories used for qualitative analysis of science textbooks and their description.

We have created boxes for each graph with a unique number for the graph. The box for the graph contains the particulars of the parameters for the given graph. We have presented some of these graphs in Chapter 3, in this appendix we present all of the graphs that we have analysed.

A.2.1 Class 7

In this section we see the graphs appearing in Class 7 Science textbook (NCERT, 2007b). There are a total of eight graphs appearing in this textbook. Seven of these appear in Chapter 13 *Motion and Time*. Graphs are seen here mostly in the context of motion, though as we find there are many places where we can use graphs for improving the understanding of concepts at hand.

Graph 1



Date	Maximum temperature
03-08-06	26.0°C
04-08-06	23.5°C
05-08-06	25.0°C
06-08-06	22.0°C
07-08-06	25.5°C
08-08-06	23.3°C
09-08-06	24.4°C

Fig. 7.2 *Graph showing the variation of maximum temperature during 03 to 09 August 2006*

Figure A.1: A sample graph from the science textbook. This graph appears in Class 7, Chapter 7, on Page 70.

Function

The first graph in Class 7 textbook, (here Figure A.1) shows the variation of maximum temperature for the seven days in August for the city of Shillong, in the state of Meghalaya. Here the graph is introduced in a way under the assumption that the students already know about graphs.

Reference

The graph is referred to in the main text with the figure number, but there is no elaboration on the nature of the graph or its interpretation.

> Look at the graph given below which shows the maximum temperature recorded during 03 August 2006 to 09 August 2006 at Shillong, Meghalaya (Fig. 7.2).

Data The table next to the graph shows the data used, the source for this data is not provided. Though the data has a 0.1 degree accuracy, the plot has only five divisions per degree hence 0.2 degree accuracy.

Legend, Axes

The labelling on the axes is missing. The scales or units are not mentioned anywhere in the graph.

Close-to-life

The situation depicted in the graph is close-to-life, as the variation in daily temperature, is a phenomenon which is experienced by us every day.

Design Aspects

The graph has too many horizontal lines, which might be useful in finding the temperature. However, the number of grid lines can be reduced.

Being the first graph, the introductory aspects related to reading of the graph are entirely missing. Information on how to read the graph, what are its salient features, how to interpret the variation in the graph and how to infer from the graph is completely missing in the main text. The graph itself has data for maximum temperature from many days and does not show any particular trend. Neither are any particular questions asked about the graph. The next paragraph that follows mentions maximum and minimum temperature, and also the probable times during a day when these temperatures might be reached. A graph depicting the variation of temperature over the course of a single day, and the supporting text would have done a better job and made the point about the variation of temperature during the day. We think that the poor treatment given to this graph is a missed opportunity to introduce graphs in a close-to-life context like the daily variation of temperature.

Figure A.2 shows an alternative graph that we offer to illustrate the points for daily maximum and minimum temperature. Table A.23 shows the data which was used to draw this graph. For the variation of the maximum temperature on the annual scale, Figure A.4 illustrates how the minimum and maximum temperature varies through the year. The data for this figure is from Tables 7.2 and 7.3 in Class 7 science textbook, (pg. 71). (Table A.3 here).

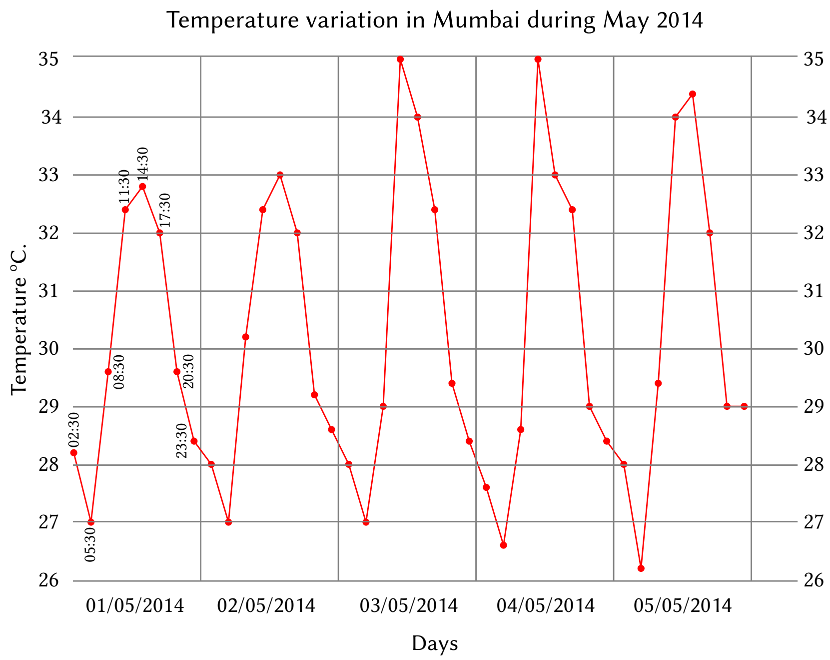


Figure A.2: A graph showing the temperature variation in Mumbai in the first week of May 2014. The observations were taken eight times every day with 3 hour intervals. The readings were taken at 02:30, 05:30, 08:30, 11:30, 14:30, 17:30, 20:30 and 23:30 hours. Data from http://rp5.in

In the textbook, the students are told to analyse the data given in Tables 7.2 and 7.3 (shown in Figure A.3 here). The textbook has following to say about the use of tables:

> In Table 7.2 and 7.3, we have given the climatic condition at two places in India. The mean temperature for a given month is found in two steps. First, we find the average of the temperatures recorded during the month. Second, we calculate the average of such average temperatures over many years. That gives the mean temperature. The two places are: Srinagar in Jammu and Kashmir, and Thiruvananthapuram in Kerala.

This paragraph explains the concept of mean temperature in the tables. Then in the next para-

Date/Time	02:30	05:30	08:30	11:30	14:30	17:30	20:30	23:30
01/05/14	28.2	27.0	29.6	32.4	32.8	32.0	29.6	28.4
02/05/14	28.0	27.0	30.2	32.4	33.0	32.0	29.2	28.6
03/05/14	28.0	27.0	29.0	35.0	34.0	32.4	29.4	28.4
04/05/14	27.6	26.6	28.6	35.0	33.0	32.4	29.0	28.4
05/05/14	28.0	26.2	29.4	34.0	34.4	32.0	29.0	29.0
06/05/14	28.2	28.0	29.6	33.0	34.2	32.0	30.0	30.2

Table A.23: Variation of temperature during the day in Mumbai for the first week in May 2014. The temperature is in collected eight times a day in three-hour intervals. Figure A.2 uses this data.

graph, the textbook says:

> By looking at Tables 7.2 and 7.3 we can *easily see* the difference in the climate of Jammu & Kashmir and Kerala. We can *see* that Kerala is very hot and wet in comparison to Jammu & Kashmir, which has a moderately hot and wet climate for a part of the year. (emphasis added)

It is not clear by looking at the tables how the difference in the climate can be *easily* seen. Perhaps one is reminded of the quote from Farquhar and Farquhar (quote at the beginning of Appendix B) when this claim is made. When the same data given in the two tables are plotted (Figure 3.10), the difference in the climate of the two places stands out.

We provide the analysis of Figure A.4 as follows. In general, the trend that we observe is that the range of temperature in Thiruvananthapuram is moderate between 22 to 32 °C throughout the year. Srinagar on the other hand has significant variation in temperature through the year.

Table 7.2 Srinagar (Jammu & Kashmir)

Information about climate

Month	Mean temperature °C		Mean total rainfall (mm)
	Daily minimum	Daily maximum	
Jan	-2.3	4.7	57
Feb	-0.6	7.8	65
Mar	3.8	13.6	99
Apr	7.7	19.4	88
May	10.7	23.8	72
Jun	14.7	29.2	37
July	8.2	30.0	49
Aug	17.5	29.7	70
Sep	12.9	27.8	33
Oct	6.1	21.9	36
Nov	0.9	14.7	27
Dec	-1.6	8.2	43

Table 7.3 Thiruvananthapuram (Kerala)

Information about climate

Month	Mean temperature °C		Mean total rainfall (mm)
	Daily minimum	Daily maximum	
Jan	22.2	31.5	23
Feb	22.8	31.9	24
Mar	24.1	32.6	40
Apr	24.9	32.6	117
May	24.7	31.6	230
Jun	23.5	29.7	321
July	23.1	29.2	227
Aug	23.2	29.4	138
Sep	23.3	30.0	175
Oct	23.3	29.9	282
Nov	23.1	30.3	185
Dec	22.6	31.0	66

Figure A.3: Tables 7.2 and 7.3 for Figure A.4 from pg 71. Chapter 7, Class 7, science textbook.

We now provide a month-by-month description of how the graph is to be read.

Jan-Feb

Starting in January we see that the temperature in Thiruvanathapuram is in the range of 22 to 32 °C, while at the same time in Srinagar the temperature is much lower in the range of −5 to 5 °C. In the case of rainfall during these two months Srinagar experiences *more* rainfall than Thiruvananthapuram.

March-April

During March and April the temperature rises in Srinagar, moving from sub-zero to positive. While in Thiruvananthapuram the minimum temperatures rise slightly. In case of rainfall in March Srinagar still has more rainfall than Thiruvananthapuram. However, in April Thiruvananthapuram has more rain than Srinagar.

May-June

During May-June the temperature continues to rise in Srinagar with the maximum crossing 20 °C and almost reaching 30 °C in June. While in Thiruvananthapuram there is a drop in temperature by a couple of degrees, (this is perhaps related to the rainfall due to the onset of monsoon). In May Thiruvananthapuram gets second highest rainfall in the year (~ 230 mm), while Srinagar gets lower rainfall (snowfall?) than the previous months. In June monsoon is at the peak in Thiruvanathapuram the rainfall is above 300 mm, but at the same time in Srinagar, the rainfall is modest, while the temperature is on the rise.

July-August

During July the rainfall is lower than in June in Thiruvananthapuram. The mean temperature difference is minimum in Thiruvananthapuram, while it is maximum in Srinagar (8.2 to 30 °C). Also in July Srinagar experiences the highest mean maximum temperature, making it the hottest month of the year. In August the rainfall drops significantly in Thiruvananthapuram, while in Srinagar it increases. During August the daily minimum temperature in Srinagar is highest.

Sept-Oct

With the onset of September the mean temperatures begin to fall in Srinagar, while in Thiruvananthapuram the temperature is almost the same as previous months. The rainfall increases in Thrivananthapuram, while in Srinagar it decreases. In October, the general decline in mean temperatures continues in Srinagar while in Thiruvananthapuram there is not much change in the temperature. The minimum temperature goes below 10 °C in October in Srinagar, which indicates that the winter is setting in. The second monsoon peaks during October in Thiruvananthapuram, while Srinagar registers little rainfall.

Nov-Dec

During November - December the temperatures in Thiruvananthapuram rises slightly

> from the months before. In Srinagar the winter has set in, the mean minimum temperature going close to 0 °C in November, while going to negative in December. The rainfall in Thiruvanantahpuram decreases sharply in December while in November it still has rainfall above 150 mm after the October monsoon peak. In Sringar the rainfall is lowest during November and increases in December and following months.

We provided this detailed exposition to show how to read graphs such as these, especially when presenting them in the textbooks. Expositions like this in which the narrative have the following advantages:

1. Provides a way of reading graphs to the learner.
2. Integrates the text and the graph with the data provided.
3. Provides help on reading salient features from the data and their meaning.
4. Provides connections to the physical meaning of the various features of the graph.
5. Poses questions that can be asked and answered from the graph.

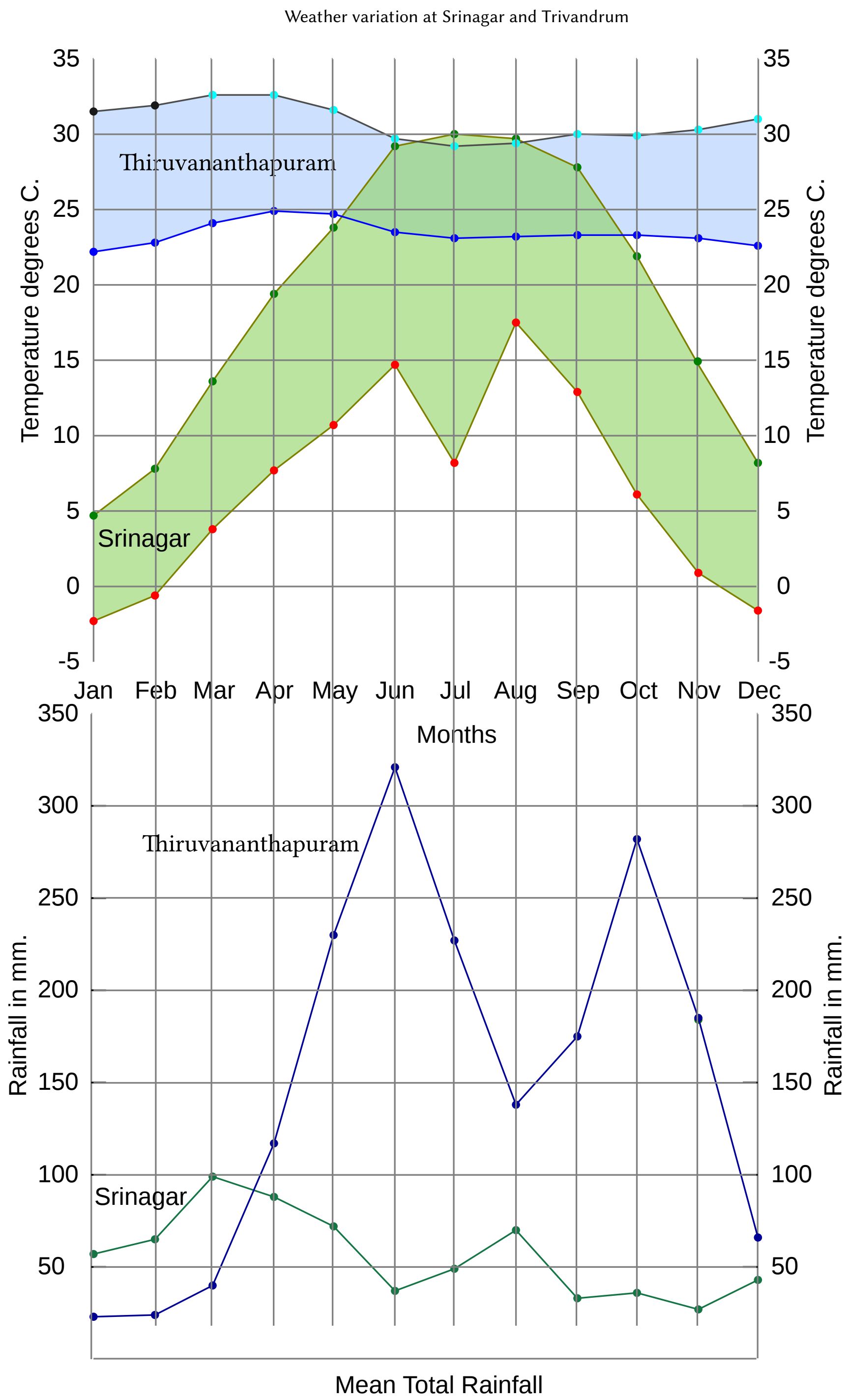


Figure A.4: A graph showing the annual mean rainfall and maximum and minimum temperatures in Srinagar and Thiruvananthapuram. The data for this graph is in Figure A.3.

Data table for Graphs

Table 13.5 Odometer reading at different times of the journey

Time (AM)	Odometer reading	Distance from the starting point
8:00 AM	36540 km	0 km
8:30 AM	36560 km	20 km
9:00 AM	36580 km	40 km
9:30 AM	36600 km	60 km
10:00 AM	36620 km	80 km

Figure A.5: A table containing readings from odometer of a car for analysing. From Class 7 science textbook p.151.

Function
The Chapter on *Motion and Time* in Class 7 in science textbooks introduces graphs to the students in the context of solving a problem.

Reference
The problem involves a travelling bus, with odometer readings available for specific times.

> Looking at the Table, Boojho teased Paheli whether she can tell how far they would have travelled till 9:45 AM.

The question posed is to find the distance travelled by 09:45 AM.

Data A table (Figure A.17) gives the data used for the problem. The textbook does not elaborate the method for obtaining the data.

Legend, Axes
Not applicable.

Close-to-life
People travel every day by various means of transport.

Design Aspects
Table can be made more readable by use of alternating coloured rows. The units for entries in the columns can be given at the column heads and not repeated with every entry. Vertical lines should be avoided while drawing a table. Mori (2007) gives excellent guidelines for drawing beautiful tables.

To answer the question "Looking at the Table, Boojho teased Paheli whether she can tell how far they would have travelled till 9:45 AM." A line graph should be constructed, as per the instructions. In the three graphs that follow, (Figures A.6, A.8, A.10 here), we see that the students are introduced to different *types* of graphs namely bar graph, pie-chart and line graph.

Graph 2

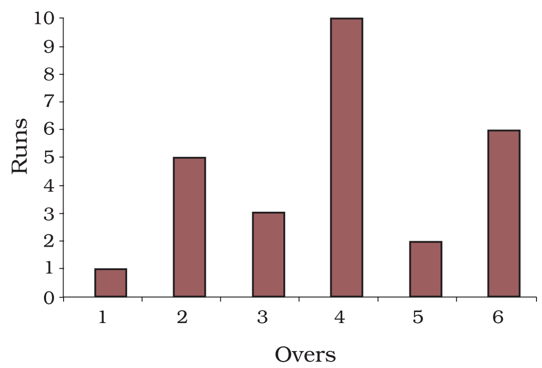


Fig. 13.8 *A bar graph showing runs scored by a team in each over*

Figure A.6: An example of a bar graph, showing runs scored in an over. From Class 7 science textbook p.151.

Function	Presented as an example of a bar graph.
Reference	The bar graph is only referenced in the main text once as: The type of graph shown in Fig. 13.8 is known as a bar graph.
Data	The data used in the graph is not given.
Legend, Axes	The axes are labelled and have units.
Close-to-life	The data presented has close-to-life context in form of score of a cricket match score.
Design Aspects	The graph presented is free from chart-junk.

The concept of graphs is introduced as if the students have not encountered graphs before. There is no effort to link it to the study of graphs that the students have already done in other subjects like mathematics and the social sciences (Geography). For example, in case of mathematics, we see bar charts introduced previously in Class 5 (Figure A.7). There are also questions asked about these bar graphs which require that the students study these graphs and answer them:

(1) Which city is hottest on June 1?

(2) Which city is coldest on December 1?

(3) Which city shows little change in temperature on the two days - 1 June and 1 December? (p. 165, Class 5, Mathematics textbook.)

There is also an associated activity for the students to collect data about the temperature of a city from TV or newspapers and plot it."On any one day, choose any three cities and record their temperatures from the TV or newspaper. Make a bar chart in your notebook and ask your friends a few questions about it. See if they understand your chart! (pg. 165, Class 5, Mathematics textbook.)"

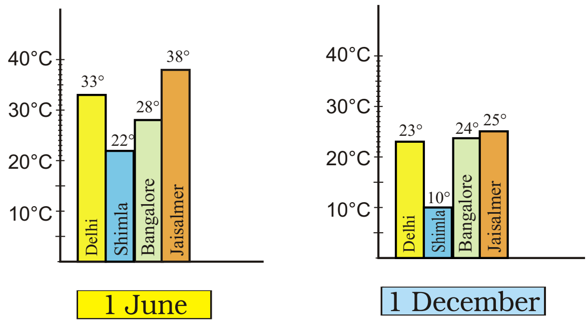


Figure A.7: Figure from Class 5, Chapter 12, page number 165 in Mathematics textbook. The two bar graphs show temperatures in four different cities, namely Delhi, Shimla, Bangalore and Jaisalmer at two different times in the year. Students are asked questions to be answered by studying the graphs.

However, in the bar chart in science textbook (Figure A.6) no such attempt is made. There is no further information or elaboration for the inclusion of this graph here. Neither the text details how to construct one. Sometimes, it is good to repeat a point more than once, but it is immensely beneficial if a reference is made to the earlier presence of the topic. For example, Roth et al. (2005) make a pertinent point regarding the use of repetition in the contest of captions of a graph and how it helps in constructing meaning from the graphs: " . . . *caption* illustrates that scientists do not leave uncertain how to read a line graph. It repeats information that might be gleaned from the graph alone. The effect of this redundancy, however, is to guide readers to a congruent construction of graph and text. (emphasis in original, p. 36)"

So, it is a little strange that the science textbook is not building upon the mathematical knowledge that the students have been already exposed to, but instead treats it as an entirely new concept. Even when treating as a new idea, no effort is seen to engage the students about how such graphs are made, or how to read them or what is the use case for each one of them. This exercise would have been an excellent opportunity to show the idea that how the same type of graphs can be used to depict different situations, across disciplines. We find this approach towards graphs lacking coherence across subjects.

Graph 3

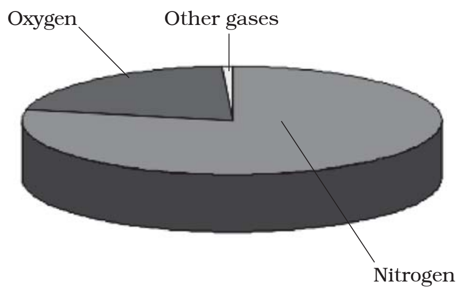


Fig. 13.9 *A pie chart showing composition of air*

Figure A.8: A pie chart showing components of air. Figure from Class 7, Chapter 13, page number 152 in the science textbook.

Function Figure A.8 we see a 3-D pie chart showing the different component gases of air.

Reference The only reference to this pie chart in the textbook is this:

> Another type of graphical representation is a pie chart (Fig. 13.9).

This graph presented here adds no value to the context or discussion.

Data Data for the graph is not given. If data was given, the students could at least redraw the chart if required.

Legend, Axes Component gases are labelled in the marked areas.

Close-to-life The air components are close-to-life.

Design Aspects Presenting the pie chart in 3-D for essentially a 2-D data can be misleading. It adds to the difficulty in reading the graph and might lead to incorrect display/reading of the values, especially when the not providing corresponding data. We provide a detailed analysis of one such graph later in this section (FigureA.53).

Another graph (Figure A.9) for the composition of air appears in the Class 7 Environmental science book in Chapter 4, page 20. The percentage values of the components are provided, making it better than the previous graph. The pie-chart is presented in the form of a musical instrument *dafali* held by one of the characters in the textbook.

A pie chart from EVS book showing components of the air.

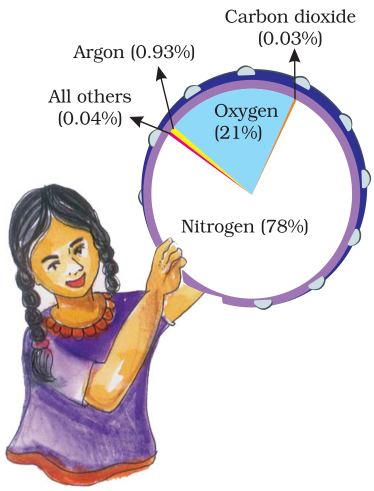


Fig. 4.1 *Constituents of Air.*

Figure A.9: A Pie chart showing components of the air. Figure from Class 7, Chapter 04, page number 20 in EVS textbook.

Function
The pie chart serves to show the components of air. But, the aim of presenting this chart is not clear from the textual narrative in the accompanying text.

Reference
The graph is referred in the main text in a single sentence. "The pie chart gives you the percentage of different constituents of air (Fig. 4.1)." No further elaboration is provided.

Data The data source is not given.

Legend, Axes
The components are colour coded.

Close-to-life
This is a close-to-life context.

Design Aspects
The pie chart is shown in the form of a musical instrument, the *dafali*, in hand of a girl. The design of the graph is unnecessarily complicated by this. This graph qualifies as chart-junk as the added elements have no value either to aid understanding of the graph or to connect it to the context of its presentation.

The graph in EVS book has a reference in the text, which explains the different component gases and their relevance. However, there is no mention of the presence of water vapour! Neither it is explained that these percentage values of the gases are for *dry* air. Water vapour is an important component of life and weather should have a mention in the chart.

The pie chart gives you the percentage of different constituents of air (Fig. 4.1).

The table below (Table A.24) shows data for air content by volume *ppmv*: parts per million by volume. When presenting a graph, the data used for creating the graph should be made available in the form of a table. The presence of data tables is helpful for the students learning to read and make graphs. Presenting the data in multiple representations aids the students to move between representations. Bell & Janvier (1981) argue that: " . . . the use of tables proved a powerful tool to study "how variables change". The results conclusively show that the table approach certainly spelt out many ideas to the extent of making possible transfers from tables to graphs. Consequently, results suggest that the use of tables should be included in our graph teaching scheme." (p. 41)

Gas	Percentage %
Nitrogen (N_2)	78.084
Oxygen (O_2)	20.946
Argon (Ar)	0.9340
Carbon dioxide (CO_2)	0.0397
Neon (Ne)	.001818
Helium (He)	0.000524
Methane (CH_4)	0.000179
Water vapour (H_2O)	.001 %–5 %

Table A.24: Percentage composition of dry atmosphere by different gases. Water Vapour contents is not included in above table. Data from Wikipedia page on Atmosphere of Earth https://en.wikipedia.org/wiki/Atmosphere_of_Earth

We present the same data in a Table A.24. Though the atmospheric data need not be presented to the students in Class 7 in such detail, nevertheless an effort can be made to include a table and source of the data.

After just showing the pie chart and the bar graph, a line graph is shown in the textbook (Figure A.10 here).

Graph 4

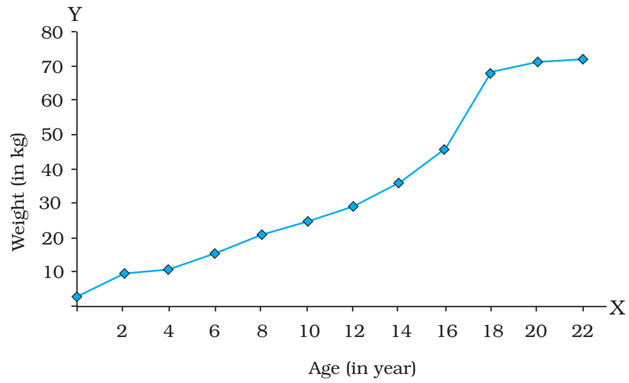


Fig. 13.10 *A line graph showing change in weight of a man with age*

Figure A.10: Graph showing weight of man as a function of age. Figure from Class 7, Chapter 13, page number 152 in science textbook.

Function
: Presented as an example of a line graph.

Reference
: This is an interesting graph which shows the weight of a man as a function of his age. The only mention in the text that this graph finds is:

> The graph shown in Fig. 13.10 is an example of a line graph.

Data The data is not provided.

Legend, Axes
: The axes are labelled and have units.

Close-to-life
: The data presented has close-to-life context in the growth of humans.

Design Aspects
: The graph presented is free from chart-junk. Grid lines (both X and Y) would have helped to improve readability.

Many interesting questions and observations are present in this graph, but as is the case with the previous two graphs, no effort is made to engage the students with the graph. Some of the interesting questions that can be asked and answered for the graph in Figure A.10 are:

§ During which two years does the weight gain is maximum?

§ During which two years the weight gain is least?

§ What is the total percentage gain in weight till 10 years, 20 years?

Two activities on the same theme, with a good scope for asking and answering questions, using real-world data are in class 8 (Figure A.22 here).

However, line graphs are not new to the students. In fact, just a few pages prior to this graph, in the same textbook, we have a line graph already shown (Figure A.1). This example, as we have shown, is another illustration of graphs appearing without any context in the text. We see this as another lost opportunity in the context of graphicacy.

We would also like bring to notice that mathematics textbooks also teach graphs in the previous classes. In mathematics, line graphs make their first appearance in Class 5. There is an activity centred around data collected from growing seeds (p. 168-169, Class 5, Mathematics). The activity also asks questions which require the students to study the graph (Figure A.11a). Though this graph is not from the science textbooks, we consider it here to display the disparity and missing links to other parts of the curriculum concerning the treatment of graphs in the science textbooks.

Graph from Class 5 Mathematics Textbook

Function
Data for the growth of plant shown in graphical representation along with questions that can be asked and answered. (Figure A.11 here.)

Reference
Activity centred around data collected from growing seeds (pg. 168-169, Class 5, Mathematics). The activity also asks questions which require the students to study the graph (Figure A.11a here). There are follow up tasks after the activity on similar lines.

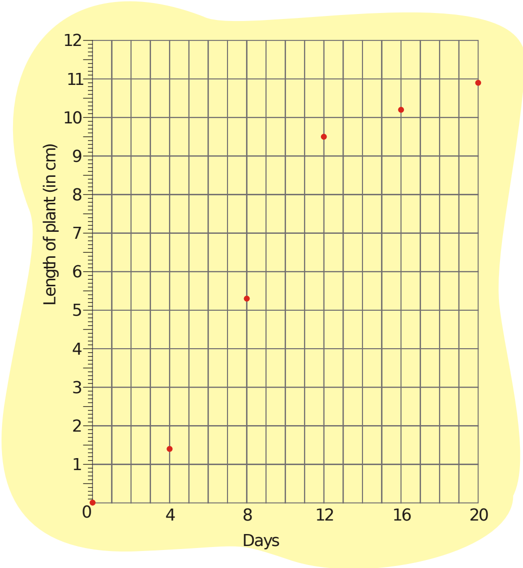
Data The data in a tabular form and the way of obtaining it is provided.

> Growth Chart of a Plant Amit sowed a few seeds of moong dal in the ground. The height of the plant grew to 1.4 cm in the first four days. After that it started growing faster.
>
> Amit measured the height of the plant after every four days and put a dot on the chart. For example, if you look at the dot marked on the fourth day, you can see on the left side scale that it is 1.4 cm high.
>
> Now look at the height of each dot in cm and check from the table if he has marked the dots correctly.

Legend, Axes
The axes are labelled and with units.

Close-to-life
Growth of plants is a close-to-life context for the students to observe and record.

Design Aspects
The graph is well made. Background colour (yellow) to the graph could be avoided.



(a)

Day	*Length of the plant (in cm)*
0	0
4	1.4
8	5.3
12	9.5
16	10.2
20	10.9

(b)

Find out from the growth chart

a) Between which days did the length of the plant change the most?

i) 0-4 ii) 4-8 iii) 8-12 iv) 12-16 v) 16-20

b) What could be the length of this plant on the 14th day? Guess.

i) 8.7 cm ii) 9.9 cm iii) 10.2 cm iv) 10.5 cm

c) Will the plant keep growing all the time? What will be its length on the 100th day? Make a guess!

There should be some discussion on the last question. Children should be encouraged to observe growth patterns of many other plants and animals.

(c)

Figure A.11: Example of a line graph based activity from Mathematics Class 5 textbook. (a) A line graph based activity in Class 5 Mathematics textbook. Figure from Class 5, Chapter 12, page number 169 in Mathematics textbook. (b) Data for the line graph activity. Observations recorded for growth of *moong dal* used to plot graph shown in Figure 3.18a. (c) Questions associated with the graph (Figure 3.18a) in the activity.

The graph above and the associated questions are good starting exercise in reading graphs. The exercise provides a connection between different representations of data, tabular and graphical. However, after this, we see that line graphs do not appear in Class 6 mathematics, except in the case of the number line. So, there is no build up from what is a good starting exercise in graphicacy.

After introducing different graph types, the textbook goes on to analyse the odometer data presented in Figure A.5. In what follows in the textbook is a step by step introduction to the plotting a line graph. The first figure shown for this is a graph paper with X and Y axes (Figure A.2.1 here).

Showing the Cartesian grid.

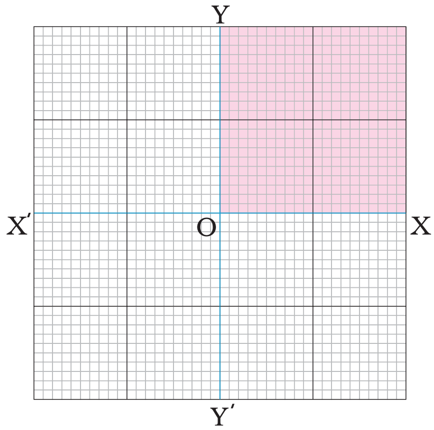


Fig. 13.11 *x-axis and y-axis on a graph paper*

Figure A.12: X and Y axes on a graph paper. Figure from Class 7, Chapter 13, page number 152 in Science textbook.

Function The figure is presented as an example of the Cartesian grid.

Reference The text elaborates on how to construct the X and Y axes on the graph paper.

> Take a sheet of graph paper. Draw two lines perpendicular to each other on it, as shown in Fig. 13.11. Mark the horizontal line as XOX'. It is known as the x-axis. Similarly mark the vertical line YOY'. It is called the y-axis. The point of intersection of XOX' and YOY' is known as the origin O. The two quantities between which the graph is drawn are shown along these two axes. We show the positive values on the x-axis along OX. Similarly, positive values on the y-axis are shown along OY. In this chapter we shall consider only the positive values of quantities. Therefore, we shall use only the shaded part of the graph shown in Fig. 13.11. (Class 7, Science textbook, pg. 152)

Data Not applicable.

Legend, Axes Only X and Y axes are shown. No legend required.

Close-to-life Cartesian grid can be introduced in a variety of close-to-life contexts like addresses in a town. But, no such effort is seen here.

Design Aspects The grid is too dense, numbers are missing on both the axes.

The next two graphs (Figure A.13) which continue after the previous three graphs, shown as different types of graphs tell the students how to construct a line graph, given the data. Here data is presented as a time-distance table as shown in Figure A.14. The textbook provides a step-by-step (pg. 152-154, Class 7 Science textbook) guide to making these graphs. There is a good correspondence between the text and the graphs.

Graph 5

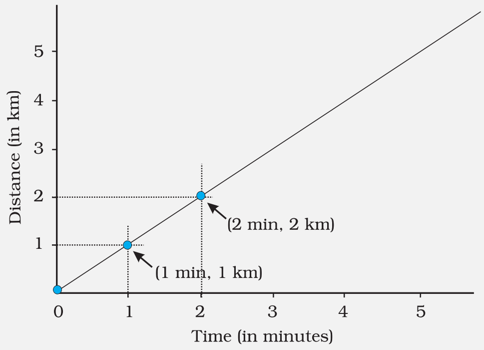


Fig 13.12 *Making a graph*

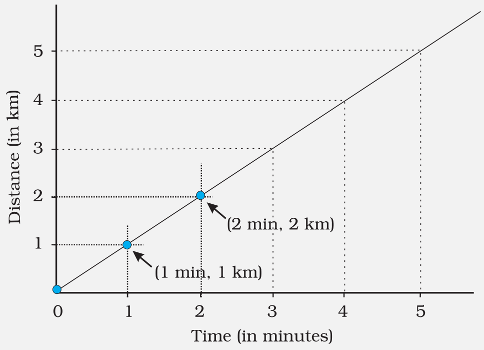


Fig 13.13 *Making a graph*

Figure A.13: Steps to explain "Making a graph". Figures from Class 7, Chapter 13, page number 153 in Science textbook. The data used in this graph is present in Figure reftime-distance-data.

Table 13.6 The motion of a car

S. No.	Time	Distance
1.	0	0
2.	1 min	1 km
3.	2 min	2 km
4.	3 min	3 km
5.	4 min	4 km
6.	5 min	5 km

Figure A.14: The motion of a car. Table from Class 7 Science textbook, pg. 152. This data is used in making the graph in Figure A.13.

Function Explaining how to plot a graph using the data in a table.

Reference There are detailed step-by-step instructions which explain the process of plotting the points on the two axes.

You can make the graph by following the steps given below:

- Draw two perpendicular lines to represent the two axes and mark them as OX and OY as in Fig. 13.11.
- Decide the quantity to be shown along the x-axis and that to be shown along the y-axis. In this case we show the time along the x-axis and the distance along the y-axis.
- Choose a scale to represent the distance and another to represent the time on the graph. For the motion of the car scales could be Time: 1 min = 1 cm Distance: 1 km = 1 cm
- Mark values for the time and the distance on the respective axes according to the scale you have chosen. For the motion of the car mark the time 1 min, 2 min, ...on the x-axis from the origin O. Similarly, mark the distance 1 km, 2 km ... on the y-axis (Fig. 13.12).
- Now you have to mark the points on the graph paper to represent each set of values for distance and time. Observation recorded at S. No. 1 in Table 13.6 shows

that at time 0 min the distance moved is also zero. The point corresponding to this set of values on the graph will therefore be the origin itself. After 1 minute, the car has moved a distance of 1 km. To mark this set of values look for the point that represents 1 minute on the x-axis. Draw a line parallel to the y-axis at this point. Then draw a line parallel to the x-axis from the point corresponding to distance 1 km on the y-axis. The point where these two lines intersect represents this set of values on the graph (Fig. 13.12). Similarly, mark on the graph paper the points corresponding to different sets of values.

- Fig. 13.13 shows the set of points on the graph corresponding to positions of the car at various times.
- Join all the points on the graph as shown in Fig. 13.13. It is a straight line. This is the distance-time graph for the motion of the car.

Data The data used for plotting is provided in the form of a table.

Legend, Axes The axes are labelled, and units are given.

Close-to-life The data is from the journey of a car, which is a close-to-life context.

Design Aspects The graphs are clean and without chart junk as such.

Once the graph is made, what is the interpretation that we can derive from the graph? For this we are told:

> If the distance-time graph is a straight line, it indicates that the object is moving with a constant speed. However, if the speed of the object keeps changing, the graph can be of *any shape.* (pg. 153, Class 7 Science Textbook, emphasis added)

The first part of the statement makes it clear to associate the line graph with a constant speed, in case of distance-time graphs. The second part of the statement is a bit problematic. What does the textbook mean by *any shape*? Is it possible that the slope of this graph is *negative*, can it be *vertical*? A better wording with an example to clarify the point would have made the remark much clear. As Tufte (2006) points in *Beautiful Evidence* this is the *First Principle for the analysis and presentation of data*

> Show comparison, contrasts, differences.

For example, consider another car, lets say **Car B**, which moves at *non-uniform* speed. The distance covered for **Car B** is recorded at the same times as the first car. Let us call it **Car A**. Let us consider some values for distance covered by **Car B** as shown in Table A.25. The graph made from this data is shown in Figure A.15.

In this, the straight line graph of **Car A** is differentiated from that of **Car B**. Conversely, if the distance-time graph of any body is a straight line then, the body is moving at a constant speed. This simple rule is helpful to determine the kind of motion that the body has, just from one look at the distance-time graph of the body. This is a good way to start graphs, but only one example and one physical situation is provided. It would be beneficial that many other physical situations be depicted using graphs. For example, in his work on motion and graphs Beichner (1994), suggests that:

Time (min)	**Car A** (distance km)	**Car B** (distance km)
0	0	0
1	1	0.75
2	2	1
3	3	2.25
4	4	2.75
5	5	3

Table A.25: The motion of two cars for comparison. Car A is moving with constant speed, while Car B is not. This difference is clearly evident in the graph made from this table, Figure A.15 below.

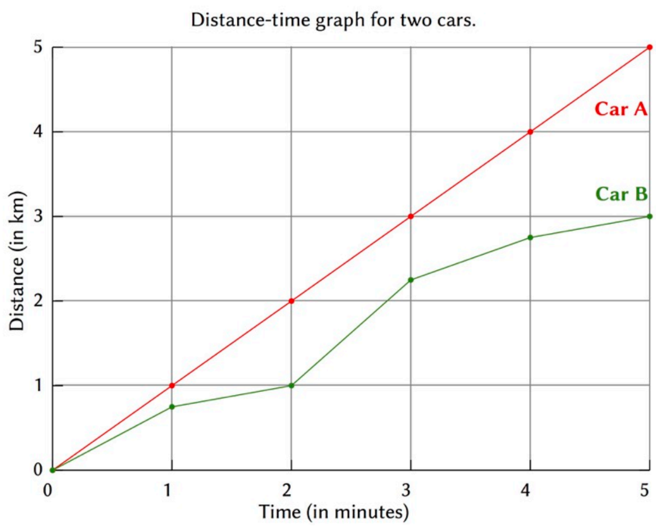


Figure A.15: The graph here represents the data shown in Table A.25. Notice that graph for **Car A** is a straight line, which means that it travels equal distances in equal amounts of time. This, in turn, means that **Car A** is travelling at a constant speed. In case of **Car B**, during each minute it is covering different amounts of distance. Hence it is not travelling at uniform speed. This is seen in the graph for **Car B**, which is not a straight line.

> Teachers should have the students examine motion events where the kinematics graphs *do not* look like photographic replicas of the motion and the graph lines *do not* go through the origin. Students should be asked to translate from motion events to kinematics graphs and back again. Instruction should also require the students to go *back and forth* between the different kinematics graphs, inferring the shape of one from another. (emphasis added, p. 755)

This would definitely help the students understand the representational nature of the graphs. The idea that abstract graphs on paper represent a physical situation or a concrete phenomenon should be reinforced through various examples. At the same time, this will also help to facilitate the idea of using multiple representations of the same data. This way one can facilitate the transition from concrete to abstract and back. Roth et al. (2005) claim that " . . . resources [associated with graphs like legends, labels etc.] assist readers in a reconstructive process that allows them to imagine 'real' situations from which these graphs have been abstracted' ' (p. 30). For example, the linear function can be used for designing activities which can lead to modelling of many situations which are close-to-life (see Chapter 4).

Continuing with the textbook, we see that the exercise of creating a graph for the Question posed on the speed of the bus from Table 13.6 in the textbook (Figure A.14 here). The making of the graph is again discussed in the textbook. For example, it distinguishes between the previous example in which scale of 1 cm was good for representing the 1 km, but the same scale cannot be used in this case.

> The total distance covered by the bus is 80 km If we decide to choose a scale 1 km = 1 cm, we shall have to draw an axis of length 80 cm. This is not possible on a sheet of paper. On the other hand, a scale 10 km = 1 cm would require an axis of length only 8 cm. This scale is quite convenient.

The choice of scales can be detrimental to the way data can be interpreted and represented. A bad choice of scale will misrepresent the data leading to erroneous observations, while a good choice of scale can make the salient features of the graph stand out. The books by Tufte (2001) and Wainer (1984) have many examples that illustrate this point. The textbook provides a few rules of thumb for choosing the scale of the graph (pg. 54. Class 7 Science textbook).

> Some of the points to be kept in mind while choosing the most suitable scale for drawing a graph are:
>
> - the difference between the highest and the lowest values of each quantity.
> - the intermediate values of each quantity, so that with the scale chosen it is convenient to mark the values on the graph, and
> - to utilise the maximum part of the paper on which the graph is to be drawn.

In addition to these rules of thumb, we feel that counter-examples illustrating the choice of bad scales, and associated problems could have been immensely beneficial. For example, see the article by Wainer (1984) titled *How to display data badly.*

Graph 6

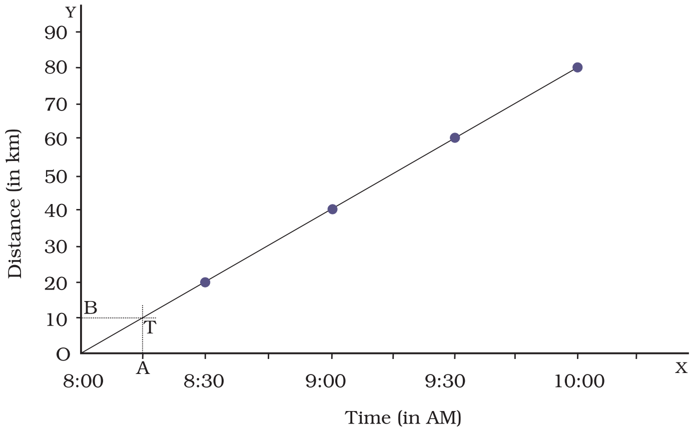


Fig. 13.14 *Distance-time graph of the bus*

Figure A.16: A distance-time graph for a bus. The data is from the table shown in Figure A.17. The Figure from Class 7, Chapter 13, page number 154 in Science textbook.

Table 13.5 Odometer reading at different times of the journey

Time (AM)	Odometer reading	Distance from the starting point
8:00 AM	36540 km	0 km
8:30 AM	36560 km	20 km
9:00 AM	36580 km	40 km
9:30 AM	36600 km	60 km
10:00 AM	36620 km	80 km

Figure A.17: Odometer reading at different times of the journey. Table from Class 7 Science textbook, pg. 151.

Function The graph draws the table of odometer values given earlier (Figure A.17 here).

Reference The graph is referred in the main text as an illustration of how graphs can give us more information than tables. The essential point being made is of *interpolation* of data, to find out required parameters not given in the table.

> Distance-time graphs provide a variety of information about the motion when compared to the data presented by a table. For example, Table 13.5 gives information about the distance moved by the bus only at some definite time intervals. On the other hand, from the distance-time graph we can find the distance moved by the bus at any instant of time. Suppose we want to know how much distance the bus had travelled at 8:15 AM. We mark the point corresponding to the time (8:15 AM) on the x-axis. Suppose this point is A. Next we draw a line perpendicular to the x-axis (or parallel to the y-axis) at point A. We then mark the point, T, on the graph at which this perpendicular line intersects it (Fig. 13.14). Next, we draw a line through the point T parallel to the x-axis. This intersects the y-axis at the point B. The distance corresponding to the point B on the y-axis, OB, gives us the distance in km covered by the bus at 8:15 AM. How much

> is this distance in km? Can you now help Paheli to find the distance moved by the bus at 9:45 AM? Can you also find the speed of the bus from its distance-time graph?

Data The data for the graph is given in Table earlier (Figure A.17 here).

Legend, Axes Both the axes are labelled, and units are given.

Close-to-life The data describes a journey of a bus. This could have been presented in narrative form.

Design Aspects The graph does not have any chart-junk. Some grid lines could have been helpful.

The textbook then poses a problem for the students to draw a similar graph (pg. 154, Class 7, Science textbook).

> Can you now draw the distance-time graph for the motion of the bus? Is the graph drawn by you similar to that shown in Fig. 13.13?

The textbook further explains the advantage of using a graph over a table. For example, it is noted that

> Table 13.5 gives information about the distance moved by the bus only at some definite time intervals. On the other hand, from the distance-time graph we can find the distance moved by the bus at any instant of time.

The question of finding the distance covered at a particular time is explained with an example. The markings of **A** and **B** that are seen in Figure A.16 relate to this discussion.

Finally, the students are asked the question which started this discussion (pg. 155, Class 7, Science Textbook):

> Can you now help Paheli to find the distance moved by the bus at 9:45 AM? Can you also find the speed of the bus from its distance-time graph?

The last question is interesting. The slope of the graph is the average speed in case of a time-distance graph. The fact that the students have already found speed using tables in the earlier part of the textbook should have been used. This would have led to approaching the concept of speed from different approaches and representations (numerical and graphical).

Graph 7

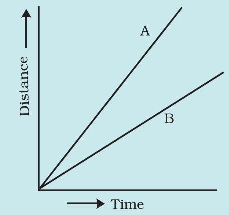


Fig. 13.15 *Distance-time graph for the motion of two cars*

Figure A.18: A distance-time graph for two cars. Figure from Class 7, Chapter 13, page number 157 in Science textbook.

Function Questions based on graphs.

Reference At the end of the chapter, we find that there are three graph based questions. The first question (Question 7 in the textbook, pg. 156) asks:

> 7. Show the shape of the distance-time graph for the motion in the following cases:
>
> (i) A car moving with a constant speed.
>
> (ii) A car parked on a side road.

Data The source of data or how the data was obtained are not provided.

Legend, Axes The axes are labelled, no units are given.

Close-to-life

Design Aspects The graph does not have chart-junk.

This question itself is excellent. It can check whether the students have understood the meaning of motion in terms of the graphical feature: the slope. The next set of questions involve interpretation of graphs (Figures A.2.1 and A.2.1 here) for answering questions about the motion that they represent. The first question (question 12 in the textbook pg. 157) reads thus:

> **12.** Fig. 13.15 shows the distance-time graph for the motion of two vehicles A and B. Which one of them is moving faster?'

The question can be answered correctly by the students, by giving the *wrong reasons*. The students might choose the answer to be A, because A is at a higher position, hence it is also faster. This is the most common error in reading graphs, the so-called *slope-height confusion* as seen in Chapter 3. Even though a correct response is chosen, reasoning may be wrong. A vehicle which is moving faster will have a greater slope, a line which is steeper on the distance-time graph. So it is important that the reasons for the answer be also asked.

Graph 8

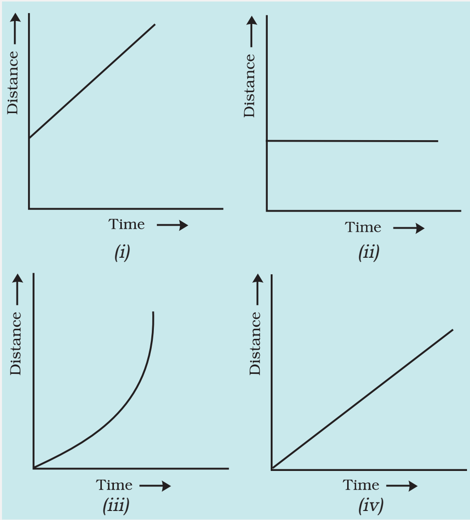


Figure A.19: Distance time graphs for a problem. Figure from Class 7, Science textbook Chapter 13, page number 157-158.

Function In the next question, there are four graphs (Figure A.2.1 here) to be studied for answering the question.

Reference The question reads:

13. Which of the following distance-time graphs shows a truck moving with speed which is not constant?

Data The source of data or how the data was obtained are not provided.

Legend, Axes The axes are labelled, no units are given.

Close-to-life The question is not set in any real-world context.

Design Aspects The graph does not have chart-junk.

The question has only one curve in the four which is non-linear (option (iii)). This question does not have good distractors. Just by looking at the options, one may choose to see the option (iii) as the correct alternative.

A.2.2 Class 8

In case of Class 8 Science textbook, we consider the graphs which appear in varied contexts.

Graph 9

9. The following Table shows the total power shortage in India from 1991–1997. Show the data in the form of a graph. Plot shortage percentage for the years on the Y-axis and the year on the X-axis.

S. No.	Year	Shortage (%)
1	1991	7.9
2	1992	7.8
3	1993	8.3
4	1994	7.4
5	1995	7.1
6	1996	9.2
7	1997	11.5

Figure A.20: Table showing total power shortage in India in different years. Figure from Class 8, Chapter 5, page number 63 in Science textbook.

Function The next exercise that we see is from Class 8, Science textbook. The exercise appears in the Chapter on *Coal and Petroleum* in which these resources are discussed. It seems that the exercise is intended for plotting the data.

Reference Data in the form of a table to be used for plotting graph, but there is no discussion on the data.

Data The source of this data is not provided.

Legend, Axes Not applicable.

Close-to-life Power crisis is experienced by many on a regular basis. This data can be contextualised very well within the setting of the learners.

Design Aspects Not applicable.

When one plots the data from the problem the resulting graph is as seen in Figure A.21.

Apart from plotting the data, the idea behind this exercise is difficult to understand. However, if the intended aim is to make the students aware of the power crises, then some investigative questions need to be asked. Many leading and exploratory questions could have been asked with this data. For example, What was the reason for the sudden decrease in 1994-95? What could be the reason for the surge in the power shortage in the post-1995 years? Is this in anyway connected the material in the chapter? What is the power situation now (post-1997)? However, the textbook makes no use of any of these and instead remains at the first level of graph usage as described by Bertin (2011).

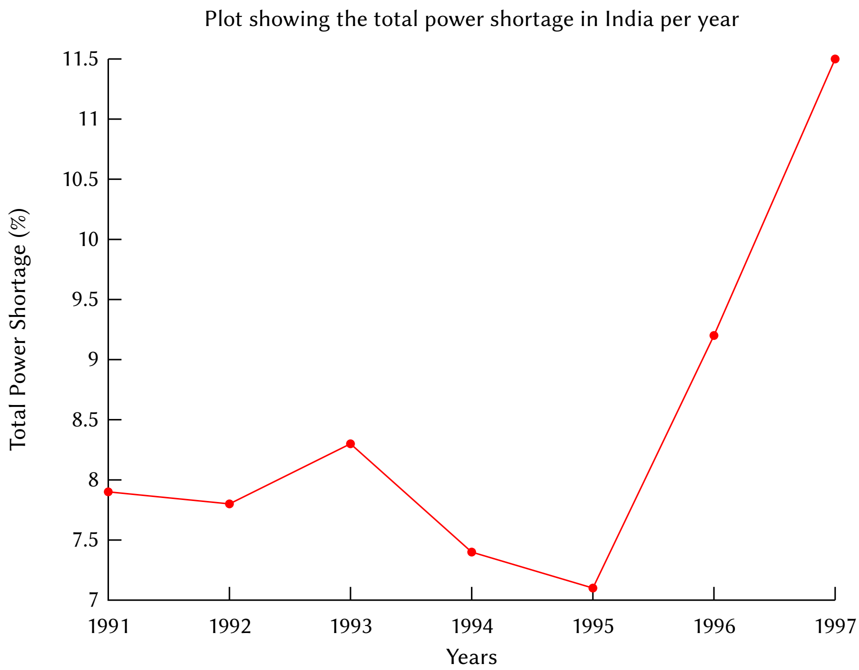


Figure A.21: Graph showing total power shortage in India in different years. Data from Class 8, Chapter 5, page number 63 in Science textbook (Figure A.20) here.

Graph 10

Activity 10.2

Use the data given in Activity 10.1 to draw a graph. Take age on the X-axis and per cent growth in height on the Y-axis. Highlight the point representing your age on the graph. Find out the percentage of height you have already reached. Calculate the height you might eventually reach. Tally your graph with the one given here (Fig. 10.1).

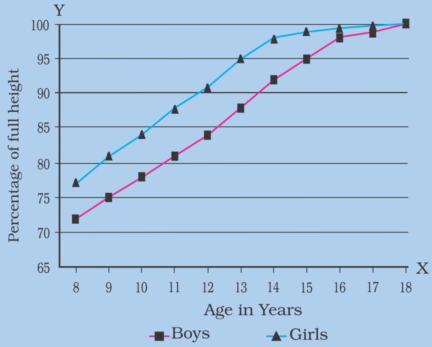


Fig. 10.1 : Graph showing percentage of height with age

Age in Years	% of full height	
	Boys	Girls
8	72%	77%
9	75%	81%
10	78%	84%
11	81%	88%
12	84%	91%
13	88%	95%
14	92%	98%
15	95%	99%
16	98%	99.5%
17	99%	100%
18	100%	100%

Figure A.22: A graph showing percentage height with age. Graph showing percentage height with age. Figure from Class 8 Science textbook, Chapter 10, page number 115. The data for this graph is in the table shown in the table on the right.

Function The activity asks the students to plot the data given in an earlier activity. The activity also asks some questions to be answered from the graph drawn by the students.

Reference The graph is provided so that the students can check if they have drawn the graph correctly.

Data The data for the graph is provided in an activity earlier.

Legend, Axes Axes are labelled and units are provided. The legends for points showing boys and girls are marked.

Close-to-life The activity asks the students to plot their own height-age on the graph, to see where they are placed. This also asks few interpolatory and exploratory questions to be answered. The graph relates to differential growth in males and females which is close-to-life for the age group in Class 8, as the students are themselves undergoing these changes.

Design Aspects The legends could have been placed next to the line graphs.

We think it is a good activity as it allows individual learners to find answers for their own data. The questions asked requires the learners to use their own age as a starting point. The questions span all three levels of graph use described by Bertin. For example,

Level 1 Point your own age on the graph,

Level 2 Find out the % height you have achieved, and,

Level 3 Calculate the height you might eventually reach.

This can be a good group activity when the students exchange their data with others in the class. Work done on a similar theme is reported by Ainley (1995), with use of computers with first graders. The results of the study indicated that the children were able to respond intuitively to the idea of growth using graphs and were successful in plotting, reading values from the graph and interpolating them.

Graph 11

10. The table below shows the data on likely heights of boys and girls as they grow in age. Draw graphs showing height and age for both boys and girls on the same graph paper. What conclusions can be drawn from these graphs?

Age (Years)	Height (cm)	
	Boys	Girls
0	53	53
4	96	92
8	114	110
12	129	133
16	150	150
20	173	165

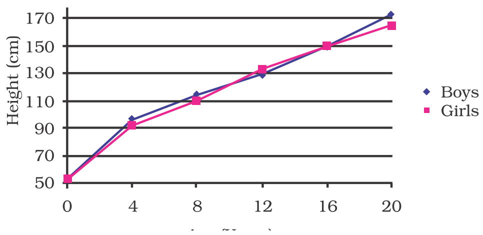


Figure A.23: A graph showing weight of man as a function of age. Figure from Science Class 8, Chapter 10, pg. 126.

Function The graph occurs in the Exercises of Chapter 8 (Figure A.22). There is a table in the question which shows the comparative height of boys and girls at different ages.

Reference The data from the table is plotted on a graph. The students have to draw the graph from the table.

Data Data is given in the form of a table. How the data was obtained is not provided.

Legend, Axes Axes are labelled, and units are provided.

Close-to-life This is close-to-life.

Design Aspects The graph does not have chart-junk. Using light grey lines for the grid is recommended.

This is a good activity as it allows the learners to look beyond the given data and infer patterns from the data.

A.2.3 Class 9

The Science textbook of Class 9 (NCERT, 2007c) has the maximum number of graphs in the sample of science textbooks in this analysis. Most of the graphs appear in Chapter 8 on Motion. The first graph that appears in this chapter is (Figure A.24 here) a number line used to represent distance travelled. Points on the line are used to represent the landmarks in the journey and help make the students distinguish between the concepts of distance covered and displacement.

Graph 12

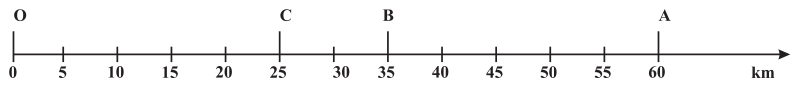


Fig. 8.1: Positions of an object on a straight line path

Figure A.24: Describing motion using a line graph. Figure from Class 9, Chapter 8, page number 99 in Science textbook.

Function The graph is used to explain motion along a straight line. The textbook uses an example of a journey starting from the origin *O* to distinguish between the concepts of distance covered and displacement. Some reference to the number lines they studied in the earlier mathematics class should be done.

Reference There are a couple of references that are found in the textbook for this graph.

> The object starts its journey from O which is treated as its reference point (Fig. 8.1). Let A, B and C represent the position of the object at different instants. At first, the object moves through C and B and reaches A. Then it moves back along the same path and reaches C through B. (pg. 99)

After this, the journey is analysed in terms of distance covered, and a definition of displacement is given. Then the students are asked this question:

> Can the magnitude of the displacement be equal to the distance travelled by an object? Consider the example given in (Fig. 8.1). (pg. 99)

Integration The graph is well integrated into the overall narrative of the text. The narrative takes care to refer to the graph to understand the meaning in terms of arithmetic operations and the physical meaning of the symbols.

Data The data is not given, separately and is not in the context of any narrative. Data is provided as a part of the text and the graph. Could have been better done if the data was provided in a table and then plotted along the axes as required.

Legend, Axes The *X-axis* is marked with the unit of kilometre.

Close-to-life The example is an abstract one, in which there is no close to life context. A better example would have been a context to which the students can relate. For example, a journey which starts from a town and covers some towns and back to the starting town can form a good narrative which the students can relate.

Design Aspects The graph does not have any chart junk in the current form.

Graph 13

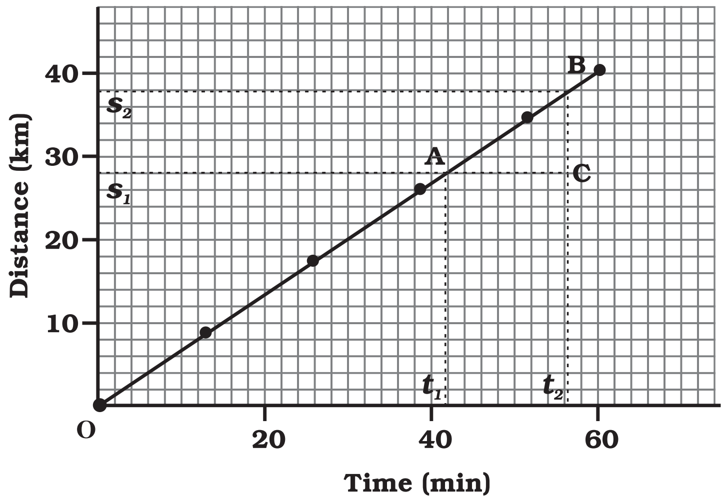


Fig. 8.3: Distance-time graph of an object moving with uniform speed

Figure A.25: A distance-time graph of an object moving with uniform speed. Figure from Class 9, Chapter 8, page number 104 in Science textbook.

Function The graph is used to represent motion in a straight line and to describe a method to find the speed from the distance-time graph.

Reference The graph is woven into the narrative of the text.

> Thus, for uniform speed, a graph of distance travelled against time is a straight line, as shown in Fig. 8.3. The portion OB of the graph shows that the distance is increasing at a uniform rate.

Also the next paragraph shows how to find the speed from the graph (p. 104)

> We can use the distance-time graph to determine the speed of an object. To do so, consider a small part AB of the distance-time graph shown in Fig 8.3. Draw a line parallel to the x-axis from point A and another line parallel to the y-axis from point B. These two lines meet each other at point C to form a triangle ABC. Now, on the graph, AC denotes the time interval $(t_2 - t_1)$ while BC corresponds to the distance $(s_2 - s_1)$. We can see from the graph that as the object moves from the point A to B, it covers a distance $(s_2 - s_1)$ in time $(t_2 - t_1)$. The speed, v of the object, therefore can be represented as
>
> $$v = \frac{(s_2 - s_1)}{(t_2 - t_1)} \qquad (8.4)$$

This particular part of the text is very well integrated with the graph. Some numerical examples with a context would have been helpful.

Data The data for the graph is not provided in the form of a table, neither it is presented in any context.

Legend, Axes Both the axes are labelled, and units are given. The $X - axis$ has the unit of Time in minutes, while the $Y - axis$ has the unit of Distance in kilometres. Legends are not given for the graph.

Close-to-life The example is very abstract and does not relate to the everyday experience of the learners. A better context which the learners can relate to from their everyday experiences could have been useful.

Design Aspects The graph could have been made less cluttered with sparser grids for both the axes. The grid lines should be lighter in colour so that it does not interfere with the perception of the graph. For example, see Figure A.26 below.

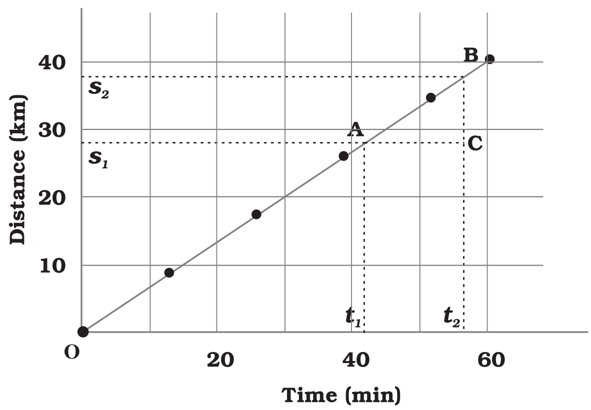


Fig. 8.3: Distance-time graph of an object moving with uniform speed

Figure A.26: A distance-time graph of an object moving with uniform speed. Redrawn figure from Class 9, Chapter 8, page number 104 in Science textbook.

Graph 14

Table 8.2: Distance travelled by a car at regular time intervals

Time in seconds	Distance in metres
0	0
2	1
4	4
6	9
8	16
10	25
12	36

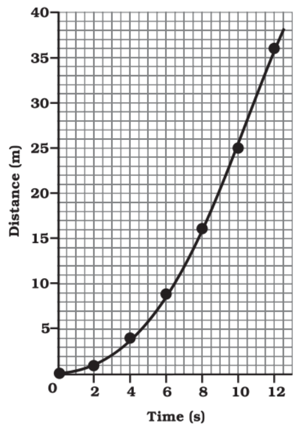


Fig. 8.4: *Distance-time graph for a car moving with non-uniform speed*

Figure A.27: A distance-time graph showing non-uniform speed. Figure from Class 9, Chapter 8, page number 105 in Science textbook. The data used for plotting the graph is shown in the table.

Function The graph shows the non-linear character of the Distance-Time graph in case of a non-uniform speed of an object.

Reference The graph is referred to in the main text and is compared to the previous graph representing motion with uniform-speed Figure A.25 here. This is a good comparison as the nature of the two motions is cleared. The textbook reads

> The distance-time graph for the motion of the car is shown in Fig. 8.4. Note that the shape of this graph is different from the earlier distance-time graph (Fig. 8.3) for uniform motion. The nature of this graph shows a non-linear variation of the distance travelled by car with time. Thus, the graph shown in Fig 8.4 represents motion with non-uniform speed.

The graph that comes out is a parabola, which can also be seen by looking at the numbers in the data table. Each of the velocity given is a square and is related to the seconds by the relation $v = 2 \times t^2$. It would have been desirable that the students are reminded of the fact that the relationship that they see the data table and the resulting graph is an example of the quadratic relationship that they already know from the mathematics textbooks. However, no such attempt is made to make this bond between the two subjects and the idea that it is the same mathematical entity that is being presented remains hidden. Perhaps presenting the students with the above equation will also provide a context for comparing the equation with the equation for the uniform motion.

Another idea that can be well connected in case of the uniform motion is that the distance covered is directly and linearly proportional to the time taken to cover that distance. In case of non-uniform motion, this direct proportion takes a non-linear form. Representing the motion algebraically will give the students one more way of looking at it.

Adding linkages to the mathematics concept which the students are already aware of will greatly help in making sense of both the concepts in a better way.

Data The data is presented in the form of a table, shown in Figure A.25.

Legend, Axes The axes are correctly labelled with units.

Close-to-life The non-uniform motion is more common than uniform motion in real life.

Design Aspects The graph could be made better with the removal of the grid. The redrawn figure is shown in Figure A.28.

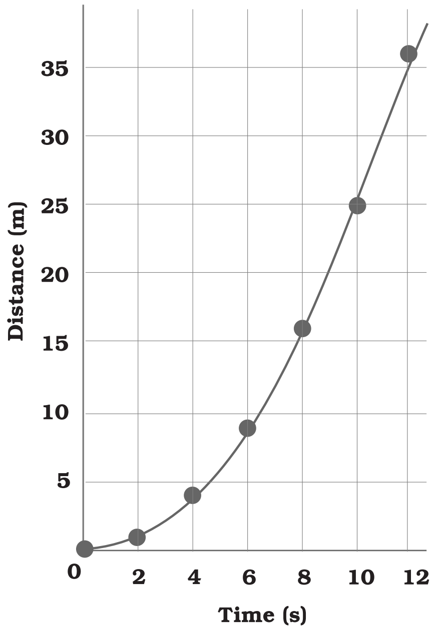


Fig. 8.4: Distance-time graph for a car moving with non-uniform speed

Figure A.28: A distance-time graph of an object moving with an uniform speed. Redrawn figure from Class 9, Chapter 8, page number 104 in Science textbook.

Since the point of the last two figures was to compare uniform and non-uniform motion, the two graphs should be presented together. This would lead to a direct comparison of the two kinds of motion that the students have to compare. Such a combined graph which has both is shown in Figure A.15. This graph enables direct comparison of the two cases of motion. This design recommendation also follows from studies in perceptual research pertaining to graphs (Cleveland & McGill, 1984). Position along the common scale is the easiest perceptual feature in the reading of graphs.

In the next two examples, we look at Velocity-Time graphs. In these graphs, a link is made to the interpretation of the area under the graph for distance covered. These graphs include uniform as well as non-uniform speeds.

Graph 15

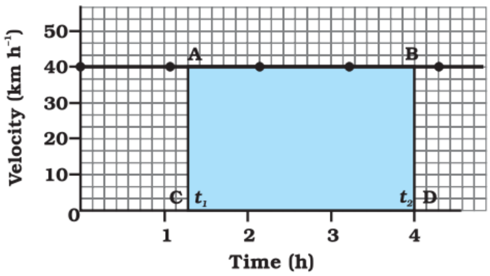


Fig. 8.5: *Velocity-time graph for uniform motion of a car*

Figure A.29: A distance time graph showing uniform speed. Figure from Class 9, Chapter 8, page number 105 in Science textbook.

Function The graph depicts the motion of a car which is travelling at 40 km/h. The coloured region in the graph is the region for which we want to find the distance covered.

Reference The graph is well integrated with the text. The entire section 8.4.2 talks about this graph.

> If the object moves at uniform velocity, the height of its velocity-time graph will not change with time (Fig. 8.5). It will be a straight line parallel to the x-axis. Fig. 8.5 shows the velocity-time graph for a car moving with uniform velocity of 40 km h^{-1}.
>
> ...
>
> To know the distance moved by the car between time t_1 and t_2 using Fig. 8.5, draw perpendiculars from the points corresponding to the time t_1 and t_2 on the graph. The velocity of 40 km h^{-1} is represented by the height AC or BD and the time $(t_2 - t_1)$ is represented by the length AB. So, the distance s moved by the car in time $(t_2 - t_1)$ be expressed as
>
> $$\begin{aligned} s &= AC \times CD \\ &= [(40 \text{ km h}^{-1}) \times (t_2 - t_1) \text{ h}] \\ &= 40\,(t_2 - t_1) \text{ km} \\ &= \text{area of the rectangle ABDC (shaded in Fig. 8.5).} \end{aligned}$$

The physical interpretation of the region below the curve, in this case the distance covered, is made clear.

Data The data is in form of a statement in the text. There is no context for the presentation.

Legend, Axes The graph is labelled, and the units are present on the axes correctly.

Close-to-life ??

Design Aspects The grids in the background could be removed, such a redrawn graph is shown below in Figure A.30.

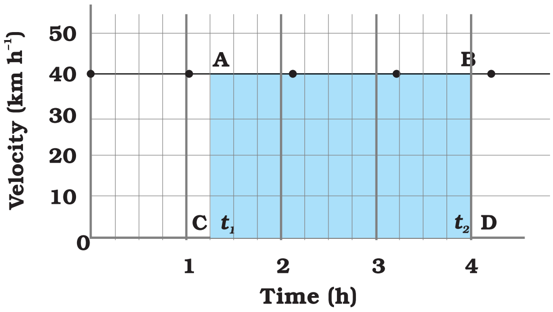


Fig. 8.5: Velocity-time graph for a car moving with uniform velocity of 40 km h^{-1}. The coloured portion represents the distance covered by the car between times t_1 and t_2.

Figure A.30: Figure redrawn from Class 9, Chapter 8, page number 105 in Science textbook.

The next graph is very similar to the previous one. In this case the data is provided in form of a table (table in Figure A.31 here).

Graph 16

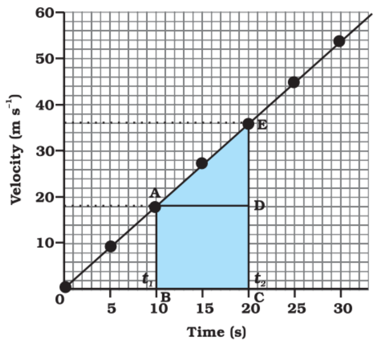


Fig. 8.6: *Velocity-time graph for a car moving with uniform accelerations.*

Table 8.3: Velocity of a car at regular instants of time

Time (s)	Velocity of the car ($m\ s^{-1}$)	($km\ h^{-1}$)
0	0	0
5	9	2.5
10	18	5.0
15	27	7.5
20	36	10.0
25	45	12.5
30	54	15.0

Figure A.31: Velocity-time graphs from text showing uniformly accelerated motion. The table shows the data used for the graph showing uniformly accelerated motion. Figure from Class 9, Chapter 8, page number 106 in Science textbook.

Function This graph and the next two graphs (graphs in Figure A.33 here) serve as an extension of the previous graph in which uniform speed is considered. In this graph, non-uniform speed is considered. Rather than focusing on the first graph in detail, analysis of a graph which has both positive and negative acceleration (for example second graph in Figure A.33) could have been more fruitful. A good discussion on these lines is present in the first section of *Questions and Answers in School Physics* by Tarasov and Tarasova (Tarasov & Tarasova, 1973).

Reference The graph is well integrated with the text. The narrative refers to the graph to press home two points: motion with uniform acceleration has a straight line velocity-time graph and the area under the curve of this graph is the distance covered.

> In this case, the velocity-time graph for the motion of the car is shown in Fig. 8.6. The nature of the graph shows that velocity changes by equal amounts in equal intervals of time. Thus, for all uniformly accelerated motion, the velocity-time graph is a straight line.
>
> ...
>
> If the car would have been moving with uniform velocity, the distance travelled by it would be represented by the area ABCD under the graph (Fig. 8.6). Since the magnitude

of the velocity of the car is changing due to acceleration, the distance s travelled by car will be given by the area ABCDE under the velocity-time graph (Fig. 8.6). That is,

$$\begin{aligned} s &= \text{area ABCDE} \\ &= \text{area of the rectangle ABCD + area of the triangle ADE} \\ &= AB \times BC + \frac{1}{2}(AD \times DE) \end{aligned}$$

In the case of non-uniformly accelerated motion, velocity-time graphs can have any shape.

Fig. 8.7(a) shows a velocity-time graph that represents the motion of an object whose velocity is decreasing with time while Fig. 8.7 (b) shows the velocity-time graph representing the non-uniform variation of the velocity of the object with time. Try to interpret these graphs.

Data The data for the uniformly accelerated motion is given in the form of a table (Figure A.31 here.) The values in the table do not seem realistic, (A speed of 2.5 km/h for a car?). A better-contextualised example should have been provided. For the other two graphs (Figure A.33) no data is provided. It would have been better if at least for one of them, only the data provided, and the students were to *plot* the data and also *analyse* the graph.

Legend, Axes All three graphs have units on both the axes.

Close-to-life Since non-uniform motion is much more common in real life, and some effort should have been put to make connections between the physical experience of motion with acceleration and the graphs presented here. Some very simple examples could have been thought and contextualised in this regard.

Design Aspects As with the other graphs in this section the background grid is too dense. Modified versions of the same graphs are shown in Figure A.32.

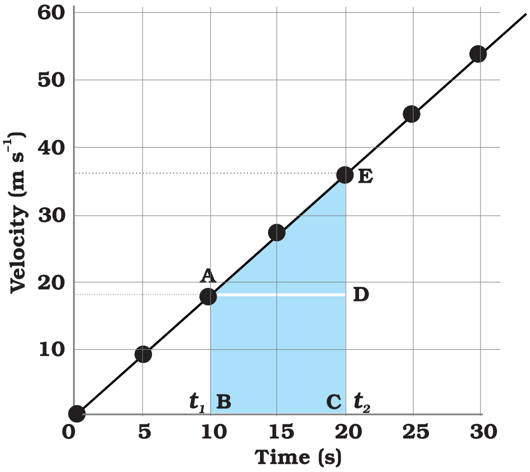


Fig. 8.6: Velocity-time graph for a car moving with uniform accelerations.

Figure A.32: Redrawn: Velocity time graphs from text showing non-uniformly accelerated motion. Figure from Class 9, Chapter 8, page number 106 in Science textbook.

Graph 17

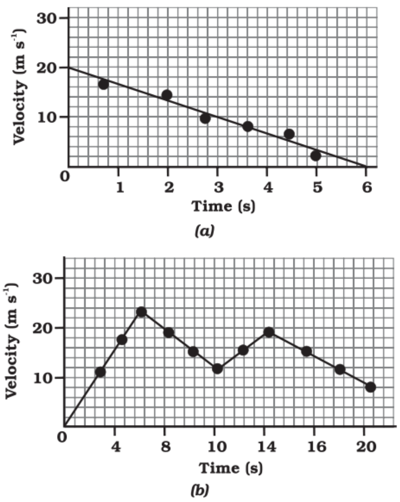


Fig. 8.7: *Velocity-time graphs of an object in non-uniformly accelerated motion.*

Figure A.33: Velocity time graphs from text showing non-uniformly accelerated motion. Figure from Class 9, Chapter 8, page number 106 in Science textbook.

Function The graphs are presented as examples of non-uniformly accelerated motion.

Reference Both the graphs are referred in the main text.

> Fig. 8.7(a) shows a velocity-time graph that represents the motion of an object whose velocity is decreasing with time while Fig. 8.7 (b) shows the velocity-time graph representing the non-uniform variation of the velocity of the object with time. Try to interpret these graphs.

Data The data for the graphs is not provided.

Legend, Axes Both the graphs have units on both the axes.

Close-to-life The data for the examples could have been contextualised to experiences of non-uniform motion in real life. Currently, the examples are not close-to-life. For example, see the monograph by Gerhart & Nussbaum (1966).

Design Aspects The background grid is too dense. It can be reduced for a cleaner graph. A redrawn version is shown in Figure A.34.

Though the caption of the graphs says that they represent graphs in "non-uniformly accelerated motion", in the first graph this is not the case. The first graph is a graph of uniformly decelerating motion. This is clearly a case of an **incorrect** graph being presented. Comparing this graph to the one of uniform acceleration (Figure A.31 here), the students might get a misconception that uniform acceleration can result only from *increasing* speeds and not from *decreasing* speeds.

The text asks the students to "interpret the graphs", some pointed questions, which can be answered by analysing the graph, should have been added. For example, how can motion depicted in the second graph be described?

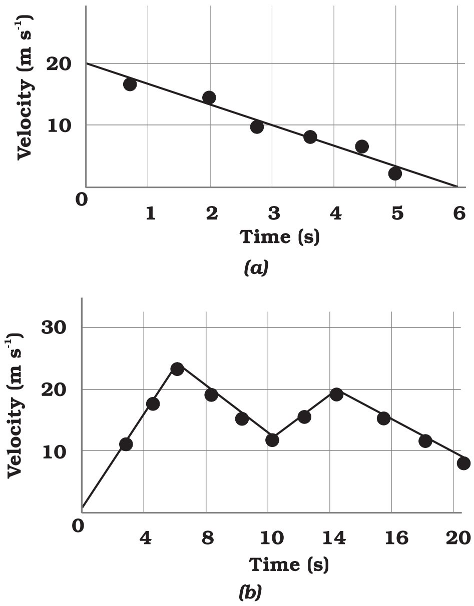


Fig. 8.7: Velocity-time graphs of an object in non-uniformly accelerated motion.

Figure A.34: Redrawn: Velocity time graphs from text showing non-uniformly accelerated motion. Figure from Class 9, Chapter 8, page number 106 in Science textbook.

Graph 18

Activity 8.10

- Feroz and his sister Sania go to school on their bicycles. Both of them start at the same time from their home but take different times to reach the school although they follow the same route. Table 8.5 shows the distance travelled by them in different times

Table 8.5: Distance covered by Feroz and Sania at different times on their bicycles

Time	Distance travelled by Feroz (km)	Distance travelled by Sania (km)
8:00 am	0	0
8:05 am	1.0	0.8
8:10 am	1.9	1.6
8:15 am	2.8	2.3
8:20 am	3.6	3.0
8:25 am	–	3.6

- Plot the distance-time graph for their motions on the same scale and interpret.

Activity 8.9

- The times of arrival and departure of a train at three stations A, B and C and the distance of stations B and C from station A are given in table 8.4.

Table 8.4: Distances of stations B and C from A and times of arrival and departure of the train

Station	Distance from A (km)	Time of arrival (hours)	Time of departure (hours)
A	0	08:00	08:15
B	120	11:15	11:30
C	180	13:00	13:15

- Plot and interpret the distance-time graph for the train assuming that its motion between any two stations is uniform.

Figure A.35: Data for Activity 8.9 (p. 106) and 8.10 (p. 107). Figure from Chapter 8, in Class 9 Science textbook.

Function The activity is for plotting and interpreting distance-time graph from the data given. There are two Activities, namely Activity 8.9 (Figure A.35) and Activity 8.10. In these two activities data is presented to the students in the form of tables. Real-world context is also provided. In case of the first activity, the data is that of a train timetable. The exercise for the students is to "plot and interpret" the distance-time graph. Instead of leaving it to the students to interpret the graph, some specific questions that can be answered from such a graph should have been present.

Reference The construction of graph is set within an activity as described above.

Data **Activity 8.9:** Data is given in the table in the form of distance and time of arrival and departure of trains, no source for data is given. **Activity 8.9:** Data is given in the table in the form of distance travelled and time taken by two people. Students have to process the time data in the table to construct the graph. As a variation, the students could have been asked to time their own travel (in bicycle or train) (of known distance) moreover, do a subsequent analysis. In both the activities, the students have to process the data in the table to construct the graph.

Legend, Axes Not applicable. Students have to choose the scale of the axes.

Close-to-life Both the context, **Activity 8.9:** train travel and **Activity 8.10:** travelling by bicycle are close-to-life context. Both of them could have been contextualised in the form of a narrative.

Design Aspects The tables can be redesigned with less use of colours and colouring of alternate rows for better readability.

The graph that we consider next is used to derive the three equations of motion, namely:

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$2as = v^2 - u^2$$

This appears in the section *8.5.1 Equation for Velocity-Time Relation* p.107.

Graph 19

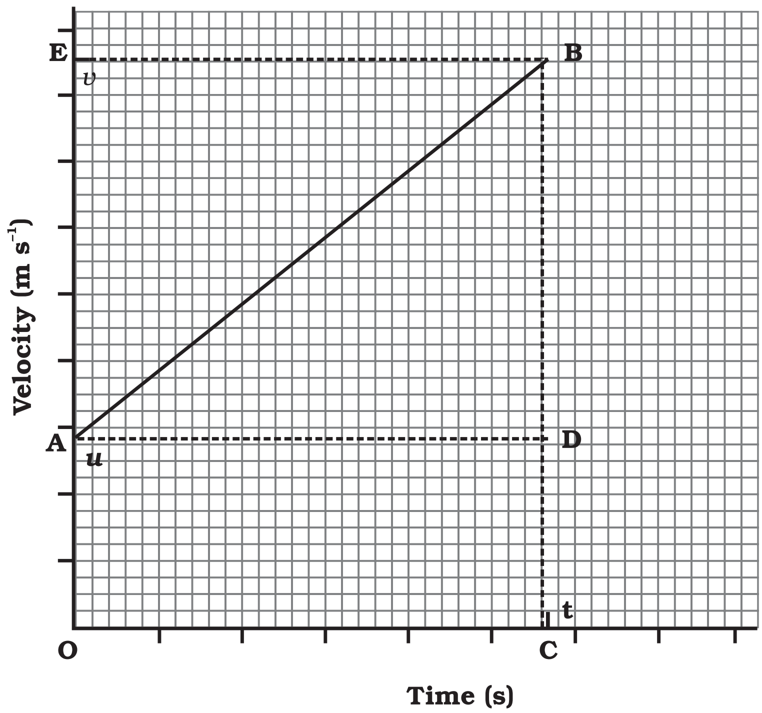


Fig. 8.8: Velocity-time graph to obtain the equations of motion

Figure A.36: A velocity time graph from the textbook. Figure from Class 9, Chapter 8, page number 107 in Science textbook.

Function The graph of a uniformly accelerated motion is used to derive the equations of motion. Using geometry from the graph, algebraic relations are derived. Additional constructions like perpendicular lines are used for the derivations.

Reference The graph is well integrated with the text and referred to while deriving each of the equations of motion. The points and segment lengths in the graph are used in the derivation. For example,

- Points A and E as initial and final velocities
- Length BC is the time interval under consideration
- Length BD is the change in velocity

- Area OABC is the distance travelled

Also, there is a reference to the previous graph which shows uniformly accelerated motion but without an initial velocity. The addition of initial velocity, in this case, is essentially moving the graph up along the Y-axis. Perhaps this should have also been done with the distance-time graphs as a generic principle.

Data The data for the graph is not given.

Legend, Axes Both the axes are labelled, and units are given. The constructed extra points have mention in the text. Perhaps a graph showing stepwise constructions of the points and line segments might have been useful.

Close-to-life The entire exercise does not try to provide any context for the derivation.

Design Aspects As in the previous cases the dense grid in the background obscures the dotted lines which are the constructions on the graph. A redrawn version of the graph is shown below in Figure A.37.

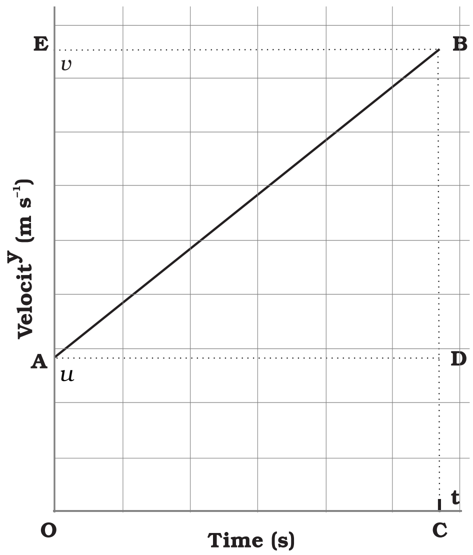


Fig. 8.8: Velocity-time graph to obtain the equations of motion

Figure A.37: Redrawn figure from Class 9, Chapter 8, page number 107 in Science textbook.

The next two graphs are in the problems which are posed at the end of the chapter. The first of the graphs (Figure A.38 here) has

Graph 20

6. Fig 8.11 shows the distance-time graph of three objects A,B and C. Study the graph and answer the following questions:

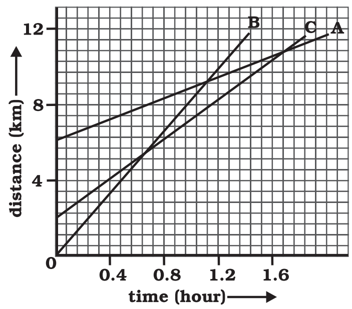


Fig. 8.11

(a) Which of the three is travelling the fastest?
(b) Are all three ever at the same point on the road?
(c) How far has C travelled when B passes A?
(d) How far has B travelled by the time it passes C?

Figure A.38: A graphical problem on motion with distance-time graph. Figure from Class 9, Chapter 8, page number 112 in Science textbook.

Function The graph serves as a context for a problem. The questions that are asked to probe the students' understanding of the physical interpretation of the graphs. For example, to answer the first question, namely which of the three is travelling the fastest, the student will have to know the physical interpretation of slope of the line as speed in case of a distance-time graph.

Reference the problem is referred to in the main text. A set of 4 questions are asked on the basis of this graph.

Data The data for the graph is not provided.

Legend, Axes Both the axes are labelled, and units are given.

Close-to-life There is no context in which this problem is set. It would have been better if the three objects were set in a context which the students could relate to.

Design Aspects The dense grid in the background could be made fainter to emphasize the lines A, B and C.

Graph 21

8. The speed-time graph for a car is shown is Fig. 8.12.

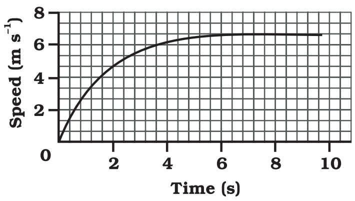


Fig. 8.12

(a) Find how far does the car travel in the first 4 seconds. Shade the area on the graph that represents the distance travelled by the car during the period.
(b) Which part of the graph represents uniform motion of the car?

Figure A.39: A graph on motion from a textbook problem in Class 9, Science textbook. Figure from Class 9, Chapter 8, page number 113 in Science textbook.

Function This graph serves as a context for a problem at the end of Chapter 8. The students need to know the meaning of the area under the graph as distance covered in case of the speed-time graphs to answer the questions. The questions asked in the graph are interpretative in nature. To answer these questions, the students need to understand the physical meaning of the features of the speed-time graph. The first question asks the students to find the distance from the graph and then to show this distance using shading a region in the graph.

The unit of area in a speed (m/s) - time (s) graph is distance. Thus the area under the curve represents the distance covered. This point covered in a previous example (Figure A.31 here) in the text. The example makes it amply clear that the area in the graph represents distance covered.

The second part of the question asks which part of the graph represents the uniform motion of the car. This part again is interpretative. In this case, the nature of the graph is important. This aspect is also covered in the text on page 105 (Science, Class 9), in the same chapter. It says,

> If the object moves at uniform velocity, the height of its velocity-time graph will not change with time (Fig. 8.5 (Figure A.29 here)). It will be a straight line parallel to the x-axis.

These type of questions which deal with the nature of the graphs and the physical meaning of the various features on the graphs are helpful in understanding the phenomena the graph represents. The example could have been made close-to-life by using a context to set the problem.

Reference There are two questions asked on the basis of the graph.

> The speed-time graph for a car is shown is Fig. 8.12 (Figure A.39 here).

(a) Find how far does the car travel in the first 4 seconds. Shade the area on the graph that represents the distance travelled by car during the period.

(b) Which part of the graph represents the uniform motion of the car?

Data The data or how it was obtained is not provided.

Legend, Axes Both the axes are labelled and are with units.

Close-to-life The problem could have been made a close-to-life one with some context about the travelling of the car.

Design Aspects The denseness of the grid can be removed, with some actual points on the graph.

Graph 22

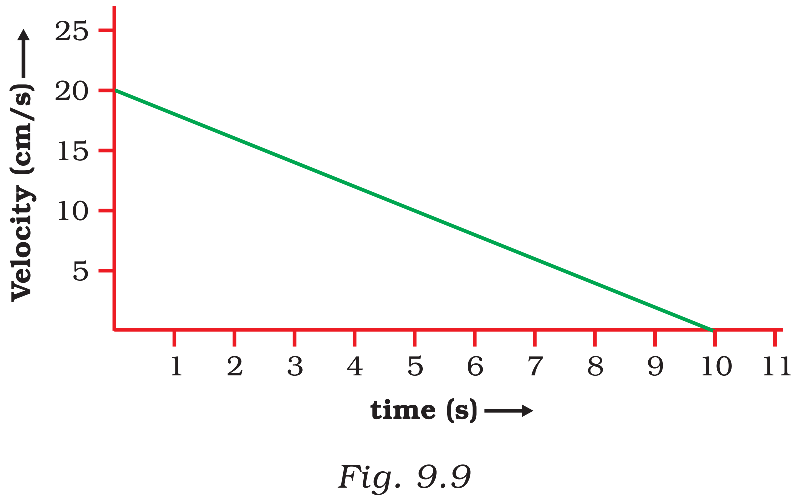


Figure A.40: .
Figure from Class 9, Chapter 9, page number 121 in Science textbook.

Function The graph is a part of a problem in which the students have to read values from the graph to solve the problem. Figure A.40 is a problem from the Chapter 9, Class 9 Science textbook, Figure 9.9 in the textbook. In this problem, the students are presented with a Time vs Velocity graph for a ball moving along a straight line on a table and are supposed to find out the force exerted on the ball by the table.

> **Example 9.5** The velocity-time graph of a ball of mass 20 g moving along a straight line on a long table is given in Fig. 9.9 (Figure A.40 here).

How much force does the table exert on the ball to bring it to rest?"

Reference The graph is referred to in the main text. A physical interpretation of negative sign in the answer is given.

Data The source of the data and how it was obtained are not given.

Legend, Axes The axes are labelled, and units are given.

Close-to-life The ideas of acceleration and deceleration are experienced every day. The problem could have been set in a daily life situation like moving of a ball, and by providing the different measurements to make the graph.

Design Aspects There is no chart-junk, the graph can be made more readable by adding grid lines. Grid lines should be for aiding the reading of graph, and there should be a balance between the readability and non-readability.

The solution to this problem is given in the textbook. The required force F is given by the equation

$$F = m \times a$$

where m and a are mass and acceleration respectively. The graph allows us to find the accelaration a, as the initial and the final velocity at start and end time are known. Thus initial velocity u =

$20\,\text{cm/s}$ and final velocity is $v = 0\,\text{cm/s}$, and this change happens in $10\ \text{s}$. So the acceleration is

$$\begin{aligned} a &= \frac{v-u}{t} \\ &= \frac{(0-20)\ \text{cm/s}}{10\ \text{s}} \\ &= -2\ \text{cm/s}^2 \end{aligned}$$

So the required force is $F = 20\text{g} \times -2\ \text{cm/s}^2 = -0.0004\text{N}$. In the solution to the exercise in the textbook it is noted that:

> The negative sign implies that the frictional force exerted by the table is opposite to the direction of motion of the ball.

Apart from this, there is no interpretation of the problem in terms of the graph given, or the physical situation involved. Many leading and exploring questions about the motion of the ball can be asked and answered from this graph.

For example, can we just by looking at the graph tell that the acceleration will be *negative*? It turns out we can. The acceleration of a body is positive if it is increasing its speed over time, whereas it is negative if the speed is decreasing. This is the physical interpretation of the sign of acceleration. How is this related to a *Time vs Velocity* graph? The slope of the line in this graph is the required acceleration. If the line has a negative slope, then the acceleration is negative, while if the line has a positive slope the acceleration is positive. This simple rule enables one to understand many problems associated with motion physically.

The next set of graphs in Chapter 12 of class 9 appear in the context of sound. There are three graphs in this chapter which depict different parameters of the sound waves.

Graph 23

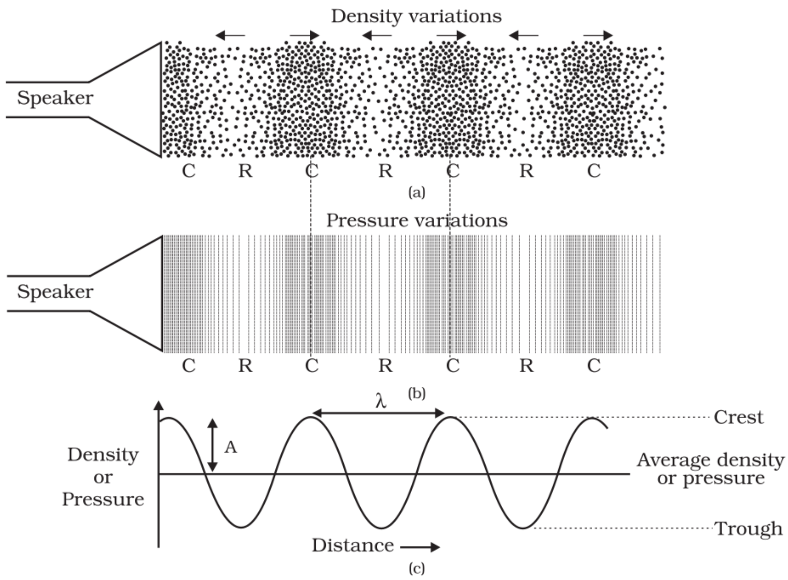


Fig. 12.8: *Sound propagates as density or pressure variations as shown in (a) and (b), (c) represents graphically the density and pressure variations.*

Figure A.41: A graph showing the variation of density during propagatoion of a sound wave. Figure from Science Class 9, Chapter 12, pg. 164.

Function The graphs shown in Figures A.41, A.43 and A.45 appear in Chapter 12 *Sound* in grade 9 textbook. This set of figures are graphical representations of the sound waves.

Reference The graphs are referred to in the text, but a full integration of different parts of the graph with the textual content is not present.

Data The source of data and the way it was obtained is not given.

Legend, Axes The axes have labels, but no units are used.

Close-to-life We experience different sounds daily. This a is close-to-life context.

Design Aspects There is no chart-junk, but the graph can be made more readable by certain modifications (Figure A.42).

In Figure A.41 the physical variation of density of air or pressure in air associated with a sound wave is represented graphically. There are three parts to this graph. Parts *12.8(a)* and *12.8(b)* show the density and pressure variation in the medium as a sound wave travels through it. Though the figure shows a speaker as the source of the sound wave, there is no reference to the source of sound (the speaker in this case) in the main text. Instead, the text directly refers to part *12.8(c)* which is the actual graphical representation. Instead of building the graphical sinusoidal graph with a physical basis of variation of density and pressure, it starts with the graphical form as a given.

> A sound wave in graphic form is shown in Fig. 12.8(c), which represents how density and pressure change when the sound wave moves in the medium. (*p. 163, Class 9, Science*)

Thus the textbook chooses an approach in which the abstracted result, the graphical form of a sound wave, in this case, is presented *before* the actual physical observations are taken into the picture. Observations about what happens in the medium with regards to density and pressure when the sound wave travels first should have come first in the explanations. This could be used logically to form the graphical form shown in part *(c)* of the figure. However, no such effort is made, rather the parts *(a)* and *(b)* though have a wealth of information are not directly referred to in the text concerning their content. They are referred to as below:

> Fig. 12.8(a) and Fig. 12.8(b) represent the density and pressure variations, respectively, as a sound wave propagates in the medium. (Class 9 Science, p. 163)

Since the two parts (*a*) and (*b*) do contain the actual physical observations, they should be used to form part *c* logically. Another thing to note is that there is a line which connects parts *a* and *b* where there are compressions in the medium. A better way to represent this entire figure would be to show vertical lines connecting all three parts. This would make clear that the three graphs are referring to the same physical space on the *X-axis*. Right now there seems to be an ambiguity in this as a line connects only the first two parts. Instead, all the compressions and rarefactions should be mapped on the graphical form to show its physical basis (Figure A.42). There is a label called *A* in part *(c)*, the immediate explanation is not given in the main text. The explanation that *A* is the amplitude appears on next page (*p. 165*) with discussions about the pitch, frequency in between. The explanation for this could have been given as a continuity to the discussion about frequency λ, which is defined with respect to the compressions and rarefactions. This would make the referencing in the later figures easier. No units are given for any of the physical quantities involved.

A redrawn version of this figure is shown in Figure A.42.

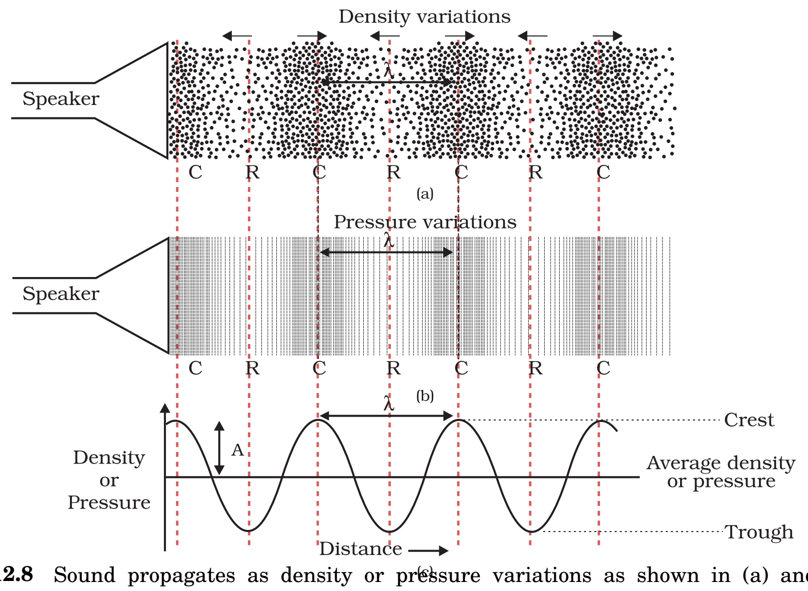


Fig. 12.8 *(redrawn)* Sound propagates as density or pressure variations as shown in (a) and (b), (c) represents graphically the density and pressure variations. The troughs and crests in (c) correspond to compressions (C) and rarefactions (R) in (a) and (b). The wavelength λ is the distance between two consecutive crests (or compressions) or troughs (or rarefactions). The amplitude A is the magnitude of maximum disturbance on either side of the mean value.

Figure A.42: A figure explaining the compressions and rarefactions for sound waves. Redrawn from Class 9, Chapter 12, page number 70 in Science textbook.

Graph 24

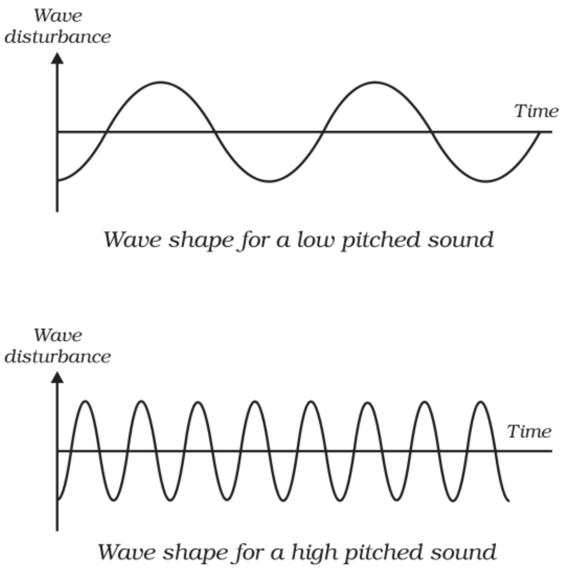


Fig. 12.9: *Low pitch sound has low frequency and high pitch of sound has high frequency.*

Figure A.43: Graph showing comparison of high and low pitched sounds. Figure from Science Class 9, Chapter 12, pg. 165.

Function In Figure A.43 the idea of *pitch* is represented graphically. The upper part of the figure shows a wave with low pitch and the lower one with a high pitch. The *X-axis* represents time in this case, while the *Y-axis* represents "Wave disturbance". No units are given to either of the axes.

Reference The figure is referred to in the main text as under:

> The faster the vibration of the source, the higher is the frequency, and the higher is the pitch, as shown in Fig. 12.9. (*p. 165, Class 9, Science*)

Data The data source and how the data was obtained is not provided.

Legend, Axes The axes have labels, but numbers are missing. In graphs, like this one, where we want to emphasise the *nature* of the graph and its interpretation, we need not have numerical data.

Close-to-life We experience different sounds daily. We experience various types of sounds in real-life. Hence, this is close-to-life context. However, the way the example is presented in the text lacks an experiential narrative.

Design Aspects The graphs do not have chart-junk. However, a combination of the two graphs which have to be compared can be helpful in emphasising the points made in the text.

As in the previous graph (A.42)the reader is supposed to make a comparison between the two parts of the figure to notice the difference in the waveforms in the given space. For this purpose, an equally spaced vertical grid would make the comparison much more accessible. We can add points to such a grid and show that till a certain point a wave with high pitch covers these many rarefactions while a wave with low pitch covers less number of rarefactions. This way there is a concrete resource for the comparison. In the given figure there is no resource for the comparison. This again makes use of the perceptual research in graph comprehension (Cleveland & McGill, 1984).

Another way to draw this graph would be to superimpose the two waveforms into a single graph. This would remove the constraint of comparing the two waveforms located at different positions in the page. In this way the fact that the amplitude of the waveforms can also be seen to be the same, even when their frequency is different. Some real-life examples of sources of sound which produce high and low pitch sounds should be given to make the idea of the pitch more concrete. For example string and percussion instruments. This redrawn version of the graph is shown in the Figure A.44.

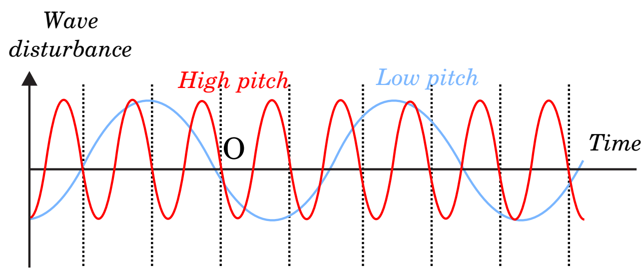


Fig. 12.9 *(redrawn)* Low pitch sound has low frequency and high pitch of sound has high frequency. In this figure the sound wave shown in red has higher frequency, while the sound wave shown in blue has lower frequency. By the time at point O in the figure, the red wave already has three crests while the blue wave has completed just one crest. Thus in unit time for red wave there are more compressions and rarefactions passing through a fixed point (O) than blue wave. Note that both the waves have same amplitude in this case, but we can also have waves with different frequencies and different amplitudes.

Figure A.44: A graph showing a comparison of high and low pitched sounds. The figure was redrawn from Class 9, Chapter 12, page number 165 in Science textbook.

Graph 25

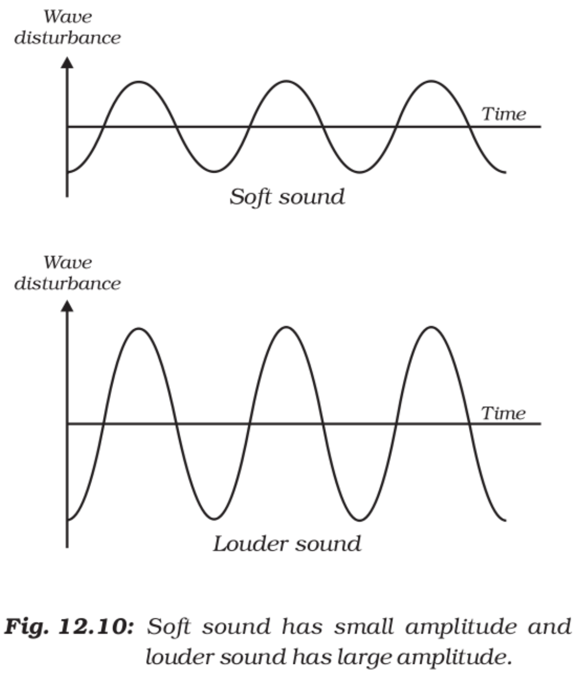


Figure A.45: Graph showing difference in amplitudes of soft and loud sounds. Figure from Class 9, Chapter 12, page number 165 in Science textbook.

Function Figure A.45 shows the third graph in Chapter 12 of Class 9 that appears in the context of sound. This graph is very similar to the earlier graph in Figure A.43, but in this case, the amplitude of the sound wave is emphasised. The *X-axis* represents time in this case, while the *Y-axis* represents "Wave disturbance". No units are given to either of the axes.

Reference The figure is referred to in the main text as under:

> Fig. 12.10 shows the wave shapes of a loud and a soft sound of the same frequency. (*p. 165, Class 9, Science*)

Data Data source is not provided.

Legend, Axes The axes are labelled, but no units are given.

Close-to-life We experience different sounds daily. This is a close-to-life context.

Design Aspects The graphs do not have chart-junk. However, a combination of the two graphs which have to be compared can be helpful in emphasising the points made in the text.

The meaning of amplitude *A* from part *(c)* of Figure A.41 is explained in the textbook. However, interestingly the figure itself has no label for amplitude *A*. As in the previous figure, this graph could be vastly improved by superimposing the two graphs using the same *X-axis*. A grid should be provided for easy comparison of the amplitude and time values. This will make the comparison along the *Y-axis* much easier. In the current version of the figure, the reader needs to make the comparison of graphical values which are situated at different positions on the page. By putting the graph in this way, we can easily see that sound waves with different amplitudes can have same frequencies. The redrawn version of the graph is shown in the Figure A.46.

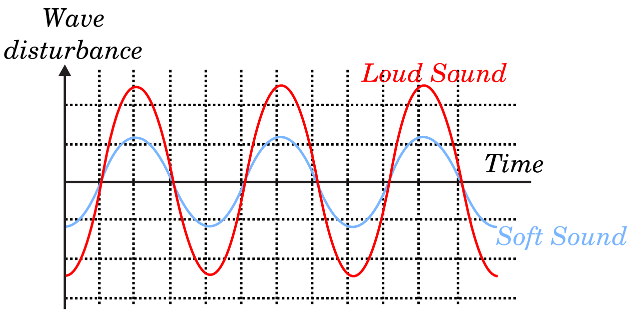


Fig. 12.10 *(redrawn)* Soft sound has small amplitude and louder sound has large amplitude. In this figure the sound wave shown in red has higher amplitude frequency, while the sound wave shown in blue has smaller amplitude. Note that both the waves have same amplitude so their troughs and crests coincide. For each trough and crest note that the maximum amplitude for red wave is larger than the blue wave. For example, during crests the blue wave is little over one unit on the grid, while the red wave is well over two units. We can also have sound waves which are different in frequncy and amplitude.

Figure A.46: Graph showing difference in amplitudes of soft and loud sounds. Figure redrawn from Class 9, Chapter 12, page number 165 in Science textbook.

Some real-life examples and activities related to wave motion and sound which bring out these points should have been provided in the textbooks.

Another graph which shows the different parameters with different waves should be shown. For example, a graph showing two waves with different frequency and amplitude. Right now the graphs two graphs seem to be of the same kind: with equal frequency but different amplitude, and with equal amplitude but different frequency. One such graph is shown in Figure A.47.

*

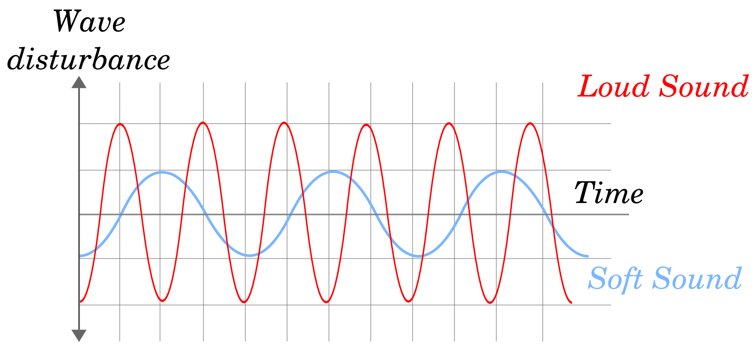


Figure A.47: A graph showing the comparison of the amplitude and freuqency in two different waves.

A.2.4 Class 10

The final textbook that we consider in this part of the analysis is the Class 10 textbook. The textbook has just four graphs in total. The graphs appear in the context of Ohm's Law, pH Values, distribution of energy sources and pollution levels in the river Ganga.

Graph 26

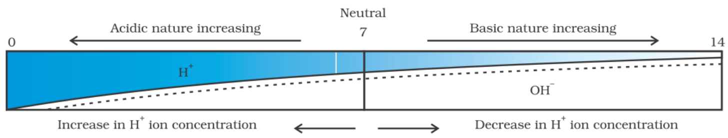


Figure 2.6 *Variation of* pH *with the change in concentration of* H^+(aq) *and* OH^-(aq) *ions*

Figure A.48: A graph showing relative strengths of acids and bases regarding pH as a function of the change in concentration of H^+ (aq) and OH^- (aq) ions. Figure from Science Class 10, pg. 25.

Function The graph shown in Figure A.48 appears in section 2.3 **HOW STRONG ARE ACID OR BASE SOLUTIONS?**. The graph is an indicator of the pH value of the solution as a function of the concentration of H^+ and OH^- ions. The blue colour in the form of a gradient is used as an indicator for the acidic and basic nature. The full blue on the left-hand side of the figure represents the maximum acidic value (pH = 0). The colour gradually changes to white which represents the maximum basic value (pH = 14). The change in colour is also linked to the changes in the H^+ and OH^- ions.

Reference In the textbook there is a sentence which refers to this figure number:

> Generally paper impregnated with the universal indicator is used for measuring pH. One such paper is shown in Fig. 2.6.

However, this seems to be a printing mistake, as the paper measuring the pH appears on the next page in Figure 2.7 in the textbook (Figure A.49 here). Thus we have a figure which is not mentioned in the main text, though there is a discussion which is linked to the figure. However, the discussion does not make any use of the figure just below. The readers it seems are supposed to make the connection to the graph below. Another peculiarity of the graph is that there is a dashed horizontal line which goes along the gradient line, it is not clear what this line represents or what is its significance. The middle vertical black line can be seen as the division between the H^+ and OH^- ions, but the purpose of the dotted line is not known.

Data The source for the data is not provided

Legend, Axes The Axes are labelled with pH values.

Close-to-life The acidic and basic nature of substances around us can be added to the chart to make it relevant to daily life. Figure A.49 shows a modified figure with pH values of a few chemicals.

Design Aspects The graph itself has elements which are not explained (dotted line).

This graph can enrich the discussion about the *pH* values immensely. One could perhaps combine the *pH* paper and this graph to show the *pH* values in colour and relation to the H^+ and OH^- ion concentration. This would reinforce the way the *pH* nature is interpreted in multiple ways. In this way, one can relate the concentration of H^+ and OH^- ions to the perceived acidic or basic nature of the substances. The combined redrawn graph is shown in Figure A.50.

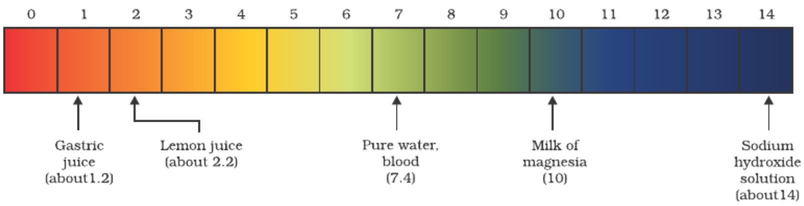


Figure 2.7 pH *of some common substances shown on a* pH *paper (colours are only a rough guide)*

Figure A.49: *pH* values of some of some common substance. Figure from Class 10, Chapter 2, page number 26 in Science textbook.

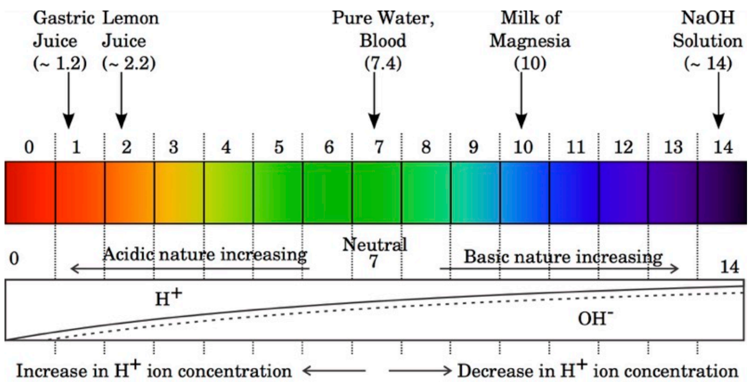


Fig. 2.6 *(redrawn)* Lower part of the figure shows the variation of pH with the change in concentration of H^+ (aq) and OH^- (aq) ions. The left hand side has pH of 0 and is most acidic, while the right hand side has pH of 14 and is most basic. The colours of a pH paper shown in the upper part of the figure roughly correspond the pH values mentioned. pH values of some of the substances are shown in the figure.

Figure A.50: Redrawn: *pH* values of some of some common substance. Figure from Class 10, Chapter 12, page number 204 in Science textbook.

As an extension to this graph, the learners can be provided with pH papers and asked to collect the pH values from common substances. The observational data can then be used to create a pH chart indicating the substance and it's pH value as shown in Figure A.51.

	Solution	Acidic	Basic	Nuetral	PH
1)	Water			✓	7
2)	Detergent		✓		10.5
3)	Chilli Water			✓	7
4)	Dettol Handwash			✓	7
5)	Tea			✓	7
6)	Milk			✓	7
7)	Thinner	✓			4
8)	Fair and lovely Winter cream			✓	7
9)	Vaseline Total Moisture			✓	7
10)	Clean Mate			✓	7
11)	Soap		✓		8
12)	Facewash			✓	7
13)	Tears			✓	7
14)	Sweat			✓	7
15)	Urine	✓			4
16)	Harpic	✓			2

Figure A.51: A table of pH values of common substances collected by a student.

Graph 27

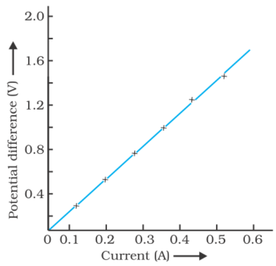


Figure 12.3
V–I graph for a nichrome wire. A straight line plot shows that as the current through a wire increases, the potential difference across the wire increases linearly – this is Ohm's law.

Figure A.52: Graph showing the values of current and potential difference for a nichrome wire. Figure from Science Class 10, pg. 204.

Function The graph shown in Figure A.52 appears in the section on Ohm's Law in the textbook. In this graph, the actual data points are shown on the graph in the form of "+" symbols. The units are provided for both the axes. Equations in the main text also accompany the graph. Also, it is reasoned why the graph has a linear form. That the current I and the voltage V being directly proportional, the constant of proportionality is equal to R.

$$V = IR$$

This makes the relationship to the graph much easier. Additionally, the measurement exercise can be performed on different materials. This will provide the students for comparison between different materials and understand the resistance in terms of the slope of the graph. This would have added to the understanding of the physical significance of the slope in the given situation. Also, the students may benefit, if some response of some *non-Ohmic* materials are shown.

Reference The graph is shown for reference to an Activity (12.1, p. 203) in the textbook. The activity reads as follows:

- Set up a circuit as shown in Fig. 12.2, consisting of a nichrome wire XY of length, say 0.5 m, an ammeter, a voltmeter and four cells of 1.5 V each. (Nichrome is an alloy of nickel, chromium, manganese, and iron metals.)
- First use only one cell as the source in the circuit. Note the reading in the ammeter I, for the current and reading of the voltmeter V for the potential difference across the nichrome wire XY in the circuit. Tabulate them in the Table given.
- Next connect two cells in the circuit and note the respective readings of the ammeter and voltmeter for the values of current through the nichrome wire and potential difference across the nichrome wire.
- Repeat the above steps using three cells and then four cells in the circuit separately.
- Calculate the ratio of V to I for each pair of potential difference V and current I.
- Plot a graph between V and I, and observe the nature of the graph.

Data The activity details how the data for the graph was obtained, but the actual data in the form of table is not provided.

Legend, Axes The axes are labelled, and units are given.

Close-to-life With all the electrical gadgets around us, electricity and Ohm's Law is a close-to-life context.

Design Aspects The graph does not have any chart-junk. Grid would have helped to the data points on the graph.

Connections with other linear graphs in the previous classes, for example, the distance-time graph depicting uniform motion should have been made. Linkages from this activity to the mathematics that the students have learned are entirely missing. The concept of direct proportion can be used in this case, which the students have already studied in the Class 9 Mathematics Textbooks.

Graph 28

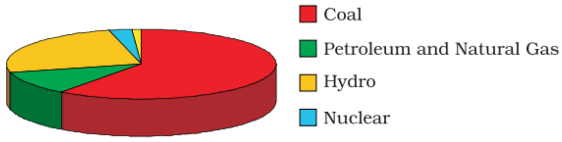


Figure 14.1
Pie-chart showing the major sources of energy for our requirements in India

Figure A.53: A piechart showing significant energy sources in India. Figure from Science Class 10, pg. 244.

Function The pie chart shown in Figure A.53 appears in the section on conventional sources of energy.

Reference The pie chart is referred to in the main text in passing:

> The fossil fuels are non-renewable sources of energy, so we need to conserve them. If we were to continue consuming these sources at such alarming rates, we would soon run out of energy! In order to avoid this, alternate sources of energy were explored. But we continue to be largely dependent on fossil fuels for most of our energy requirements (Fig. 14.1). (*p.*)

Data Source for the data is not provided.

Multiple Representations The graph does not allow for any experiences in multiple representations.

Legend, Axes The data is colour coded in different sectors of the pie-chart.

Close-to-life Energy usage is a close-to-life context.

Design Aspects The 3-D representation of 2-D data presents its own problems. In this case, we detail out the false perceptions due to the design of this graph.

The pie chart is used to make a case for alternative fuels. There is no reference given for the source of the data in the pie chart. The year is not provided for the data. Providing the year for the data sets it in a historical perspective. A time series of resources showing the trends over the years would have been much better. This would also show how newer (non-conventional) sources of energy have made an impact in the recent years. Also, the trends of whether the annual production of the resources has increased or decreased can be seen in such a graph. The pie chart here clearly shows that coal and petroleum and natural gas as the principal sources for the energy requirements. But, the exact numbers are absent. Also stimulating would have been an exercise in which the students investigate the actual usage of fuels in their own lives. When done in the classroom collaboratively, it can lead to interesting questions being asked. This exercise can be based on giving the students access to data for making conclusions based on the analysis.

The chart is in *3-D* which is not the need for it. This is aptly described by Tufte in his works as *chartjunk* or *ducks* (Tufte, 2001). Robbins (2012) elaborates on the same theme very clearly:

> These examples demonstrate that the way to read three-dimensional bar charts depends on the software used to create them. But the reader rarely knows what software was used so has little hope of reading them correctly without the values printed. Even PowerPoint and Excel, two programs that come packaged together in the same suite, use different algorithms to plot their graphs. Therefore, you should never use a three-dimensional bar chart for two variables. A properly drawn two-dimensional chart shows the same information more effectively and avoids misinterpretation. (p. 27)

To find out the angular measure in this 3-D chart is difficult due to the perspective. To find the angles and hence the percentage of the sources, we need to make the chart 2-D. The reconstructed chart, along with the approximate values of angles and percentages are shown in Figure A.54. The

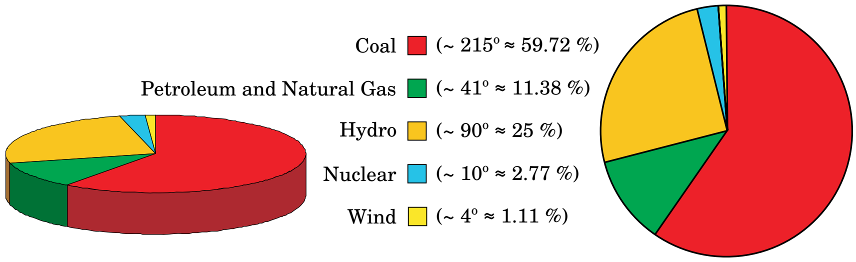


Figure A.54: A 3-D pie chart showing principal energy sources. (Figure A.53) converted to 2-D and the approximate values for percentages shown.

reconstruction of 3-D to 2-D reveals interesting statistics. The percentage values of the sources approximately are:

Coal	59.72 %
Petroleum & Natural Gas	11.38 %
Hydro	25 %
Nuclear	2.77 %
Wind	1.11 %

Since the source of this data is not given, it cannot be verified directly. The official statistic for this information can be found in the second volume of *Twelfth Five Year Plan* (Planning Commission, Government of India, 2013). Chapter 14 on energy has statistics about the energy scenario in India. Particularly in Tables 14.4 and 14.5, interesting data is given regarding the different sources of energy. In table 14.4 the trends in the supply of primary commercial energy are given regarding *million tons of oil equivalent*. That is the energy equivalent of burning million ton of oil. In Table 14.5 (Table A.26 here) the percentage share (actual, provisional and projected) of different fuels in energy supply is given. Using this data, we can see the trends in the different sectors over last decade. The percentage of the different fuels are shown in Figure A.55. Though in recent years due to a major push towards solar and wind energy projects, the projected numbers have changed towards greater percentage for renewable energy.

If we try to match values calculated from the 3D pie chart in the textbook to the values from the *Twelfth Five Year Plan*, they do not match up. Though the values that we have calculated from the 3-D pie chart are approximate, and they represent what is shown in the diagram (Figure A.53), and its significant trends.

Fuel Type	2000-01	2006-07	2011-12	2016-17	2021-22
	Actual	Actual	Provisional	Projected	Projected
Coal and Lignite	50.36	53.22	53.45	55.41	56.90
Crude Oil	37.45	33.41	31.51	26.04	23.29
Natural Gas	8.49	6.99	10.32	13.46	13.17
Hydro Power	2.17	2.53	2.17	1.79	1.73
Nuclear Power	1.49	1.24	1.57	2.26	2.95
Renewable Energy	0.04	0.22	0.98	1.43	1.97

Table A.26: The percentage share of each fuel in Total Commercial Energy Supply, source p. 134, *Twelfth Five Year Plan.*

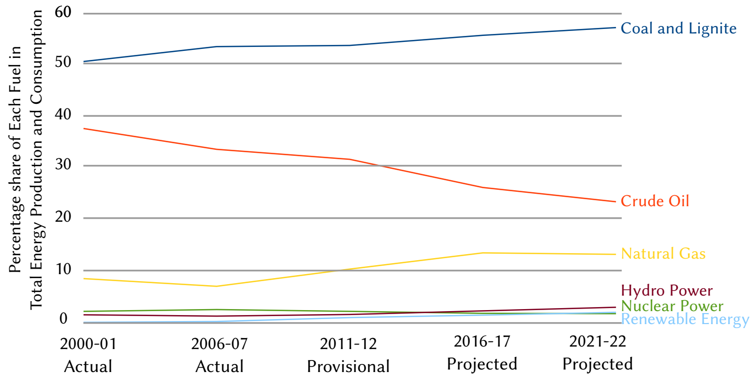


Figure A.55: Share of Each Fuel in Total Energy Production and Consumption, Data from Table 14.5 of *Twelfth Five Year Plan Vol 2.*

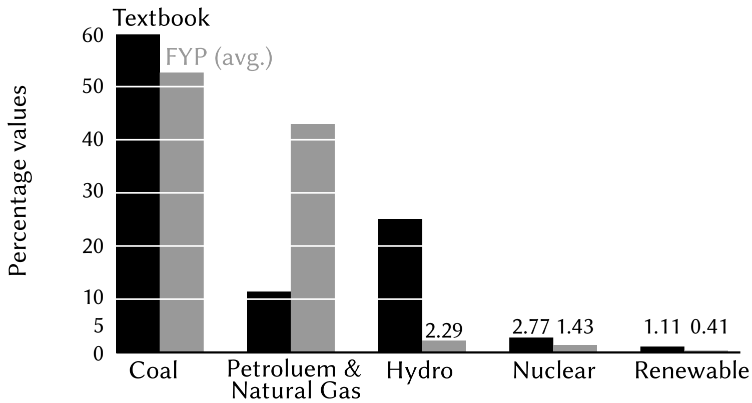


Figure A.56: Share of Each Fuel in Total Energy Production and Consumption, Data from Table 14.5 of *Twelfth Five Year Plan Vol 2.*

When we plot a graph of above data, the discrepancies are much clearer. In Figure A.56 the average value percentage of the three years shown in Table A.27 are compared against the extracted values from the textbook. In case of coal, the textbook shows ∼ 60 % while the FYP has 52 %. For Crude Oil and Natural Gas the numbers are off by a multiple of 3.5 (∼ 12 % in textbook instead of ∼ 42 % in FYP). The case for hydroelectric is even worse. In the textbook, it appears as a major player with 25 %, whereas in actual production hydroelectric is just a tenth of that at ∼ 2.3 %. In the case of nuclear energy, it is 2.77 % in the textbook, while it is ∼ 1.4 % in FYP. In case of renewable energy for the year 2000-01 the numbers are off by an order of 100 (1.11 % and 0.04 %)! Though they are

Fuel Type	Textbook	12^{th} FYP (*Avg.*)	Ratio
Coal	59.72	52.34	1.14
Petroleum & Natural Gas	11.38	42.72	0.26
Hydro	25.00	2.29	10.91
Nuclear	2.77	1.43	1.93
Renewable	1.11	0.41	2.68

Table A.27: Comparison of percentage values from the textbook (extracted) and the data from *Twelfth Five Year Plan* (FYP). The FYP has separate data on Crude Oil and Natural Gas, and here the two are added to be compatible with textbook data. Similarly, the textbook only mentions wind energy, whereas the FYP mentions a more broader category of Renewable energy. Values for 2000-01, 2006-07 are actual values, while that for 2011-12 are projected values.

closer to the average values, 1.11 % in the textbook and 0.98 % in FYP average. Looking at the table above we see that there is a gross misrepresentation of data in the pie chart. Only for coal and nuclear power the order of magnitude presentation is correct. Rest of them are off the mark.

In Figure A.55 we can clearly see the trends for the different sources of energy for the coming years. Though the dependence on crude oil is projected to decrease in the coming few years from about 37% to 23 % the dependence on coal is increasing. The total output of hydro, nuclear and renewable sources is small (all below 5%) and remains so for the projected years. With renewable energy sources being debated and discussed in a significant way with implications for the environment, we feel an immersive, data-rich activity can take place in this regard. The primary sources of the data should be made available to the students to make sense of the debates and their implications.

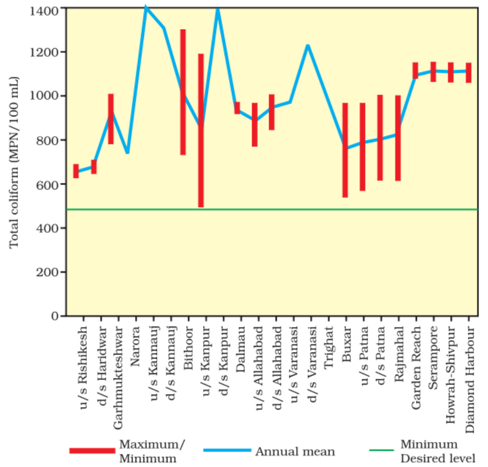
The Ti*k*Z and PGF Manual (Tantau & Wibrow, 2007) has a chapter *Guidelines on Graphics*. Among other things, it provides certain Do's, and Dont's regarding various aspects of graphics. For the 3-D pie chart, we have the following commandments Section 4.6 Plots and Charts pp. 51.):

- Do not use 3D pie charts. They are *evil*.
- Consider using a table instead of a pie chart.
- Due not apply colours randomly; use them to direct the readers focus and to group things.
- Do not use background patterns, like a crosshatch or diagonal lines, instead of colours. They distract.
- Background patterns in information graphics are *evil*.

  (emphasis in original)

Graph 29

(see Fig. 16.1). Coliform is a group of bacteria, found in human intestines, whose presence in water indicates contamination by disease-causing microorganisms.



MPN: most probable number *mL: millilitre*
u/s: upstream *d/s: downstream*

***Figure 16.1** Total coliform count levels in the Ganga (1993-1994)*

Source: Anon 1996, Water Quality – Status and Statistics (1993 & 1994), Central Pollution Control Board, Delhi, p.11.

Figure A.57: A graph showing total coliform levels in Ganga river. Figure from Class 10 Science textbook, pg. 267.

Function The graph displays (Figure A.57) the pollution levels in Ganga to make a rationale for the GAP.

Reference The graph is referred to in the main text.

Data The data source is provided, but the data is not provided in a tabular form.

Legend, Axes Legend keys are provided at the bottom of the graph. The Y axis has the count of Total coliform in MPN/100 ml. The X axis has places where the sample had been collected. The places are shown to be equidistant when they are not.

Close-to-life With water purification being one of the pressing issues in everyday life this graph presents a close-to-life context.

Design Aspects The graph could have been redrawn to indicate the physical distances of the places of data collection.

This is perhaps the most complicated figure in the entire sample of textbooks. The source of the image has been provided. The source is cited as *Water Quality - Status and Statistics (1993 & 1994), Central Pollution Control Board, Delhi.* The graph finds a mention in the main text with these lines:

> Awareness about the problems caused by unthinkingly exploiting our resources has been a fairly recent phenomenon in our society. And once this awareness rises, some action is usually taken. You must have heard about the Ganga Action Plan. This multi-crore project came about in 1985 because the quality of the water in the Ganga was very poor see Fig. 16.1). Coliform is a group of bacteria, found in human

intestines, whose presence in water indicates contamination by disease-causing microorganisms.

However, unfortunately, there is almost no discussion *about* the graph itself in the textbook. The graph has Total coliform (MPN/100 ml) on the *Y-axis*. There is no explanation given on how this count was done or what method was used to arrive at these numbers. The unit of the of the count MPN is not explained. MPN is the *Most Probable Number Index* and relates to the number of bacteria in the water. A brief explanation of this would have been helpful to situate the graph in the context of the discussion in the textbook.

The *X-axis* of the graph presents towns and cities in the course of Ganga river. For some of the places, the data from both upstream and downstream is presented. This itself could have been a good source of discussion in the main text. For example, determining whether by passing through a city or town how much does the river gets polluted? Is this related to any peculiar industries being present in those towns? Why there are peaks in certain towns (for example, Narora)and not in others (for example, Dalmau)?

Also if there is any variation in the counts during the different seasons is not clear from the graph. This can itself be an excellent question to be asked. Apart from this, there are several 'peaks' in the graph, which need explanation. For example, why does the NPR suddenly peaks at Narora is not clear. Another thing that is not clear from the graph is the geographical distances involved. The way graph has been drawn, it seems that the places on the *X-axis* are all equidistant from each other, when actually they are not. The cities in the graph span almost the entire extent of Ganga in India from Rishikesh (∼ 10 km) to Diamond Harbour (∼ 2500 km). The graph can create a false sense that the data was taken at equal distances across the length of Ganga. A better way to create the graph would be to use the actual distances from the origin of the locations in which it was taken. The MPN on the *Y-axis* should have been related to other variables near the observation sites. Also, the numbers should have been provided for the graphical values on the graph. Also providing data over the years would have added another dimension to the graph enriching it further.

Another point of note is the rationale for showing this graph. The main text says the *Ganga Action Plan* (GAP) was started in 1984, but the text reads as if the data in the graph was the rationale for this project. However, the data is from the years 1993-94. Perhaps showing the data from years before 1984 would have provided a better justification for the formation of the Ganga Action Plan (GAP).

There are many position papers providing data on the *Ganga Action Plan*. The data that is shown in the figure is part of an extensive monitoring network which is spread across the length of the river. The spatial distance covered along the length of the river is given for each of these monitoring stations. So this can serve as a variable for *X-axis*. This gives the reader an idea about the distribution of the observing stations along the length. In the data provided in these reports, along with the *Coliform*, two other significant indicators for analysing the water quality are used. One is the *Biological Oxygen Demand* (BOD) and the other is *Dissolved Oxygen* (DO). There two parameters along with the *Coliform* activity usually are indicators for the biological health of the river.

An entire narrative can be built using these data sets. (alph*) How bad was the quality of water before the *GAP* and what measures have been taken to improve it? and, (alph*) how successful it has been. The first graph of the narrative can be represented by some parameter which has seen change before and after the *GAP*.

Pre-GAP The data from the early 1980s show that the pollution in river Ganga was at alarming levels. Various indicators of pollution were beyond the prescribed limits. The Ganga Action Plan was formed with the objective of preventing the pollution of Ganga and to improve its water quality to acceptable standards.

For example the designated best use standards according to Central Pollution Control Board (*CPCB*) are as under:

Class	**Designated Best Use**			**Indicator**	
	DBU	pH	DO mg/l	BOD mg/l	Total Coliform MPN /100 ml
A	Drinking Water Source without conventional treatment but after disinfection	6.5-8.5	≥ 6	≤ 2	50
B	Outdoor bathing (Organised)	6.5-8.5	≥ 5	≤ 3	500
C	Drinking Water Source with Conventional treatment followed by disinfection	6.5-8.5	≥ 4	≤ 3 *Free Ammonia*	5000
D	Propagation of wild life and fisheries	6.5-8.5	≥ 4 *Electrical Conductivity*	1.2 mg/l *Sodium Absorption Ratio*	*Boron*
E	Irrigation, industrial cooling and controlled waste disposal	6.5-8.5	2250 ℧1/cm	26	2 mg/l

Table A.28: The Designated Best Use Classification of Inland Surface Water.

As an example, the data from pollution indicators from pre-GAP period (1982) is shown in Figure A.58.The data is from *Status Paper On River Ganga* (Alternate Hydro Energy Centre, 2009).

The GAP over the years has shown improvement in the various parameters that measure the pollution levels. Given the importance of Ganga Action Plan in this aspect should have been revealed from the graph. The graphs and the data that they come from should challenge the students into asking questions from the data, making relevant conclusions about the effectiveness of the project, significant roadblocks etcetera.

But, this is entirely missing in the glancing treatment that this single complex graph receives in the textbook.

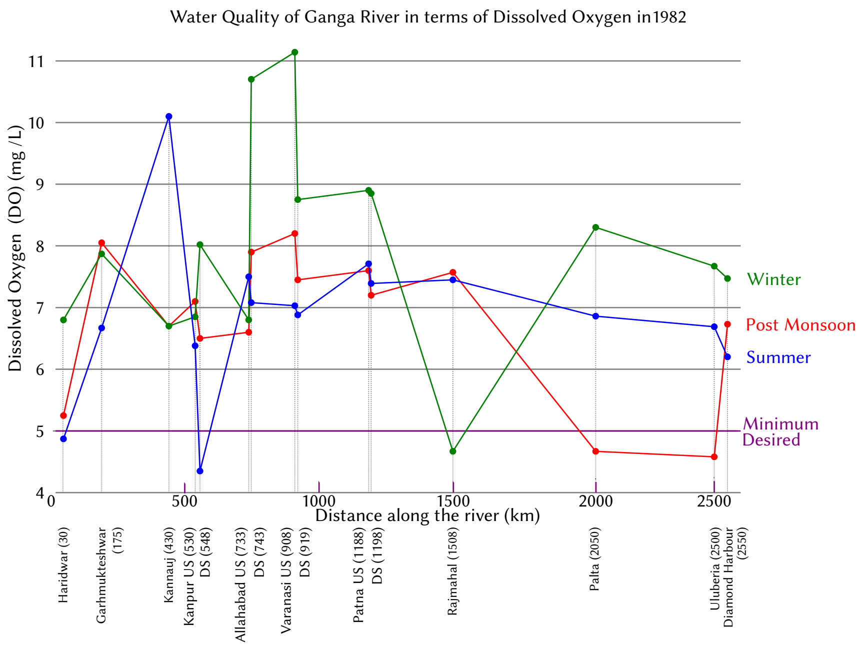


Figure A.58: Variation of Dissolved Oxygen (*DO*) with seasons in Ganga, 1982. The numbers in the bracket next to city names indicate distance in kilometres along the river.

"The graphical method has considerable superiority for the exposition of statistical facts over the tabular. A heavy bank of figures is grievously wearisome to the eye, and the popular mind is as incapable of drawing any useful lessons from as of extracting sunbeams from cucumbers."

A. B. Farquhar and H. Farquhar, *Economic and Industrial Delusions, 1891*
partially quoted in Wainer (2007)

B

A short history of graphs

Though the graphical method for representing and analysing data is commonplace today, it did not happen overnight. During the seventeenth and eighteenth centuries, the data was without exception presented in the form of tables. Presenting data in the form of tables has some advantages, when data sets are small, but creates problems when the data sets are large. Finding patterns in a table with a large data set is very difficult.

The quote at the beginning from Farquhar and Farquhar (Farquhar & Farquhar, 1891) sums up the necessity for using graphs for representing data as opposed tables. The metaphors of sunbeams for the analysed data is not hyperbole. The opinion in the quote is especially true when the number of data points is considerable. Though usage of graphs to communicate statistical and experimental data are too familiar today, but it was not always the case. Though there are references to graphs in earlier works including Greek, Arab and Indian mathematicians, the use of graphs to present and analyse data is not frequent in communicating results till the late 1700s. Funkhouser's paper titled *Historical Development of the Graphical Representation of Statistical Data* (Funkhouser, 1937) provides a comprehensive survey till the 1930s. For works which include the works after 1930's, including that of John Tukey, one can refer to the work by Friendly (2008). The treatment presented below follows largely from these two sources.

B.1 Pre-requisites for development of graphs

For graphs to become commonly used as means of displaying information some concepts and mathematical ideas are pre-requisites. One of the first conditions for the historical development of graphs is the development of a system of coordinates. The system of coordinates has been in use since the ancient times. The evidence for the first use of coordinates is in Egypt for laying out of towns. The definite idea of a coordinate system appears in the works of Greek astronomers and geographers. For example, Hipparchus (c. 140 BC) used longitude and latitude to locate stars and also to map places on the Earth. Some Greek mathematicians also used orthogonal axes in their treatment of geometric figures. So what stopped them from making use of the graphical method of representing data?

One of the main differences that are pointed out is that the Greeks were non-symbolic in their operations. That would mean that they did not see the connection between the curve and the equation representing the curve. Another difference, as pointed out by Heath, is that the Greeks did not direct their efforts to make the fixed lines as few as possible, but rather to expressing their equations between areas in as short and as simple form as possible. For example, x^2 and x^3 can be considered as area and volume, but also as curves in two dimensions. The next condition was the realisation that the variables in such a system of coordinates can be used to represent physical quantities. For usage of graphs in communicating quantitative data following conditions were to be met:

(1) The invention of a system of coordinates.

(2) The recognition of one to one correspondence between algebra and geometry.

(3) The graphic representation of the expression $y = f(x)$.

(4) Collection of numerical data resulting from observations.

(5) Translation of the numerical statistical data into graphical expression.

B.2 Some historical graphs

One of the earliest attempts to represent changing values graphically can be seen in a graph (Figure B.1) from tenth or possibly eleventh century (Funkhouser, 1936)

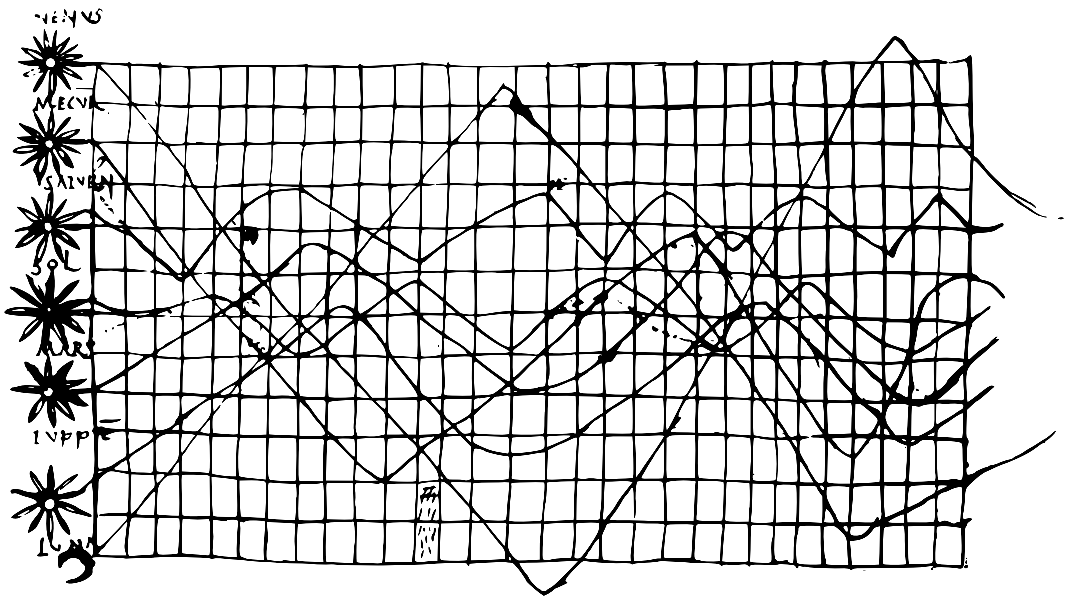


Figure B.1: One of the earliest known examples of representing changing values in graphical format (Funkhouser 1936). The horizontal axis represents time, while the vertical axis represents the amplitude of inclinations of the planets from the ecliptic.

The graph in Figure B.1 appears in an appendix to a manuscript which describes the movement of the planets. The graph is supposed to represent a plot of inclinations of the planetary orbits as a function of time. The central horizontal line represents the ecliptic, and one can see the entry for Sun (*SOL*) next to it. The horizontal lines are not evenly scaled, as the periods of planets represented cannot be matched. The vertical axis represents the width of the zodiac. Though this graphic cannot be said to represent the correct movements of the planets, but what we see here an attempt towards representing quantitative data with graphics. Another noteworthy point in this particular graphic is the use of a *grid*. Though common now, the use of grids or graph papers was not a common practice until the middle of 19^{th} century. After this, a prototype of bar graphs is seen in the work by Nicolos Oresme.

After Descartes published *Le Géométrie* in 1637, the association of mathematical variables with geometric curves was established. But, we see that for a long time the use of graphs to represent two measured physical quantities was not typical. Historically, the use of graphs for presenting data started from late eighteenth to the early nineteenth century on a large scale. Earlier tables were the preferred format for presenting the data. In record books of observations, there are pages and pages of tables. Though the tables might be useful in some places, for example, to find exact values, the bigger picture is lost out. To find any trends or patterns in large data sets is extremely difficult with the tables. One of the first persons to use graphs to communicate experimental data and to draw inferences from them was Johann Lambert (1728-1777). In his works, Lambert explicitly discusses the method of use of graphs for display of data and to find inferences from them, for example, see (Tilling, 1975).

Figure B.2 shows one of the graphs from the work of Lambert. This graph appears in his work on heat, *Pyrometrie* published in 1779.[1] The graph depicts the mean monthly temperatures at various depths inside Earth. Symbols note the months at the base of the X axis.

William Playfair (1759-1823) did the first extensive use of graphs for presenting statistical data and drawing inferences. Playfair's graphs are considered to be exemplary, and it is only after him that the power of graphical display of quantitative data and inferences from them came in vogue in science. The fact that he published several works containing various graphical formats and also elaborated in text on how to interpret them makes him one of the real pioneers in this field. Various authors have given highest laurels to Playfair's work (Hankins Thomas, 1999; Royston, 1956; Tufte, 2001; Wainer, 1990).

Playfair in introductions to his books gives the following tenets for his philosophy of graphics as summarised in Costigan-Eaves & Macdonald-Ross (1990).

1. The graphic method is way of simplifying the tedious and complex.
2. Busy men need some sort of visual aid.
3. The graphic method is more accessible than the table.

[1] An electronic version the book can be accessed at Université de Strasbourg.

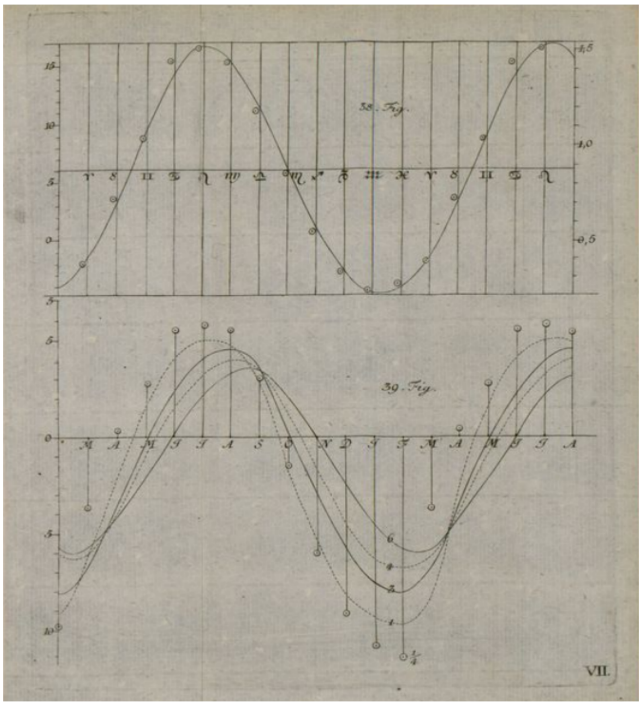


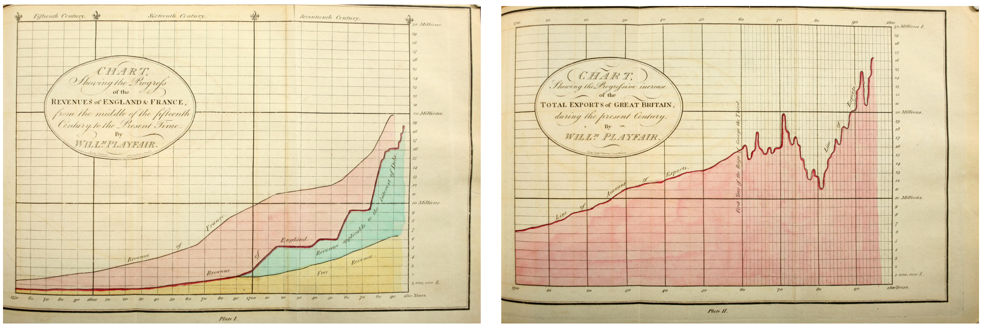
Figure B.2: A graph (Fig. No. 39) from Lambert's *Pyrometrie*. The graph depicts the mean temperature inside the Earth.

4. The graphic method appeals to the eye.

5. The graphic method appeals to the mind. (p. 320-323)

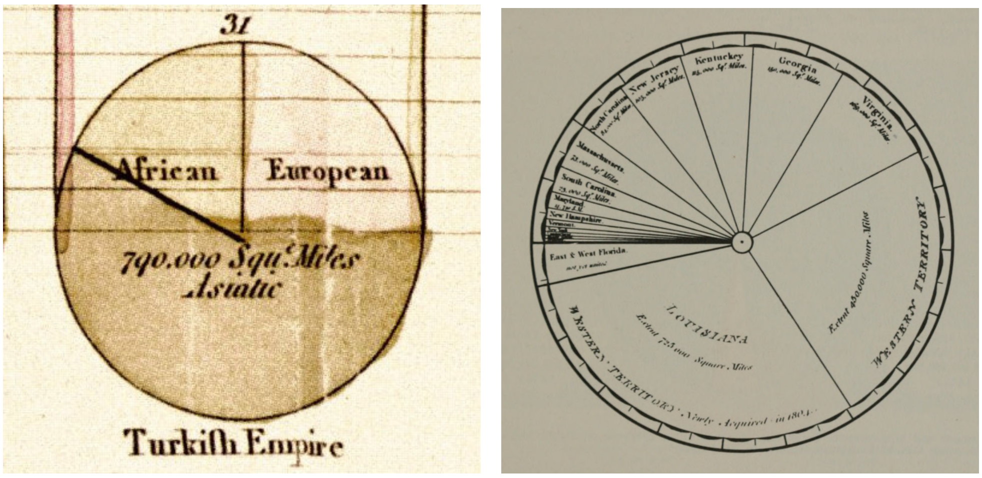
In his book graphs from his book *For the use of enemies of England: A real statement of the finances and resources of Great Britain* (Playfair, 1796) published in 1796, William Playfair produced two copperplate graphs in colour. The first graph (Figure B.3a) shows revenues of England and France over last three centuries. The graph has years starting from 1550 to 1600 on the horizontal axes. The vertical axis is represented by Millions of Pounds. The second graph (Figure B.3b) in the book shows the total exports of Great Britain from 1700 to 1796. From the year 1760 onwards a finer grid is used for the horizontal axis.

After this Playfair went on to produce his other books in which he presented statistical graphs with ingenuity. Playfair produced perhaps the first pie chart in his book *Statistical Breviary*. He drew circles to represent the areas of the countries. In the case of the Turkish empire, the circle was divided into three parts representing the areas of the empire in Africa, Europe and Asia. This pie chart is shown here in Figure B.4.



(a) (b)

Figure B.3: Two line graphs from Playfair's 1796 book. (a) Revenues of England and France over last three centuries. (b) The total exports of Great Britain from 1700 to 1796.



(a) (b)

Figure B.4: Examples of pie-charts from William Playfair's work. (a) The pie chart depicting the area of Turkish Empire in each of the continents. This figure appears as a part of a graph depicting dominant powers of Europe and their colonies in *Commercial and Political Atlas*. (b) A pie chart depicting the areas of states of the USA, made by Playfair from his book *Statistical Account of the United States of America*.

A few years later, in 1805, Playfair drew a pie chart depicting the areas of states in the United States of America for the book *Statistical Account of the United States of America*. Playfair called the pie chart as a "divided circle." (Brinton, 1939).

Playfair also introduced the idea of a bar graph in his work, *The Commercial and Political Atlas* published in 1786 has the first bar graph (Figure B.5).

After Playfair's work during the end of the eighteenth and beginning of nineteenth centuries, the use of graphs slowly percolated in depicting various statistical data in different fields.

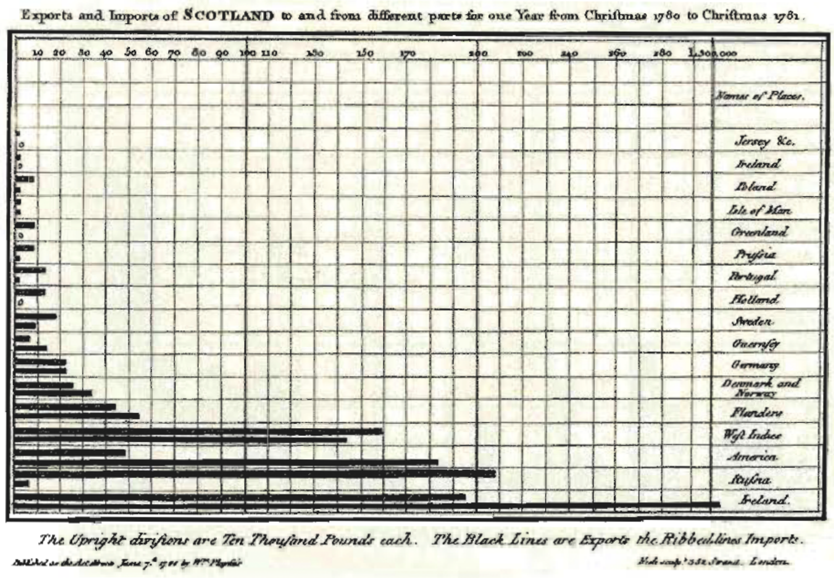


Figure B.5: The bar graph appearing the *Atlas* by Playfair.

The use of the graph paper was not the common practice till the 1850s (Funkhouser, 1936). The textbooks on science from that era explain in great detail of how to construct a graph paper with even and uneven axes. For example, in *Principles of Science* by Jevons (Jevons, 1920) there is a section titled *The Graphical Method* p. 492 under *Quantitative Induction*:

> Even if the numbers were absolutely correct and disposed at regular intervals, there is, as we have seen, no direct mode of discovering the law, but the difficulty of discovery is much increased by the uncertainty and irregularity of the results.
>
> Under such circumstances, the best mode of proceeding- is to prepare a paper divided into equal rectangular spaces, a convenient size for the spaces being one-tenth of an inch square. The values of the variable being marked off on the lowest horizontal line, a point is marked for each corresponding value of the variant perpendicularly above that of the variable, and at such a height as corresponds to the value of the variant.
>
> The exact scale of the drawing is not of much importance, but it may require to be adjusted according to circumstances, and different values must often be attributed to the upright and horizontal divisions, so as to make the variations conspicuous but not excessive. If a curved line be drawn through all the points or ends of the ordinates, it will probably exhibit irregular inflections, owing to the errors which affect the numbers. But, when the results are numerous, it becomes apparent which results are more divergent than others, and guided by a so-called sense of continuity, it is possible to trace a line among the points which will approximate to the true law more nearly than the points themselves. The accompanying figure sufficiently explains itself.

The book cites an example which uses similar methods for displaying data with graphs. This is followed by telling the reader where one can buy "engraved sheets of paper for drawing of curves," i.e. the graph papers.

By the end of the nineteenth century the use of graphs to depict observational data in various sciences became a standard practice.

B.3 Spread of graphs in natural sciences

Many instruments were developed ingeniously from the eighteenth century onwards which would automatically record graphs. The self-recording graphs, in which the measured variable was recorded on a sheet of paper as a function came in the context of weather measurements. Some of such instruments could record temperature, wind direction, humidity and rainfall. Several such instruments are discussed in the article *The Beginnings of Graphic Recording* by Hoff & Geddes (1962).

Also of interest is to note the *Watt indicator*, which plotted the pressure-volume curve (Tilling, 1975). This article discusses the earliest instruments which could automatically record data graphically. Tilling also discusses a particular "weather-clock" which was designed by Robert Hooke. Though in the early stages the graph as a product was incidental to the apparatus and most of the time no analysis was done by use of the graph itself.

By the middle of the nineteenth century, the graphic method had established itself well. The books *Animal Mechanism: A Treatise on Terrestrial and Aerial Locomotion* and *Movement* (Marey, 1874; 1885) by E. J. Marey has a substantive number of graphs displaying the variation of various quantities related to motion studies on animals. Marey also explains in detail, the ingenious mechanisms used to obtain these graphs. For example, to study the movement of the human body during running, the man is running with the graphing instruments attached to the body (Figure B.6a). Another example (Figure B.6b) from the work of Marey, shows the apparatus to record the movement of the wings of a bird.

In the Indian historical context, the books by J. C. Bose use the graphs to illustrate, present and analyse the experiments he did with plants. Figure B.7 shows two graphs from the work of Bose. The graph in the Figure B.7a is from *The Physiology of Photosynthesis* (Bose, 1924), and it compares the photosynthetic activity of Hy-

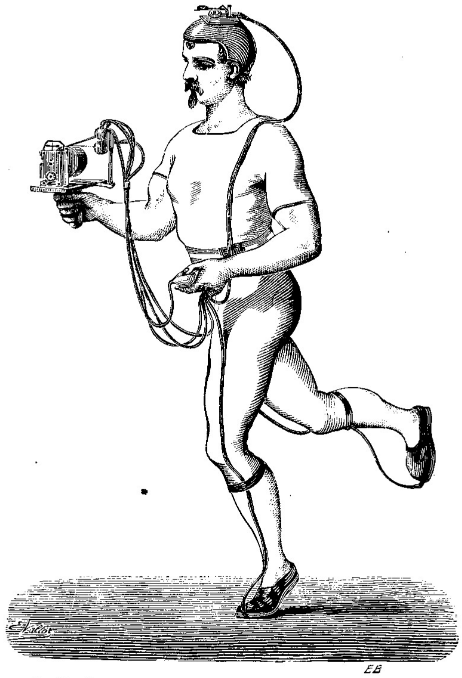


Fig. 27.—Runner provided with the apparatus intended to register his different paces.

(a)

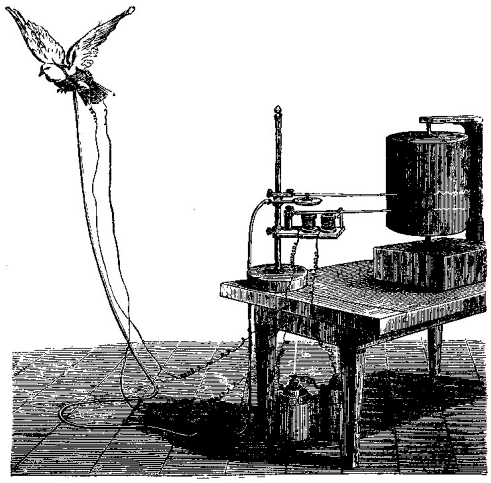


Fig. 94.—Experiment to determine by the electrical and myographical methods, at the same time, the frequency of the movements of the wing and the relative durations of its elevation and depression.

(b)

Figure B.6: . (a) The running man with apparatus to record his movement. Note that the graph paper roll is in the right hand of the runner. From *Animal Mechanism* p. 126. (b) A setup to record the movement of the wings of a bird, the cylinder on the right records the movements of the wings. From *Animal Mechanism* p. 230.

drilla under Sunlight and an artificial source of light. The graph in the Figure B.7b is from his book *Growth and Tropic Movements of Plants* (1929) and gives data for rate of growth on temperature.

We see that by the end of the nineteenth century the presence of graphs was a normal feature in scientific communication. In just over a hundred years graphs had become a central feature in science. Michael Friendly has started an online project, called the *The Milestones Project* which is an effort towards collating the history of data visualisation. The rationale for building such a project is given on the website:

> More importantly, we envisage this Milestones Project as the beginning of a contribution to historiography, on the subject of visualisation. . . . One goal is to provide a flexible, and useful multimedia resource, containing descriptions of events and developments, illustrative images, and links to related sources (web and in print) or more detailed commentaries. Another goal is to build a database which collects, catalogs, organizes, and illustrates these significant historical developments.

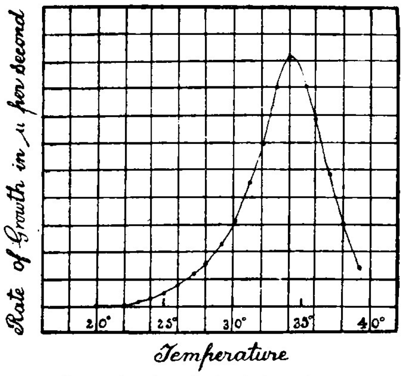


FIG. 17. Curve showing relation between temperature and rate of growth.

(a)

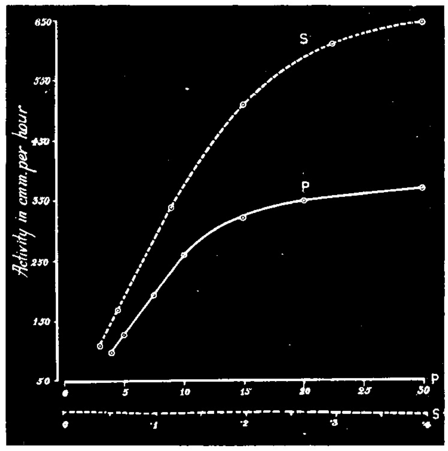


FIG. 8. Photosynthetic Curves for identical *Hydrilla* plant under Sunlight (S) and Pointolite (P)

(b)

Figure B.7: Graphs from the work of J. C. Bose, pertaining to experiments on plant growth under various circumstances. (a) Graph from the book *The Physiology of Photosynthesis*. (b) A graph from the book *Growth and Tropic Movements of Plants*.

(Friendly & Denis, 2001, p. 3)

The Milestones project provides a comprehensive archival listing for all type of historical visualisations of data.

B.4 Golden age of graphics

Some scholars studying the history of graphic representation of data call the period from the 1860s to 1890s as the "Golden Age" for graphic representation. During this era, there was a proliferation of graphic practices across domains. Funkhouser (1937) in his work says this about this period.

> The period from 1860 to 1890 may be called the golden age of graphics, for it was marked by the unrestrained enthusiasm not only of statisticians but of government and municipal authorities, by the eagerness with which the possibilities and problems of graphic representation were debated and by the graphic displays which became an important adjunct of almost every kind of scientific gathering. During this period the method was officially recognized by government agencies and became a feature of official publications. Here also is found the first reference to the graphic method as a universal language together with the opinion of more sober statisticians that the method was running away with itself. (p. 330)

Friendly (2008) in the conclusion of his article on this topic, *The Golden Age of Statistical Graphics* says:

> I have tried to show how a collection of developments in data collection, statistical theory, visual thinking, graphic representation, symbolism from cartography and technology combined to produce a `perfect storm' for statistical graphics and thematic cartography in the last half of the 19th century. Moreover, many of these early statistical maps and diagrams, drawn by hand in the pre-computer era, were able to achieve a high degree of graphical impact - what Tukey referred to as interocularity (the message hits you between the eyes). This is a lesson to bear in mind as we go forward. (p. 32)

Just around this age the proliferation of graphics in the mass media began and continues to increase exponentially. The ease of creating a graphic from given data by using "templates" in the software has resulted in an explosion in the number of graphs that are produced and consumed globally in all types of media.

The magic of graphs

To understand the zeitgeist about graphs at the beginning of the 19^{th} century we quote Henry D. Hubbard. This quote appears in the introduction of *Graphic Presentation* by Willard Cope Brinton (Brinton, 1939).

> There is a magic in graphs. The profile of a curve reveals in a flash a whole situation - the life history of an epidemic, a panic, or an era of prosperity. The curve informs the mind, awakens the imagination, convinces.
>
> Graphs carry the message home. A universal language, graphs convey information directly to the mind. Without complexity there is imaged to the eye a magnitude to be remembered. Words have wings, but graphs interpret. Graphs are pure quantity, stripped of verbal sham, reduced to dimension, vivid, unescapable.
>
> Graphs are all inclusive. No fact is too slight or too great to plot to a scale suited to the eye. Graphs may record the path of an ion or the orbit of the sun, the rise of a civilization, or the acceleration of a bullet, the climate of a century or the varying pressure of a heart beat, the growth of a business, or the nerve reactions of a child.
>
> The graphic art depicts magnitudes to the eye. It does more. It compels the seeing of relations. We may portray by simple graphic methods whole masses of intricate routine, the organization of an enterprise, or the plan of a campaign. Graphs serve as storm signals for the manager, statesman, engineer; as potent narratives for the

> actuary, statist, naturalist; and as forceful engines of research for science, technology and industry. They display results. They disclose new facts and laws. They reveal discoveries as the bud unfolds the flower.
>
> The graphic language is modern. We are learning its alphabet. That it will develop a lexicon and a literature marvelous for its vividness and the variety of application is inevitable.
>
> Graphs are dynamic, dramatic. They may epitomize an epoch, each dot a fact, each slope an event, each curve a history. Wherever there are data to record, inferences to draw, or facts to tell graphs furnish the unrivalled means whose power we are just beginning to realize and to apply. (p. 2)

This quote in a way summarises the spirit of the graphical method which was present at the beginning of the 20th century. At the beginning of the 20th century, the use of graphs for representing data and analysing had become the standard practice in all fields of inquiry which allowed collection of quantitative data.

B.5 Reflections

We see that though the conditions for the use of graphs to represent data were there during the eighteenth century, the actual use graphs for displaying data did not start for some time. This is important because the representation of physical quantities which are not areas or lengths by areas and lengths is highly counter-intuitive. The transition from concrete reality to its abstract representation in two dimensions is not easy. As Tufte (1997) remarks:

> Despite their quantifying scales and grids, maps resemble miniature pictorial representations of the physical world. To depict relations between *any* measured quantities, however, requires replacing the map's natural spatial scales with abstract scales of measurement not based on geographical analogy. To go from maps of existing scenery to graphs of newly measured and collated data was an enormous conceptual step. Embodied in the very first maps were all the ideas necessary for making statistical graphics -- quantified measures of locations of nouns in two-dimensional space -- and yet it took 5,000 years to change the name of the coordinates from *west-east* and *north-south* to empirically measured variables X and Y. (emphasis in original, p. 14-15)

The *enormous conceptual step*, as Tufte rightly calls for the transition between the concrete and abstract is a difficult one. Especially for the learners, who have no experience in dealing with graphs of abstract quantities, this can be a significant hurdle. Numerous studies relating to comprehending and constructing the graphs have shown this to be the case (for details see Chapter 2).

Ernst Haeckel's idea that *phylogeny recapitulates ontogeny* has some parallels in this case. Historically, we see, it was not straightforward for people to represent data on graphs, as the various studies show, children also face difficulties in relating to graphs. This historical fact may have wide-ranging implications for education. The graphs, their construction and understanding do not come naturally to the learners. However, the issue of graphicacy in general and graphs, in particular, is not addressed in the current educational set-up.

C

Student Handbook and Pre-Test Questionnaire

This section has the *Student Handbook* given to the students as a part of the Astronomy Summer Camp. The Handbook has the instructions for the two activities reported here: Mustard Seed Task (Chapter 5) and Sun Measurement Task (Chapter 6) along with other activities conducted during the camp.

Table of Contents

An advice to those who Measure the Universe - **Don't Panic!**

In this booklet, you will come across three experiments designed to give you a feel of scales of the Universe and a look at how different methods are applied to measure things at various scales. This guide, true to its name, actually guides you to a complete understanding of those methods and purports to make you aware of all the associated details, errors etc. Rather than being a set of instructions to repeat, it asks a lot of intuitive questions and expects you to find the answers yourself. You will probably know the answers to some; others you will find too difficult. Some, you will find, have no simple answer. This is intentional to put you into a habit of asking further questions. See what you can do without giving up. Moreover, some problems are meant to start a discussion. So do talk to your friends about them.

Good scientists tests what they can, and what they have time for. But they cannot test everything or cannot find all the answers. All the same, they enjoy speculating and wondering about a lot of other things. Altogether there are far too many problems for one to be able to tackle all of them. You will have to pick and choose. Some problems will be more interesting, or provoking, than others. There are good chances you will enjoy them.

ICON KEY

- Hands-on Suggestion. Do it !
- Ask a Question to yourself
- Note down your Observations
- Important information
- Find on the Internet

SUMMER ASTRONOMY CAMP - 2014

IUCAA Muktangan Vidnyan Shodhika

Let us measure the Universe :

A Hands-On Experiments' Reference Guide

IUCAA

Task 1

Find the average diameter of a Mustard seed

Mustard seeds are commonly found in Indian kitchens. The seeds are small and roundish; mostly black, but sometimes yellow in colour. In this task we will try to estimate and measure the average diameter of a mustard seed. Apart from the actual measurement, this task will make you aware about what Errors are, how they can occur and what kind of precautions can be taken to minimise the errors. We will also try to see whether the measurements that we make can be fitted into some mathematical pattern. In trying to understand the natural world around us, scientists try to fit their observations to mathematical patterns, which in turn enable them to predict things which are not known. We will also try to do some predictions in our experiments that we do. How we communicate about our results and measurements that we have to others is also an important part of doing science. As Science cannot be done in isolation, we will see how best to share our findings with our friends and colleagues (peers).

Observing the Seeds

Take a handful of mustard seeds and look at them with your eyes. Observe the shape of those seeds. For better contrast use a sheet of white paper as a background.

When we observe any material things we should try to estimate their size. Estimations are really helpful when at times there is no direct way of actually measuring things.

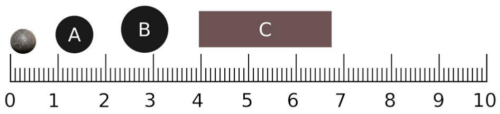
- ✍ What is the general shape of these seeds?
- ✍ Are the seeds all of the same shape and size and colour?
- ✍ Can you estimate the size of these seeds just by looking at them?

The Scale / Ruler

Take a scale or ruler which has markings on it for measurement. Most probably your scale will have two units for measurements - the inch scale and the centimeter scale.

- ◈ What are the different units of measuring length that you know?
- ◈ Why do you think that different units are used? Is it not a good idea to use same units every where?
- ◈ Why are there smaller markings on the scale (they don't even have numbers)?

- ✍ What is the smallest marking that you can find on your scale?
- ✍ In total, how many marking are there on the scale?



The First measurement

You must have previously measured something with the scale that you have.

- ◈ What are the different measurements that you have done with your scale?

- ✍ Can we start our measurements at any point on the scale, or is it necessary to use the **0** at the starting of the scale? Give reasons for your answer.

Not all objects that we want to measure are of size of the scale, some are larger than the scale and some are smaller than the smallest markings on the scale.

- ◈ What do you do when the object to be measured is larger than the scale (let's say a table)?

- ◈ What do you do when the object is smaller than the smallest marking on the scale (let's say thickness of sheet of paper)?

The quantities that we measure can either be continuous and non-standard, like the length of a table or the mass of an object or they can be discreet like the number of seeds (can we have 2.1 seeds?). A problem that can sometimes arise is that the object that is to be measured will not fit on any of the standard markings exactly.

For example: we might have an object which is more than 24 mm but does not quite reach 25 mm. What do you note in such a case: 24 or 25 mm? If we choose to write 24 mm we would be underestimating the value, and if we choose 25 mm we would be overestimating it. In such a case, we can write 25 mm but we have to say that there is an error of about 1 mm in the measurement that we have done.

Getting back to our mustard seeds...

- Can we use the scale that we have to directly measure the diameter of the mustard seed?

- Does the mustard seed fit exactly in the divisions of the scale?

- Try fitting 10 different seeds on the scale. Do all of them measure the same?

Averages

During our measurements, we may find that the seeds have a variation in their sizes. We may also observe that the seeds are not of same shape i.e. unlike the spherical shape that we had assumed them to be. Some of them maybe more flattened at one end. All this tells us that different mustard seeds may have different diameters. But, we cannot measure the diameter of each seed, since they are too many in number. Yet, do you notice that we can say something about their size? We can say that the seeds would mostly be within such & such range of size. In the next step, we measure a large number of seeds and find out the average diameter for these seeds.

- What do you understand by the average of a set of numbers?
- Is there a general formula for finding an average?
- What does an average signify?

The Second Measurement

The method that we will use is measuring many seeds at the same time. For this purpose we will arrange 5 or more seeds in a row on the scale and measure the total length of such an arrangement. This measurement we will do for 5, 10, 15, 20, 25 and 30 seeds. For each number of seeds, we will take 3 sets of readings. For each set of measurements we will change the seeds, i.e. we do not use the same seeds for measurement of 15 seeds, that we had used for 10 seeds.

- What are the advantages of using 5 or 10 seeds over just 1 seed for measurements?

- Do we need to change the seeds for making different observations? Why?

We need to be very careful while making these measurements. For example you need to make sure that there is no space left between the seeds, at the same time the seeds should also be in the same base. Also we need to make sure that the starting point of the row and the end point of the row are taken properly.

- How will you make sure of the above mentioned things?

Presenting and Graphs

- Once you are done with the taking the observations present the numbers in a table for easy reference.

Quantities which vary are called ***variables* !** In our observations we have two variables

1. the number of seeds (let us call it **n**, as it can change) and

2. the total length that you measure for the **n** seeds - **L**

In our case we are choosing the number of seeds **n** to be some exact value that we decide. This is then called an ***independent*** variable. But since the length that we measure depends on the number of seeds, it is will be called a ***dependent*** variable – dependent on what we choose **n** to be.

- Give some more examples of dependent and independent variables.

- What is the type of proportion between these two quantities? Is it a direct or indirect proportion?

Graphs are an excellent way to see whether two quantities are related. Let us see if there is relation between the above two quantities.

- How would you choose whether to plot **n** on the X-axis or the Y-axis? How about **L**?

We also have to choose scales for the two axes to fit all the data that we have collected in our experiment.

- What scales would you choose for X-axis and Y-axis to fit all your data?

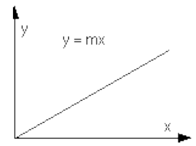
- Draw the graph, mention the scales properly.

Once the plotting of all the points is done, we proceed to check whether there is any relationship between the two quantities that we have plotted. The relationship between two quantities can be of two types, namely linear (like a straight line) and non-linear. In case of linear relationships, the points that we plot will fit/lie on a straight line. In the other case they do not do so.

- Can you draw a straight line that passes through all the points that you have plotted?

- If you can draw a line, then does this line pass through the Origin?

With each line we associate a certain number known as the ***slope*** of the line. The slope of the line determines whether the line is steep or gentle compared to the X-axis. A line that passes through the origin may be expressed as a mathematical equation,



$$y = mx$$

where, y is the dependent variable, x the independent variable and m is the slope of the line. The slope m, can take both positive and negative values.

Modelling

In our case the slope of line has a special significance. If we try to model the graph of our observations (i.e. make it look similar) to a straight line, the equation of our graph should also be similar to the equation for a straight line. We have found out that indeed the two quantities namely, the number of seeds **n** and the measured length **L** are related.

From our definition, we can see that in our case the equation takes the form $L = dn$, if **d** is the diameter of the seeds. Can this mathematical relation be applied generally? Yes.

- What similarities do you get if you compare this equation with the equation for the straight line?
- What is the slope in this case? How will you find it? Does it represent anything? Will the slope change if the line is extended up to a long distance?
- Can you see the use of a graph of real quantities to find another real number?

- From the graph and from the table of values what is the average diameter of the mustard seed that you have found? What is the percentage error?

Predicting

Science is all about finding patterns like we did above. But we should also be able to predict things and be able to test them as well.

- Using the graph can you predict what would be the length of the 400 seeds, when they are placed in a row?
- What is the value that you actually observe for the seeds on the scale? Are the two results matching?

- Suppose that all these seeds have come from the same plant, why do you think they differ in their characteristics?

Task 2

Know the Samrat Yantra

Samrat Yantra literally means the king of all the instruments. The first one was built by Maharaja Jai Singh, in 1727 in Jaipur as a part of his Astronomical observatory – the *Jantar Mantar*. That one has an accuracy of about two seconds. IUCAA has its own *Samrat Yantra* made of Steel and Granite. It has a semicircular Dial embedded through a wall. The wall is in fact a very large right-angled triangle.

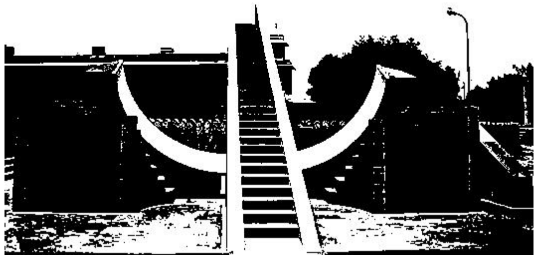


FIGURE: The Samrat Yantra in Jaipur.

In this task we will try to measure & calculate some details about the IUCAA *Samrat Yantra*. We will study how to mark time & other astronomical quantities with it. We will also familiarize ourselves with the branch of mathematics called "Trigonometry" & use some of its identities in our calculations.

Can you answer these questions looking at the IUCAA *Samrat Yantra*

- What are the things needed to find time by the Sundial?

- Does the time shown by the sundial match the time by your watch? Why?

Trigonometry Primer

Trigonometry is a part of mathematics which deals with the angles & the sides of a 'Right-Angled Triangle'. In trigonometry we relate the angles of the right-angled triangle to the ratio of the sides of the "same" triangle. It was originally developed in 3rd century BC as a tool to study Astronomy, in which angles are easier to measure than actual distances.

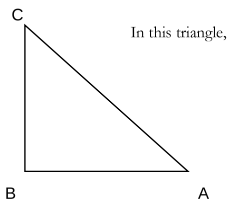
We define the most commonly used ratios to be **Sine, Cosine and Tangent**. These are also written as ***sin, cos*** and ***tan***

- Consider a right-angled triangle with three corners or vertices A, B, C. Let B be the vertex at which the angle is 90°. Now, the angle at the vertex A will have the trigonometric functions defines as follows.

$$Sine(sin) = \frac{Opposite\ Side\ to\ the\ Angle}{Hypothenuse}$$

$$Cosine(cos) = \frac{Adjacent\ side\ to\ the\ given\ angle}{Hypothenuse}$$

$$Tangent(tan) = \frac{Opposite\ side\ to\ the\ given\ angle}{Adjacent\ side\ to\ the\ given\ angle}$$



In this triangle, for the angle at vertex A, we write

$$\sin(A) = \frac{BC}{AC}$$

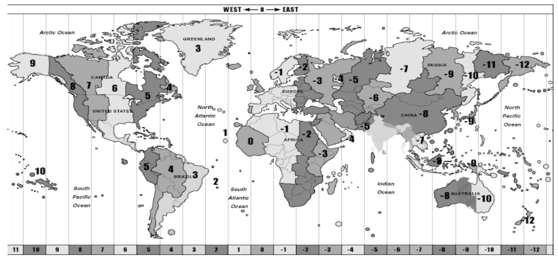
$$\cos(A) = \frac{AB}{AC}$$

$$\tan(A) = \frac{BC}{AB}$$

- 🕮 Writing ***sin (A)*** DOES NOT MEAN ***sin multiplied by A*** or ***sin × A.***
- 🕮 ***sin (A)*** is a "function" of ***A***. We can call it as "sine of A" which gives a certain value only if some value of an angle is given to *A*. The same is applicable for ***cos*** & ***tan***.
- 🕮 For all angles between *0°* to *90°* values of ***sin, cos, tan*** are unique i.e. No two angles have same values of ***sin, cos, tan***.

Is "Time" Geography or Astronomy?

As a concept of time, a day is defined as the time required for the Earth to completely rotate by 360°. Since the day is divided into 24 hours, the Earth rotates at the rate of 15° an hour. This could be noted only because of the astronomical observation that the Sun changes its position in the sky (exactly at the same rate as the Earth's rotation). The year is defined from the observations of the time taken by a given star to return to its exact sky position at a standard time by your watch. This motion of stars is due to the revolution of the Earth. Again, this concept has come from the studies & notes made by Astronomers.



Meridians & Longitudes

We study Longitudes in Geography. Originally called Meridians of longitude, these are lines drawn from the North Pole to the South Pole and are at right angles to the Equator. The "Prime Meridian" which passes through Greenwich, England, is used as the zero degree line from which measurements are made in degrees east and west. The meridians are also useful for designating time zones. Noon at any longitude is defined as the time when the Sun is directly above that meridian. To the west of that meridian is forenoon, the sun's position in the sky being Ante-Meridian and to the east is afternoon, with the sun Post-Meridian.

⯐ Why are there 360 meridians of longitudes? Why not 100 or 500?

Since different longitudes come under the Sun at different times, for a country as wide as India, the extreme east & west locations will have a great time difference between the "Noons". To avoid inconvenience due to this, we have adjusted all watches to a time of an intermediate longitude of 82.5° E longitude.

✋ Verify that this Indian Standard Time (IST) is 5.5 hours ahead of GMT.

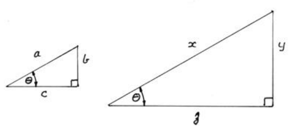
Task 3

Find the distance to diameter ratio of the Sun

In this task we will try to observe & calculate some details about our star - the Sun. The Sun is the most important object in our Solar System. It is enormous as compared to Earth. But it appears small to us because it is very far away. This is similar to a building it appearing large when close by, but growing smaller size as we move away. We know that in reality it is not shrinking. But can we measure the distance to the Sun or its actual size, without leaving the Earth? In this task we will work on a method which will give us not the actual distance or diameter, but a ratio of the two. We are going to use some geometry that we have already learnt. In particular we will make use of the properties of similar triangles.

Similar Triangles

Similar triangles have the same shape but not the same size. The corresponding angles of the two triangles are congruent, and that the corresponding sides are in the same ratio. Trigonometric functions (see Task 3) can be used to find sides of similar triangles where only one side and angle are known.



Making our own Solar Camera / Sun Measurer

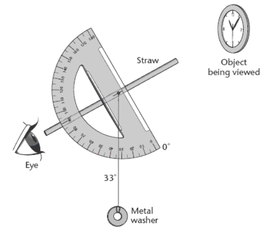
We have to make an instrument for this task which will create an image of the Sun that can be measured. Let us call it the "Solar Camera". This is basically a "pinhole" mounted on a scale. This pinhole gives us an image of the Sun which can be obtained on a screen at a certain distance from the pinhole.

Making our own Astrolabe

We need
(i) A protractor,
(ii) A straw,
(iii) A thread
(iv) A small weight
(v) Glue.

In ancient times, astronomers used a very simple tool to measure the height of a star in the sky. This instrument, the ***Astrolabe***, enabled scientists to create the first star charts. People relied on the astrolabe to determine the altitude of objects above the horizon. Using measurements made with an astrolabe, navigators could even calculate their latitude. In this activity you will make and test an astrolabe.

Make: Take your protractor and glue the straw along its central 90° line. Now stick a small string to the protractor so that it dangles exactly from the point where the zero degree and ninety degree lines coincide. Tie a small weight to the end of this string so that the string hangs vertically.

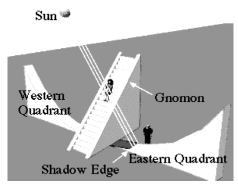


Use: Look at the top of any tall object or a star through the straw. Read the degrees which coincide with the vertical thread from the side of the astrolabe. This is the angular height of the object from your eye-line or of the star from the horizon.

- Can you find any other use of an Astrolabe or any other way of making it?

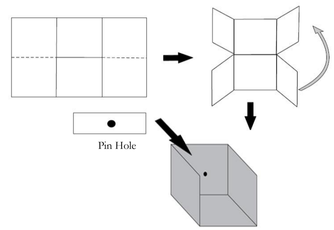
Observing the Samrat Yantra

The Samrat Yantra shows time by using the shadow of a wall (also called the ***gnomon)***, due to the Sunlight, falling on a marked dial.



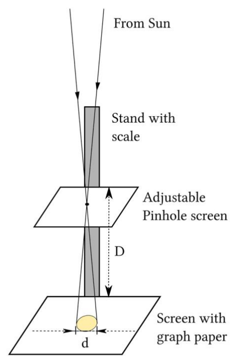
- Find longitude of Pune by noting time-difference of the dial and your watch.
- Find the angle of inclination at the base.
- Find the height of the IUCAA Samrat Yantra. Use any method that comes to your mind, but do use at least one that used the above mentioned angle & trigonometry.
- The shadow of a sundial's ***gnomon*** points to the North every time the Sun is overhead. This time is also called the local noon. Now go ahead & find the exact North for IUCAA.

We need
(i) A metre scale,
(ii) A pinhole (a sheet of metal with a small hole made with the tip of a pin),
(iii) cardboard sheets,
(iv) mm graph paper,
(v) glue and
(vi) cellotape



We will first prepare box like structures using cardboard sheets. Take the rectangular card-sheet and fold it to get six squares on the surface as shown in the figure. Then cut the sheet in the way shown and fold and paste it to make a partial box. Make two such boxes. On one of the boxes we will punch a hole on top of which we can place the pinhole. The pinhole can be made out of a discarded Aluminium can. We use the other "box" for the screen. Attach a graph paper on one side of this box (from the inside) so that we can see the dimensions of the image and read them off it easily.

When you have your pinhole box and screen box, attach them at the opposite end of a metre scale and your Solar Camera is ready!



- ✍ At which end of the scale will you fix the graph paper screen? (0cm or 100cm)
- ✍ Which coloured sheet will you prefer for making graph paper screen?
- ✍ What is so special about aluminium? Can you use any other material instead?
- ✍ What is the purpose of the pinhole? What should be the size of the pinhole?

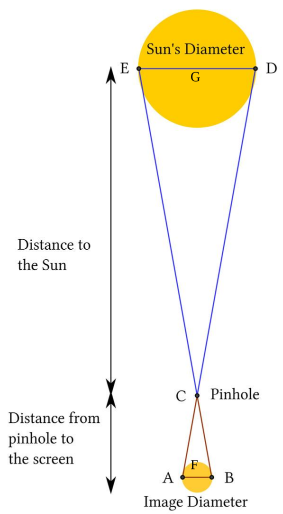
◈ Have you heard of the Pin-hole camera? Do you think it is somehow related to this setup?

The basic setup of the instrument is as shown above. The boxes are meant to prevent extra light from disturbing us and are not shown. For the experiment, we need to be able to adjust the distance between the pinhole and the screen to get an image of the Sun.

The Geometry

To find the distance-diameter ratio for the Sun, we construct similar triangles with the pinhole as one of the vertices and the diameter of the Sun and its image as bases. We can see such a construction in the figure below (this figure is obviously not to scale). In this figure, Δ ***ABC*** is formed with ***C*** the pinhole at its vertex, and ***AB*** the diameter of the image of the Sun as its base. We must make sure that the screen containing the pinhole and the screen on which we see the image are parallel. Now, if we extend the figure on the other side of the pinhole, we get another triangle, with the diameter of the Sun **ED** as its base, and the pinhole again at the vertex.

Can we prove that these two triangles are similar, i.e. Δ ***ABC*** ~ Δ ***CDE***? The task to prove this is left as an exercise for you. We will have to make some assumptions to prove the same. What are these assumptions?



Once it is proved that these two triangles are similar, we have the following ratios of the sides which are equal:

$$\frac{AC}{CD} = \frac{BC}{CE} = \frac{AB}{DE}$$

Another thing that we notice from the actual readings in the instrument, is that the lengths of the sides ***CA*** and ***CB*** in the *Δ* ***ABC*** are very large as compared to the length of side ***AB***. Now if we draw the height of *Δ* ***ABC*** from the vertex ***C***. Let the midpoint of ***AB*** be F. Then we would have ***AC ≈ CB ≈ CF***. That is to say if angle ***ACB*** is very small, ***CF*** is almost equal to ***AC*** or CB. Same would be true for the other triangle we have, namely *Δ* ***CDE***. This approximation will be only valid when ***<ACB*** is very small.

◈ How small? What is the percentage error if we make this approximation?

If we measure the height of Δ ***ABC***, which is the distance between the pinhole and the image screen in our case, we have measured the sides of Δ ***ABC*** as well. To find the base ***AB*** we have a graph paper on which we see the image of the sun, and we can read the value of the diameter directly off the graph paper.

Diameter / Distance

Now we have two observations noted - one is the height of the *Δ* ***ABC***, ***CF*** which is approximately equal to ***AC*** or ***BC***, and the second is that of the base **AB**. We have

$$\frac{AB}{AC} = \frac{DE}{CD}$$

But since $AC \approx CF$

$$\frac{AB}{CF} = \frac{DE}{CG} = \frac{\text{Diameter of the Sun}}{\text{Distance to the Sun}}$$

where ***G*** is the midpoint of ***DE***. Thus we can find the distance to diameter ratio of the Sun by measuring the two quantities with our instrument!

- What are the sources of error in this experiment?
- What are the precautions that we take while taking the readings?

We take another set of readings by changing the distance between the pinhole screen and the image screen. Take at least three readings. Make a table for writing down the values of the two observed values i.e. the distance of the pinhole from the image and the diameter of the image on the image screen. In the same table note the ratios of the two values.

- Do you get same values for the ratios in each case?

We can take yet another approach to find the ratio. We plot these values on a graph. (What would you plot on the X-axis and the Y-axis?) Choose the scales properly so that all of the readings are displayed in the graph.

- Can we pass a straight line through the points on the graph?
- Can you relate the slope of this line to the ratio of the diameter to distance for the Sun?

On Page 15 you will find a table of 'Scales of the Solar System', where various sizes and distances are given. Combined with the ratio that we have calculated, you can now use it to determine either the Diameter of the Sun or its Distance from the Earth.

- Calculate the Diameter of the Sun
- Calculate its Distance from the Earth.
- Use the table to design a Solar System Model with an Earth of 1 m diameter.

Points to Ponder

There are several more questions that you can think about:

- What is the best time to perform this experiment?
- What are the differences between measuring diameter of a mustard seed and the Sun?
- Can you find the diameter of the moon using the same method?
- Can you think of any other method for finding the diameter of the Sun?
- Is the distance between the Earth from the Sun always same?
- What are the sources of error in this experiment and how will you minimise them?
- What are the assumptions taken when doing this experiment?

Internet Resources

- An interactive Time Zone Map: timeanddate.com/time/map
- A sky simulation software: stellarium.org
- Monthly Sky Maps: skymaps.com
- Information about Solar system: spaceweather.com
- Daily Astronomy Picture: apod.nasa.gov
- Online Science Projects: zooniverse.org/projects

SCALES OF THE SOLAR SYSTEM

Name	Radius in km	Scaled Diameter in meters	Distance from Sun in km	Scaled Distance in meters
Sun	695000		-	
Moon	1738		384000 km from Earth	
Mercury (बुध)	2440		57910000	
Venus (शुक्र)	6052		108200000	
Earth (पृथ्वी)	6378	1	149600000	
Mars (मंगळ)	3397		227940000	
Jupiter (गुरु)	71492		778330000	
Saturn (शनि)	60268		1429400000	
Uranus (युरेनस)	25559		2870990000	
Neptune (नेप्चून)	24766		4504300000	

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NOTES

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Credits (this booklet was made in 2013 with the help of the following people)

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NOTES

C.1 Pre-Test Questionnaires

This section has the Pre-test questionnaires the students were administered. The setting of these two questionnaires was in the context of the *Astronomy Summer Camp*. The students took the Pre-test on the first of the camp. Some of the responses from these tests featured in the discussions for Mustard Seed Task (Chapter 5) and Sun Measurement Task (Chapter 6).

IUCAA Astronomy Summer Camp

Questionnaire for Basic Astronomy and Physics Understanding

Note: Answer ALL questions with whatever you think is the answer. But do think.

Short Answer type:

1. A friend tells you the earth is fixed in space and that the sun revolves about it. Which one of the following facts contradicts his hypothesis?
   a. Each day the sun rises in the east and sets in the west.
   b. During the night the stars appear to move.
   c. The sun makes one complete trip among the stars in one year.
   d. Eclipses of the sun sometimes occur.
   e. none of the above

2. ALL EXCEPT ONE of the following statements are true. Which is the exception?
   a. The earth moves fastest when it is nearest to the sun.
   b. The earth's orbit lies in a plane which passes through the sun.
   c. A line drawn from the sun to the earth sweeps over the same area from March 21 to March 23 as it does from December 21 to December 23.
   d. The sun is at the exact center of the earth's orbit.
   e. The earth's orbit around the sun is an ellipse.

3. To ancient observers, the principal difference between the planets and the stars was that the planets appeared
   a. brighter.
   b. more like the earth.
   c. to wander among the other stars.
   d. closer to the earth.
   e. to travel around the sun.

4. Which of the following correctly places the Earth, Jupiter, Mars, the Moon and the Sun in order of increasing mass?
   a. Moon, Earth, Mars, Sun, Jupiter
   b. Moon, Mars, Earth, Jupiter, Sun
   c. Mars, Earth, Moon, Jupiter, Sun
   d. Moon, Mars, Earth, Sun, Jupiter
   e. Moon, Earth, Jupiter, Mars, Sun

5. Two sacks of marbles are hung one meter apart. Which of the following would approximately double the gravitational force between the two sacks?
   a. Double the number of marbles in one sack.
   b. Double the number of marbles in both sacks.
   c. Move them closer, to one-half the separation.
   d. Move them further apart, to twice the separation.
   e. Move them further apart, to four times the separation.

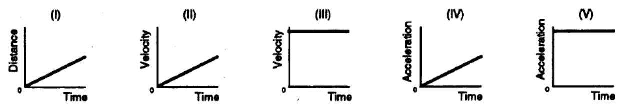
Long Answer type:

6. What are the ancient and modern evidences which lead us to believe that
   a. the Earth is round?
   b. the Earth is spinning?

7. How could you use the shadow cast by a vertical stick on horizontal ground (let's call it "the gnomon") to find the local noon?

8. Draw a rough diagram to show the relative positions of Sun, Earth and Moon to explain the phases of the Moon.

9. Consider the following graphs showing the motion of a spaceship.



   Noting the different axes, which of these represent(s) motion at constant velocity & why?
   a. I, II, and IV
   b. I and III
   c. II and V
   d. IV only
   e. V only

10. How can you measure approximate distance between sun and earth, moon and earth?

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