An Experiment on the Formation of Rainbows

RAJESH B. KHAPARDE AND H.C. PRADHAN
Homi Bhabha Centre for Science Education
Tata Institute of Fundamental Research
V. N. Purav Marg, Mankhurd
Mumbai 400088, India

(rajeshkhaparde@gmail.com)

ABSTRACT
We present here an undergraduate level experiment which illustrates in a college laboratory, the formation of rainbows of different orders on the basis of refraction, internal reflection and dispersion of light incident on a suspended drop of liquid. These rainbows are observed directly by the eye and their angular positions are measured using the telescope and the circular scale of a prism spectrometer. Thus, the variation of total deviation of the incident light for different orders with the order of the rainbow is studied in case of a water drop. We then obtain rainbows with different liquids and measure the total deviation for the second order rainbow and determine the refractive index of the unknown liquid ‘A’ graphically.
Introduction

A rainbow is probably one of the most spectacular natural light show observed in the sky. A number of scientists and mathematicians, including Aristotle, Bacon, Theodoric, Descartes, Newton, Young, Airy and Mie have worked on the explanation of various observations on rainbows. A rainbow is produced, when sunlight falls and gets diverted to the eye of the observer due to a large number of water drops in the sky, on a rainy day. We can observe various colors in the sky at particular angles. This is due to the fact that sunlight is made up of a range of colors, and the light of different colors is refracted at different angles, when it passes from one medium into the other. Thus ‘dispersion’ occurs as the sunlight propagates through the drop. In the case of a primary (first order) rainbow, the sunlight incident on a raindrop is refracted at the front surface, internally reflected at the back surface and refracted again as it emerges into the air, as shown in Figure 1.

Figure 1. Ray diagram for the formation of a primary rainbow with the sunlight.

A secondary (second order) rainbow occurs in the same manner as the primary rainbow, but it is due to two internal reflections. In nature we can observe only primary and secondary rainbows. Higher order rainbows are never seen since they are weaker than (i) the background sky brightness, (ii) the light reflected from the outside surface of the drops and (iii) the light transmitted through the drops with no internal reflections.

To understand the formation of rainbows, one should actually study the refraction, internal reflection, dispersion and total deviation of white light by a spherical water drop. In the present experiment, we study the formation of multiple order rainbows using a suspended drop of a liquid. (Note that, here we use the term ‘rainbow’ to mean a ‘rainbow like spectrum and effect’ and not strictly the ‘rainbow’ in the sky.)
Objectives

A) To understand the refraction, internal reflection and dispersion of light due to a suspended water drop, which give rise to multiple order rainbows and study the variation of total deviation of the incident light with the order of the rainbow.

B) To determine the refractive index of an unknown liquid ‘A’ by studying second order rainbow formed due to suspended drops of different liquids.

Theory

When white light is incident on a suspended drop of liquid, the suspended drop will produce different order rainbows as a result of two refractions and \(K\) \((K = 1, 2, 3, \ldots)\) internal reflections of light. The first order rainbow corresponds to one internal reflection. The second order rainbow corresponds to two internal reflections. Thus, the \(K^{th}\) order rainbow corresponds to \(K\) internal reflections.

The geometrical theory of the formation of a rainbow requires knowledge of the manner in which a pencil of parallel rays is refracted and internally reflected in a spherical drop of water. Consider the ray diagram as shown in Figure 2 for the first order \((K = 1)\) rainbow formed by a spherical drop of liquid with refractive index \(\mu\).

![Ray diagram for the first order rainbow.](image)

Let RABCS be the path of a ray of incident monochromatic light, which emerges out of a spherical drop of liquid at C after suffering two refractions at A and C and one internal reflection at B.

Let \(i\) and \(r\) be the angle of incidence and the angle of refraction at A respectively, \(r\) also represents the angles of incidence at B and C. Since the angle of incidence inside the drop at C is \(r\), the angle of emergence at C will be \(i\). The deviation of the ray due to refraction at A and C are \((i - r)\) each. The deviation of the ray
due to the internal reflection at B is \((180 - 2r)\). Thus the total deviation is

\[
\phi_i = 2(i - r) + (180 - 2r)
\]

For the second order rainbow instead of one internal reflection, there are two internal reflections as shown in Figure 3.

\[\begin{align*}
\phi & = 180 + \theta \\
S & \quad R \\
& \quad A \\
& \quad \theta \\
& \quad B \\
& \quad D \\
& \quad C
\end{align*}\]

Figure 3. Ray diagram for the second order rainbow.

The deviation due to 2 internal reflections will be \(2(180 - 2r)\). Similarly for the \(K^{\text{th}}\) order rainbow (corresponding to \(K\) internal reflections) the deviation will be \(K(180 - 2r)\). As there are only two refractions at the point of incidence and emergence, the total deviation \(\phi_K\) suffered by the incident ray after \(K\) internal reflections will be,

\[
\phi_K = 2(i - r) + K(180 - 2r)
\]

We can write,

\[
\phi_K = K(180) + 2i - 2r(K + 1)
\]

(2)

For the minimum value of \(\phi_K\), \(\frac{d\phi_K}{di} = 0\). Note that \(\frac{d\phi_K}{di} = 0\) represents the condition of minimum deviation, is clear from the fact that if \(\frac{d^2\phi_K}{di^2}\) is calculated, it is found to be positive.

From Equation (2), we have,

\[
\frac{d\phi_K}{di} = 2 - 2\frac{dr}{di} - 2K\frac{dr}{di}
\]

\[
\frac{d\phi_K}{di} = 2 - 2(K+1)\frac{dr}{di}
\]

The condition,

\[
\frac{d\phi_K}{di} = 0 \Rightarrow 2 - 2(K+1)\frac{dr}{di} = 0
\]

Hence,
\[
\frac{dr}{di} = \frac{1}{(K+1)} \tag{3}
\]

If \( \mu \) is the refractive index of the material of the drop for a particular wavelength, then from Snell’s law,

\[
\mu = \frac{\sin i}{\sin r} \tag{4}
\]

Hence,

\[
\sin i = \mu \sin r \tag{5}
\]

We can also write,

\[
r = \sin^{-1}\left(\frac{\sin i}{\mu}\right) \tag{6}
\]

The equation giving the relation between the total deviation \( \phi_K \) for the \( K^{th} \) order rainbow, which relates to the angle of incidence \( i \) and the refractive index \( \mu \), can be written as (substituting Equation (6) in Equation (2)),

\[
\phi_K = K(180) + 2i - 2(K+1)\sin^{-1}\left(\frac{\sin i}{\mu}\right) \tag{7}
\]

Now, let us derive the expression for the angle of incidence for different order rainbows. For this, differentiating Equation (5) with respect to \( i \), we obtain,

\[
\cos i = \mu \cos r \frac{dr}{di}
\]

Substituting for \( \frac{dr}{di} \) from Equation (3),

\[
\cos i = \frac{\mu}{K+1} \cos r
\]

\[
= \frac{\mu}{K+1} \sqrt{1-\sin^2 r}
\]

\[
= \frac{\mu}{K+1} \sqrt{1-\sin^2 i} \tag{7}
\]

Squaring both sides, we can obtain,

\[
(K+1)^2 \cos^2 i = \mu^2 - \sin^2 i = \mu^2 - (1 - \cos^2 i)
\]

\[
\Rightarrow \quad (K+1)^2 \cos^2 i = \mu^2 - 1
\]

\[
\Rightarrow \quad (K^2 + 2K)\cos^2 i = (\mu^2 - 1)
\]

Hence,

\[
\cos i = \frac{\sqrt{\mu^2 - 1}}{K(K+2)} \tag{8}
\]

Thus, for

\[
K = 1, \quad \cos i = \sqrt{\frac{\mu^2 - 1}{3}}
\]

\[
K = 2, \quad \cos i = \sqrt{\frac{\mu^2 - 1}{8}}
\]

\[
K = 5, \quad \cos i = \sqrt{\frac{\mu^2 - 1}{35}}
\]

Equation (8) gives the relation between the refractive index of a liquid and the angle of incidence for different order rainbows. Note that the each order of rainbow is due to white light incident on the drop at a well-determined angle of incidence, which is different for each order of the rainbow. Each order of rainbow contains all the colors of the spectrum of incident white light.

**Apparatus**

A modified prism spectrometer, an incandescent lamp mounted inside a wooden
box, a power supply for the incandescent lamp, a holder for the syringe, four small syringes, four small beakers, four small petri dishes, a magnifying reading torch, a spirit level, a thick black paper and small amount of water, glycerine, clove oil and an unknown liquid ‘A’

Experimental Setup

A prism spectrometer is an optical instrument, which may be used to observe the spectrum, measure the angles and hence study reflection, refraction, dispersion, diffraction, etc. It consists of four parts, a collimator, a telescope, a prism table and a circular scale disc. The collimator is used to obtain a parallel beam of light. The telescope is used to collect and observe the parallel light. The prism table is used to mount optical components like, prism, grating, etc. The circular scale disc is used to measure and record the position of the telescope arm and thus it measures the angle through which the telescope is moved. The circular scale disc has three scales, a circular main scale and two identical vernier scales. The vernier scales are fixed to the prism table and the circular main scale moves with telescope arm.

Figure 4. Photograph of the complete experimental setup.

In this experiment, we use a spectrometer in which the regular arrangement of the collimator, telescope and the prism table is modified. In the ‘modified’ spectrometer, the mounting base of the collimator arm is extended to mount the collimator little away from the circular base (as shown in Figure 4), which helps to increase the range of movement of the telescope. The collimator has a regular lens arrangement and slit at the front. An extra adjustable slit may be mounted externally at the back of the collimator. The extra slit will allow us to block the light falling on one side of the liquid drop so that only half the liquid drop is illuminated. Here, the telescope does not have the regular lens arrangement, instead the eyepiece is replaced by a pinhole and the objective is replaced by a lens of suitable focal length, to obtain a magnified image of the drop. The circular scale disc is kept as it is, but its black cover is removed.
A high power (150 W, 24 V) incandescent lamp (tungsten filament lamp) is provided as a source of white light. This lamp is mounted on a stand with adjustable height, inside a wooden box. A small cooling/exhaust fan is fixed inside the wooden box to avoid any damage due to excessive heating. A specially designed AC power supply is made available to provide the necessary power to the lamp. This power supply has one common and six fixed output terminals. The power supplied to the lamp and hence the intensity of the light emitted by the lamp may be changed by connecting the lamp to different fixed output terminals of the supply. This is helpful in clearly observing the rainbows, as one may need to have a higher intensity of the incident white light for higher order rainbows.

A specially designed syringe holder is provided to clamp a syringe. This holder is fixed on the prism table (as shown in Figure 4). The holder is designed such that when the syringe is clamped properly and the drop is obtained, the drop is seen from all the angular positions of the telescope and that, too, exactly at the center of field of view. A set of four small syringes (2 ml) is provided to obtain, at the nozzle, a steady drop of the given liquids. Four beakers and petri dishes are provided. Beakers are used to store different liquids. Petri dishes can be placed on the prism table and are used to collect the liquid drops, falling from the nozzle of the syringe. Four different liquids, namely water, glycerine, clove oil and an unknown liquid 'A' are provided. A magnifying reading torch is made available to help in reading the circular scale. Thick black paper is supplied to block the stray light falling on the drop. A spirit level may be used to level the prism table.

**Useful Data**

1) Refractive index of water = 1.33
2) Refractive index of glycerine = 1.47
3) Refractive index of clove oil = 1.53

**Warning**

1) Avoid observing the intense beam of light, emitted by the source, directly or through the telescope for a long time. This may strain or harm the eye.

2) Clamp the syringe on its holder carefully, so that the syringe is held properly and can be seen through the telescope at various angular positions.

3) Use the set of petri dish, beaker and syringe, which correspond to the same liquid.

4) Light reflected directly from the outer surface of the drop may produce bright white glare spots that may hinder your observations. You will have to take care of this and identify the rainbows carefully.

5) The adjustments and the measurements have to be performed in a fairly dark room.

6) You may use only one (out of two) vernier scale of the spectrometer for recording the readings. Use the same vernier scale throughout the experiment.

**Procedural Instructions**

**Part A**

Initially spend some time and try to understand the use of the modified spectrometer, its main parts and the modifications. Determine the least count of the given modified spectrometer. Level the prism table of the spectrometer using the spirit level. Adjust the collimator to give a nearly parallel beam of light. Take some water in the syringe and clamp the syringe on its holder, which is fixed to the prism table. Keep the petri dish on the prism table to collect the drops of water, which may fall from the nozzle of the syringe. Obtain a steady drop of fairly
large size at the nozzle of the syringe. Adjust the position of the syringe such that the drop is seen at the center of the field of view of the telescope for all angular positions.

Align the source, collimator, suspended drop and telescope, such that the drop is fully illuminated by the parallel beam of light. This should be carried out with both the collimator slits fully open. You may have to adjust the height of the prism table or source for the above alignment. Once this alignment has been carried out, partly close the externally mounted back slit of the collimator so that only the right half of the water drop is illuminated by the white light.

Keep the telescope, the collimator and the drop in a straight line, so that the intense white light coming from the central region of the drop may be seen. Note this angular position of the telescope and treat it as the direct reading for further measurements. As said earlier, this drop will produce rainbows of different orders, which can be observed at different angular positions around the drop. The angular positions of rainbows of different orders (for water) should be as shown in Figure 5.

![Diagram of experimental arrangement for obtaining rainbows with a water drop.](image)

Figure 5. Schematic diagram of the experimental arrangement for obtaining rainbows with a water drop.

Now, move the telescope in the clockwise direction and observe the bright first order rainbow \((K = 1)\) first by the eye and then by the telescope. This is the most intense rainbow in which all the colors from red to violet can be clearly seen. Observe the red and violet colors and note the sides/edges/angles at which they are seen. Adjust the telescope such that the bright red line of the spectrum can be seen. Record the spectrometer reading for this angular position of the telescope and determine the total angle of deviation \(\phi_1\).

Repeat the above for the second order rainbow \((K = 2)\) and determine the total angle of deviation \(\phi_2\). Observe the second order rainbow carefully, which may be little less intense than the first order. Note the sides/edges/angles at which you see the red and violet ends of the rainbow. You will find them exactly opposite to those in the first order rainbow.

Repeat the above for higher orders up to at least the fifth order \((K = 5)\). (In this case you may have to take the maximum possible...
intensity of the incident light.) Determine the total angle of deviation $\phi_K$ for the rainbows of different orders. (Note: Before taking readings for each order, adjust the telescope to see the red light of the spectrum.) Plot a graph of $\phi_K (K = 1, 2, 3, \ldots)$ against $K$ for the rainbows of different order formed due to a water drop.

**Part B**

Replace the syringe and petri dish containing water, by the syringe and petri dish containing glycerine. Obtain a drop of glycerine at the nozzle of the syringe and adjust the syringe to see the drop through the telescope. Measure $\phi_2$ for glycerine. Repeat the above steps and measure $\phi_2$ for the clove oil and the given unknown liquid 'A'. Plot a graph of $\cos(\phi_2/6)$ against $1/\mu$ for water, glycerine and clove oil. (Note that for $\mu = 1$ (air), $\phi_2 = 0$. This can be taken as one of the points for plotting the above graph.) Obtain the best-fit straight line graph through these points. Knowing the value of $\phi_2$, determine the value of $\mu$ for the liquid 'A' from the graph.

**Observations and Results**

**Part A**

The experimental setup was arranged to observe rainbows of different orders with a drop of water. The direct reading of the incident white light was recorded and the angular positions of different order rainbows were measured. Thus the total deviation $\phi_K$ for each order of rainbow was determined. A graph of total deviation $\phi_K$ versus the order of the rainbow $K$ was plotted as shown in Figure 6. This graph was a straight line as predicted from Equation (7).

![Graph of total deviation vs order of rainbow](image)

**Figure 6.** Graph of the total deviation $\phi_K$ versus the order of the rainbow $K$ for a drop of water.
Part B

Now, different order rainbows were observed with the other liquids, namely, glycerine, clove oil and the unknown liquid 'A'. The total angle of deviation $\phi_2$ for only the second order rainbow was measured for each liquid. A graph of $\cos(\phi_2/6)$ versus $(1/\mu)$ was plotted as shown in Figure 7. The graph was a straight line ($y=Px+Q$). The slope $P$ of the straight line was 0.858 and the Y-intercept $Q$ was 0.144. Thus, the refractive index of the unknown liquid 'A' was found out to be 1.45.

![Figure 7. Graph of $\cos(\phi_2/6)$ versus $1/\mu$ for different liquids.](image)

Learning Outcomes of the Experiment

This experiment involves skill and illustration types of experimental activities. Here, detailed procedural instructions are provided to the students. This experiment emphasizes the development of conceptual understanding and experimental skills. We describe below important aspects of conceptual understanding and experimental skills which we expect the students to get introduced to and develop through this experiment.

Conceptual Understanding

Reflection, refraction and dispersion of light due to a drop of a liquid, angle of incidence/refraction, Snell's law, wavelength of light, refractive index of a medium, total deviation of light, conditions of minimum deviation, formation of different order rainbows, etc.

Experimental Skills

1) Leveling and using the modified spectrometer (skill of alignment and use of a new instrument).
2) Reading the circular and vernier scales and determining the least count of a prism spectrometer (skill of recording).
3) Arranging the collimator for parallel light (skill of alignment).
4) Adjusting the drop in the center along the collimator and the telescope (skill of alignment).
5) Illuminating half of the liquid drop and reducing unnecessary glare (skill of alignment).
6) Identifying the exact angular position of rainbows of different orders (skill of observing).
7) Measuring the angular position of rainbows of different orders from the circular scale of the spectrometer (skill of measurement).
8) Locating the rainbows and noting the readings without disturbing the arrangement (skill of control).
9) Plotting the linear graphs (skill of graph plotting).

Possible Modifications

We present below a list of possible modifications, which may be tried out and incorporated in the existing experiment.

1) One can use a variety of liquids and study the formation of rainbows of different orders. Thus one can measure the total deviation and determine the refractive index.
2) With a water drop, and a sufficiently intense source of light one can study rainbows of higher orders. People have even observed the rainbow upto the 15th order, in a suspended water drop. Thus one can measure the angle of deviation for various colors for rainbows of different orders. One can even determine the refractive indices for various colors.
3) One can use a low power He-Ne laser source (or a sodium vapor lamp) and study the refraction and reflection of monochromatic light through a suspended liquid drop.

4) Using the same setup, with extra optical filters, one can study the refraction and reflection of light through a suspended drop for different colors. Thus one can measure the angle of deviation for different colors and determine the refractive index.
5) One can demonstrate the formation of a complete circular rainbow in the laboratory with a white light source.
6) One can use a bigger syringe or a glass rod with a polished end to obtain a bigger drop of the given liquid. (Note that the size of the drop will be limited by the surface tension.) This will improve the visibility and reliability of measurements.
7) One can obtain the rainbow using a sprinkler, spray, fountain or one can even use very small glass spheres, used for indicators/reflectors. With an appropriate arrangement of the source, one can obtain the primary (first order), secondary (second order) and even higher order rainbows. One can even obtain a $360^\circ$ complete circular rainbow with an appropriate experimental arrangement.
8) One can use glass or acrylic spheres to obtain the rainbow and determine the angle of deviation and hence the refractive index of glass or acrylic.
9) Using a suitable photodetector, one can study the relative intensity of light obtained for rainbows of various orders and thus estimate the loss of intensity of light during multiple reflections inside the liquid drop.
10) One can study the formation of rainbows with suspended liquid drop of various sizes. One can measure the angle of deviation for drops of different sizes and show that the angle of deviation is independent of the size of the drop. This is the reason, why one can observe a rainbow in the sky at a particular angle in spite of the water drops having different sizes.
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References


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