

## Trends in Mathematics Education Research: The Example of Algebra Education

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In this lecture, I will survey some recent trends in mathematics education, by examining the evolution of the study of the teaching and learning of algebra. Even though the field of research in mathematics education is small by comparison with the major disciplines in science and the humanities, it is now too large to make strong claims about trends and findings, so a narrowing is essential. Algebra education is a good choice because it has been a particularly active field and I can draw on the outcomes of the recent 12th ICMI study on “The Future of the Teaching and Learning of Algebra” (see Stacey, Chick & Kendal, 2004 and Chick, Stacey, Vincent & Vincent, 2001) where many of the world’s leading algebra educators worked together to identify major trends and consider the way forward. More importantly, algebra education is a good choice because many of the major concerns of mathematics education as a whole also impact on algebra education.

I see mathematics education as essentially a practical discipline, where the underlying goal is always to promote better learning of mathematics by students. Of course there are many subtleties of what mathematics should be learned and why, in whose interest is it that students learn, how achievement is measured etc, but the discipline of mathematics education is underpinned by a faith that a good education in mathematics benefits both the individual and society. These benefits include shared values (e.g. empowerment of the individual, the value of logical thought, etc) as well as practical outcomes such as the support of scientific and business endeavours.

Mathematics education is an interesting discipline because it draws on many other fields of study. To answer its central question of “how to teach mathematics better” requires understandings from many other disciplines (e.g. psychology, human development, sociology, mathematics, philosophy and epistemology) and many areas of general education (pedagogy, curriculum studies, policy, etc). All of these are also relevant to algebra education. Mathematics education also draws its research methods from these and other disciplines, and the need to take stock of the plethora of approaches is an ongoing area of concern. The status of the find-

ings in mathematics education is mixed. Some results describe in a very deep way the basic interaction of the human brain and mathematical ideas. These results are likely to be observable in any society and across time. As the oldest human study, stretching back millennia, mathematics retains an identifiable core of fundamental processes by which it is developed. One example particularly relevant to algebra is the way in which many concepts have their roots in actions and processes, which become encapsulated as mathematical objects. We see these changes in students’ growing conceptions (e.g. of algebraic functions) and we know that teaching needs to ease such transitions. After three decades of work identifying some of these fundamental processes, algebra education can now go forward on a strong base. At the other extreme, some findings of mathematics education are insightful and important, but are mostly relevant to a particular situation at a particular time. Studies of the impact of the recent changes to the algebra curriculum in some Western countries, where traditional manipulation was suddenly markedly reduced, are like this. Much of the best research has something to say of immediate interest, as well as contributing lasting insights.

Trends in algebra education research, like other areas of mathematics education, are influenced by factors external and internal to the field. A group of external factors have led to the “massification” of secondary school education, whereby it is now the norm in many countries, that most students complete secondary education, and this education includes algebra. Algebra is seen as a “gateway” to higher mathematics, because it provides the language in which generalisations are expressed. Consequently, having students learn algebra is important for the production of “knowledge-workers” as well as being important for social equity. But algebra is difficult, and instead of being a gateway, it can easily be a wall that blocks students’ paths. This leads to a reconsideration of the goals of algebra, to identify what are its essential components and to a search for improved teaching methods. Traditionally, algebra had been mainly seen as symbol manipulation, but most graduates of such curricula have no appreciation of why this knowledge is important. On-

going reconceptualisation of the core of algebra has, for example, elevated the importance of graphs and functions, at the expense of solving and rearranging expressions. The ability to deal with graphs and a basic concept of function are more likely to be seen as “basic” now than moderately complicated symbolic manipulation. The new technologies have also impacted very strongly on algebra education research. As communication technologies, these impact on all of education (e.g. distance learning, data from the internet, new means of presentation etc). But as information technologies, these impact centrally on the way in which mathematics is done. Mathematics at every level, from the work of the shopkeeper to the mathematician, has always struggled to make calculations easier, and we now have tools that can perform all of the standard routines known by an undergraduate at the press of a button. This provides exciting new opportunities for teaching, especially through the possibilities of teaching with multiple representations of algebraic ideas. It also provides a serious challenge to existing curriculum: what is the role of the machine and what is the role of pencil-and-paper skill? To answer this question, we need to be very clear about goals for algebra education, about what it means to understand and to develop new pedagogies to meet the new situation.

Algebra education research is also impacted by trends that are internal to the educational research community. At a simple level, the growth in international exchange has opened up appreciation of the possibilities for curriculum, teaching and assessment. Algebra education was perceived as something common around

the world, but the differences are now known to be large on all dimensions - the degree of formalism, the amount of manipulation, the place of functional thinking, the use of technology, the age of introduction etc. There are many alternative successful approaches.

Important intellectual movements from other disciplines also impact on algebra education. For example, studying the history of mathematical ideas has led to the identification of particular cognitive obstacles for students (e.g. related to the ways in which letters are used), and consequently to teaching approaches that assist students cross the barriers. On a more theoretical level, the role of algebra as “the language of mathematics” has been studied from the point of view of semiotics (the science of symbols) and linguistics (the science of language).

Research into algebra education is a lively field, aiming to engage with social needs and intellectual advances. However, its success should be judged by the extent to which it can promote algebra as a lively, engaging and worthwhile subject for an increasingly large number of students.

## References

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