Math Wars and the Epistemic Divide in Mathematics

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1. Problem

(a) Why do school (K-12) students find mathematics especially difficult?[1] (b) What is a good way to ameliorate these difficulties? (c) Would the new technology of computation *fundamentally* change the *content* of mathematics?

2. Earlier attempts

The "new" math curriculum of the 1960's and 1970's sought to align mathematics teaching with the formalist approach to mathematics, finalized in the 1930's. More recently, the reformed constructivist "new new math" curriculum, endorsed by the US Education Department in 1999, but rejected as "fuzzy math" and "no correct-answer math" by its opponents,[2] has sparked off a huge dispute called the "Math War".

3. Our analysis

Learning difficulties peculiar to mathematics are the root cause of this dispute, and this paper locates the root cause of these learning difficulties in an *epistemic schism* within mathematics. The quarrel about *what* and *how* mathematics should be taught reflects fundamentally divergent perceptions of what mathematics *is*—and that ought to be decided *not* by mathematical authority, but by recourse to history and philosophy.

4. Re-examination of history and philosophy of mathematics.

Two key principles: (a) History and philosophy go together: distorted history distorts the philosophy of the subject. (b) The learning process mirrors the historical evolution of a subject, in a "fast-forward" mode: learning difficulties reflect actual historical difficulties.

Detoxifying the current history of mathematics. What is the actual history? When state and church combine, history becomes an instrument of religious politics—as also happened for the last 1700 years. Hence, the "historical" claim: mathematics originated in "Greece" and was further developed mostly in Europe. This racist fantasy (Bernal, 1991) is based on papal *fatwas* ("Doctrine of Christian Discovery"), and excessively flimsy evidence. (Raju, forthcoming) Current philosophy of mathematics hence *defines* mathematics as something that imitates the "Greek" method of proof—as subsequently (unwittingly) aligned to the requirements of Christian rational theology by Russell, Hilbert et al. This definition is supported by the claim of universality—which is bogus, since proof varies with logic, which varies with culture (e.g. Buddhist and Jaina logic), and is not empirically certain (Raju, 2001; Raju, 1999). Hence, also, we must accept as legitimate variations in the notion of mathematical proof across cultures, as in the Indian notion of *pramāṇa*, which involves the empirical (and irrespective of Western taboos against the empirical in mathematics).

The multicultural origins of mathematics. Actually, Europe inherited not one but *two* streams of mathematics with distinct philosophical orientations: (i) an Egyptian – "Neoplatonic" mathematics that was spiritual, anti-empirical, proof-oriented, and explicitly religious, and (ii) an Indo-Arabic mathematics that was pro-em-

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[1] Ignoring possible variations across cultures we will go with the readily available data.
[2] including Field medallists and Nobel prize winners, *Washington Post*, 18 Nov 1999.

pirical, and calculation-oriented, with practical objectives. Much mathematics taught at the K-12 level is of Indo-Arabic origin: (1) arithmetic, (2) algebra, (3) trigonometry, and (4) calculus. Europeans, however, failed to recognize the distinct epistemic settings of the two streams of mathematics, and sought to assimilate both under one "universal" mathematics. This led to the real math wars.

First math war: Arithmetic. The import of the algorismus (elementary arithmetic algorithms), from India to Europe, via Arabs and Florentine merchants, led to the first math war, lasting some 500 years, from Pope Sylvester II (d. 1003 CE) to the 16th c. CE victory of algorismus over abacus. Apart from zero (place value system), zeroing or discarding non-representables (*ṣūnya*) was a key problem.

Second math war: Calculus. Indian methods of proof and rounding led naturally to infinitesimal techniques ca. 1400 CE, when Aryabhata's finite differences were extended to infinite and "Taylor" series expansions for the sine, cosine, and arctangent functions, to calculate precise trigonometric values—imported into Europe by Mercator, Clavius, Stevin, etc. via the missionaries established in Cochin from 1500 CE, and since required for the European navigational problem and the (related) Gregorian calendar reform of 1582 (Raju, 2001). The Indian infinitesimal techniques were again incompatible with European ideas of "universal" mathematics, so that Cavalieri, Fermat, Pascal, Gregory, Newton, and Leibniz etc. in the 17th c. while reproducing exactly the Indian infinite series, tried to "prove" them in the "Euclidean" way—with complete lack of success as is clear from Berkeley's criticism, for example. This second math war over "infinitesimals" lasted some three centuries until Dedekind's semi-formalisation of real numbers in 1872.

Third math war: Computers. Like algorismus and calculus, computers have again greatly enhanced the ability to calculate for practical application, but in a way regarded as epistemologically insecure (Raju, 2001). Abacus has now gone to Kindergarten and calculus has descended to K-12: analogously, computer packages, like my CALCODE, can now be used to teach nonlinear ordinary differential equations to K-12 students.

Phylogeny is ontogeny. In a nutshell, European historical difficulties in assimilating mathematics-as-calculation all relate to its perceived epistemological insecurity. This "epistemological clash" arose from an unacceptable theology of mathematics-as-'Greek'-proof. Analogously, the formal justification for K-12 mathematics (via mathematical logic $\rightarrow$ axiomatic set theory $\rightarrow$ algebraic structures $\rightarrow$ number systems $\rightarrow$ mathematical analysis) is out of the reach of most K-12 students. Hence, on the present paradigm, K-12 students are condemned to learn mathematics in an epistemologically insecure way, reflecting exactly the European history of difficulties with imported mathematics.

Correction. The situation can be corrected by changing over to the notion of mathematical proof within which the relevant mathematics originated.

Prognosis. Computers will subversively transform mathematics as arithmetic and calculus did earlier. With concern boiling over the issue of calculators vs memorized algorithms, US math warriors have failed to notice that computer arithmetic uses (floating point) numbers that *must* disobey most of the formal mathematical "laws" about numbers, such as the associative law (Raju, 2001). Further, next generation "quantum" computer may negate the very logic naively assumed as "universal" in the present-day (Hilbertian) notion of formal mathematical proof!

5. Conclusions

(0) We need to reject racist history which has fantasized a monocultural origin for mathematics, and has built the philosophy of mathematics around this fantasy.

(a) The present-day learning difficulties of much K-12 mathematics reflect the real historical difficulties that Europe encountered in the last thousand years in forcibly trying to fit practical mathematics-as-calculation, imported from India via Arabs, into the theological paradigm of mathematics-as-proof.

(b) The right way to ameliorate learning difficulties in K-12 mathematics is, hence, to teach practical mathematics-as-calculation, with its non-"Euclidean" epistemology, as fundamentally different from aesthetic/-theo-logic-al mathematics-as-proof (Raju, 2004). Accepting the empirical in mathematical proof, would help teach K-12 mathematics in an epistemologically secure way.

(c) Computers make manifest the existence of non-representables in any practical mathematical calculation, forcing a shift away from the formal understanding of numbers. Future computer technology may also probably transform the understanding of logic.

References

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