will master how to ask the right questions to get to the thoughtful answers. Their metacognitive abilities could be heightened and they could use this ability while learning other topics in mathematics.

Findings will be analyzed in terms of thinking processes and reasoning skills with both the topic at hand and also with the mathlets mentioned. A triangulation of the data from both quantitative gain scores and from the qualitative interview results will be carried out.

References

A View on Active Learning in Mathematics through Historical Context

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In recent times there was a consensus that students should learn through inquiry and through the construction of their own mathematics (Davis, 1991; Harel & Papert, 1990; NCTM, 1989). The same situation remains nowadays. The view of the learner has changed from that of a passive recipient of knowledge to that of an active constructor of knowledge. We take into account that current learning perspectives incorporate three important assumptions (Anthony, 1996):

- learning is a process of knowledge construction, not of knowledge recording or absorption;
- learning is knowledge-dependent; people use current knowledge to construct new knowledge;
- the learner is aware of the processes of cognition and can control and regulate them.

From a constructivist perspective it is easier for a student, under appropriate arrangement of teaching, to...
act as an architect, to reveal the truth and construct new knowledge, than to learn ready-made knowledge without understanding its origin, meaning and interrelations. In other words, “learning is a process of construction in which the students themselves have to be the primary actors” (von Glasersfeld, 1995).

At the same time, using history of mathematics is most naturally integrated in active learning processes. (By learning activities of students we mean their investigational work, problem solving, small group work, collaborative learning and experiential learning.) While solving a certain problem, every student has been proposed to investigate “mathematical situation” of it with his/her own priorities for further inquiry of that problem. Like Brown and Walter (1990), we consider “situation”, an issue, which is a localized area of inquiry with features that can be taken as given or challenged and modified, but with its neighbourhood in historical-mathematical sense: when a certain problem was posed for the first time, who was the author, whether that author proved/solved a problem on his/her own, who of other mathematicians was interested in it, for what reasons, how long a problem was an unsolved one, etc (Yevdokimov, 2004).

We would like to consider the possibility of using principles of active learning in teaching mathematics through such historical context. The aim of our research was to show that students can learn effectively through appropriately designed historical environment. It is important to note that we did want neither stimulate students’ using ICT support (e.g. dynamic geometry environment) nor restrict them in it. However, our choice was not taken by chance. Focusing students’ attention on historical context of mathematical content we would like, on the one hand, to contribute that students’ imagination would be absorbed in that time, when certain property was revealed or certain concept was proposed by mathematicians, from Ancient World to nowadays. On the other hand, we would like to develop students’ abilities to analyze mathematical content from today’s point of view and perceive evolutionary development of mathematical concepts and different properties throughout the centuries.

We proposed a flexible structure of units for active learning geometry supported with materials from the history of mathematics. Every unit consisted of three parts: preliminary, basic and advanced ones. In the preliminary part of every unit tasks were given in the usual form:

“What are the properties for mathematical object Y”? And in the advanced part of every unit most of the tasks were given in the generalised inquiry form:

“Find properties of something” (without indication of a mathematical object, but concerning given “mathematical situation”).

Flexibility of the units was provided with numerous links between tasks and materials of different units and easy transition from one unit to another without strict instruction in learning, i.e. unit-by-unit. The main mission for teachers involved in research was their assisting students in discovering any property, which should be done by the students themselves.

The key question of students’ inquiry work in all parts of the unit was the following: where, when and for what of mathematical objects a student would apply a certain mathematical property for proving and solving and whether it would be necessary to apply that property generally in that case. This question is connected with an Active Fund of Knowledge of a Student (AFKS) in the given area of mathematics. As AFKS we called student’s understanding of definitions and properties for some mathematical objects of that domain and skills to use that knowledge (Yevdokimov, 2003).

We would like to emphasize that historical context in active learning of mathematics has invaluable importance. In our opinion, it is really an effective way – to learn and teach new mathematical content through historical context. Moreover, we found out that active learning of a certain area of mathematics through historical context enriches AFKS in the same area of mathematics and contributes to development of creative thinking of students.

References
This paper examines the notion of transparency in learning and understanding mathematics. The notion of transparency may be associated with mathematical objects as well as thought processes. It carries two related meanings: Seeing-through and being visible. It has been addressed with respect to learning mathematics in a number of different contexts. In dealing with the roles and nature of mathematical examples, Mason & Pimm (1982) discuss the idea of a generic example, which basically is a specific example that conveys a more general case. That is, a generic example is transparent to a general case. The authors (ibid) suggest that often students fail to see in an example what the teacher had in mind. In particular, a generic example that is meant to demonstrate a general case or principle may be perceived by the students as a specific instance, overlooking its generality. When this is the situation, we consider the example to be non-transparent (or opaque) to the learner. Movshovitz-Hadar’s (2002) approach to transparent proofs can be viewed as an extension of this kind of transparency. A transparent proof, according to Movshovitz-Hadar (ibid), is a proof of a particular case that is “small enough to serve as a concrete example, yet large enough to be considered a non-specific representative of the general case. One can see the general proof through it because nothing specific of the case enters the proof”.

In the context of generating counter-examples Peled and Zaslavsky (1997) distinguish between counter-examples that only disprove a statement, and those that also reflect an explanatory feature regarding how it was generated and why it, in fact, refutes the statement. The latter can be seen as transparent counter-examples.

Another aspect of transparency in mathematics education is related to thought processes. More specifically, to the extent to which teachers’ authentic mathematical thought processes are made transparent to the learners. Schoenfeld (1983) raises this point in the following excerpt:

“Part of the difficulty in teaching mathematical thinking skills is that we’ve gotten so good at them (especially when we teach elementary mathematics) that we don’t have to think about them; we just do them, automatically. We know the right way to approach most of the problems that will come up in class. But the students don’t, and simply showing them the right way doesn’t help them avoid all the wrong approaches they might try themselves. For that reason we have to unravel some of our thinking, so that they can follow it.” (ibid, p. 8).

In this sense, unraveling one’s thinking is actually making it a transparent process to the learner.

The above discussion of the notion of transparency focuses on its meaning in terms of seeing-through. We also consider transparent objects and properties in terms of being visible. The proposed presentation will elaborate on the notion of transparency in mathematics education, and adopt it to examine a number of studies in which the extent to which an object or process is transparent plays a role in mathematical understanding.

Among the examples that will be presented are the following:

**Transparent algorithms:** There are several deep mathematical principles that are embedded in the basic arithmetic algorithms. Apparently, the algorithms are not

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**Transparent Objects and Processes in Learning Mathematics**

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